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# The Level, Slope, and Curve Factor Model for Stocks: Evidence, Theory, and Explanation

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# The Level, Slope and Curve Factor Model for Stocks: Evidence, Theory and Explanation

Charles Clarke, PhD

University of Connecticut, [2016]

The reported number of firm characteristics that predict stock returns is growing at a rapid pace. This dissertation offers a reorganization of this exploding space.

In the first chapter, I use regressions to aggregate the explanatory power of many anomalies into one proxy for expected returns. I find that sorting on this proxy creates large spreads in average returns and large alphas when compared to the leading factor models. The procedure allows me to evaluate the marginal economic significance of each anomaly. Asset growth, net stock issues and momentum are the strongest anomaly variables. Anomaly importance varies across size groups, but size provides relatively little explanatory power. I use principal components analysis to show that a strong multifactor structure underlies the spreads created from my one dimensional sort. In the second chapter, I develop a method to extract only the priced factors from stock returns. The first step estimates expected returns based on characteristics.

The second uses the expected returns to form portfolios. The last step uses principal components to extract factors from the portfolio returns. The procedure isolates and emphasizes the comovement across assets that is related to expected returns as opposed to firm characteristics. It produces three factors—level, slope and curve—which perform as well or better than other leading models. Horse races show that other leading factors add little to the model. The factors have macroeconomic risk interpretations.

The third chapter reevaluates the Consumption Capital Asset Pricing Model's ability to price the cross-section of stocks. With a few adjustments that generate more informative tests by increasing test power, I find that the simple linearized CCAPM often matches key features of the cross-section: the consumption risk premium is positive and significant, the zero beta rate is near zero and insignificant, and the CCAPM captures much of the variation across average portfolio returns. A key stylized fact emerges that many interesting "anomalies" share the characteristic that high expected return portfolios tend to have higher covariance with consumption.

The Level, Slope and Curve Factor Model for Stocks: Evidence, Theory and Explanation

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A Dissertation

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APPROVAL PAGE

Doctor of Philosophy Dissertation

The Level, Slope and Curve Factor Model for Stocks: Evidence, Theory and Explanation

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# Chapter 1

## The Economic Significance and Factor Structure of Combining Anomalies to Estimate Expected Returns

*In the beginning, there was chaos. Practitioners thought that one only needed to be clever to earn high returns. Then came the CAPM. Every clever strategy to deliver high average returns ended up delivering high market betas as well. Then anomalies erupted, and there was chaos again. . . Fama and French brought order once again with size and value factors. . . Alas, the world is once again descending into chaos. Expected return strategies have emerged that do not correspond to market, value, and size betas.*

*John Cochrane (2011)*

### 1.1 Introduction

When stocks can be sorted in a way that creates average returns that are not accounted for by prevailing asset pricing models, we call the result an anomaly. The most resilient finding in the empirical asset pricing literature may be that no model of expected returns can explain the rapidly increasing number of stock price anomalies. Armed with a tremendous amount of data, finance researchers stand ready to reject any new theory with a new set of anomalies. Reviewing the literature, Harvey et al. (2015) count over three hundred anomalies, and Goyal (2012) concludes that no study to date conducts a comprehensive analysis of the joint impact of these anomalies. Fama and French (2008) explore several anomalies and find that each provides independent information for expected returns. This paper explores the importance or economic significance of each anomaly, focusing on economic significance as opposed to statistical significance.

How much alpha do asset pricing anomalies reveal when combined in a group? For instance, the momentum anomaly (Jegadeesh and Titman 1993) and the accruals anomaly (Sloan, 1996) each

individually produce 12 percent annual returns, but do they jointly produce a 24 percent annual alpha when combined? Momentum may subsume some of the economic importance of the accruals anomaly or vice versa. We need to answer this question across a growing array of anomalies, but as Cochrane (2011) points out, the standard method of sorting portfolios on characteristics and looking at average returns or risk adjusted alphas is becoming too unwieldy.

To grapple with this problem, Fama and French (2008) show that cross-sectional regressions in the style of Fama and MacBeth (1973) can be used to analyze characteristics jointly. I use characteristic regressions to sort stocks into a one dimensional array of portfolios. The procedure is simple. First, estimate Fama-MacBeth regressions of returns on all relevant anomaly variables. Next, sort the stocks into portfolios using the fitted values of the regressions (ie. expected returns). The resulting array of portfolios summarizes the joint predictive power of the anomaly variables.

This sorting procedure does an extremely good job of separating high expected return and low expected return stocks, thus generating a wide return spread. When sorted into ten portfolios, the high return tenth decile minus low return first decile hedge portfolio generates an average annual return of 18 percent and an annual Sharpe ratio of 0.87. In twenty-five portfolios, the high minus low hedge portfolio generates an average annual return of 30 percent and an annual Sharpe ratio of 1.07. Only a portion of these returns are captured by risk adjustment.

The alpha relative to the CAPM of hedge portfolios from sorts into ten and twenty-five portfolios is nearly as large as the original return spread, and impervious to the risk adjustment. The Fama and French three factor model yields annual risk adjusted alphas of 14 percent and 25 percent for ten and twenty-five portfolios, respectively. Only the Carhart model with momentum as a factor makes a dent in the return spread, decreasing the alphas to 6 percent and 15 percent, respectively. These alphas still retain a high level of statistical significance.

While these portfolios are not explained by leading factor models, a principal components analysis reveals that they have a strong factor structure. The analysis reveals level, slope and curvature factors similar to those found in bond returns and bond yields. These portfolios show tremendous comovement that is primarily explained by the first three uncorrelated principal components. Thus, the anomalies may support an Arbitrage Pricing Theory approach to asset pricing even if they present anomalies to commonly used factor models. The underlying stocks may instead be exposed to common factors that are not captured by the leading factor models.

I use this one dimensional sorting procedure to parse out the economic significance of each anomaly. I measure the economic significance of an anomaly by its contribution to creating a spread in average returns after controlling for other anomalies. I examine the spread in average returns before and after adjusting for risk with standard factor models. Results show that asset growth, net stock issues and momentum are the most important anomalies in the full model. Earlier work by Fama and French (2008) suggests that the asset growth anomaly has somewhat weak statistical significance that is not always consistent across size sorts. But, omitting asset growth from the sorting regressions lowered the spreads across average returns by 29 percent, which suggests that asset growth has large economic significance. On the other side, the size effect has no economic significance after controlling for the other important return predictors.

Regression sorted portfolios were first used by Haugen and Baker (1996) to study a large class of predictors, though unlike this study, many of the predictors had only short term predictive power. Haugen and Baker focus on the out of sample predictability of the anomalies studied. Hanna and Ready (2005) show that the portfolios have high turnover and most of the profits are eroded by transactions costs. Lewellen (2015) studies these out of sample Fama MacBeth regressions and finds they are well-specified. He studies the same set of anomalies used in this study and in Fama and French (2008), as well as an additional set of predictors that have limited to no predictive power. Unlike the previous papers, this paper focuses on the economic significance of the various regression specifications presented in the literature. I replicate the out of sample sorts in Lewellen (2015) in this paper to focus on the relative performance of out of sample regressions to in sample regression. This paper is the first to explore which anomalies are the most important in generating return spread, and the cost of out of sample as opposed to in sample results. Additionally, this paper is the first to show the covariance structure of anomaly sorted portfolios and explore the implications.

Fama and French (2006) find that size and book to market account for most of the economic significance among the return predictors that they consider. The authors sort stocks into two portfolios and look at the return spread created. I look at sorts into two, ten, twenty-five and one hundred portfolios, and look at the spread between the highest and lowest predicted return portfolios. A lot of important information is potentially lost by sorting into only two portfolios. Some of the most important predictors may be washed out. I find support for this claim as sorting

into more portfolios leaves size and book to market at times unable to account for even 30 percent of the full model spreads in returns. Sorting into more portfolios leaves each portfolio with more idiosyncratic risk—the cost of getting a larger spread in returns. The return spread is just a tradeable hedge portfolio created by going long the predicted high return stocks and short the predicted low return stocks, while finer sorts increase the expected returns of the hedge portfolio, they also increase its volatility. Sharpe ratios are a natural way to weigh the costs and benefits of finer sorts. In the full model, sorting into ten portfolios raises the Sharpe ratio of the hedge portfolio 30 percent compared to using only two portfolios. A sort into twenty-five portfolios raises the Sharpe ratio over 60 percent. At 100 portfolios, the increase in the spread in returns represented by the hedge portfolio continues to rise, but the Sharpe ratios tend to fall relative to sorts into twenty-five portfolios. The increase in the volatility of the hedge portfolio more than offsets the increase in returns.

The full model shows substantial gain over size and book to market used alone. A portion of the gain can be attributed to using separate regressions across size groups following Fama and French (2008). This approach allows parameter estimates to differ for different size groups. Anomalies that are important in a full regression may be strongest and most prevalent in micro capitalization stocks that are costly to trade and difficult to arbitrage. Additionally, these micro-cap stocks, being small, will have only a small impact on value weighted portfolios. Combining regressions across size groups with sorting into portfolios allows us to assess whether statistically significant predictors of returns are also economically significant. Statistical significance can arise because a small effect is extremely accurately measured or because it is concentrated in a certain size group. In addition, an anomaly variable that is significant on its own may add almost no new information when other anomaly variables are also considered. Intuitively, sorting into portfolios allows one to measure how influential a particular anomaly variable is in separating high expected return stocks from low expected return stocks. If an anomaly variable has no new information, omitting it from a sorting procedure will have no effect on the return spread of regression sorted portfolios.

Allowing regression parameters to differ across size groups has an important effect on return sorts. When stocks are sorted into ten or twenty-five value weighted portfolios, using just one regression with all the anomaly variables only captures about 50 percent of the spread created by the full model. Even with a limited number of predictors, (e.g. only size and book to market),

allowing parameters to differ across size groups has large effects.

With the exception of Lewellen (2015), previous work on sorting regressions has focused on full sample parameter estimates. These estimates have two main drawbacks. First, if parameters vary over time, the parameters will at times overstate and at other times understate an anomaly’s true affect, which may hurt the regression sorts. Second, full sample regressions may depend on information in the post formation period to form high quality sorts. Since this information is not available at the time of the sort, the sorted portfolio is not really tradeable. If a full sample including the post formation period is required for good parameter estimates, the full sample results will overstate the economic importance of anomaly variables.

In a related paper, Lewellen (2015) shows that Fama-MacBeth regressions work well out of sample and portfolios formed on out of sample Fama-MacBeth estimates create large spreads in return. This paper focuses on the economic significance of out of sample performance. I explore out of sample portfolios to ask how much predictability is lost out of sample relative to the full sample.

I address these issues with sorts based on rolling regressions and “no peeking” regressions. Both sorts only use information available at the time that the portfolios are created. The rolling regressions restrict the window to the 60 months prior to portfolio estimation, allowing parameter estimates to change over time. The no peeking regressions use all information available up until the time of the portfolio formation. Thus, everything used in both procedures would be available to a trader at the time of portfolio formation. No peeking regressions perform almost as well as the full sample regressions, undermining the view that regression sorts suffer considerably from look ahead bias.

The paper is organized as follows. Section II describes the data and variables. Section III presents the full model and uses it to produce sorts. Section IV explores the economic significance of individual anomalies. Section V uses rolling and no peeking regression to explore the effect of limited information sets on the regression sorts. Section VI conducts a principal components analysis and discusses the strong factor structure of the sorted portfolios.

## 1.2 Data and Variables

The sample period covers July 1963 until December 2012. I obtain return data from the Center for Research in Security Prices (CRSP) and accounting data from Compustat. I restrict the sample to NYSE, NASDAQ and AMEX stocks (share code 10 and 11). I exclude financial firms (Standard Industry Classification codes of 6000 to 6999). I adopt the Fama and French (2008) variable definitions for firm characteristics (in detail in the data Appendix). I measure each anomaly variable at the beginning of July using the last fiscal year’s accounting data, except for momentum, which is defined monthly. The relevant anomaly variables are precisely defined in the data appendix and include: size, book to market, momentum, net stock issues, accruals, investment, and profitability.<sup>1</sup>

Many asset pricing studies ask how a characteristic is associated with stock returns. The purpose of this study is to ask how a characteristic helps us distinguish a high expected return stock from a low expected return stock. Because each firm is a bundle of many characteristics, this procedure uses a group of characteristics and studies both how they together predict returns, as well, as how each variable contributes to predicting returns. In order to achieve this I employ regressions of one month ahead returns on several stock characteristics.

Each Fama-MacBeth cross-sectional regression has the following form:

$$XRet_{i,t+1} = \beta_0 + \beta_1 LogSize_{i,t} + \beta_2 LogB/M_{i,t} + \beta_3 Mom_{i,t} + \beta_4 zeroNS_{i,t} + \beta_5 NS_{i,t} + \beta_6 negACC_{i,t} + \beta_7 posACC_{i,t} + \beta_8 dA/A_{i,t} + \beta_9 posOP_{i,t} + \beta_{10} negOP_{i,t} + e_{i,t+1} \quad (1.1)$$

The stock return in excess of the risk free (XRet) rate for each stock in the following month is regressed on log firm size (LogSize), log book to market (LogB/M), momentum (Mom), a dummy if no stock was issued (zeroNS), net stock issues (NS), negative accruals (negACC), positive accruals (posACC), asset growth (dA/A), positive operating profit (posOP) and negative operating profit (negOP). Fama and French (2008) find that stocks of different size groups (micro, small and large) have different exposures to characteristic predictors, so I estimate the regression above separately for each size group allowing the parameter estimates to differ across these groups.

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<sup>1</sup>Size is attributable to Banz (1981a), book to market to Rosenberg et al. (1985), Chan et al. (1991), and Fama and French (1992), momentum to Jegadeesh and Titman (1993), and net stock issues to Daniel and Titman (2006) and Pontiff and Woodgate (2008) following earlier work by Fairfield et al. (2003) and Loughran and Ritter (1995). Accruals is attributable to Sloan (1996), profitability to Haugen and Baker (1996), Cohen et al. (2002) and Novy-Marx (2013a), and investment to Fairfield, Whisenant, and Yohn (2003) and Titman, Wei, and Xie (2004).

The literature has employed a variety of characteristics and information sets for these regressions, but little work has been done on whether and how much these choices matter. For instance, Lewellen (2015) studies these characteristics in a rolling and no peeking manner, but doesn't separate firms into size groups as does Fama and French (2008). In this study, I will go through several choices for characteristics and information sets and explore how much the choices matter.

These sorts generate large spreads in portfolio returns. Table 1.1, Panel A shows the resulting one dimensional sorts into two, ten, twenty-five, and one hundred portfolios. These portfolios are formed with returns in excess of the risk free rate. The average returns for the lowest return portfolio in each group declines as the sort gets finer and finer. The lowest value weighted return portfolio of a sort into centiles is -1.03 percent per month, which is much lower than the 0.43 percent from sorting into only two portfolios. The equal weighted portfolios follow a similar pattern and produce even more extreme sorts. The sort also works well at finding high return stocks. The high expected return value weighted portfolios earn a minimum of 0.99 percent and a maximum of 1.72 percent, increasing in spread as portfolios are diced finer and finer.

The next row, labeled spread, shows the return of a hedge portfolio built by going long high return stocks and short low return stocks to form a zero cost portfolio. Forming this hedge portfolio with decile sorts creates a spread of 1.41 percent per month in value weighted sorts and 2.00 percent in equal weighted sorts. The finer we dice portfolios the larger this spread grows, but because forming more portfolios requires fewer stocks in each portfolio, the volatility of the hedge portfolio grows as the number of portfolios increases. A natural way to assess the value of taking on the additional risk is to use the Sharpe ratios of the resulting hedge portfolios, to weight the gain in average returns against the increase in volatility. For the value weighted portfolios, sorting into twenty-five portfolios results in the highest Sharpe ratio. The equal weighted portfolios show a much flatter pattern of Sharpe ratios with a sort into ten, twenty-five and one hundred portfolios producing similar results.

An obvious question is, how much of these return spreads are the result of risk captured by leading asset pricing models? Panel B answers this question by regressing each hedge portfolio return against the factors in the CAPM, the Fama and French three factor model, and the Carhart (1997) four factor model. The alphas are very large and statistically significant. The value weighted and equal weighted decile sorts correspond to an 18.01 percent and 28.93 percent annualized risk

adjusted return over the CAPM, respectively. The CAPM absorbs very little of the spread in value weighted returns created by the sorts, and the CAPM alpha is consistently larger than the spread in equal weighted return.

The Fama and French (1993) three factor model absorbs only a small portion of the spread in returns. In value weighted decile sorts, the alpha is 22.6 percent lower than the original 10 portfolio spread and 16 percent lower than the twenty five portfolio spread. The factor model performs the best when portfolios are separated into only two portfolios as in Fama and French (2006b), absorbing 43 percent of the spread in value weighted returns. Thus, the success of size and book to market in explaining anomalies presented in that paper is sensitive to sorting stocks into a small number of portfolios. The Fama and French model explains little of the spread in equal weighted sorts; the alpha is 12 percent lower than the spread in two portfolios and 4 percent lower than the spread in one hundred portfolios.

The Carhart (1997) four factor model that includes momentum as a factor performs better than the CAPM and three factor model, foreshadowing that momentum is one of the most economically significant anomalies. The Carhart model explains all of the return spread in a value weighted sort into two portfolios and almost two thirds of the original spread in decile sorts. Yet, the remaining alpha in ten and twenty-five portfolio sorts corresponds to a statistically significant annual risk adjusted return of 5.66 percent and 14.98 percent, respectively. The Carhart model does considerably worse on the equal weighted sorts with Sharpe ratios remaining close to 0.40 for sorts into to ten or more portfolios and an annual alpha for the twenty-five high minus low portfolio of almost 25 percent.

### **1.3 Economic Significance of Asset Pricing Anomalies**

The sorts on anomaly variables in Section III do a good job of separating high return stocks from low return stocks. Only a portion of this spread is explained by the leading asset pricing models. With a general procedure using several anomaly variables to generate a one dimensional sort into portfolios, we can now examine the economic significance of important anomalies.

Using sorts into two portfolios, Fama and French (2006) find that sorts based on regressions with size and book to market on the right hand side capture most of the spread in returns. Adding



profitability, accruals and asset growth lead to modest gains of .05 percent in value weighted and 0.12 percent in equal weighted returns per month. Table 1.2 replicates their base model and explores whether their finding still holds for the finer sorts used in this paper. Unlike the full model presented in Table 1.1, only size and book to market are used in the regression, and the model is not separately estimated for micro, small and large capitalization stocks to allow different regression slopes for anomalies across size groups. If a sorting procedure works well, it will generate a large spread in returns. If a sorting variable adds a lot of useful information, then omitting that variable will cause a large decrease in returns. Comparing the full model in Table 1.1 with the limited model in Table 1.2, we can evaluate the importance of anomalies beyond size and book to market.

Table 1.2 shows that the baseline model of Fama and French (2006) fails to capture a considerable amount of the predictable spread in returns. The table shows for ten, twenty-five and one hundred portfolios the returns on the low return portfolios are much higher, and the high return portfolios are much lower. Only the sort into two portfolios shows a small change that masks a large relative change. The lower panel shows the change in the return spread generated both in absolute and relative percentage terms. The value weighted returns spread falls, in absolute terms, 0.25 percentage points for sorts into two portfolios, 0.76 percentage points for sorts into ten portfolios, 1.62 percentage points for sorts into twenty-five portfolios and 1.80 percentage points for sorts into one hundred portfolios. In percentage terms, the fall is from a minimum of 44% to a maximum of 73%. Thus, using only book to market and size to sort will miss almost three fourths of the spread in returns. All the Sharpe ratios are considerably smaller, from 35 percent smaller for two portfolio sorts to a maximum of 68 percent smaller for twenty-five portfolio sorts.

Creating return spreads is an important goal of portfolio sorts because a spread in returns is necessary to test asset pricing models. Yet, the gold standard Fama and French (1992) 25 size and book to market sort only creates a maximum spread in value weighted returns over this sample period of 0.92 percent, compared to the 2.21 percent return spread across twenty-five portfolios created with the full model in this paper. While one dimensional sorting with many explanatory variables is uncommon in the finance literature, Cochrane (2005) suggests that it should be the preferred method.

As the number of anomalies becomes large, multidimensional sorts become unwieldy. A sort into terciles across three anomalies generates 27 portfolios. Adding a fourth variable generates 81

portfolios, while adding a fifth generates 243. In the 1960s, the number of available stocks in CRSP is only 800, while that number rises to 5000 in the 1990s. Thus, each portfolio would have between 3 to 20 stocks and have considerable idiosyncratic risk.

An alternative approach used by Hou et al. (2014) and others is to test each asset pricing models with a series of one dimensional sorts. It produces statements like, “model X explains 15 of 20 anomalies.” This can overstate a model’s power, however, because many of the 15 anomalies that it explains could be versions of the same underlying anomaly. A joint test would be less unwieldy and more comprehensive. My regression-based approach accounts for the unique parts of the anomalies, and then asks a model to price many anomalies at once. Because it is possible to trade many anomaly strategies at once, it is essential that our models be able to price combined strategies.

As was shown in Table 1.1, Panel B, regression sorts have the benefit of yielding an easily interpretable alpha on a high minus low hedge portfolio that summarizes an asset pricing model’s ability to explain the return spreads across portfolios. Comparing the performance of two different models across a series of one dimensional tests on individual anomalies has no obvious conclusion unless one model dominates the other.

Having shown that using only size and book to market in returns misses a substantial amount of the predictable variation in average returns across firms, I explore what parts of the full model are driving the result. Fama and French (2008) find that the parameters in the multivariate predictive regressions differ across stocks of different market capitalization. Separating stocks into micro, small, and large size groups, they show that parameter estimates are significantly different across size portfolios. They do not explore how important this separation is or the impact on the spread in returns.

In Table 1.3, I explore the importance of allowing coefficient estimates to vary across size groups. Table 1.3 shows the results of sorting portfolios using three separate regressions for micro, small and large stocks, but only size and book to market as regressors. If the spreads in Table 1.3 are nearly as large as the full sample spreads displayed in Table 1.1, then spreading stocks into groups and allowing the coefficients on anomaly variables to vary over the groups is driving the important variation across returns. If the spreads in Table 1.3 are very similar to the spreads in Table 1.2, then allowing coefficients to vary across size groups has little economic importance for the size and

book to market variables. Panel A shows the pattern observed in Table 1.1 where spread rises as stocks are grouped into an increasing number of portfolios. One difference is that, unlike in Tables I and II, the Sharpe ratio for the value weighted sorts is highest when sorted into one hundred portfolios, as opposed to peaking for sorts into twenty-five portfolios.

Panels B and C compare the results to the other models. Panel B shows that separate regressions for the three size groups considerably increase the ability of the regression model, to generate large spreads. The spread in twenty-five value weighted portfolios is 0.43 percent per month higher, an increase of 73 percent. The increase is weakest for the ten portfolios, where the spread increases by 0.08 percent per month, an increase of 13 percent. The panel also shows that a separate regression for each size group is more important for value weighted sorts than equal weighted sorts. Equal weighted sorts will be dominated by the microcap stocks, and the regression results of using only one regression across all stocks will also be dominated by small stocks. Thus, equal weighted sorts are less dependent on the information lost from the procedure.

Panel C shows that using separate regressions captures more of the return spread, but still falls short of the full model. The return spreads for value weighted sorts into ten and twenty-five portfolios are 0.68 percent and 1.19 percent lower than in the full model, a relative decline of 48 percent and 54 percent, respectively. Thus, the additional anomalies momentum, net stock issues, accruals, asset growth and profitability taken together are contributing economically significant predictive power over firm returns. An investor using the additional knowledge of the firm characteristics will earn an additional 17 percent in annual returns on the high minus low hedge portfolio formed from a twenty five portfolio sort. The contribution to equal weighted portfolios is less extreme. Omitting the additional explanatory variables misses approximately 20 percent of the return spread across all sorts.

Lastly, I ask, which of these anomalies is contributing to the return spread in an important way. Fama and French (2008) find that all of the anomalies are statistically significant in a predictive regression, but the question remains, how much is left to be gained by using this information? I take the approach of starting with the full model and systematically dropping each individual anomaly from the regression. If an anomaly is important, then the return spread generated by sorting on the information from the predictive regressions will substantially decrease. If an anomaly is unimportant the spread will remain unchanged, as the information imbedded in the anomaly is

either inconsequential or absorbed by the other anomaly variables. The results show the economic significance of an anomaly variable in the presence of the other predictors.

Table 1.4 shows the results of this approach using the value weighted sorts into twenty-five portfolios. The results show that asset growth is the strongest anomaly; omitting asset growth causes the sorted return spread to fall 29 percent. Intriguingly, asset growth presents some problems in the Fama and French (2008) paper. The coefficient on asset growth is not always significant in the multivariate regressions. The difference here is that Table 1.4 asks for the economic significance of an anomaly variable as opposed to its statistical significance. Statistical significance asks whether the coefficient is measured precisely enough to differentiate its value from zero, while here we ask if the information in an anomaly is easily absorbed by other predictors and if using the anomaly has a large effect on our results (McCloskey, 1985). Net stock issues and momentum follow as having very important predictive value. Omitting one of these variables causes the spread in returns across twenty-five value weighted portfolios to fall about 20 percent. Book to market follows with a 12 percent fall in the return spread after its omission. Dropping accruals and profitability has very little effect on the return spread, while size has no measured effect.

## 1.4 Rolling and No Peeking Regressions

The regression sorts presented thus far use the full sample of returns to sort stocks into portfolios. One rationale to support this approach is that the market (in aggregate) understands and correctly prices the appropriate correlation between an anomaly variable and future returns, but this correlation is unknown to the researcher. Full sample regressions use all available information to estimate the correlation correctly. Yet, these portfolios are not tradable and suffer from look-ahead bias. Information in the post portfolio formation period is used for parameter estimation, which is in turn used to sort stocks into portfolios. Additionally, the parameters may not be constant over time. If anomaly variables reflect mispricing they may be arbitrated away over time.

I present two specifications that overcome these concerns. Both approaches use only information available at the time of portfolio formation to estimate parameters and to create portfolios. The first, rolling regression, uses only information in the 60 months prior to the portfolio formation date. This procedure allows parameters to change slowly over time. The cost is that there is less data

to precisely estimate the parameter on each anomaly variable. If the parameters have considerable time variation, the procedure may produce larger spreads than the full sample procedure. Perhaps an anomaly has been erased by arbitragers completely over time. In this case, the full sample approach forms the full sample parameter estimate on the whole sample, which causes the first part of the sample to understate the correct parameter estimate and the second part of the sample to overstate the parameter estimate.

A second procedure, which I call “no peeking regression,” uses the entire sample available before the portfolio formation period. Thus, parameter estimates use all the information available at the time of the sort. If the parameter estimates are stable across time, this method should outperform the rolling regression method. The gap between this method and the full sample method in Table 1.1 gives a sense of how important look ahead bias is in forming portfolios. For both approaches, I require at least 60 months of data to estimate a parameter, so I compare them to the full sample regressions across an identical time period omitting the first 60 months.

Table 1.5 presents the rolling regression approach. The results show that rolling regressions perform worse than the full sample regressions. The value weighted spreads are 20 percent to 37 percent lower and the monthly Sharpe ratios 34 percent to 40 percent lower. If the parameters vary over time, the effect is more than offset by the precision lost in the measurement of the parameter. The equal weighted portfolios are less affected, with declines in spread of 11 percent to 19 percent, and declines in monthly Sharpe ratios of 26 percent to 35 percent.

Table 1.6 presents the results for the no peeking regression approach. The no peeking regressions perform much better than the rolling regressions and almost as well as the full sample regressions. Sorts into ten and twenty-five portfolios lose only 5 percent and 8 percent of their spread in returns and only 13 percent and 6 percent of their monthly Sharpe ratios, respectively. The maximum loss of 20 percent in return spread for 100 portfolios is lower than any of the losses for rolling regression sorts. The effect is not as strong in equal weighted portfolios, especially in regards to the return spreads created by their hedge portfolios, but their Sharpe ratios do much better in general with about half the loss of rolling regression sorts. The results in Table 1.6 assuage concerns that the previous results in the paper are primarily due to look ahead bias.

## 1.5 Factor Structure of One Dimensional Sorts

The results that were presented in Table 1.1 show that these anomaly variables, when sorted together into a one dimensional sort, are not explained by the Fama and French three factor model, nor by the addition of a momentum factor. Nevertheless, the portfolios may have a strong factor structure that is not captured by traditional factor models. In this section, I use principal components analysis (PCA) to examine the comovement of the one dimensional portfolios. PCA uses an eigenvalue decomposition of the covariance matrix to construct a set of orthogonal linear factors. Each factor explains a decreasing amount of total variance, such that the most variance is explained by the first factor. PCA reduces a large dimensional covariance matrix into the most important components by the potential to explain relatively complicated comovements with a small number of factors.

Previous research using principal component analysis, such as Connor and Korajczyk (1986, 1988), has often focused on a large sample of individual stocks and shown a limited ability to find interesting common movement. The market factor dominates. Zhang (2009) is an exception. He uses principal components analysis on stocks presorted by size and book to market. PCA then finds the SMB and HML factors that were previously hidden. By sorting stocks on a characteristic, he was able to reinforce the common movement related to that characteristic.

But the characteristic factors, HML and SMB, had already been well established, as well as many characteristic factors. After a return spread across some characteristic is found, its relatively easy to find a common factor with a high minus low sort. This has long been considered a success of the APT. Rather than use one of many characteristics, I extend the approach to cover portfolios sorted by expected returns. Now rather than uncovering the factor structure related to characteristics, PCA will uncover the factor structure of expected returns, the only characteristic that really matters.

Table 1.7 shows the results of a principal components analysis on twenty-five value weighted portfolios sorted using the full model regressions. The table shows that 90 percent of the comovement of the portfolios is explained by three uncorrelated factors. A plot of the weights that comprise the first three principal components shown in Figure I reveals the level, slope and curvature factors also common to bond returns.

The first factor explains 74 percent of the portfolio variance. This factor corresponds to the market portfolio; it is comprised of near equal weightings of all twenty-five test portfolios. It captures how stocks in general tend to move together. The second factor, the slope factor, explains 7 percent of the portfolio variance. This factor captures that high and low return stocks tend to move together. The second factor has a correlation coefficient of 0.8 with the high minus low hedge portfolio examined in Table 1.1.

Unlike the high minus low portfolio, the second factor puts a non-zero weight on all the portfolios. It is long the highest return stocks with decreasing intensity until it is short the low returning stocks at an increasing intensity creating a sloping pattern. The second factor shows that a significant portion of the difference in the way high return stocks move relative to low return stocks is explained by their exposure to a common risk factor.

The third factor, curvature explains 3 percent of portfolio variance. The curvature factor is formed by short positions in the extreme portfolios and long positions in the middle portfolios. The curvature pattern signifies that the third factor affects the highest and lowest return stocks in a similar fashion and in an opposite fashion than medium return stocks. Curvature patterns are not common in empirical asset pricing. Baker and Wurgler (2006) find a curvature pattern in response to investor sentiment among book to market firms. They argue that the low return growth stocks and the high return value stocks are both difficult to value and thus respond positively to shocks to sentiment. In a theoretical model McQuade (2013), shows that low return growth firms and high return value firms may both respond positively to spikes in volatility. The growth firms have growth options, while the value firms have options to default and the value of the options increases. The average firm responds negatively to volatility as it represents a deterioration of the investment opportunity set.

In bond yields, Litterman and Scheinkman (1991) find the slope factor corresponds to a steepening of the yield curve, while the curvature factor corresponds to the bending shape of the yield curve. By analogy the slope factor in these anomaly portfolios corresponds to shocks that simultaneously create high returns in the extreme high return portfolios and low returns in the extreme low return portfolios, while curvature corresponds to the changing magnitude of this effect.

Why does this method of portfolio sort have a strong factor structure that seems to condense to three main factors, when traditional methods tend to find an increasing large number of factors?

The answer may be in the Arbitrage Pricing Theory of Ross (1976). Suppose APT holds, so that the return on a security,  $X_i$ , is the sum of its expected return and its loadings on priced and unpriced factors:

$$X_i = E_i + \beta_{1,i}F_1 + \beta_{2,i}F_2 + \dots + \beta_{N,i}F_N + \phi_{1,i}G_1 + \phi_{2,i}G_2 + \dots + \phi_{N,i}G_N + \epsilon_i$$

Here, without loss of generality, I separate the priced factors (F) from the unpriced factors (G). An unpriced factor has a zero risk premium. An example might be industry factors. Technology stocks will tend to all move together through time, but if that comovement is not representative of a systematic risk, then it will not be priced in the aggregate.

Thus, the loadings on the unpriced factors ( $\phi$ ) do not enter into the expected return of the asset.

$$E[X_i] = E_i = \lambda_1\beta_{1,i} + \lambda_2\beta_{2,i} + \dots + \lambda_N\beta_{N,i}$$

The risk premiums of priced factors ( $\lambda$  and the asset's loadings on those factors ( $\beta$ ) determine the expected return of the assets. The anomaly variables in this paper confer information about the expected returns of the underlying stocks, and consequently the stock's loadings on the priced risk factors. A value weighted portfolio then becomes an asset with loadings equal to the value weighted betas and phis of all the assets in the portfolio. The sorting procedure seems to be sorting on the expected returns and thus the loadings on priced factors, whereas characteristics seem to sort on the loadings of both priced and unpriced factors. To take an example, stocks sorted on momentum may commove for both priced and unpriced reasons, but sorting on several anomalies averages out the unpriced factors which appear to differ across anomalies and reinforces the priced factors which the anomalies seem to share. The sorting procedure seems to focus on the covariance structure that matters for pricing.

## 1.6 Conclusion

Regression sorts produce large and important spreads in portfolio returns that are not absorbed by risk adjustments using common factor models. These results are robust to look-ahead bias. The



most important variables to create these spreads are asset growth, net stock issues and momentum. Book to market has intermediate importance, while accruals and profitability have limited importance. Size is not an important predictor. The sorted portfolios present a strong factor structure with an identifiable level, slope and curvature, much like bond returns sorted by maturity. The sorting method allows researchers to identify the factors that matter most to asset pricing and separate them from the unpriced factors.

Table 1.1: One Dimensional Portfolio Spreads

Panel A shows returns in excess of the risk free rate of value and equal weighted portfolios formed by the full model regressions. The model estimates separate regressions for micro, small and large stocks using size, book to market, momentum, net stock issues, asset growth and profitability on next period returns. The fitted regression is then used with time  $t$  data to predict returns at time  $t+1$ . Predicted returns are used to sort into the number of specified portfolios. The Sharpe ratios are the spread divided by the standard deviation of hedge portfolio returns. Panel B shows the alpha of leading models regressed on the hedge portfolios formed by taking the highest predicted return portfolio minus the lowest predicted return portfolio

Panel A

	<u>Value-Weighted Portfolios</u>				<u>Equal-Weighted Portfolios</u>			
Portfolios	2	10	25	100	2	10	25	100
Low	0.43	-0.08	-0.65	-1.03	0.51	-0.13	-0.35	-0.84
High	0.99	1.33	1.56	1.72	1.32	1.87	2.09	2.63
Spread	0.56	1.41	2.21	2.75	0.81	2.00	2.44	3.47
Sharpe	0.19	0.25	0.31	0.27	0.35	0.45	0.45	0.44

Panel B

	Value-Weighted Portfolios				Equal-Weighted Portfolios			
CAPM $\alpha$	0.51	1.39	2.24	2.80	0.88	2.14	2.58	3.60
$t(\alpha)$	4.22	5.97	7.72	6.70	9.60	12.10	11.80	11.08
CAPM $\alpha$ Sharpe	0.17	0.24	0.32	0.27	0.39	0.50	0.48	0.45
FF $\alpha$	0.32	1.09	1.86	2.47	0.71	1.80	2.22	3.31
$t(\alpha)$	2.94	5.03	6.69	5.92	8.50	10.90	10.57	10.15
FF $\alpha$ Sharpe	0.12	0.21	0.27	0.24	0.35	0.45	0.43	0.42
Carhart $\alpha$	-0.06	0.46	1.17	1.85	0.50	1.46	1.86	3.02
$t(\alpha)$	-0.82	2.60	4.80	4.55	6.86	9.56	9.27	9.23
Carhart $\alpha$ Sharpe	-0.03	0.11	0.20	0.19	0.28	0.39	0.38	0.38

Table 1.2: One Regression with Only Size and Book to Market

The table shows the returns in excess of the risk free rate of portfolios formed using only one regression for all stocks of one month ahead excess returns on size and book to market. The second panes shows the change in spread from the full model in Table 1.1 as well as the change in the Sharpe ratio of the extreme high minus extreme low hedge portfolio.

<b>Portfolios</b>	<b>Value-Weighted Portfolios</b>				<b>Equal-Weighted Portfolios</b>			
	2	10	25	100	2	10	25	100
Low	0.43	0.38	0.37	0.22	0.59	0.27	0.20	-0.01
High	0.74	1.03	0.96	1.17	1.20	1.61	1.79	2.01
Spread	0.31	0.65	0.59	0.94	0.60	1.34	1.59	2.02
Sharpe	0.12	0.14	0.10	0.11	0.28	0.31	0.31	0.27
$\Delta$ Spread	-0.25	-0.76	-1.62	-1.80	-0.21	-0.66	-0.85	-1.44
$\Delta$ Spread	-44%	-54%	-73%	-66%	-26%	-33%	-35%	-42%
$\Delta$ Sharpe	-0.07	-0.11	-0.21	-0.16	-0.07	-0.14	-0.14	-0.17
% $\Delta$ Sharpe	-35%	-43%	-68%	-59%	-21%	-31%	-31%	-39%

Table 1.3: Regressions Across Size Groups with Size and Book to Market

Panel A shows portfolios sorted by separate regressions for micro, small and large capitalization stocks of next month excess returns on size and book to market as predictor variables. Panel B compares the results to the results in Table 1.2 with only one regression for all sized stocks. Panel C compares the results with the full model presented in Table 1.1.

**Panel A**

	<u>Value-Weighted Portfolios</u>				<u>Equal-Weighted Portfolios</u>			
Portfolios	2	10	25	100	2	10	25	100
Low	0.44	0.43	0.28	-0.02	0.57	0.31	0.14	-0.05
High	0.90	1.17	1.30	1.78	1.22	1.88	2.16	2.73
Spread	0.46	0.74	1.02	1.81	0.65	1.57	2.02	2.78
Sharpe	0.15	0.15	0.18	0.21	0.28	0.31	0.31	0.27

**Panel B: Comparison With Size and Book to Market in One Regression**

$\Delta$ Spread	0.15	0.08	0.43	0.86	0.05	0.23	0.43	0.75
% $\Delta$ Spread	47%	13%	73%	92%	7%	17%	27%	37%

**Panel C: Comparison With Full Model Using All Anomalies**

$\Delta$ Spread	-0.10	-0.68	-1.19	-0.94	-0.16	-0.44	-0.42	-0.69
% $\Delta$ Spread	-18%	-48%	-54%	-34%	-20%	-22%	-17%	-20%

Table 1.4: Anomaly Significance

Table 1.1V presents the full model regression from Table 1.1 reestimated, each time omitting one of the anomaly variables. The regressions are formed into twenty-five portfolios. The table presents the change in spread from the full model in Table 1.1.

Anomaly	$\Delta$ Spread	% $\Delta$ Spread
Asset Growth	-0.65	-29%
Net Stock Issues	-0.47	-21%
Momentum	-0.43	-20%
Book to Market	-0.28	-12%
Accruals	-0.14	-6%
Profitability	-0.05	-2%
Size	0.01	0%

Table 1.5: Rolling Regression Sorts

This table presents rolling regressions using the full model with all anomaly variables and separate regressions across micro, small and large capitalization stocks. Regressions use the last 60 months of data ending with returns in period  $t$  and anomaly variables in period  $t-1$  to form parameter estimates. These parameter estimates are used to form portfolios. Presented are returns in excess of the risk free rate in the following month,  $t+1$ . The changes are relative to the full sample accross the same time periods.

	<b>Value-Weighted Portfolios</b>				<b>Equal-Weighted Portfolios</b>			
Portfolios	2	10	25	100	2	10	25	100
Low	0.38	-0.17	-0.55	-0.88	0.50	-0.04	-0.33	-0.73
High	0.70	0.99	1.00	0.89	1.15	1.57	1.83	2.17
Spread	0.32	1.16	1.55	1.77	0.65	1.61	2.16	2.90
SD	3.12	6.97	7.72	10.21	2.70	5.60	6.93	9.26
Sharpe	0.10	0.17	0.20	0.17	0.24	0.29	0.31	0.31
$\Delta$ Spread	-0.19	-0.19	-0.58	-0.88	-0.13	-0.38	-0.28	-0.50
% $\Delta$ Spread	-37%	-14%	-27%	-33%	-17%	-19%	-11%	-15%
$\Delta$ Sharpe	-0.07	-0.07	-0.10	-0.09	-0.09	-0.15	-0.13	-0.11
% $\Delta$ Sharpe	-40%	-31%	-34%	-34%	-28%	-35%	-29%	-26%

Table 1.6: No Peeking Regression Sorts

This table presents No Peeking regressions using the full model with all anomaly variables and separate regressions across micro, small and large capitalization stocks. Regressions use the entire data set available at the time of portfolio formation. Thus, parameter estimates have no look ahead bias.

	<b>Value-Weighted Portfolios</b>				<b>Equal-Weighted Portfolios</b>			
Portfolios	2	10	25	100	2	10	25	100
Low	0.40	-0.07	-0.66	-0.82	0.48	-0.10	-0.36	-0.80
High	0.84	1.21	1.30	1.29	1.17	1.56	1.70	1.82
Spread	0.44	1.27	1.96	2.11	0.69	1.65	2.06	2.62
SD	2.97	6.08	6.90	8.96	2.21	4.40	5.47	7.85
Sharpe	0.15	0.21	0.28	0.24	0.31	0.38	0.38	0.33
$\Delta$ Spread	-0.07	-0.07	-0.18	-0.54	-0.09	-0.34	-0.38	-0.79
% $\Delta$ Spread	-14%	-5%	-8%	-20%	-11%	-17%	-16%	-23%
$\Delta$ Sharpe	-0.02	-0.03	-0.02	-0.03	-0.02	-0.06	-0.06	-0.09
% $\Delta$ Sharpe	-13%	-13%	-6%	-10%	-6%	-14%	-14%	-22%

Table 1.7: Principal Component Analysis

The table presents a Principal Components Analysis of the twenty-five portfolios formed using the full model in Table 1.1.

<u>Component</u>	<u>Eigenvalue</u>	<u>Variance Explained</u>	<u>Cumulative</u>
Component 1	691.82	74.39%	74.39%
Component 2	67.17	7.22%	81.62%
Component 3	31.19	3.35%	84.97%
Component 4	13.36	1.44%	86.41%
Component 5	12.52	1.35%	87.75%
Component 6	11.82	1.27%	89.02%
Component 7	10.13	1.09%	90.11%
Component 8	9.34	1.00%	91.12%
Component 9	9.02	0.97%	92.09%
Component 10	8.50	0.91%	93.00%



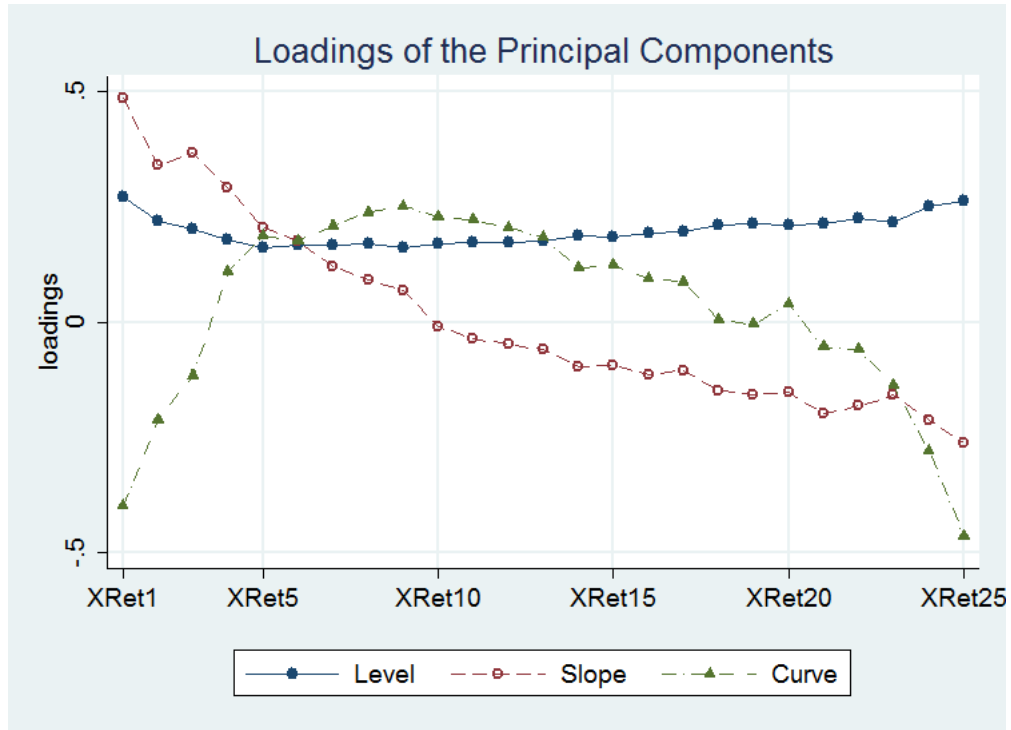


Figure 1.1: PCA Weights

The figure shows the loadings of each of the first three principal components of twenty-five anomaly portfolios.

## Chapter 2

# The Level, Slope, and Curve Factor Model for Stocks

### 2.1 Introduction

The number of potential new asset pricing factors is exploding. Some of these factors are priced and some are not. A priced factor exposes the investor to systematic risk and comes with a risk premium. An unpriced factor represents common movement across stocks unaccompanied by systematic risk or a risk premium. The standard approach is to first identify a common factor, and then to test whether this factor is priced.

In contrast, I describe a method to search specifically for the priced common factors in stock returns. First, sort stocks by expected returns using several different return predictors. Second, use principal components to extract the common factors. The first stage produces portfolios sorted in one dimension on expected returns. The second stage isolates comovement that is priced in the cross-section of stocks by searching for common factors in portfolios already sorted by expected returns.

In order to separate priced from unpriced common factors, the method relies on sorting expected returns using many different predictors. Sorting stocks by only book to market in step one produces a sort on expected returns, but it is unlikely to isolate priced risk factors, because portfolios sorted on book to market have both priced and unpriced common movement (Daniel and Titman (1997)

and Gerakos and Linnainmaa (2014)). To overcome this obstacle, I sort on expected returns estimated from multiple regressions that combine signals from many different predictor variables. The procedure isolates factors with nonzero prices of risk, and separates these factors from their unpriced counterparts.

Using this method, I show that extracting common factors from these portfolios extracts three factors with the familiar level, slope and curve pattern that can be extracted from returns of bond portfolios sorted by maturity (Litterman and Scheinkman, 1991). The level factor is highly correlated with the market factor, but the slope and curvature factors are distinct from commonly used factors.

I find that the Level, Slope and Curve model outperforms the Fama and French (1993, 2014a) three factor and five factor models, and performs similar to three other leading models: a four factor model with momentum of Carhart (1997), the four factor model of Hou et al. (2014), and the four factor model of Novy-Marx (2013a). In addition, I use “horse races” to test whether adding one of the other anomaly-based factors to the Level, Slope and Curve model adds any explanatory power. I find that additional factors add very little to the Level, Slope and Curve model.

Finally, I test whether the Level, Slope and Curve factors relate to deeper economic models. I show that the three factors proxy for priced risk in accordance with the Intertemporal Capital Asset Pricing Model. Factor returns correspond to changes in the investment opportunity set. In addition, the portfolios sorted on expected return have large spreads in their covariance with annual consumption growth, a prediction of the Consumption Capital Asset Pricing Model (CCAPM). A standard linearized CCAPM explains almost all of the spread in average returns across the portfolios. Since the Level, Slope and Curve model explains almost all of the variance in the expected return sorted portfolios, another interpretation is that the model represents high frequency factors mimicking changes in expected future consumption growth.

This procedure to extract priced factors makes an important contribution to the literature on the cross-section of stocks. First, Cochrane (2011) calls for a reorganization of the factor structure of stock returns to decide which factors are the most important, and which factors should we be writing deeper macroeconomic models to explain? The procedure searches for the most economically important risk factors that are *priced* in the cross-section, and finds that the cross-section can be summarized by three important factors. This result stands in stark contrast to Green

et al. (2014) who find staggering multidimensionality in stock returns, and suggest a factor model with twenty-four factors. Their approach follows the more traditional procedure of constructing portfolios of stocks sorted by characteristics, but this approach allows unpriced factors to creep into the results.

Second, this paper bridges a gap between empirically generated factor models and theoretically generated factor models. The models of Fama and French (1993), Carhart (1997), and Novy-Marx (2013a) are all empirically motivated factor models. The prediction of Arbitrage Pricing Theory of Ross (1976) is that expected returns should be accompanied by common factors. Novy-Marx (2013a) explains the motivation, “While I remain agnostic here with respect to whether these factors are associated with priced risks, they do appear to be useful in identifying underlying commonalities in seemingly disparate anomalies.” Unfortunately, the general procedure is prone to concerns about data mining. It is not clear precisely when, after observing alphas on sorted portfolios, one should add another return factor.

The traditional response to data mining concerns is to lean more heavily on theoretical motivations. The five factor model of Fama and French (2014) and the Q-factor model of Hou, Xue and Zhang (2014) are both advocated on theoretical grounds. But the specific theoretical motivations are different and will likely continue to be contentious. Hou et al. (2015) criticizes the Fama and French (2014a) model on theoretical grounds and Novy-Marx (2015) criticizes the Q-factor model on theoretical grounds. More importantly, models based on these specific theories are less useful to researchers developing distinct theoretical insights to deepen our understanding of the cross-section of returns. Cochrane (2011) calls not for a perfect theory to end all debate, but rather a synthesis of data, a parsimonious description of the important factors in the cross-section that theory is meant to explain.

This paper offers a theoretically motivated search for empirical factors. This procedure bridges theory and empirics to identify the salient facts that deeper models should be trying to explain. The Arbitrage Pricing Theory *predicts* that expected returns should be associated with a factor structure. Kozak et al. (2015) show that empirical factor models don’t have the ability to distinguish between rational and behavioral explanations of the cross-section of returns. This highlights a great strength of this approach. By using only the law of one price and the absence of arbitrage, the procedure synthesizes the most important factors from both rational and behavioral models,

without taking a stand on particular models.

Third, a strength of this paper is its description of the factor structure of returns that is not centered around firm characteristics. While much has been learned by characteristic-based sorts and described by characteristic-based factors, it is at least conceivable that the common movement across stocks is not principally caused or described by firm characteristics. In this paper, characteristics are just useful signals for identifying latent factors. The Level, Slope and Curve Model offers a lens through which to view the factor structure of the cross-section without appealing to characteristics.

There are limitations to this approach. The procedure may not find all priced factors. There may be important priced factors that are only important to a small number of stocks or there may be factors that affect many stocks, but have very small prices of risk. The procedure is also limited by the predictors used in the first stage. A strong enough predictor could change the factors found in this paper. But the method has great resilience to these changes, since a new predictor would have to be strong *in the presence of other predictors* (Fama and French, 2014b). Thus, identifying priced factors using this procedure makes it less likely that future researchers will write consumption based asset pricing models explaining false factors. The method also provides some protection against the datamining concerns of Kogan and Tian (2013). Harvey et al. (2015) show datamining may be a concern in the first stage, linking characteristics to expected returns, but this datamining need not be associated with the strong factor structure produced in the second stage.

The benefit of this approach is that it narrows the factor space once again. A primary goal of reducing a set of portfolio returns to a much smaller set of common factors is data reduction (Cochrane, 2011). After all, we do not require a theory that explains all of the comovement of the hundreds of assets used in this paper; one that explains the common factors that price them would be very useful. Since this method shows some robustness to new anomalies, theoretical work to explain the statistically extracted factors is less likely to go off course. The underlying factor structure is more stable. A disciplined empirical approach to generating priced factors can help narrow the focus of new theoretical work from many disparate characteristics to a few priced factors. While unpriced common factors may be interesting in their own right, they are unlikely to be central components connecting asset price movements to business cycle movements. They are not likely to be central puzzles in the intersection of macroeconomics and finance.

## 2.2 Literature Review

I draw on the large literature of firm characteristics that predict returns (often called anomalies). There are several papers that have used multiple regression on many predictive characteristics. Haugen and Baker (1996) use regressions on many variables to sort stocks by predicted next period returns. Fama and French (2008) show that many characteristic variables contain separate and distinct information that varies across size groups. Lewellen (2015) shows that these variables also predict returns out of sample.

Each predictor individually has information about expected returns, and portfolios sorted on each individual predictor likely has unpriced common variation. But as long as the unpriced variation is not perfectly correlated across predictors, the common signal of expected return will be reinforced and the noise will be averaged out. Zhang (2009) demonstrates this logic using principal components analysis to find factors similar to the Fama and French (1993) small minus big (size) and high minus low (value). When using principal components to extract common factors from individual stocks, he finds no evidence of common variation due to differences in size and book to market (Connor and Korajczyk, 1986, 1988), but when using principal components analysis on a 10 by 10 portfolio sort on size and book to market, he recovers the common variation due to the size and book to market factors. Sorting on a common characteristic reinforces the common signal. The difference in my approach is that I use expected return as the common signal rather than a firm characteristic. By sorting on expected returns, I reinforce the priced factors and average out the unpriced factors, allowing me to uncover the priced factors through principal components analysis.

The method can both grow with and show robustness against the growth and inclusion of new anomaly variables. Including an additional anomaly with predictive power will help generate an even sharper estimate of the true underlying factors, but it would take a very strong one to dramatically alter the factors. The anomaly must have large explanatory power even after controlling for several other strong anomalies.

Often, authors choose factors in response to the observation that there is a spread in average returns across stocks. The spread is accompanied by common movement across stocks, and the authors test to see if the comovement is priced in the cross-section. Fama and French (1993) saw persistent differences in average returns created by a double sort on size and book to market, and

reasoned that the spread in value versus growth stocks holding size constant, and the spread in small minus large stocks holding book to market constant, could explain returns by proxying for latent priced factors. Fama and French (1996) show the model successfully prices other anomalies such as sales growth and long-term reversal.

In a general sense, the method and goal of this paper are the same. I start with anomaly variables that generate large spreads in return, then identify comovement across the portfolios caused by a factor structure. Finally, I determine whether these factors can price other assets as the Arbitrage Pricing Theory predicts. The difference is that I am looking only for common factors that explain expected returns. I am not looking for common factors related to firm characteristics that may be priced or may have a priced component.

While the three factor model of Fama and French (1993) provided a workable replacement for the Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965), the number of anomalies has continued to grow. The response has traditionally been to add additional factors to describe portfolio spreads unexplained by the three factor model, such as the momentum factor of Carhart (1997), the liquidity factors of Pastor and Stambaugh (2003) and Sadka (2006), and the volatility factor of Ang et al. (2006). More recently, authors have questioned the three factor model more fundamentally and produced factor models combining different anomalies, such as the four factor models of Novy-Marx (2013a) and Hou et al. (2014), and later the five factor model of Fama and French (2014a). All of these factors are formed from sorts on firm characteristics or firm betas. This paper is the first to extract factors from portfolios sorted by expected returns using many characteristics.

## 2.3 Factor Structure of Anomalies

The Arbitrage Pricing Theory of Ross (1976) posits that returns are generated by an asset's loadings on common factors and an idiosyncratic term. If the APT holds, the return on security  $i$ ,  $X_i$ , is the sum of its expected return ( $E_i$ ) and its loadings multiplied by priced and unpriced factors:

$$X_i = E_i + \beta_{1,i}F_1 + \beta_{2,i}F_2 + \dots + \beta_{N,i}F_N + \phi_{1,i}G_1 + \phi_{2,i}G_2 + \dots + \phi_{N,i}G_N + \epsilon_i$$

Without loss of generality, I separate the priced factors (F) from the unpriced factors (G). An

unpriced factor has a zero risk premium, for example industry factors. Idiosyncratic risk is captured by a mean zero error term ( $\epsilon$ ).

The loadings on the unpriced factors ( $\phi$ ) do not enter into the expected return of the asset.

$$E[X_i] = E_i = \lambda_1\beta_{1,i} + \lambda_2\beta_{2,i} + \dots + \lambda_N\beta_{N,i}$$

Only the risk premiums of the priced factors ( $\lambda$ ) and an asset's loadings on those factors ( $\beta$ ) determine its expected return.

In the Arbitrage Pricing Theory, expected return itself is the characteristic of interest. Using this as motivation, I sort stocks by their expected returns, in order to reinforce the priced comovement across stocks and wash out the unpriced comovement. The one dimensional sort strengthens patterns created by the comovement of the portfolios related to expected returns, and weakens the patterns created by common factors with zero risk premiums. If there is only one predictor of expected returns, such as book to market, then my ability to isolate priced movements from unpriced movements is minimal. But by utilizing the expansive anomaly literature, I can sort on expected returns from several different sources. This creates large spreads in expected returns both in and out of sample.

## 2.4 Data and Variables

The sample runs from July 1964 until December 2014. The variable definitions are identical to Fama and French (2008) with two exceptions. Reacting to Novy-Marx (2013a) and Ball et al. (2015), Fama and French (2014a) argue that operating profit is a more robust predictor of average returns in the cross-section than return on book equity, and Aharoni et al. (2013) show that asset growth at the firm level is a better and more theoretically motivated predictor than asset growth per share. Thus, I slightly alter the Fama and French (2008) regressions to reflect these insights and to match the definitions used in Fama and French (2014a).

Returns are monthly holding period returns obtained from the Center for Research in Security Prices (CRSP) and adjusted for delisting return when available. The accounting data is from Compustat, which has survivor bias before 1962. Since I require two years of accounting data to form some characteristics, my sample begins with stock returns in 1964. The sample includes only



common equity securities (share code 10 and 11) for firms traded on NYSE, NASDAQ or AMEX. Additionally, I drop financial firms (Standard Industry Classification codes of 6000 to 6999) and stocks trading below \$1. All anomaly variables are measured at the end of June using the last fiscal year's accounting data, except for momentum, which is defined monthly. The precise variable definitions can be found in Fama and French (2008) and include: size, book to market, momentum, net stock issues, accruals, investment, and profitability.<sup>1</sup>

## 2.5 One Dimensional Portfolio Sorting Procedure

In order to sort stocks by expected returns, I use a procedure that forms portfolios using many firm characteristics as predictors. Fama and French (2006a) provide a logical three step procedure to do this. First, run Fama-MacBeth cross-sectional regressions of one month ahead firm-level returns on current values of the anomaly variables. Second, use the coefficient estimates from the regressions to predict the one month ahead return for each stock. Third, sort stocks into portfolios based on the predicted returns.

The goal of the procedure is to yield a portfolio sort that creates a wide spread in average returns using only information available to the investor at the time they form their portfolios. An economically significant predictor will account for a relatively large portion of the spread. Clearly, I must be explicit when I define an investor's information set. Fama and French (2006a) use parameter estimates from the full sample in order to sort stocks into portfolios. A rationale for this approach is that the whole time series best reflects the contribution of each anomaly to expected returns. Alternatively, I could use regressions only on past data to form sorts or rolling regressions that capture time varying betas as in Haugen and Baker (1996) and Lewellen (2015). Since my goal is to identify the factors, rather than trading on them, I use the full sample for my main tests, but I also show out of sample results and find that the level, slope and curve factors are not very sensitive to the choice of information set.

Each cross-sectional regression takes the following form:

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<sup>1</sup>Size is attributable to Banz (1981b), book to market to Rosenberg et al. (1985), Chan et al. (1991), and Fama and French (1992), momentum to Jegadeesh and Titman (1993), and net stock issues to Daniel and Titman (2006) and Pontiff and Woodgate (2008) following earlier work by Ikenberry et al. (1995) and Loughran and Ritter (1995). Accruals is attributable to (Sloan, 1996), profitability to Haugen and Baker (1996), Cohen et al. (2002) and Novy-Marx (2013a), and investment to Fairfield et al. (2003), Titman et al. (2004) and Cooper et al. (2008).

$$XRet_{i,t+1} = \beta_0 + \beta_1 LogSize_{i,t} + \beta_2 LogB/M_{i,t} + \beta_3 Mom_{i,t} + \beta_4 zeroNS_{i,t} + \beta_5 NS_{i,t} + \beta_6 negACC_{i,t} + \beta_7 posACC_{i,t} + \beta_8 dA/A_{i,t} + \beta_9 posOP_{i,t} + \beta_{10} negOP + e_{i,t+1} \quad (2.1)$$

The stock return in excess of the risk free (XRet) rate for each stock in the following month is regressed on log firm size (LogSize), log book to market (LogB/M), momentum (Mom), a dummy if no stock was issued (zeroNS), net stock issues (NS), negative accruals (negACC), positive accruals (posACC), asset growth (dA/A), positive operating profit (posOP) and negative operating profit (negOP). Fama and French (2008) find that stocks of different size groups (micro, small and large) have different exposures to characteristic predictors, so I run the regression above separately for each size group allowing the parameter estimates to differ across these groups.

These sorts are very effective at generating a spread in portfolio returns. Figure 2.1 shows the results of the sort for each portfolio. Predicted returns, represented by the line, are produced from the fitted values of the regressions for each stock combined into a value-weighted portfolio. Average excess returns, represented by dots, are the average value-weighted returns for each portfolio.

Table 2.1 displays the summary statistics of the twenty-five sorted portfolios. Each portfolio characteristic is formed by the value-weighted average (using beginning of the month market equity) of each stock in the portfolio. Thus, the portfolios are not dominated by the plentiful, but tiny micro cap stocks. The sorting method creates a large spread in average excess returns (XRet) similar to the spread in predicted returns ( $\widehat{XRet}$ ). All the multiple regression characteristics, except for size (JME), show monotonically increasing or decreasing patterns in expected returns with the sign predicted by previous research. Momentum (Mom) and investment (dA/A) show especially strong patterns, the difference between the extreme low return and extreme high return portfolios are two or more standard deviations. Net stock issues (NS) also shows a strong trend, but it is concentrated in the low return, high net issue portfolios. Accruals (A/BE) and book-to-market (B/M) both create spreads of less than one standard deviation between the high and low return portfolios. Lastly, size has a somewhat curved sort. The June month end market equity increases to a maximum at portfolio five and then decreases from portfolio five to the highest return portfolio twenty-five, consistent with the size effect. The extreme low return portfolios aren't especially dominated by small stocks. The value-weighted June market equity is still larger than half the other portfolios.

I also include two characteristics not included in the regression, return on equity (ROE) and idiosyncratic volatility (IVOL). There is not a clear trend for return on equity except to note that it is only negative at the extremes. Idiosyncratic volatility, the average daily realized volatility in the current (not preceding) month, shows a curved pattern related to, but not identical to size. Idiosyncratic volatility is largest at the each extreme and monotonically decreases from both extremes until it reaches the minimum value at portfolio nine.

In the last column, I show the sort on average excess returns from the “No Peeking” sorts (XRet-NP). I run the same Fama-MacBeth regressions using ten years to form an initial estimate and then re-estimating the regressions with an ever expanding window. The first month of portfolio formation is July 1974 using data on characteristics and returns from July 1964 to June 1974. The window expands to include each new month of data on returns and characteristics to form portfolios over the remaining sample. The spread in returns formed by the no peeking regressions is nearly as large as the full sample regressions, only 20 basis points smaller (7% smaller than the full sample spread).

## 2.6 Factor Structure of One Dimensional Sorts

The next step is to extract common factors from these portfolios. I use principal components analysis (PCA), which uses an eigenvalue decomposition to identify common factors across portfolios. By construction, the method extracts linear combinations of the test asset returns that explain the structure of the covariance matrix (Tsay, 2005). This approach translates the comovement between the test assets from a covariance matrix to uncorrelated factors. Each factor is formed as a set of weights on the test portfolios. The first factor explains the largest amount of the covariance between the portfolios. The second factor explains the next largest amount that is not captured by the first factor and so on. In total, the factors describe the entire covariance structure between test assets.

When used on a large sample of individual stocks, as in Connor and Korajczyk (1986, 1988), PCA has little power to extract useful factors from stock returns (Brennan et al., 1998), but Zhang (2009) uses portfolios to recover the underlying comovement across stocks related to their characteristics. His insight is that portfolios sorted on firm-level characteristics strengthen the

patterns in stock returns related to the firm-specific pattern. Patterns in returns unrelated to the characteristics cancel out. This paper extends that insight by sorting the portfolios on expected returns, rather than the firm-level characteristics. Using PCA on these portfolios isolates the common factors that determine expected returns.

I use PCA on the twenty-five portfolios sorted from low to high by expected returns using the anomaly regressions. Table 2.2 shows the results of the principal components analysis. The table presents the first ten components, the respective eigenvalues and variance explained. The first three components explain 86% of the variance of the portfolios. The first component explains 74% of returns, the second explains 9% of returns, and the third explains 3% of returns. In Figure 2.2, I present the weightings of the first three components.

The first factor resembles a general market portfolio because it approximately equally weights all 25 portfolios. The factor has a correlation of .95 with the CRSP value weighted market index used in all popular factor models. This is a “level” factor as it represents comovement with the overall level of the market. Stocks tend to rise and fall together.

The second factor is long low expected return stocks and short high expected return stocks. Weights decrease monotonically from long to short. This “slope” factor captures the feature that high expected return stocks all tend to move together and opposite low expected returns stocks, which symmetrically are also moving together. Since the slope factor is going long low expected return stocks and short high expected return stocks, on average it has a negative realization.

Most factors already identified in the finance literature are slope factors. The hml factor captures the tendency of growth stocks to move opposite of value stocks, while the smb factor captures the tendency for small stocks to move opposite of large stocks. Other examples include slope factors for momentum, profitability, investment, volatility, and liquidity.

My slope factor is different in that it captures common movement using all of the characteristics at once. The underlying characteristic of interest is expected returns and not a firm-level proxy for expected returns. While each firm-level characteristic offers some information about expected returns, portfolios built on characteristics alone may share a large degree of common movement that isn’t related to expected returns.

The last “curve” factor is short the extreme low and high return portfolios and long the middle portfolios. The curvature factor shows that extreme stocks tend to move together. If the curvature

factor has a positive realization, both very high and very low expected return stocks will have relatively low returns and the stocks with moderate expected returns will have relatively high returns. Altogether, the factors bear a striking resemblance to the bond factors found by Litterman and Scheinkman (1991).<sup>2</sup> Lord and Pelsser (2007) show that level, slope and curvature characterizes a robust fact about the variance-covariance matrix. Since any factor model can be written as a one factor model, the important point is not the number of factors that principal components produces, but that the factors yield a stable description of the variance-covariance matrix (Roll (1977) and Hansen and Richard (1987)).

Principal components are identified down to a scalar transformation of the factors, so without effecting any of the results I scale each factor to make the interpretation more intuitive. To reinforce a portfolio interpretation, Campbell et al. (1997) suggest dividing by the sum of the loadings on each factor, so that the weights sum to one. That works well for the first factor, but creates a very unintuitive hedge fund for the slope factor. Since the factors are excess returns, the slope factor would represent borrowing \$1 at the risk-free rate, and investing over \$7 long and over \$6 short. Since the slope factor is negative, the hedge portfolio is long low return stocks and short high return stocks and losing money at a very rapid pace. Instead I adopt a different definition of the slope and curve factors by limiting the factors to 100% short. This choice is made in the spirit of Fama and French (1993) who define their high-low factors by investing \$1 short and \$1 long. The choice of scalar is somewhat arbitrary and made only to aide the interpretation of the factors.

In order to ascertain if the principal components analysis uncovers a true common signal, I also use the same method for sorts on ten portfolios and 100 portfolios. The results also show the level, slope and curve patterns. In Table 2.3, I show the correlations of the first five components using sorts on 10, 25 and 100 portfolios. The results show a very strong correlation among the first three components, regardless of the number of portfolios used. The lowest correlation is always between the component extracted from 100 portfolios and the component extracted from 10 portfolios, and for the first three components, the correlation is .993, -.955, and .842, respectively.<sup>3</sup> The fourth and fifth components are not nearly as correlated across sorts. For the fourth, the 100 and 10 portfolio sorts only share a correlation of .150. For the fifth component, the 100 and 25 sorts share

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<sup>2</sup>Lustig et al. (2011) find level and slope factors in portfolios formed on the carry trade.

<sup>3</sup>Since the components are describing variance and are only estimated up to a scalar transformation, a sign change in the correlation coefficient has no content.

the smallest correlation of .065. I exclude the fourth and higher factors from this study. While I make no attempt to rule out the possibility that the fourth and fifth factors represent some form of priced risk, there is at a minimum an issue with measuring the signals precisely. I only include the first three factors in this study in order to get strong common signals that are not dependent on the sorting procedure.

The factors are very stable across subsamples. Table 2.4 shows the results of splitting the sample into two halves and conducting principal components on each subsample. The first sample runs from July 1964 to March 1988, while the second sample runs from April 1988 to December 2012. Each subsample shows the level, slope and curve factors. Dividing the sample allows for two out of sample tests. Because I run the regression and PCA separately in each subsample, I can compare the factors formed with first half weights with the factors formed with the second half weights. Thus, I have two out of sample tests. An investor forming a factor model in the first half can be taken out of sample to the second half. Alternatively, the factor model formed in the second half can be tested out of sample in the first half of the data. In the first half of the sample, when the end of sample components are out-of-sample, the in and out of sample level, slope and curve factors have correlations of 1.00, -0.92, and 0.73. In the second half of the sample, the correlations are 1.00, -0.96, and 0.82. Evidently, the factor structure of these portfolios is very stable.

These factors differ from Fama and French's three factors. Table 2.5 shows the correlation of the level, slope, and curvature factors with several other proposed factors. None of the three extracted factors has a correlation above 0.25 with HML. SMB is correlated at a moderate level of 0.46 with the level factor and 0.49 with the curve factor, echoing somewhat the shape in the characteristics in Table 2.1, but has correlations below 0.40 with slope and curve. The slope factor has a correlation of 0.77 with momentum, the strongest correlation in the table. The profitability factors, RMW, ROE and PMU have low correlations with the slope factor and with the curve factor. Investment shares a low correlation with level, slope, and curve. None of the three factors are very correlated with liquidity. The slope and curve factors are different than the factors already represented in these leading models.

## 2.7 Time-series Asset Pricing Tests

The Arbitrage Pricing Theory predicts that the wide spread of excess returns created by sorting stocks into portfolios based on their expected returns will be explained by each portfolio's loadings on common factors. Table 2.6 shows the results of time series regressions of the portfolio returns in excess of the risk free rate on the first one, two, three and four principal components. Because the factors are uncorrelated, the pattern in betas are captured by the loadings shown in Figure 2.2. The table shows the alphas, t-statistics and R-squareds from each of the four time series regressions on each of the twenty-five portfolios.

In the third column,  $\alpha_1$  shows that the large spread in returns is not captured by the first factor. This regression is almost identical to the traditional CAPM, so while it is not surprising that the alphas are not captured, it is interesting that much of the alpha shifts to the short leg. Over 60% of the alpha on the high return portfolio is explained by the first factor. The level loadings in Figure 2.2 actually mask a somewhat significant variation in market betas across portfolios. The extreme portfolios have loadings of 0.26, and the middle portfolios have 0.17, which form a barely perceptible curved pattern in the figure, but represent a 50% increase from the low beta middle portfolios to the high beta extremes. This curved pattern in the level beta creates the result in column three, helping the first factor capture the alpha on the high beta, high return portfolio, and increasing the alpha on the extreme low return portfolio.

The fourth column shows that the second factor explains a large portion of the one factor alpha. The alpha on the extreme high portfolio is slightly negative and insignificant. While the alpha on the extreme low portfolio has fallen over 50% from the one factor model, it remains statistically significant. The average R-squared of the twenty-five regressions rises from 76% to 83% with the addition of the slope factor, while the average alpha falls from 0.40% per month to 0.19% per month. Column five shows that adding the curve factor increases the average R-squared to 86% and decreases the alphas to 0.17% per month. The fourth factor adds little additional R-squared, and while it seems to decrease some alphas, it follows a somewhat suspect zig-zag pattern that was shown earlier to be somewhat unstable.

The GRS Tests show that all four specifications are strongly rejected, not unlike Fama and French (1993). Importantly, the three factor model captures a large portion of the spread in average

returns, and a large portion of the variance of the twenty-five portfolios. Perhaps unsurprisingly, the low return portfolio proves much more difficult to price than the high return portfolio given that an arbitrager must take a short position to profit off of these portfolios.

## 2.8 Time Series and Cross-sectional Asset Pricing Tests and Comparisons with Leading Models

If the APT holds and this method succeeds at extracting priced factors, the model predicts a relationship between expected returns and factor loadings. In this section, I perform a number of asset pricing tests in order to compare the Level, Slope and Curve model to leading factor models. Using a variety of test portfolios, I compare the model to the Fama and French three factor, four and five factor models, as well as the four factor models of Novy-Marx (2013a) and Hou et al. (2014).<sup>4</sup> I also compare the Level, Slope and Curve model to the no peeking expanding window version of itself, which is first available in July 1974.

The goal of this paper is to introduce a new method to summarize the cross-section of stock returns. To this end, I will test the Level, Slope and Curve model against a large variety of benchmark models with both time series and cross-sectional test designs to try to give a comprehensive view of how well the model captures important features of the cross-section and how that relates to other leading models. Because all the factors are tradeable, the time series tests impose the requirement that the factors price themselves without error. I compare each model’s ability to price a range of assets and ultimately summarize the results by calculating the Hansen and Jagannathan (1997) distance for each model.

The cross-sectional tests relax this assumption allowing the price of risk to deviate from the average in sample return of the factor. Lewellen et al. (2010) point out a number of problems with cross-sectional asset pricing tests, especially when only twenty-five portfolios of size and book to market are used for test assets. If the test portfolios have a strong factor structure, the cross-sectional asset pricing tests may not be informative. They show that including many diverse test assets relaxes the factor structure and creates more informative asset pricing tests.

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<sup>4</sup>I would like to give special thanks to each author for generously sharing their factors. I obtained the Fama and French factors (including momentum) from Ken French’s data library. I obtained the Novy-Marx four factor model from his data library. Chen Xue shared the Q-Factor model through email correspondence.



In my tests, I include a diverse set of portfolios in order to relax the factor structure and attain more informative results. These cross-section tests have three testable implications. The R-squared of the cross-sectional regression should be close to 1, as the assets should be priced by the factors. The constant term should be close to zero, as the constant return represents the zero-beta rate, which should be near the risk free rate. The coefficients of the cross-sectional regressions should be near the average return on the factors, as the coefficient should equal the cross-sectional risk premium.

### 2.8.1 Factors and Test Assets

The Fama and French three factor model (Fama and French, 1993) uses the market portfolio and two hedge portfolios, one long high book to market stocks and short low book to market stocks (HML) and the other long small stocks and short large stocks (SMB). The Fama and French four factor model, also called the Carhart (1997) model, adds a momentum factor (MOM), long stocks that have risen over the last 12 months and short stocks that have fallen over the last 12 months. The Fama and French (2014a) five factor model excludes momentum and includes a factor long low investment stocks and short high investment stocks (CMA) and a factor long high profit stocks and short low profit stocks (RMW). Since both the five factor models and three factor models use multidimensional sorts to form factors, the SMB and HML differ across the two models. I use the appropriate version for each. The Novy-Marx four factor model uses the market portfolio combined with hedge portfolios of industry adjusted value (HML), momentum (UMD) and gross profitability (PMU). The Hou, Xue and Zhang four factor model uses the market portfolio combined with hedge portfolios on size (SIZE), investment (INV) and profitability measured by return on equity (ROE). In the asset pricing tests, the use of the Novy-Marx and Hou, Xue, Zhang model restrict the sample to end December of 2012.

For test assets, I use two groups, one consisting of 119 portfolios of stocks and bonds, and the other consisting of 32 “hedge” portfolios formed as high minus low, long/short portfolios formed from the extreme deciles of stock characteristic sorts. For the 119 portfolios, I use ten portfolios formed by the results of the “No Peeking Dissecting Anomalies” regressions in Section 4, twenty-five portfolios sorted on size and book-to-market, 10 portfolios sorted on momentum, returns on five treasury bonds (1 year, 5 year, 10 year, 20 year and 30 year), forty-nine industry portfolios, ten

portfolios formed on operating profit and ten portfolios formed on investment (asset growth).<sup>5</sup>

The second group of test assets consists of 32 hedge portfolios formed by decile sorts on 32 anomalies by Novy-Marx and Velikov (2016).<sup>6</sup> Each hedge portfolio is formed by sorting on a characteristic and then taking a long position in one extreme decile and an equal short position in the other extreme decile. The portfolios are formed on sorts on size, gross profitability, value, value-profitability (combined), accruals, asset growth, investment, Piotroski's f-score, net issuance, return on book equity, failure probability, value-profitability-momentum (combined), value-momentum (combined), idiosyncratic volatility, momentum, standardized unexpected earnings, earnings surprise, industry momentum, industry relative reversals, industry relative reversals combined with industry momentum, short-term reversals, and low volatility industry relative reversals. Detailed descriptions of each hedge portfolio are available in the appendix of Novy-Marx and Velikov (2016). The diverse set of portfolios serve to give a holistic look at how well the models capture the cross-section of expected returns.

### **2.8.2 The Level, Slope and Curve Model vs. The Fama and French Three Factor Model**

First, I compare the level, slope and curve model with the Fama and French three factor model using time series tests. Figure 2.3 displays graphically the results of time series tests of each of 119 test portfolios on the respective three factor models. In each panel, on the X-axis is the predicted return of the model, i.e., the betas times the average return on the factor for each test asset. The Y-axis is the average return. The pricing error (alpha) is the vertical distance of each graphed test asset from the 45-degree line. Perfect pricing with zero alphas would put each test asset on the 45-degree line.

The right panel, the results with the Fama and French three factor model, captures the chaos that has befallen the cross-section (Cochrane, 2011). The left panel shows that without increasing the number of factors the Level, Slope and Curve model captures a large amount of the spread in average returns, effectively reorganizing the cross-section. A summary statistic for the graphical relationships is the cross-sectional R-squared (while still imposing the time series restrictions),

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<sup>5</sup>I obtain the portfolios formed on size and book to market, momentum, industries, operating profit, investment and the Fama and French factors from Ken French's website. I obtain the bond portfolio returns from CRSP.

<sup>6</sup>I obtain these portfolios from Robert Novy-Marx's data library and I thank the authors for making them available.

which is equivalent to the squared correlation of average returns and predicted returns. The R-squared for the Fama and French three factor model is 12%, while it is 56% for the Level, Slope and Curve model.

Figures 2.4 through 2.10 break apart this picture, showing each group of assets individually. Figure 2.4 shows the pricing errors of the two models tested against the 10 “No Peeking Dissecting Anomaly” portfolios formed using expanding window regressions of returns on seven firm characteristics. The Level, Slope and Curve model captures almost all the spread in average returns.<sup>7</sup> The Fama and French three factor model shows fairly large pricing errors, with just a little ability to price some of the high expected return portfolios.

Figure 2.5 shows the initial success of the Fama and French three factor model, pricing twenty-five portfolios built on size and book to market. Each test asset is labeled by its sorted portfolios with the size quintile listed first (smallest to largest) and the book to market quintile listed next (growth to value). The small growth portfolio (SBM11) clearly confounds both factor models. While the Level, Slope and Curve model captures a large spread in the average returns, it leaves alphas on the extreme value portfolios (SBM15, SBM35) and doesn’t price the second to smallest growth portfolio (SBM21) as well as the Fama and French three factor model.

Figure 2.6 shows a stark contrast between the two models. The Level, Slope and Curve model captures all of the spread in momentum and then some, slightly reversing the momentum anomaly. The return to high past return stocks (MOM10) is not quite high enough to justify its factor loadings. The Fama and French three factor model actually has pricing errors larger than the spread in average returns.

Figure 2.7 shows the two models for portfolios formed on 49 industries. While there is less spread in average returns to price in the industry portfolios (notice the decreasing axes), the Level, Slope and Curve model has smaller pricing errors than the Fama and French three factor model. The average absolute alpha is 15 and 21 basis points for the two models, respectively.

Figure 2.8 shows the two models tested on U.S. Treasuries ranging from one to thirty years in duration. The Level, Slope and Curve model shows a clear spread in predicted returns that begins to line up with average returns. The spread comes from significant positive loadings on

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<sup>7</sup>Despite the seeming success of the model, the GRS test of Gibbons et al. (1989) rejects the model on these ten portfolios with a p-value on the order of  $10^{-7}$ . With the understanding that all the models are to some degree misspecified, I for the most part omit the GRS tests.

the curve factor for long-term bonds, suggesting that long-term bonds behave similar to large, low volatility stocks. The Fama and French three factor model doesn't produce much of any spread in the predicted returns of treasury bonds.

Figure 2.9 shows the two models tested on ten portfolios formed on operating profit. Most of the alphas are actually created by the models as the spread in the average returns from the most profitable decile to least profitable decile is only 26 basis points. That the lowest profit stock portfolio (OP1) has a high level and market beta serves to confound both models, and while it sticks out as an outlier in the left panel, average returns on the other nine deciles line up reasonable well with predicted returns. In the right panel, both low and high profit stocks confound the three factor model, such that the ranking of predicted returns is nearly the inverse of the ranking of average returns.

The ten investment portfolios in Figure 2.10 show similar results for both models. The average absolute alpha for both models is 10 basis points. The high investment portfolio (INV1) isn't quite captured by either model with the two models leaving -26 and 22 basis points unexplained, respectively. The horizontal spread between the high investment portfolio (INV1) and low investment portfolio (INV10) shows that the Level, Slope and Curve model is generating a spread in predicted returns. In this case, a slightly larger level beta for the high investment portfolio is being offset by a much larger slope beta.

### 2.8.3 Level, Slope and Curve Model Vs. Leading Factor Models

The Level, Slope and Curve Model performs very well versus the Fama and French three factor model. The comparison is important because of the preeminence of the model's status as the default method for risk adjustment over the last two decades, and also because the two models have the same number of factors. I also test the Level, Slope and Curve model versus more recent models that use additional factors to price assets. I find that the Level, Slope and Curve model, despite having fewer factors, performs comparably and often better than other leading models.

Panel A of Table 2.7 shows the results of time series tests on the set of 119 test portfolios comparing the Level, Slope and Curve model (LSC) and the out of sample Level, Slope and Curve No Peeking model (LSC-NP), to the Capital Asset Pricing Model (CAPM), the Fama and French three factor model (FF3), the Carhart model (CAR), the Fama and French five factor model, the

Novy-Marx four factor model (RNM) and the Hou, Xue, and Zhang four factor model (HXZ). The first row shows that the average absolute alpha of the Level, Slope and Curve model is the lowest of all models at 15 basis points, and only equaled by the Carhart model. The No Peeking LSC model has only slightly higher alphas of 16 basis points. The CAPM has the highest average absolute alphas of 23 basis points, followed by the Carhart and Hou, Xue, Zhang models at 20 basis points.

The second row, in panel A shows the number of t-statistics significant at the 5% level of confidence. While not a rigorous comparison of the models, this serves to capture what researchers mean by “model X explains Y anomalies,” especially in the next set of assets formed on anomaly hedge portfolios. The LSC and LSC-NP models leave the least significant alphas. The next row shows the average R-squared on the time series regression. This isn’t a test of the model. The correct model of expected returns need not explain time series variation, but it may be interesting none the less to know how well each model captures the time series variation.

The next row shows the cross-sectional R-squared of the time series tests. That is, it is a summary statistic for the percent of variation in average returns captured by the models predicted returns. It acts as a summary statistic for the evidence presented graphically in Figures 2.3 through 2.10. The LSC and LSC-NP models both explain the largest spread in average returns with R-squared of 56% and 51%. The next highest are the Carhart model at 48% and the Novy-Marx model at 43%.

I summarize the results of the time series test by computing the Hansen and Jagannathan (1997) distance (HJ). The HJ distance is defined as:

$$HJ = \sqrt{\alpha'(E[RR']^{-1})\alpha}$$

The alphas are the the pricing errors in the time series regressions and the middle term is the inverse of the second moment matrix of the test asset returns, that will be estimated by its sample counterpart. Unlike the GRS test, the HJ distance doesn’t use the covariance matrix of the estimated pricing errors for inference. A model can’t lower it’s HJ distance just by enlarging its standard errors. The GRS test often easily rejects very good models for not being perfect, and fails to reject very bad models that have large and imprecise pricing errors. Scaling the alphas by the same second moment matrix for all the models makes the HJ distance well suited to compare

models, even acknowledging that all the models may be misspecified to some degree.

The bottom half of panel A shows the HJ distances for each model estimated on the 119 test portfolios. I report the HJ distance for all the assets, as well as for the 114 stock portfolios, omitting the bond portfolios. The LSC model has a smaller HJ distance than five of the six leading models. Only the Novy-Marx model produces a lower HJ distance. The no peeking version performs better than four of the six models with only the Fama and French five factor model out performing it. The next two rows show that the superior performance of the other models is driven by the bond portfolios. Within the subset of stock portfolios, both the LSC and LSC-NP model have lower HJ distance than all of the other leading models.

Panel B displays the results for the cross-sectional tests. For the cross-sectional tests, I follow the Black et al. (1972) two-step approach. First, I estimate the full-sample betas of each test asset on the level, slope and curve factors using time series regressions, then I regress the average returns on the estimated betas. The regression estimates the risk premium associated with each factor. If the risk premium is significantly different from zero, the factor is priced. A high R-squared indicates that the spread in average returns is explained by the spread in the betas of the test assets on the common factors. I estimate the model with an intercept term. The model predicts the intercept term should be close to zero as the zero-beta rate should be close to the risk free rate. Since an arbitrageur would have to borrow at the risk-free rate and buy a zero beta portfolio to profit from a spread in the risk-free and zero beta rate, the two rates will only be equal if the arbitrageur can borrow at the risk free lending rate (Brennan, 1971). Because the error terms may be cross-sectionally correlated, I report coefficients and t-statistics using Fama and MacBeth (1973) regressions. I report the R-squared statistics from the ordinary least squares regressions.

The first two columns in Panel B of Table 2.7, show the results of the two step procedure for the LSC models. For the LSC model, each factor risk premium is large and statistically significant. The first factor, which is highly correlated with the market portfolio, generates a factor risk premium of 0.50% monthly, statistically significant at the 10% level and reasonably close to the historical excess return of the level factor over the time period of .89%. The second factor generates a risk premium of -0.93% per month, statistically significant at the 1% level and close to the factors average return of -1.35%. The third factor, curvature, is associated with a risk premium of 0.41% monthly, statistically significant at the 5% level and similar to the factor's historical average excess

return of 0.52%. Because the test assets are excess returns, the APT implies that the constant term should be close to zero. The constant term estimate is 0.31% and also statistically significant. This implies a difference in the zero-beta rate and the risk free rate of 31 basis points monthly or 3.78% annually. The 56% R-squared implies that the model captures a large amount of the cross-sectional spread in risk. The no peeking model performs comparably with similar risk premiums measured with similar precision. The average returns on the factors in the no peeking model are 0.87%, -0.99% and 0.52%, differing slightly from the full sample version.

On several important dimensions, the LSC models perform well compared to the other leading models in the cross-sectional tests. The factors always produce statistically significant risk premiums (at least at the 10% level), which no other model accomplishes. The level factor has a risk premium different than zero, which five of the size models don't accomplish. Even though all of the models have factors represented by test assets sorted on similar characteristics, only the momentum factor consistently produces a risk premium. HML only produces a risk premium in Novy-Marx's model, while the size factor has a positive price of risk in two of the four models that employ it. Investment is significant at the 1% level for the Fama and French five factor model, but not significant in Hou, Xue and Zhang's Q-factor model. Profitability is accompanied by a risk premium in the Hou, Xue, Zhang model, but not in Novy-Marx's model or the five factor model.

Finally, I summarize the cross-sectional tests with Hansen and Jagannathan distances for each model. Kan and Robotti (2009) develop a method to compare models using the HJ distances and produce asymptotically valid confidence intervals under the assumption that factors and returns are multivariate elliptically distributed.<sup>8</sup>

The last six results for panel B show the results for the tests of cross-sectional HJ distance. Again, I report results for the whole sample and the subset of stocks. The first row shows the HJ distance for each model when the prices of risk are chosen to minimize the HJ distance. The next row is the difference between the LSC model and the comparison models. Positive squared distances mean the LSC model has a lower HJ distance. The third row gives a p-value for the hypothesis test that the two models are equal.

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<sup>8</sup>The method calls for estimating the factor risk premiums to minimize the HJ distance, which in general, differs from the ordinary least squares estimate (Kan et al., 2013). I thank the authors for making the code available on Cesare Robotti's and Raymond Kan's website for the related empirical papers Gospodinov et al. (2015) and Gospodinov et al. (2014) that (among many other contributions) demonstrate the method.

The CAPM, Fama French three factor model and the Carhart model all have higher HJ distances than the LSC model, statistically significant at the 1% level. The Fama French five factor model, which benefits substantially from allowing its risk premiums to differ from the average returns on the factor is not statistically distinguishable from the LSC model. The Novy-Marx model is the only model that outperforms the LSC model over the whole sample, with a statistically significant lower HJ distance, but again the distance evaporates in the stock subsample. The Hou, Xue and Zhang model shows the opposite pattern, under performing the LSC model in the full sample, but not statistically distinguishable in the subsample.

Broadly, the results for the 32 hedge portfolios presented in Table 2.8 are similar. In panel A, the LSC model and its no peeking counterpart have the lowest average absolute alphas with an average of 28 basis points. The two models also have the lowest number of anomalies left “unexplained” with only 8 and 10 statistically significant t statistics. The low number is not just due to explaining large average returns on hedge portfolios, sometimes it is due to not creating large alphas where there are little to no average returns to explain. The Novy-Marx model seems particularly susceptible to this with average absolute alphas even higher than the CAPM and more significant anomalies. The hedge portfolios of asset turnover and idiosyncratic volatility make good examples. The hedge portfolio on asset turnover returns -1 basis point over the sample, but due to it’s high loading on PMU has a predicted return of 72 basis points for an alpha of -73 basis points. A 26 basis point average return on portfolios sorted by idiosyncratic volatility is turned into an -88 basis point alpha due to high loadings on HML and PMU that aren’t offset by a market beta of -0.5.

The average R-squared of the time series tests are lowest for the LSC model, showing that unlike a pure APT model, it doesn’t need to explain time variation to effectively price assets. The next row shows the two models capture by far the most spread in average returns with their predicted returns. At the bottom of panel A, the time series HJ distances are smallest for the LSC models. The last row shows the squared differences in the HJ distances. That all the numbers are positive demonstrates all six benchmark models have higher HJ distances than the LSC model.

Panel B shows that again the three factors in the LSC models all generate risk premium estimates different than zero at a confidence level of at least 5%. Similar inconsistencies arise as before with the factor risk premiums in the other models. Market beta only carries a positive risk premium



statistically different than zero in two of the benchmark models. Value and investment never generate risk premiums and profitability only carries a risk premium in one of the three models it appears in. The zero beta rates are lowest for the LSC models, but are at reasonable magnitudes for all the models. The cross-sectional R-squared is highest for the full sample LSC model, but the no peeking version is bested by the Carhart and Novy-Marx model. The cross-sectional test effectively strips both models down to the market factor and momentum. Because the cross-sectional R-squared is adjusted for the number of factors, three of the models end up with negative values.

The HJ distances, differences in squared differences, and p-values summarize the cross-sectional results. Only the HJ distance for the Novy-Marx model is ever so slightly lower than the LSC model, a difference possibly due to sample error. The no peeking version has a smaller HJ distance than the remaining models. A hypothesis test on the differences rejects the hypothesis that the differences in the LSC model and the Fama and French three factor model, the Carhart model, and the Fama and French five factor model is due to sampling error at the 5% level of significance.

#### **2.8.4 Horse Races of All Factors**

A central question remains, which factors are important in explaining the cross-section of returns? Which factors provide marginal explanatory power in the presence of other factors? The goal of this paper is to organize the many disparate characteristics and factors into a parsimonious factor model of expected returns. If all or many of the factors in the literature can be boiled down to a much more parsimonious representation, the space left to explain with theory is dramatically reduced. If the LSC model is a better representation of the latent factor structure in returns, then it should drive out other factors. Other factors may just be some combination of level, slope and curve and potentially several unpriced factors.

I follow the procedure in Cochrane (2005) to conduct factor horse races. I run ordinary least squares regressions with returns on each individual asset pricing factor. When the estimated coefficient is added to a cross-sectional asset pricing test with other factors, the resulting coefficient estimate yields the marginal significance of the factor. If a factor is insignificant, it adds little explanatory power to the model. In the spirit of a fair race, I use as the benchmark the no peeking version of the LSC model, though the results change very little using the full sample version of the model. Tables 2.9 and 2.10 present the results starting with the three factor LSC model.

Table 2.9 shows the resulting horse race using the 119 test portfolios. The leftmost column shows the results with the LSC model. First, I add the four non-market factors from the Fama and French five factor model HML, SMB, CMA and RMW. When univariate betas of the factors are added to the model, none of the factor risk premiums are significantly different than zero. While the HML and SMB factors from the three factor model differ somewhat from their five factor counterparts, the same results hold (not shown). The momentum factor added next is also driven out by Level, Slope and Curve. The next three factors from the Novy-Marx model UMD, PMU and HML similarly don't help price the 119 test portfolios. Lastly, neither ROE nor INV from the Hou, Xue and Zhang model add to the LSC model.

Examining the Level, Slope and Curve risk premiums shows some interesting patterns. First, in all specifications, the slope beta has a risk premium significantly different than zero at the 1% level. The curve factor seems to compete with RMW and INV as it no longer has a significant price of risk when those factors are added. Lastly, market beta (level) is driven out when SMB is added and when ROE is added. No factor adds much to the reported adjusted R-squared, adding no more than 1%.

Table 2.10 shows the same horse race using the 32 hedge portfolios of the extreme deciles of characteristic sorts. Again, the addition of the four Fama and French factors change the results very little as all three LSC factors remain significant and no Fama and French factor is significant. The pattern persists for momentum, the Novy-Marx factors and the Q-factors. The level risk premium is fairly resilient, only competing with the two investment factors CMA and INV. The slope factor remains significant in eight of the ten tests, but the factor risk premium is no longer significant when the two momentum factors are added. The curve factor competes with the profitability factors and SMB. None of the additional factor increase the adjusted R-squared more than 1%. Eight of the ten additional factors decrease the adjusted R-squared.

Across both tests, the LSC model performs very well. The three factors are consistently priced in both tests. Additional factors add little explanatory power to the LSC model. The slope factor is clearly related to momentum and the curve factor somewhat related to sorts on profitability and size as was evident in Tables 2.1 and 2.5. Overall, the horse races suggest that the LSC model largely succeeds at the goal of summarizing the key features of the cross-section of stock returns.

## 2.9 ICAPM Interpretation

While the methodology in this paper is general enough to find pricing factors consistent with a wide range of pricing models, the asset pricing literature has stressed interpretations of empirical factor models through the lens of the Intertemporal Capital Asset Pricing Model of Merton (1973), at least since Fama and French (1996) suggest this interpretation for their three factor model. While I don't stress an ICAPM interpretation above any other, the relationship between the level, slope and curve factors to well established state variables is still of great interest. Petkova (2006) shows a simple way to embed pricing factors into an ICAPM in the style of Campbell (1996). First, she sets up a Vector Autoregression model to capture the relationship between the state variables and the market return, as well as the predictability of each state variable. Then, she tests whether changes in the pricing factors proxy for innovations in the economic state variables.

I specify the following VAR model:

$$\begin{bmatrix} R_{M,t} \\ DIV_t \\ TERM_t \\ DEF_t \\ RF_t \\ SVAR_t \\ Lev_t \\ Slp_t \\ Cur_t \end{bmatrix} = A \begin{bmatrix} R_{M,t-1} \\ DIV_{t-1} \\ TERM_{t-1} \\ DEF_{t-1} \\ RF_{t-1} \\ SVAR_{t-1} \\ Lev_{t-1} \\ Slp_{t-1} \\ Cur_{t-1} \end{bmatrix} + u_t$$

The first term  $R_{M,t}$  is the excess return on the market defined as the value-weighted return on the CRSP index less the risk-free rate from Ken French's data library. The remaining state variables are dividend to price, the term spread, the default yield and stock variance. These state variables are obtained from the Goyal and Welch data library on Amit Goyal's website.<sup>9</sup>

The dividend to price is defined as the log of the trailing sum of the 12 month dividends minus the log month end value of the CRSP index. The term spread is the *U.S. Yield on Long-*

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<sup>9</sup>Special thanks to Amit Goyal and Ivo Welch for making this data available and keeping it updated. The data is available at [<http://www.hec.unil.ch/agoyal/>].

term *United States Bonds* series from NBER's Macroeconomy database minus the *3-Month Treasury Bill: Secondary Market Rate* from the research database at the Federal Reserve Bank at St. Louis (FRED). The default spread is the difference between the yield on BAA- from FRED and the long-term U.S. Yield (defined identically to the term spread). The stock variance variable is the sum of squared daily returns on the S&P 500 for the month.<sup>10</sup> Additionally, the excess returns on the Level, Slope and Curve factors are included in the VAR system as potential state variables. The error term  $u_t$  is a vector of innovations, unpredicted changes in state variables. The question is whether the Level, Slope and Curve factors are good proxies for these unexpected innovations in state variables.

Following Petkova (2006), I orthogonalize each innovation to the excess return on the market, and scale the innovation so the variance is equal to the market. Table 2.11 shows the results of the innovations in predictive variables regressed on the Level, Slope and Curve factors. The t-statistics are corrected for heteroskedasticity and autocorrelation with Newey-West regression using five lags. Table 2.11 shows that Level, Slope and Curve are all significantly correlated with innovations in dividend yield. The slope factor and curve factors are negatively associated with innovations in dividend yield. Because a decrease in the dividend yield is associated with lower future returns, the slope factor does best (other things equal) when expected future returns are low. Recall the slope factor is long low return stocks and short high return stocks, so that an increase in the slope factor means low return stocks are doing better versus high return stocks. Low return stocks respond positively to decreases in the dividend yield (deterioration in the investment opportunity set). This suggests a good beta, bad beta interpretation for the slope factor. After controlling for the market return, low return stocks do well when future expected returns are relatively low (good beta). High return stocks do relatively better, when future expected returns are relatively high (bad beta).

None of the factors seem to be associated with large moves in the term spread or risk-free rate. The curve factor is associated with an increase in the default spread. The curve factor has greater returns when the default spread increases. The curve factor is long larger, often more profitable and less volatile stocks. These stocks do relatively better, when the default spread is higher. Thus, this finding is similar to Petkova (2006) that SMB has a negative association with the default spread.

Lastly, increases in the slope and curve factors are associated with increases in monthly stock

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<sup>10</sup>Detailed definitions are available at Amit Goyal's website [<http://www.hec.unil.ch/agoyal/docs/AllTables2013.pdf>].

variance. Low return stocks do relatively well when future stock variance is higher than expected. Because high stock variance is relatively bad for the investment opportunity set, low return stocks again act as a hedge for negative shocks to investors.<sup>11</sup> The curve factor's exposure implies that large, low volatility stocks outperform small volatile stocks when there is a shock to market volatility. Taken together, the slope and curve factors seem to capture risk relative to the marginal investor's opportunity set consistent with an ICAPM interpretation.

## 2.10 Consumption Based Asset Pricing

A considerable body of theoretical work argues that returns across assets should be explained by an asset's covariance with investor consumption. Assets that covary positively with consumption increase the investor's consumption volatility and therefore should have lower prices and higher returns than assets that covary negatively with consumption, and act as insurance for the investor.<sup>12</sup> Empirical research has found little support for the consumption based approach.<sup>13</sup> In contrast to the thrust of the literature, Jagannathan and Wang (2007) find that annual returns matched with annual consumption growth measured in the fourth quarter provides surprisingly successful evidence in favor of the linearized consumption based model (CCAPM). The Fama and French 25 size and book to market portfolio show a spread in average returns largely explained by each portfolio's covariance with consumption.

I follow their methodology for the twenty-five Dissecting Anomaly portfolios. Figure 2.11 displays the results. Average returns, on the Y-axis, are regressed on consumption betas estimated from time-series regressions on the X-axis. The resulting regression line is displayed, with most of the portfolios and factors fitting tightly around the line. The adjusted  $R^2$  is 79%, matching the smooth linear pattern in the graph. The largest outlier is the low expected return portfolio, the same portfolios that seems to confound factor pricing in Table 2.6. This portfolio has a low and slightly negative consumption beta, but its average return is especially low at -14% per year, creating an alpha of -10%.

The X-intercept is not statistically significant -3.5% (t-statistic of -1.06). The coefficient esti-

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<sup>11</sup>(Campbell et al., 2012) report a similar finding in a different setting.

<sup>12</sup>Rubinstein (1976), Lucas Jr (1978) and Breeden (1979)

<sup>13</sup>Hansen and Singleton (1982), Hansen and Singleton (1983) and Hansen and Jagannathan (1997)

mate is statistically different from zero suggesting that consumption is priced in the cross-section. The equity premium puzzle remains as the risk premium on consumption growth is 2.98% (t-statistic 5.90). The cross-sectional premium is largely explained by each asset's covariance with consumption. From this perspective, the level, slope and curve factors are consumption mimicking portfolios. The sorts on expected returns also create large spreads in consumption betas. Level, slope and curve then act as summary statistics for these sorted portfolios.

Much of the failure of the CCAPM may be related to a small number of stocks. The one dimensional characteristic sorts pre-dominantly used in the literature may be poor proxies of expected returns with much less stable covariance with consumption. The success of the CCAPM suggests that the Level, Slope and Curve model not only provides factors consistent with Arbitrage Pricing Theory, the resulting factors support a strong relation between risk and return as measured by macroeconomic data.

## 2.11 Conclusion

This paper develops a new method for extracting the priced factors in the cross-section of stock returns. The first step uses cross-sectional regressions on many predictive variables to sort stocks into portfolios from high return to low return. The second step uses principal components to extract factors from these portfolios. The goal of this approach is to sort portfolios on expected returns and then extract factors related to expected returns. The resulting factors are level, slope and curve as the loadings form this familiar pattern over the test assets. I perform asset pricing tests using the Level, Slope and Curve model compared to several leading models. I find that the model performs very well, despite having only three factors. Horse races show that adding additional factors adds very little additional explanatory power. The factors have compelling relationships with the ICAPM and CCAPM, suggesting a deeper relation between the Level, Slope and Curve Model and systematic risk.

Table 2.1: Average Excess Returns and Characteristics for Twenty-five Portfolios Formed Using Sorts on Estimated Expected Returns

Portfolios are formed each month using predicted returns estimated from cross-sectional Fama-MacBeth regressions of each firm's return in excess of the risk-free rate on size, book to market, momentum, net stock issues (and a dummy for 0), accruals (split into positive and negative), asset growth and operating profit (split into positive profit and a negative profit dummy). The regression is run each month, separately for big, small and micro cap stocks defined with size breakpoints of 50% and 20% of NYSE market equity. Firms are then sorted into twenty-five value weighted portfolios based on the predicted returns from the regressions. The table shows the excess returns, predicted returns and characteristics, all value weighted. Value weighted ROE and stock volatility are also reported, but not used in portfolio formation. The last column shows the excess returns from portfolios formed using the same methodology performed in an expanding window fashion.

Portfolios	XRet	$\widehat{\text{XRet}}$	JME	B/M	Mom	dA/A	A/BE	NS	OP	ROE	Vol	XRet NP
1	-1.12	-0.76	9786	0.44	-0.17	0.69	0.10	0.32	-0.08	-0.30	3.04	-0.90
2	-0.18	-0.26	16519	0.44	-0.08	0.44	0.05	0.19	0.15	-0.08	2.42	-0.14
3	0.25	-0.03	20735	0.46	-0.04	0.30	0.05	0.10	0.23	0.04	2.17	0.31
4	0.25	0.12	24073	0.47	0.00	0.21	0.03	0.05	0.27	0.08	1.97	0.42
5	0.48	0.23	24614	0.51	0.05	0.17	0.02	0.04	0.30	0.12	1.85	0.43
6	0.57	0.31	23686	0.55	0.09	0.14	0.02	0.02	0.32	0.14	1.78	0.52
7	0.53	0.38	21601	0.58	0.13	0.12	0.02	0.01	0.33	0.14	1.75	0.51
8	0.77	0.44	20122	0.61	0.18	0.10	0.01	0.01	0.33	0.15	1.73	0.74
9	0.71	0.50	17535	0.64	0.21	0.10	0.01	0.01	0.33	0.14	1.75	0.69
10	0.87	0.55	16133	0.67	0.25	0.09	0.01	0.01	0.33	0.14	1.77	0.82
11	0.88	0.60	14062	0.69	0.29	0.09	0.01	0.00	0.34	0.15	1.81	0.76
12	0.85	0.65	11715	0.71	0.32	0.08	0.01	0.00	0.34	0.14	1.84	0.88
13	0.87	0.70	9923	0.74	0.36	0.08	0.01	0.00	0.35	0.15	1.89	0.70
14	0.97	0.75	8487	0.76	0.39	0.08	0.01	0.00	0.41	0.08	1.95	0.77
15	1.07	0.80	7054	0.77	0.42	0.07	0.00	0.00	0.36	0.15	2.01	1.02
16	0.90	0.86	6954	0.78	0.45	0.07	0.03	0.00	0.49	0.12	2.06	0.88
17	0.94	0.91	5929	0.79	0.49	0.07	0.07	0.00	0.62	0.22	2.13	0.91
18	1.07	0.97	5140	0.81	0.53	0.07	0.00	0.00	0.41	0.17	2.19	1.01
19	1.39	1.03	3591	0.84	0.56	0.06	-0.02	0.00	0.37	0.11	2.24	1.14
20	1.23	1.10	2955	0.86	0.59	0.06	0.11	0.00	0.82	0.20	2.29	1.05
21	1.17	1.18	2260	0.89	0.65	0.05	0.00	-0.01	0.52	0.03	2.39	1.21
22	1.32	1.26	1859	0.92	0.73	0.05	-0.03	-0.01	0.46	0.15	2.48	1.22
23	1.33	1.37	1615	0.97	0.86	0.04	-0.05	-0.01	0.47	0.12	2.61	1.30
24	1.68	1.53	2128	1.05	1.07	0.02	-0.09	-0.01	0.50	0.12	2.80	1.41
25	1.73	1.89	3418	1.16	1.52	-0.03	-0.37	-0.02	0.84	-0.04	3.14	1.67

Table 2.2: Principal Components Analysis on Twenty-Five Expected Return Sorted Portfolios

The table shows principal components analysis of the realized returns of 25 portfolios formed on one dimensional sorts of estimated expected returns. Portfolios are formed each month using predicted returns estimated from cross-sectional Fama-MacBeth regressions of each firm's return in excess of the risk-free rate on size, book to market, momentum, net stock issues (and a dummy for 0), accruals (split into positive and negative), asset growth and operating profit (split into positive profit and a negative profit dummy). The regression is run each month, separately for big, small and mirco cap stocks defined with size breakpoints of 50% and 20% of NYSE market equity. Firms are then sorted into twenty-five value weighted portfolios based on the predicted returns from the regressions.

Component	Eigenvalue	Variance Explained	Cumulative
Component 1	668.16	74.18%	74.18 %
Component 2	81.19	9.01%	83.19 %
Component 3	27.42	3.04%	86.23 %
Component 4	13.99	1.55%	87.79 %
Component 5	11.44	1.27%	89.06 %
Component 6	10.41	1.16%	90.21 %
Component 7	8.47	0.94%	91.15 %
Component 8	8.04	0.89%	92.05 %
Component 9	7.93	0.88%	92.93 %
Component 10	6.48	0.72%	93.65 %



Table 2.3: Correlation of Principal Components for the First Five Components Estimated Varying the Number of Portfolios

In each panel, the table shows the correlation of each of the first five principal components with the identical principal component formed using a different number of sorted portfolios. Portfolios are formed each month using predicted returns estimated from cross-sectional Fama-MacBeth regressions of each firm's return in excess of the risk-free rate on size, book to market, momentum, net stock issues (and a dummy for 0), accruals (split into positive and negative), asset growth and operating profit (split into positive profit and a negative profit dummy). The regression is run each month, separately for big, small and micro cap stocks defined with size breakpoints of 50% and 20% of NYSE market equity. Firms are then sorted into ten, twenty-five and one hundred value weighted portfolios based on the predicted returns from the regressions.

<b>First Component</b>			
	10 Portfolios	25 Portfolios	100 Portfolios
10 Portfolios	1.000		
25 Portfolios	0.998	1.000	
100 Portfolios	0.993	0.997	1.000
<b>Second Component</b>			
	10 Portfolios	25 Portfolios	100 Portfolios
10 Portfolios	1.000		
25 Portfolios	0.976	1.000	
100 Portfolios	-0.955	0.980	1.000
<b>Third Component</b>			
	10 Portfolios	25 Portfolios	100 Portfolios
10 Portfolios	1.000		
25 Portfolios	0.908	1.000	
100 Portfolios	0.842	0.946	1.000
<b>Fourth Component</b>			
	10 Portfolios	25 Portfolios	100 Portfolios
10 Portfolios	1.000		
25 Portfolios	0.487	1.000	
100 Portfolios	0.150	0.619	1.000
<b>Fifth Component</b>			
	10 Portfolios	25 Portfolios	100 Portfolios
10 Portfolios	1.000		
25 Portfolios	0.065	1.000	
100 Portfolios	0.124	0.578	1.000

Table 2.4: Correlation of First Three Principal Components Estimated from the Beginning and End of Sample

Principal Components Analysis estimates weights for each portfolio to form each component. These weights multiplied by the portfolio return each month sum to create the factor return. The beginning of sample component estimates use only the first half of the data to estimate the weights. The end of sample component estimates use only the second half of the data to form the weights. The for both beginning and end of sample estimations, whole sample factor realizations are generated. The panels show the correlations of the factor realizations separately on the first half and second half of the sample.

<b>Beginning of Sample</b>						
	Beg 1	Beg 2	Beg 3	End 1	End 2	End 3
Beg 1	1.00					
Beg 2	0.00	1.00				
Beg 3	0.00	-0.00	1.00			
End 1	1.00	0.03	-0.02	1.00		
End 2	0.40	-0.92	-0.04	0.37	1.00	
End 3	0.64	-0.08	0.73	0.62	0.29	1.00
<b>End of Sample</b>						
	Beg 1	Beg 2	Beg 3	End 1	End 2	End 3
Beg 1	1.00					
Beg 2	0.25	1.00				
Beg 3	-0.48	-0.06	1.00			
End 1	1.00	0.30	-0.50	1.00		
End 2	0.05	-0.95	0.12	0.00	1.00	
End 3	0.03	-0.03	0.82	-0.00	-0.00	1.00

Table 2.5: Correlation of First Five Principal Components with Other Factors

The table shows cross-correlation of the first five principle components to the market factor, HML, SMB, Momentum, Profitability (PMU), ROE, INV and Liquidity.

Variables	PC 1	PC 2	PC 3	PC 4	PC 5
Mkt-RF	0.95	0.20	0.14	0.01	0.06
SMB	0.46	-0.30	-0.49	-0.09	-0.19
HML	-0.35	-0.17	0.22	-0.04	0.02
RMW	-0.31	0.00	0.41	0.11	0.04
CMA	-0.38	-0.30	0.16	-0.05	0.00
MOM	-0.05	-0.77	0.14	-0.02	0.04
HML*	-0.17	-0.23	0.13	-0.01	-0.06
UMD*	-0.11	-0.70	0.09	0.00	0.05
PMU*	-0.33	-0.11	0.25	0.08	-0.02
ROE	-0.22	-0.26	0.36	0.01	0.10
INV	-0.37	-0.28	0.26	-0.05	0.08
Liq-T	-0.03	0.04	-0.01	-0.01	-0.01

Table 2.6: Time Series Regressions of Twenty-five Expected Return Sorted Portfolios on the Extracted Principal Components

The table shows regression results for the twenty-five portfolios sorted by estimated expected returns regressed on the extracted principal components. Time series regressions are estimated for each portfolio on the first one, two, three and four components. The alphas, t-statistics and R-squared for each regression are displayed. The bottom of the table shows average absolute alpha, the Gibbons et al. (1989) test statistic and the associated p-value. Portfolios are formed each month using predicted returns estimated from cross-sectional Fama-MacBeth regressions of each firm's return in excess of the risk-free rate on size, book to market, momentum, net stock issues (and a dummy for 0), accruals (split into positive and negative), asset growth and operating profit (split into positive profit and a negative profit dummy). The regression is run each month, separately for big, small and mirco cap stocks defined with size breakpoints of 50% and 20% of NYSE market equity. Firms are then sorted into twenty-five value weighted portfolios based on the predicted returns from the regressions.

Port	Ret	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$t_1$	$t_2$	$t_3$	$t_4$	$R_1^2$	$R_2^2$	$R_3^2$	$R_4^2$
1	-1.12	-2.23	-0.94	-0.65	-0.26	-9.72	-6.39	-5.66	-3.01	0.62	0.87	0.92	0.96
2	-0.18	-1.07	-0.18	-0.02	-0.08	-6.34	-1.32	-0.15	-0.67	0.65	0.84	0.87	0.87
3	0.25	-0.58	0.39	0.48	0.14	-3.27	2.65	3.21	1.19	0.59	0.82	0.83	0.88
4	0.25	-0.47	0.30	0.22	0.19	-3.25	2.39	1.74	1.42	0.62	0.83	0.83	0.84
5	0.48	-0.18	0.37	0.23	0.19	-1.48	3.37	2.38	1.78	0.65	0.78	0.82	0.82
6	0.57	-0.11	0.35	0.22	0.11	-1.02	3.65	2.55	1.17	0.72	0.82	0.85	0.86
7	0.53	-0.15	0.17	0.02	0.00	-1.42	1.77	0.18	-0.06	0.75	0.80	0.85	0.85
8	0.77	0.08	0.32	0.15	0.07	0.88	3.18	1.96	0.93	0.78	0.81	0.87	0.88
9	0.71	0.05	0.23	0.05	0.05	0.57	2.37	0.63	0.66	0.78	0.79	0.87	0.87
10	0.87	0.19	0.16	0.00	0.01	2.16	1.75	-0.06	0.13	0.80	0.81	0.87	0.87
11	0.88	0.19	0.09	-0.07	-0.04	2.16	1.05	-0.98	-0.53	0.81	0.82	0.87	0.88
12	0.85	0.15	0.02	-0.13	-0.13	1.67	0.19	-1.76	-1.71	0.83	0.83	0.88	0.88
13	0.87	0.15	0.00	-0.14	-0.06	1.72	-0.05	-1.66	-0.69	0.81	0.82	0.86	0.86
14	0.97	0.21	-0.05	-0.13	-0.19	2.39	-0.57	-1.67	-2.02	0.82	0.85	0.86	0.87
15	1.07	0.33	0.08	-0.01	-0.02	3.65	0.95	-0.10	-0.30	0.82	0.85	0.86	0.86
16	0.90	0.11	-0.19	-0.26	-0.21	1.16	-2.08	-2.95	-2.20	0.82	0.85	0.86	0.86
17	0.94	0.14	-0.14	-0.20	-0.18	1.42	-1.40	-2.17	-1.91	0.82	0.85	0.85	0.85
18	1.07	0.22	-0.18	-0.18	-0.04	2.10	-1.93	-2.02	-0.44	0.81	0.86	0.86	0.87
19	1.39	0.52	0.10	0.10	0.20	4.74	0.99	1.04	1.91	0.81	0.86	0.86	0.86
20	1.23	0.37	-0.03	-0.06	0.04	3.52	-0.36	-0.67	0.41	0.81	0.87	0.87	0.87
21	1.17	0.30	-0.22	-0.18	-0.12	2.69	-2.23	-1.94	-1.27	0.78	0.87	0.87	0.87
22	1.32	0.41	-0.07	-0.02	0.13	3.35	-0.62	-0.21	1.16	0.77	0.84	0.84	0.85
23	1.33	0.44	0.02	0.13	0.22	3.57	0.20	1.14	2.07	0.78	0.83	0.84	0.84
24	1.68	0.65	0.09	0.30	-0.25	4.53	0.76	2.58	-2.92	0.74	0.80	0.84	0.94
25	1.73	0.66	-0.03	0.31	0.30	3.80	-0.22	2.66	2.48	0.71	0.80	0.89	0.89
$ \alpha $		0.40	0.19	0.17	0.13					0.76	0.83	0.86	0.87
GRS		5.43	3.20	2.74	2.67								
p-val		[0.00]	[0.00]	[0.00]	[0.02]								

Table 2.7: Comparing Level, Slope and Curve to Leading Factor Models With 119 Test Portfolios

<b>Panel A: Time Series Results</b>								
	LSC	LSCNP	CAPM	FF3	CAR	FF5	RNM	HXZ
Avg $ \alpha $	0.15	0.16	0.23	0.20	0.15	0.20	0.20	0.18
$ t  > 1.96$	19	18	40	38	24	38	27	26
Avg $R^2$	0.70	0.69	0.66	0.72	0.74	0.74	0.70	0.72
CS (TS) $R^2$	0.56	0.51	0.04	0.12	0.48	0.26	0.43	0.39
HJ - All	0.892	0.902	0.945	0.927	0.909	0.900	0.879	0.914
Diff $HJ^2$	-	[0.018]	[0.098]	[0.063]	[0.031]	[0.014]	[-0.023]	[0.039]
HJ - Stocks	0.824	0.828	0.856	0.846	0.838	0.833	0.834	0.846
Diff $HJ^2$	-	[0.008]	[0.055]	[0.037]	[0.023]	[0.016]	[0.017]	[0.038]
<b>Panel B: Cross-Sectional Test</b>								
Factor	LSC	LSCNP	CAPM	FF3	CAR	FF5	RNM	HXZ
$\beta$ Level	0.50* (1.82)	0.49* (1.79)						
$\beta$ Slope	- 0.93*** (-3.88)	- 0.83*** (-3.51)						
$\beta$ Curve	0.41** (2.02)	0.51** (2.47)						
$\beta$ Mkt			0.19 (0.72)	0.10 (0.39)	0.36 (1.48)	0.21 (0.87)	0.47* (1.86)	0.26 (1.05)
$\beta$ HML				0.24 (1.56)	0.20 (1.28)	0.01 (0.04)	0.19** (2.03)	
$\beta$ SMB				0.18 (1.20)	0.14 (0.92)	0.27* (1.83)		0.34** (2.18)
$\beta$ MOM					0.78*** (3.50)		0.49*** (3.33)	
$\beta$ INV						0.42*** (3.27)		0.14 (1.12)
$\beta$ PROF						0.03 (0.27)	0.05 (0.63)	0.37** (2.39)
Cons	0.31*** (2.96)	0.31*** (2.90)	0.51*** (3.65)	0.53*** (4.42)	0.30*** (2.75)	0.39*** (3.56)	0.20* (1.82)	0.35*** (3.26)
$R^2$	0.56	0.55	0.03	0.17	0.53	0.38	0.58	0.42
HJ All	0.891	0.902	0.945	0.927	0.909	0.897	0.869	0.906
Diff $HJ^2$		[0.019]	[0.099]	[0.064]	[0.032]	[0.011]	[-0.039]	[0.025]
p-val		(0.00)	(0.00)	(0.00)	(0.00)	(0.17)	(0.01)	(0.01)
HJ Stocks	0.795	0.801	0.828	0.813	0.805	0.796	0.789	0.799
Diff $HJ^2$	-	[0.009]	[0.054]	[0.030]	[0.017]	[0.002]	[-0.008]	[0.007]
p-val	-	(0.02)	(0.00)	(0.02)	(0.01)	(0.54)	(0.44)	(0.22)

 $t$  statistics in parentheses\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Table 2.7: Comparing Level, Slope and Curve to Leading Factor Models With 119 Test Portfolios

The table shows regression results for both time series and cross-sectional tests of eight leading models on 119 test portfolios. Panel A shows for the 119 time series regressions, the average absolute alpha, the total t-statistics significant at the 5% level, the average  $R^2$ , the cross-sectional  $R^2$  when the factor risk premiums are forced to fit the in sample factor means, the Hansen and Jagannathan (1997) distance and the differences in the Hansen and Jagannathan (1997) distances. Panel B shows the cross-sectional test results. The full sample of betas are estimated in the first stage. In the second stage, the factor risk premiums are estimated using regression in the style of Fama and MacBeth (1973). Also reported the cross-sectional adjusted  $R^2$ , the HJ distance, the difference in the squared HJ distance of each model from the Level, Slope and Curve model, and the p-value of that difference. The eight leading models are the Level, Slope and Curve model, the Level, Slope and Curve model formed with an out of sample expanding window, the Capital Asset Pricing Model, the three factor model of Fama and French (1993), the Carhart model that adds momentum, the five factor model of Fama and French (2014a), the four factor model of Novy-Marx (2013a), and the four factor model of Hou et al. (2014). The test portfolios include ten portfolios formed by the results of the “No Peeking Dissecting Anomalies” regressions in Section 4, twenty-five portfolios sorted on size and book-to-market, 10 portfolios sorted on momentum, returns on five treasury bonds (1 year, 5 year, 10 year, 20 year and 30 year), forty-nine industry portfolios, ten portfolios formed on operating profit and ten portfolios formed on asset growth. The sample period covers July 1974 to December 2012.

Table 2.8: Comparing Level, Slope and Curve to Leading Factor Models With 32 Hedge Portfolios

<b>Panel A: Time Series Results</b>								
	LSC	LSCNP	CAPM	FF3	CAR	FF5	RNM	HXZ
Avg $ \alpha $	0.28	0.28	0.42	0.47	0.32	0.36	0.47	0.30
$ t  > 1.96$	8	10	15	18	17	13	20	13
Avg $R^2$	0.23	0.21	0.06	0.25	0.35	0.36	0.30	0.34
CS (TS) $R^2$	0.28	0.25	0.00	0.00	0.17	0.00	0.11	0.12
HJ	0.621	0.624	0.627	0.632	0.625	0.653	0.676	0.644
Diff $HJ^2$		[0.004]	[0.008]	[0.014]	[0.006]	[0.041]	[0.072]	[0.030]
<b>Panel B: Cross-Sectional Test</b>								
Factor	LSC	LSCNP	CAPM	FF3	CAR	FF5	RNM	HXZ
$\beta$ Level	1.04*** (3.06)	0.80** (2.28)						
$\beta$ Slope	-0.63** (-2.41)	-0.53** (-2.02)						
$\beta$ Curve	0.78** (2.59)	0.61*** (2.07)						
$\beta$ Mkt			-0.02 (-0.05)	-0.14 (-0.36)	0.87** (2.57)	-0.21 (-0.56)	0.70* (1.91)	0.53 (1.55)
$\beta$ HML				0.05 (0.30)	0.08 (0.50)	-0.08 (-0.45)	0.07 (0.73)	
$\beta$ SMB				0.03 (0.19)	-0.01 (-0.05)	0.11 (0.69)		0.24 (1.46)
$\beta$ MOM					0.64*** (2.78)		0.36** (2.42)	
$\beta$ INV						-0.06 (-0.47)		0.08 (0.67)
$\beta$ PROF						0.00 (0.02)	0.01 (0.13)	0.31** (2.15)
Cons	0.12** (2.09)	0.14*** (2.68)	0.28*** (5.36)	0.26*** (4.99)	0.17*** (3.43)	0.28*** (5.37)	0.17*** (3.70)	0.17*** (3.53)
$R^2$	0.28	0.19	-0.03	-0.09	0.23	-0.16	0.20	0.05
HJ	0.601	0.608	0.622	0.619	0.610	0.617	0.601	0.609
Diff $HJ^2$		[0.009]	[0.026]	[0.022]	[0.012]	[0.020]	[-0.000]	[0.010]
p-val		(0.03)	(0.06)	(0.05)	(0.05)	(0.02)	(0.68)	(0.11)
<i>t</i> statistics in parentheses								
* p<0.10, ** p<0.05, *** p<0.01								

Table 2.8: Comparing Level, Slope and Curve to Leading Factor Models With 32 Hedge Portfolios

The table shows regression results for both time series and cross-sectional tests of eight leading models on 32 test portfolios. Panel A shows for the 32 time series regressions, the average absolute alpha, the total t-statistics significant at the 5% level, the average  $R^2$ , the cross-sectional  $R^2$  when the factor risk premiums are forced to fit the in sample factor means, the Hansen and Jagannathan (1997) distance and the differences in the Hansen and Jagannathan (1997) distances. Panel B shows the cross-sectional test results. The full sample of betas are estimated in the first stage. In the second stage, the factor risk premiums are estimated using regression in the style of Fama and MacBeth (1973). Also reported the cross-sectional adjusted  $R^2$ , the HJ distance, the difference in the squared HJ distance of each model from the Level, Slope and Curve model, and the p-value of that difference. The eight leading models are the Level, Slope and Curve model, the Level, Slope and Curve model formed with an out of sample expanding window, the Capital Asset Pricing Model, the three factor model of Fama and French (1993), the Carhart model that adds momentum, the five factor model of Fama and French (2014a), the four factor model of Novy-Marx (2013a), and the four factor model of Hou et al. (2014). Each of the 32 portfolios is formed by sorting on a characteristic and then taking a long position in one extreme decile and an equal short position in the other extreme decile. The portfolios are formed on sorts on size, gross profitability, value, value-profitability (combined), accruals, asset growth, investment, Piotroski's f-score, net issuance, return on book equity, failure probability, value-profitability-momentum (combined), value-momentum (combined), idiosyncratic volatility, momentum, standardized unexpected earnings, earnings surprise, industry momentum, industry relative reversals, industry relative reversals combined with industry momentum, short-term reversals, and low volatility industry relative reversals.



Table 2.9: Horse Race Using 119 Portfolios

Factor	LSC	+HML	+SMB	+CMA	+RMW	+MOM	+UMD	+PMU	+HML*	+ROE	+INV
$\beta$ Level	0.49* (1.79)	0.54* (1.78)	0.44 (1.37)	0.60* (1.78)	0.55* (1.84)	0.45* (1.68)	0.44 (1.60)	0.57* (1.87)	0.51* (1.81)	0.43 (1.52)	0.59* (1.76)
$\beta$ Slope	-0.83*** (-3.51)	-0.83*** (-3.47)	-0.79*** (-2.90)	-0.77*** (-3.02)	-0.84*** (-3.54)	-1.10*** (-2.22)	-1.02*** (-2.36)	-0.81*** (-3.33)	-0.81*** (-3.39)	-0.90*** (-3.45)	-0.78*** (-3.08)
$\beta$ Curve	0.51** (2.47)	0.43* (1.91)	0.54** (2.13)	0.41* (1.84)	0.40 (1.57)	0.55** (2.38)	0.54** (2.42)	0.46* (1.86)	0.44** (2.10)	0.61** (2.15)	0.37 (1.48)
$\beta$ HML		0.11 (0.56)									
$\beta$ SMB			0.06 (0.26)								
$\beta$ CMA				0.12 (0.70)							
$\beta$ RMW					0.09 (0.52)						
$\beta$ MOM						-0.28 (-0.59)					
$\beta$ UMD							-0.14 (-0.52)				
$\beta$ PMU								0.05 (0.05)			
$\beta$ HML*									0.07 (0.68)		
$\beta$ ROE										-0.12 (-0.53)	
$\beta$ INV											0.12 (0.68)
Cons	0.31*** (2.90)	0.32*** (3.07)	0.32*** (3.01)	0.32*** (3.05)	0.32*** (2.99)	0.33*** (3.20)	0.33*** (3.19)	0.31*** (2.90)	0.33*** (3.14)	0.32*** (3.00)	0.33*** (3.19)
$R^2$	0.55	0.55	0.55	0.56	0.55	0.55	0.55	0.55	0.56	0.55	0.56

$t$  statistics in parentheses

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Table 2.9: Horse Race Using 119 Portfolios

I regress monthly excess returns of 119 test assets on each factor in time series regressions from July 1974 to December 2012. I then regress the average excess return on each test asset on the estimated betas for the Level, Slope and Curve No Peeking model and the univariate factor betas from the time series regressions in a cross-sectional regression with Fama-MacBeth standard errors. The factors include HML, SMB, CMA and RMW from Fama and French (2014a), momentum, UMD, PMU and HML\* from Novy-Marx (2013a), and ROE and INV from Hou et al. (2014). The test portfolios include ten portfolios formed by the results of the “No Peeking Dissecting Anomalies” regressions in Section 4, twenty-five portfolios sorted on size and book-to-market, 10 portfolios sorted on momentum, returns on five treasury bonds (1 year, 5 year, 10 year, 20 year and 30 year), forty-nine industry portfolios, ten portfolios formed on operating profit and ten portfolios formed on asset growth. The sample period covers July 1974 to December 2012.

Table 2.10: Horse Race Using 32 Hedge Portfolios

Factor	LSC	+HML	+SMB	+CMA	+RMW	+MOM	+UMD	+PMU	+HML*	+ROE	+INV
$\beta$ Level	0.8** (2.28)	0.77** (2.03)	1.04** (2.48)	0.66 (1.60)	0.84** (2.22)	0.81** (2.33)	0.83** (2.36)	0.81** (2.09)	0.78** (2.12)	0.84** (2.35)	0.66 (1.55)
$\beta$ Slope	-0.53** (-2.02)	-0.53** (-2.02)	-0.66** (-2.13)	-0.58** (-2.07)	-0.54** (-2.03)	-0.23 (-0.41)	-0.30 (-0.62)	-0.52** (-2.00)	-0.53** (-2.02)	-0.48* (-1.75)	-0.57** (-2.06)
$\beta$ Curve	0.61** (2.07)	0.61** (2.06)	0.34 (1.09)	0.57** (2.12)	0.50 (1.45)	0.46 (1.54)	0.47* (1.66)	0.60** (2.06)	0.59** (2.21)	0.43 (1.31)	0.61** (2.08)
$\beta$ HML		-0.05 (-0.22)									
$\beta$ SMB			-0.26 (-0.93)								
$\beta$ CMA				-0.1 (-0.61)							
$\beta$ RMW					0.06 (0.28)						
$\beta$ MOM						0.29 (0.57)					
$\beta$ UMD							0.15 (0.53)				
$\beta$ PMU								0.00 (0.05)			
$\beta$ HML*									0.03 (-0.25)		
$\beta$ ROE										0.10 (0.53)	
$\beta$ INV											-0.10 (-0.55)
Cons	0.14*** (2.68)	0.15*** (3.15)	0.17*** (3.68)	0.16*** (3.38)	0.14*** (2.74)	0.16*** (3.46)	0.16*** (3.43)	0.14*** (2.75)	0.15*** (3.23)	0.16*** (3.29)	0.16** (3.38)
$R^2$	0.19	0.17	0.20	0.18	0.17	0.18	0.19	0.16	0.17	0.18	0.18

*t* statistics in parentheses

\* p&lt;0.10, \*\* p&lt;0.05, \*\*\* p&lt;0.01

Table 2.10: Horse Race Using 32 Hedge Portfolios

I regress monthly excess returns of 119 test assets on each factor in time series regressions from July 1974 to December 2012. I then regress the average excess return on each test asset on the estimated betas for the Level, Slope and Curve No Peeking model and the univariate factor betas from the time series regressions in a cross-sectional regression with Fama-MacBeth standard errors. The factors include HML, SMB, CMA and RMW from Fama and French (2014a), momentum, UMD, PMU and HML\* from Novy-Marx (2013a), and ROE and INV from Hou et al. (2014). Each of the 32 portfolios is formed by sorting on a characteristic and then taking a long position in one extreme decile and an equal short position in the other extreme decile. The portfolios are formed on sorts on size, gross profitability, value, value-profitability (combined), accruals, asset growth, investment, Piotroski's f-score, net issuance, return on book equity, failure probability, value-profitability-momentum (combined), value-momentum (combined), idiosyncratic volatility, momentum, standardized unexpected earnings, earnings surprise, industry momentum, industry relative reversals, industry relative reversals combined with industry momentum, short-term reversals, and low volatility industry relative reversals. The sample period covers July 1974 to December 2012.

Table 2.11: Innovations in State Variables Regressed on Level, Slope and Curve

In a first stage vector autoregression, I use five state variables to predict the next month return on the market and the realizations of the five state variables. I regress the innovation from each state variable in the VAR model on the Level, Slope and Curve factors. The state variables are dividend to price, term spread, default spread, the risk-free rate and one month stock variance.

Dep. Variable	$a_0$	Level	Slope	Curve
$u_{DIV}$	-0.20 (-0.66)	0.10** (2.58)	-0.22*** (-5.30)	-0.41*** (-6.20)
$u_{TERM}$	0.01 (-0.05)	0.01 (0.30)	-0.01 (-0.13)	-0.07 (-0.91)
$u_{DEF}$	-0.04 (-0.23)	-0.04 (-0.78)	0.01 (0.27)	0.21*** (3.22)
$u_{RF}$	0.08 (0.45)	-0.00 (-0.08)	0.04 (0.82)	-0.07 (-1.00)
$u_{SVAR}$	0.09 (0.34)	-0.03 (-0.19)	0.09** (2.18)	0.12*** (2.68)

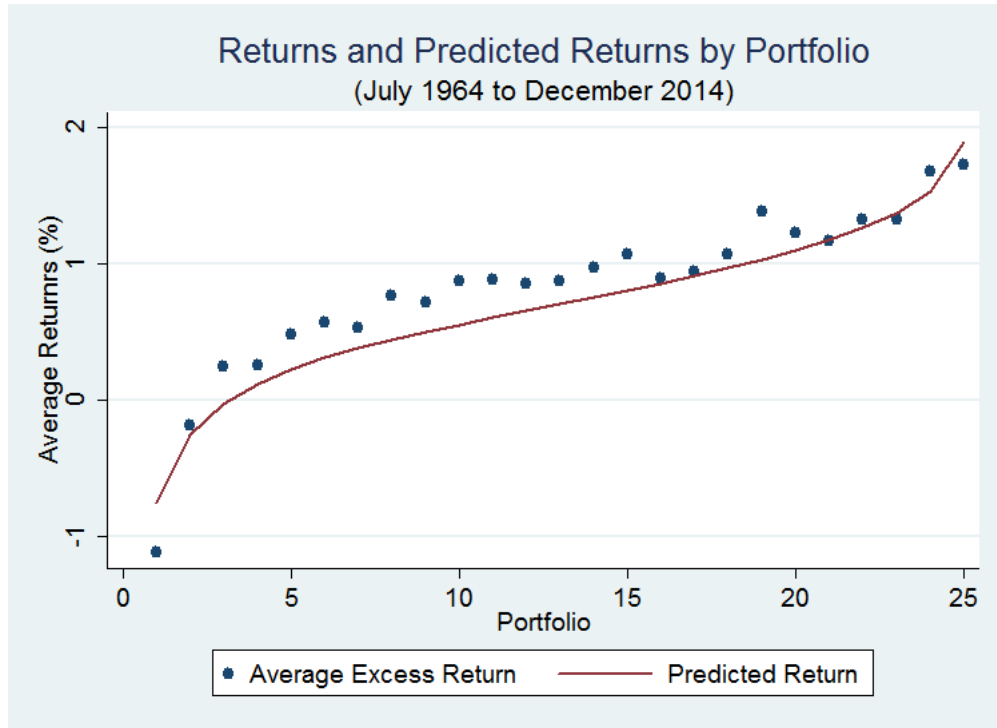


Figure 2.1: Twenty-Five Dissecting Anomaly Portfolios

The figure shows average returns and predicted returns for twenty-five portfolios built on seven asset pricing anomalies. Portfolios are formed each month using predicted returns estimated from cross-sectional Fama-MacBeth regressions of each firm's return in excess of the risk-free rate on size, book to market, momentum, net stock issues (and a dummy for 0), accruals (split into positive and negative), asset growth and operating profit (split into positive profit and a negative profit dummy). The regression is run each month, separately for big, small and micro cap stocks defined with size breakpoints of 50% and 20% of NYSE market equity. Firms are then sorted into twenty-five value weighted portfolios based on the predicted returns from the regressions. Predicted return is the value-weighted average fitted value in each regression. Average return is the value-weighted average return on each portfolio.

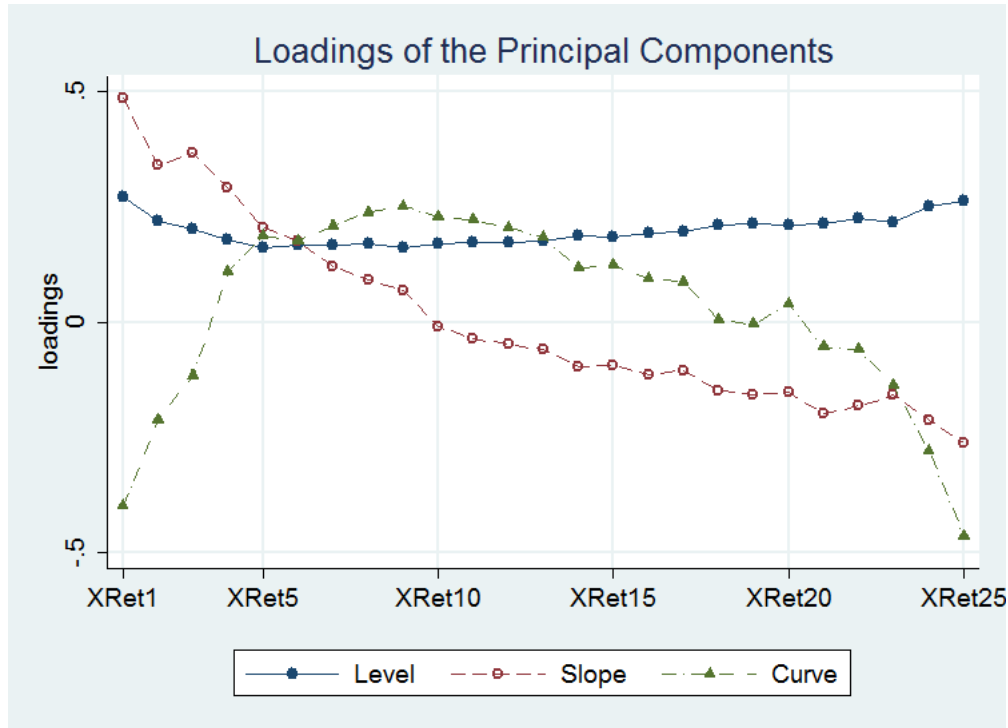
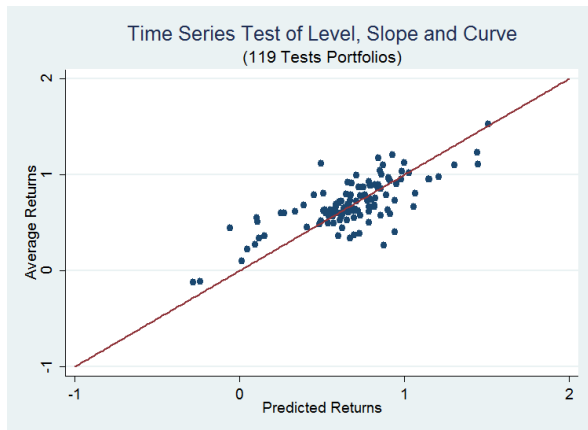
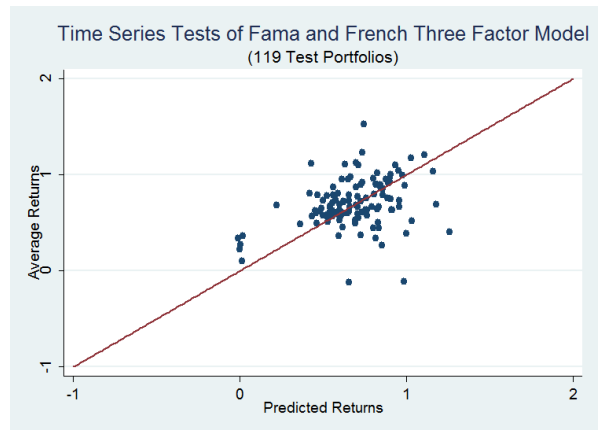


Figure 2.2: PCA Weights

The figure shows the loadings of each of the first three principal components of twenty-five anomaly portfolios. Portfolios are formed each month using predicted returns estimated from cross-sectional Fama-MacBeth regressions of each firm's return in excess of the risk-free rate on size, book to market, momentum, net stock issues (and a dummy for 0), accruals (split into positive and negative), asset growth and operating profit (split into positive profit and a negative profit dummy). The regression is run each month, separately for big, small and micro cap stocks defined with size breakpoints of 50% and 20% of NYSE market equity. Firms are then sorted into twenty-five value weighted portfolios based on the predicted returns from the regressions. I estimate the Principal Components Analysis using the monthly returns on the twenty-five portfolios from July 1964 until December 2012.



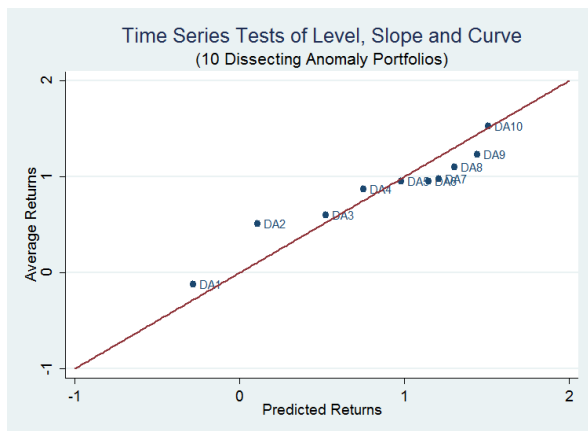
(a) LSC



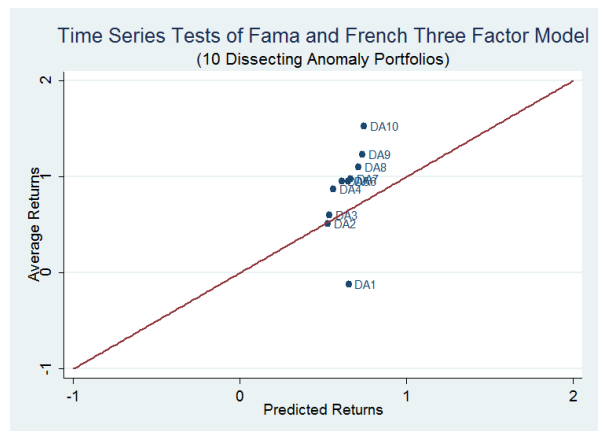
(b) FF3

Figure 2.3: Level, Slope and Curve vs. Fama and French Three Factor Model on 119 Test Portfolios

The figure shows the results of the times series regressions of the Level, Slope and Curve model and the Fama and French three factor model on 119 portfolios formed on seven anomaly variables. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample. The vertical distance from the observation to the 45 degree line is the estimated pricing error (alpha).



(a) LSC

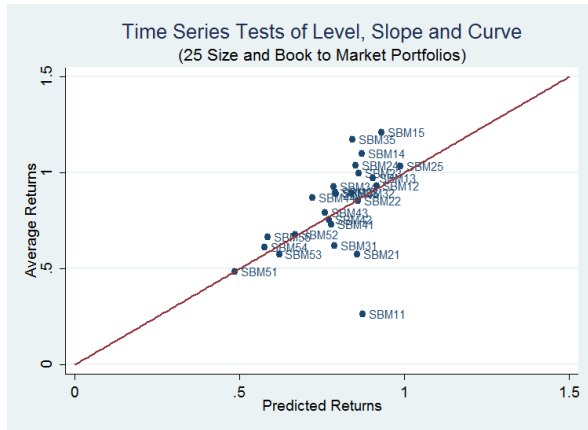


(b) FF3

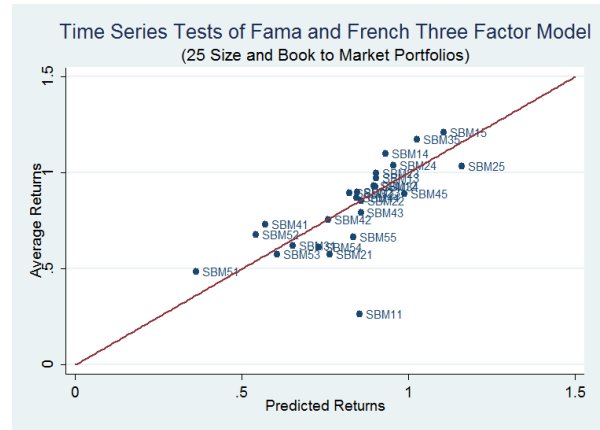
Figure 2.4: Models vs. 10 Dissecting Anomaly Portfolios

The figure shows the results of the times series regressions of the Level, Slope and Curve model and the Fama and French three factor model on 10 portfolios formed on seven anomaly variables using the predictions on multivariate regressions. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample. The vertical distance from the observation to the 45 degree line is the estimated pricing error (alpha).





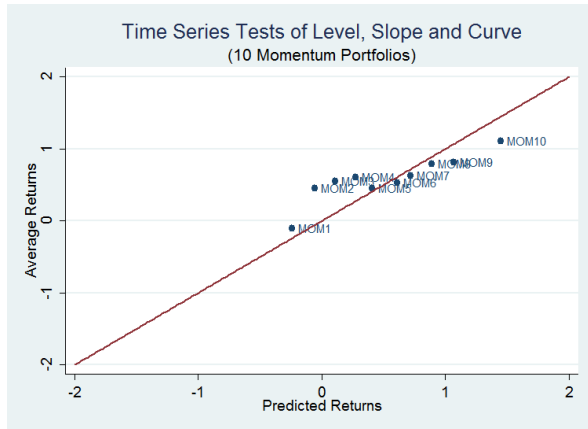
(a) LSC



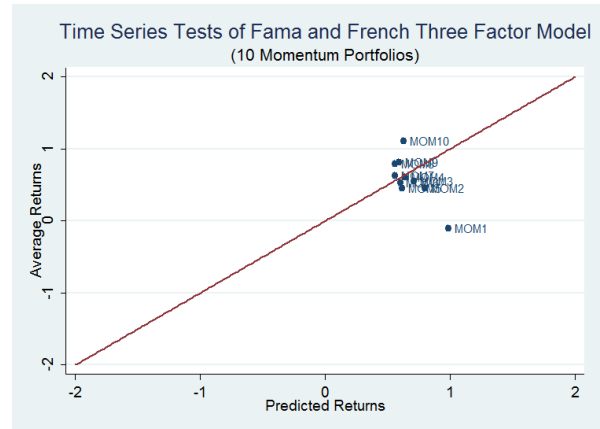
(b) FF3

Figure 2.5: Models vs. 25 Size and Book to Market

The figure shows the results of the times series regressions of the Level, Slope and Curve model and the Fama and French three factor model on 25 portfolios formed on size and book to market. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample. The vertical distance from the observation to the 45 degree line is the estimated pricing error (alpha).



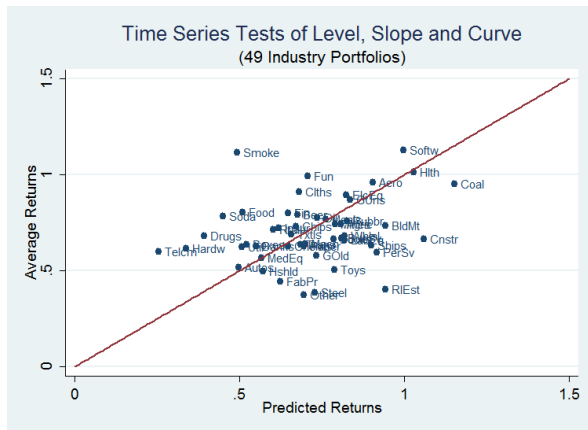
(a) LSC



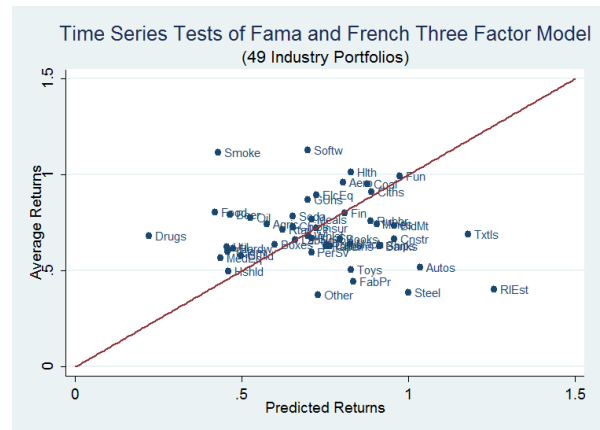
(b) FF3

Figure 2.6: Models vs. 10 Momentum Portfolios

The figure shows the results of the time series regressions of the Level, Slope and Curve model and the Fama and French three factor model on ten portfolios formed on momentum. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample. The vertical distance from the observation to the 45 degree line is the estimated pricing error (alpha).



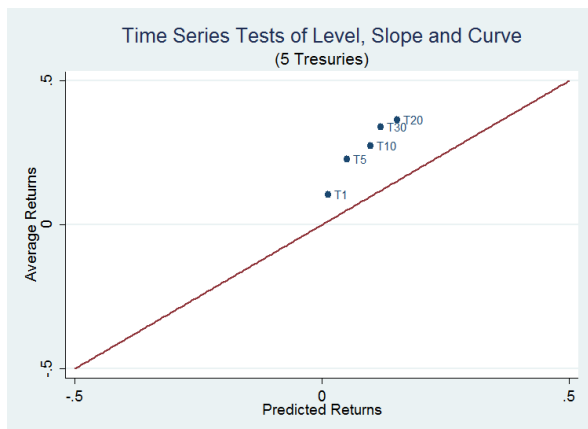
(a) LSC



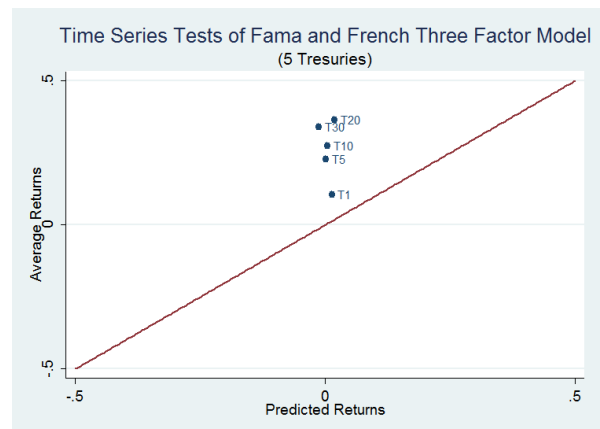
(b) FF3

Figure 2.7: Models vs. 49 Industry Portfolios

The figure shows the results of the times series regressions of the Level, Slope and Curve model and the Fama and French three factor model on 49 industry portfolios formed on the Fama and French industry definitions. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample. The vertical distance from the observation to the 45 degree line is the estimated pricing error (alpha).



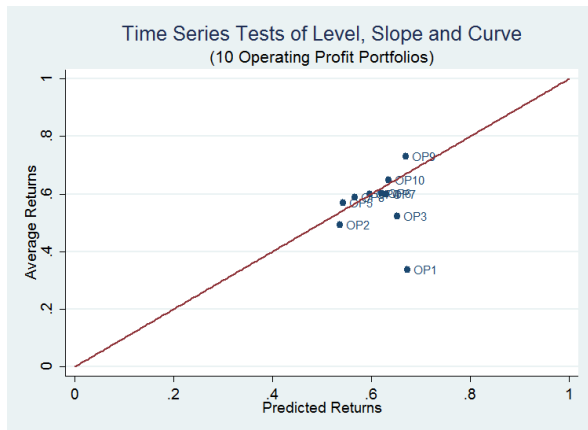
(a) LSC



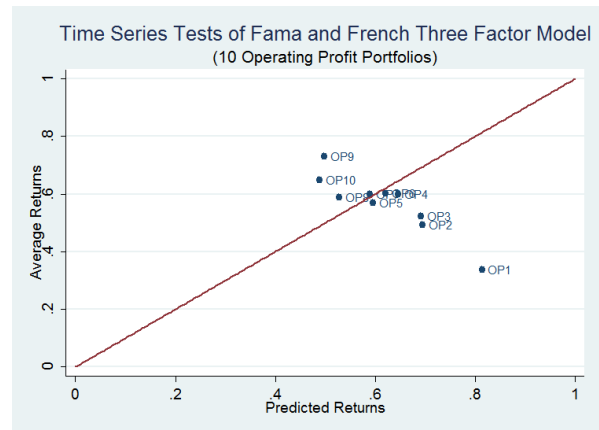
(b) FF3

Figure 2.8: Models vs. 5 Bond Portfolios

The figure shows the results of the time series regressions of the Level, Slope and Curve model and the Fama and French three factor model on five treasury bond returns of different maturities, 1 year, 5 year, 10 year, 20 year and 30 year. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample. The vertical distance from the observation to the 45 degree line is the estimated pricing error (alpha).



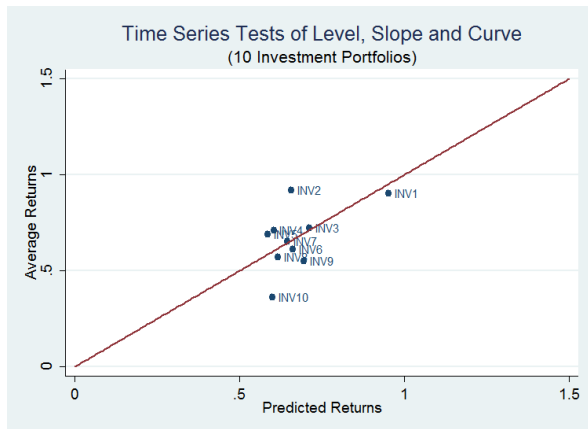
(a) LSC



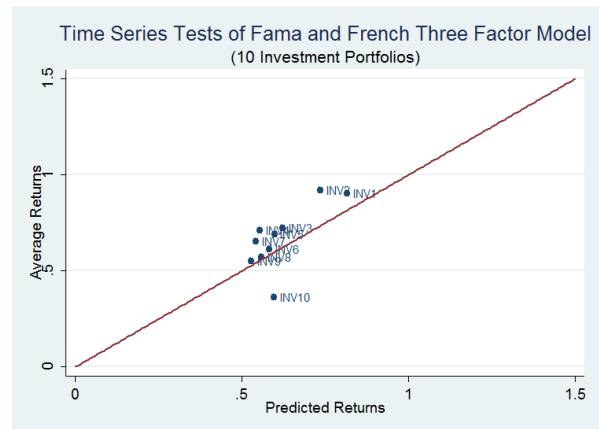
(b) FF3

Figure 2.9: Models vs. 10 Operating Profit Portfolios

The figure shows the results of the time series regressions of the Level, Slope and Curve model and the Fama and French three factor model on five treasury bond returns of different maturities, 1 year, 5 year, 10 year, 20 year and 30 year. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample. The vertical distance from the observation to the 45 degree line is the estimated pricing error (alpha).



(a) LSC



(b) FF3

Figure 2.10: Models vs. 10 Investment Portfolios

The figure shows the results of the time series regressions of the Level, Slope and Curve model and the Fama and French three factor model on five treasury bond returns of different maturities, 1 year, 5 year, 10 year, 20 year and 30 year. The X axis is the model predicted excess return. The Y axis is the average return of the portfolio over the sample. The vertical distance from the observation to the 45 degree line is the estimated pricing error (alpha).

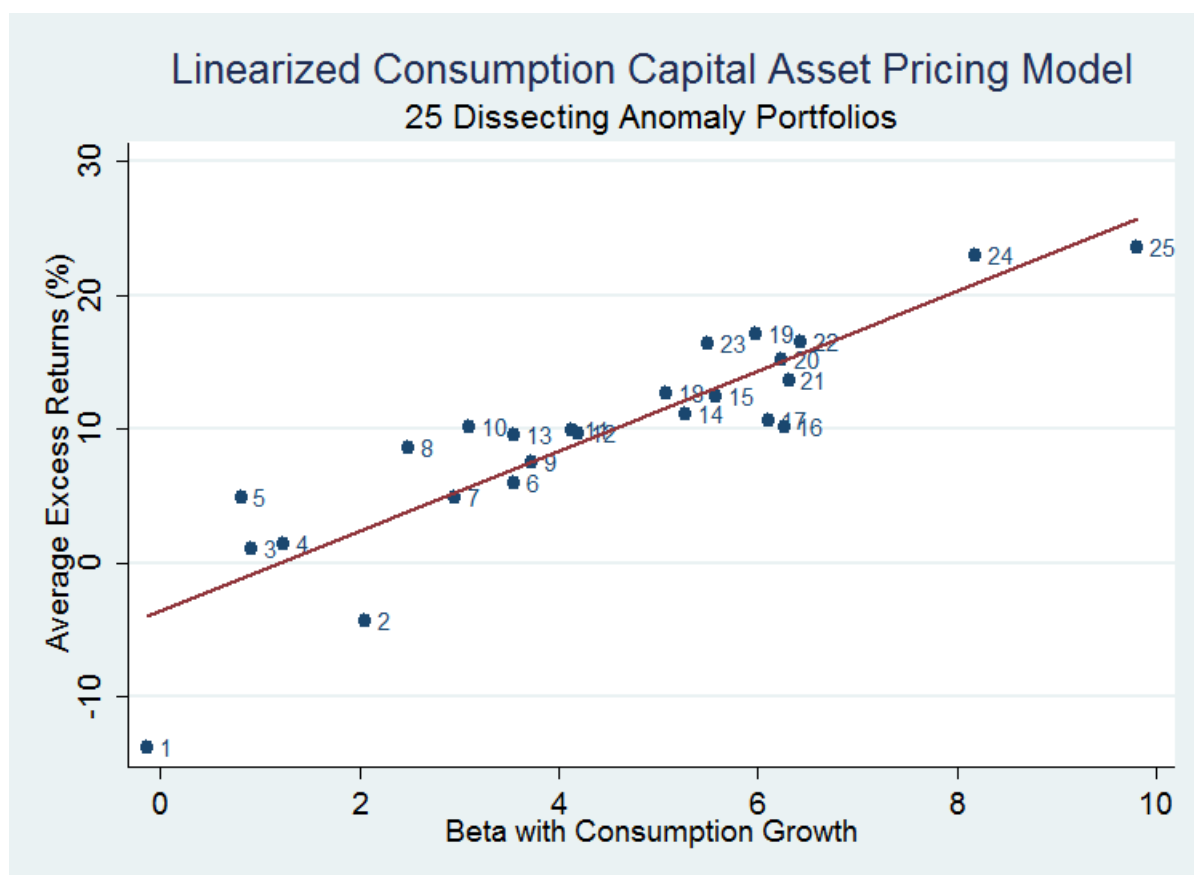


Figure 2.11: Consumption Based Asset Pricing

The figure shows the Linearized Consumption Capital Asset Pricing Model. The test assets are 25 portfolios of annual excess returns sorted from high to low returns using the Dissecting Anomalies predictors. Consumption is measured as real nondurable consumption plus services per capita, and consumption growth is the change from  $Q_4$  to  $Q_4$  of the calendar year, which matches the January to December annual returns. Each asset is graphed with its consumption beta along with a line of best fit.

## Chapter 3

# Reviving the CCAPM

### 3.1 Introduction

Using portfolios sorted in one dimension on several firm characteristics (anomalies), I ask the Consumption Capital Asset Pricing Model (CCAPM) to explain a large spread in expected returns creating more powerful tests than previous studies. This large spread in expected return coincides with a large spread in consumption risk, a prediction of the CCAPM. Therefore, this paper shows that the CCAPM works quite well in explaining expected returns, and that those exploiting anomalies to earn what appear to be high risk-adjusted returns instead bear considerable consumption-based risk.

A savvy, quantitative investor, aware of many of the leading anomalies, seeking to build a high expected return portfolio must assume a high covariance with consumption growth. The top decile of a one dimensional sort on value, profitability and momentum has a covariance with quarterly consumption growth 73% higher than the bottom decile. The highest expected return decile of portfolios formed on size, value, momentum, investment, profitability, accruals and net stock issues has a covariance with quarterly consumption growth over three times higher than the lowest expected return decile.

The CCAPM explains a large portion of expected returns, when anomalies are combined. In cross-sectional regressions, ten portfolios built on value, profitability, and momentum regressed on consumption growth generate R-squareds of 66% at quarterly horizons and 77% at annual horizons, while ten portfolios combining seven anomalies generates R-squareds of 76% at quarterly horizons

and R-squareds of 87% at annual horizons. The risk premium on consumption growth is significant in all the specifications. Combining anomalies produces large spreads in expected returns coinciding with large spreads in consumption betas. In comparison, twenty-five portfolios sorted on size and book to market regressed on consumption growth generate R-squareds of 1% at a quarterly horizon and R-squareds of 20% at annual horizons.

Firm size and book to market have become so influential in both academic work and industry that combining many characteristics and sorting into one dimension seems strange, but while the execution in this paper is new, the idea is not and, in fact, is textbook finance in the most literal since.<sup>1</sup> Firm size and book to market generate spreads in average returns. These sorts quite plausibly generate spreads in expected returns, but we should expand the set of believable proxies for expected returns. We have strong evidence in multiple samples that value, profitability and momentum combine to create even larger spreads in expected returns. Adding predictors to a one dimensional sort creates a larger spread in average returns and consequently, a more informative asset pricing test.

My study explores the results in Jagannathan and Wang (2007) by presenting results using both the change in total annual consumption as well as the change in quarterly consumption. Accumulating consumption over the whole year suffers from a time aggregation problem (Breedon et al., 1989). The procedure implicitly assumes the consumption from a given year is all consumed on the last day of the year, so that returns from January 1st to December 31st of 2001 can be aligned the change in consumption from 2000 to 2001 (Campbell, 2003). But, agents consume during the entire year. Taking the annual change from one quarter say  $Q_1$  to the same quarter of the following year mitigates this problem somewhat. The change in consumption from  $Q_4$  to  $Q_4$  may inappropriately treat consumption earned in October by summing over the quarter, but considerably less so than annual aggregation that inappropriately aggregates consumption from January to September.

Jagannathan and Wang (2007) find that  $Q_4$  to  $Q_4$  is special and suggest the end of the year is a special time when agents make investment and consumption decisions concurrently, causing the

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<sup>1</sup>As Cochrane (2001) explains, “In testing a model, it is exactly the right thing to do to sort stocks into portfolios based on characteristics related to expected returns...In fact, despite the popularity of the Fama-French 25, there is really no fundamental reason to sort portfolios based on two-way or larger sorts of individual characteristics. You should use all the characteristics at hand that (believably!) indicate high or low average returns and simply sort stocks according to a one-dimensional measure of expected returns.”

CCAPM to hold. I find the special nature of  $Q_4$  to  $Q_4$  consumption growth is limited to the Fama and French 25 portfolios. Fixing time aggregation problems with  $Q_X$  to  $Q_X$  consumption growth does produce better CCAPM results, but  $Q_4$  isn't particularly special for portfolios sorted one dimensionally on several anomaly variables. Expanding the set of test assets lessens the evidence of the “lazy investor” hypothesis of Jagannathan and Wang (2007), but strengthens the evidence that  $Q_X$ - $Q_X$  consumption is a better way to address time aggregation than total annual consumption.

Additionally, I also test the CCAPM on an expanded set of test assets. These include seventy portfolios formed from decile sorts on size, book to market, momentum, investment, profitability, accruals and net stock issues. I find that consumption risk is priced in the cross-section and the zero beta rate is generally not significantly different than zero. When taken as whole these “anomalies” fit the basic prediction of the CCAPM, higher expected return portfolios tend to be associated with higher covariance with consumption. Even momentum, in some ways the most glamorous of all the anomalies, generates a considerable spread in consumption betas. High past returns stocks (winners) have higher consumption betas than low past return stocks (losers).

The balance of the evidence suggests that the Consumption CAPM has more support in the cross-section than previously found. Much of the accumulated evidence of weak support for the CCAPM may come from relatively weak tests (Bryzgalova, 2014). This paper suggests ways to strengthen the tests in relatively straightforward ways by increasing the spread in expected returns, increasing the spread in consumption betas and increasing the diversity of test assets considered. This paper explores the baseline CCAPM, and the results show the high price of consumption risk and high risk aversion coefficients needed to match the large equity premium in the data (Mehra and Prescott, 1985). But Lettau and Ludvigson (2009) show that for a large class of the next generation models that generate higher equity premiums, the CCAPM remains a reasonable approximation in a world where those models obtain. The evidence consistent with the simple linearized version of the CCAPM presented here, then speaks to this larger class of models as well.

## 3.2 Data and Test Assets

### 3.2.1 Consumption Growth

I measure consumption as real non-durables plus services per capita. The nominal series for non-durables and services are available from National Income and Product Account (NIPA) Table 2.3.5. I deflate non-durables and services separately each quarter using price deflators available on NIPA table 1.1.9. I sum the real series and divide by the total population available in NIPA table 2.1. Consumption growth ( $\Delta C_{t,t+1}$ ) in 1966 is consumption in 1966 divided by consumption in 1965. Annual consumption growth creates time aggregation problems. Matching 1966 consumption growth with 1966 returns implicitly assumes that all of 1965's consumption was consumed at the very end of 1965, when in actuality it is consumed throughout 1965. To deal with time aggregation, following Jagannathan and Wang (2007), I also form consumption growth on a quarter by quarter basis ( $Q_X - Q_X$ ).  $Q_1 - Q_1$  consumption growth for 1966 is the consumption in the first quarter of 1966 divided by consumption in the first quarter of 1965. I match this  $Q_1 - Q_1$  consumption growth with annual portfolio returns from April 1965 to March 1966. With this construction the measurement period of consumption growth more closely matches the corresponding portfolio returns.

### 3.2.2 Cumulative Returns

For quarterly excess returns, I compound monthly returns and subtract the quarterly return on a three month treasury bond. For annual excess returns, I compound monthly returns and subtract the annual return on a one year bond. Annual returns are January to December and matched to annual consumption in the same year. Quarters end in March, June, September and December and are also matched to the concurrent consumption series.

### 3.2.3 Value, Profitability and Momentum Portfolios:

The value, profitability and momentum portfolios come from Robert Novy-Marx's website<sup>2</sup> and are constructed using rank sorts on the three anomalies. Each month, firms are sorted by each of three characteristics, and ranked from 1 to N. The three ranks are then summed. Stocks ranked close to

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<sup>2</sup>Special thanks to Novy-Marx and Velikov (2016) for sharing their data available at <http://rnm.simon.rochester.edu/data/lib/index.html>



one have relatively high market to book values (growth), low profits, and low past returns. Stocks ranked close to N have low market to book values (value), high profits, and high past returns. The stocks are combined into ten value-weighted portfolios using NYSE cut points.

### 3.2.4 Dissecting Anomaly Portfolios:

So called Dissecting Anomaly portfolios are formed from regressions in the style of Fama and French (2008). I run regressions separately for large, small and micro cap stock returns on seven anomaly variables, size, book to market, momentum, operating profit, accruals, net stock issues, and asset growth.

$$Ret_{i,t+1} = \beta_0 + \beta_1 Size_{i,t} + \beta_2 B/M_{i,t} + \beta_3 Mom_{i,t} + \beta_4 zeroNS_{i,t} + \beta_5 NS_{i,t} + \beta_6 negACC_{i,t} + \beta_7 posACC_{i,t} + \beta_8 dAtA_{i,t} + \beta_9 posOP_{i,t} + \beta_{10} negOP_{i,t} + e_{i,t+1} \quad (3.1)$$

For each stock, i, the regression produces a predicted return:

$$Ex\widehat{Ret}_{i,t+1} = \hat{\beta}_0 + \hat{\beta}_1 LogSize_{i,t} + \hat{\beta}_2 LogB/M_{i,t} + \hat{\beta}_3 Mom_{i,t} + \hat{\beta}_4 zeroNS_{i,t} + \hat{\beta}_5 NS_{i,t} + \hat{\beta}_6 negACC_{i,t} + \hat{\beta}_7 posACC_{i,t} + \hat{\beta}_8 dAtA_{i,t} + \hat{\beta}_9 posOP_{i,t} + \hat{\beta}_{10} negOP_{i,t} \quad (3.2)$$

All of the firm characteristics are known at time t. The betas form weights that transform these characteristics into a predicted return. Because the weights are estimated, they are not known to investor at time t. Therefore, I also estimate “no peeking” regressions that only form the betas using past data. For the no peeking regressions, I use ten years of data, July 1964 to June 1974, to estimate the initial betas. Then I incorporate new information as it becomes available by using a window that expands forward in time. The betas are known at time t as if an investor were to adopt this strategy in real time. The expanding window methodology can be taken literally as a quantitative trading strategy with all information known at time t. The full sample methodology treats expected return as a latent variable, unobservable to the researcher, and uses the entire sample to maximally sort stocks by expected returns given their characteristics. The two specifications produce similar results.

### 3.3 Cross-Sectional Regressions

Credit for the Consumption CAPM falls to Breeden (1979), Lucas Jr (1978) and Rubinstein (1976). The CCAPM predicts that higher consumption betas correspond to higher excess portfolio returns. I test the CCAPM with the two stage approach of Black et al. (1972). In the first stage, I estimate consumption betas by running time series regressions of each excess portfolio return on the change in consumption growth on corresponding periods.

The time-series regressions are:

$$Ret_{i,(t,t+1)} = \beta_0 + \beta_c \Delta C_{t,t+1} + \epsilon_{i,t+1}$$

The return period is matched to the consumption growth period with the end of period timing convention, so Q<sub>1</sub>-Q<sub>1</sub> consumption would be matched with April to March portfolio returns. When using whole year or quarterly consumption growth, the end of period convention dictates that I match annual returns, January through December of 1966, to consumption growth measured as total consumption in 1966 divided by total consumption in 1965.

In the second stage, I regress the average returns on all portfolios on the estimated betas to estimate the consumption risk premium or on the estimated covariances with consumption to give the coefficient the interpretation of a risk aversion coefficient.

The cross-sectional regressions are:

$$\overline{Ret}_i = \lambda_0 + \lambda_1 \beta_c + \alpha_i$$

Because the portfolio returns will tend to be correlated across a given time period, I use regressions in the style of Fama and MacBeth (1973). Because the consumption betas are generated in first stage regressions, I also report t-statistics using the Shanken (1992) corrections for generated regressors.

### 3.4 Results

Table 3.1 shows the CCAPM tested on four groups of test portfolios at both the quarterly and annual frequencies. The first set is 25 portfolios sorted in two dimensions on size and book to

market, the second is ten portfolios sorted in one dimension on value, profitability and momentum. The third set is ten portfolios formed by one dimensional sorts on expected returns estimated from regressions on seven firm characteristics formed from accounting data in the form of Fama and French (2008). I call these Dissecting Anomaly portfolios. The fourth set of ten portfolios also uses a one dimensional sort on expected returns estimated from regressions, but uses an expanding window methodology to form portfolios using estimates with no look ahead bias. I reserve ten years of data to estimate initial coefficients for the expected return of each stock, then each following month the sample expands and I repeat the process, so that coefficients are completely determined by past values. I call these Dissecting Anomaly, No Peeking portfolios

The second stage of the two-stage regression involves the asset's covariance with consumption (rather than the consumption beta), so that the coefficient can be interpreted as the coefficient of risk aversion for an agent with CRRA utility. Table 3.1 shows that the CCAPM tested on the Fama and French twenty-five portfolios performs poorly. The R-squared is 5% at the quarterly frequency and only 20% at the annual frequency. Consumption risk is not priced significantly differently than zero at the quarterly or annual frequency. Theory predicts a zero beta rate near zero, but at both frequencies, the zero beta rate is economically large and significantly different than zero. The quarterly zero beta rate of 1.42% represents an annual rate of 5.80%, and the rate estimated at the annual frequency is even larger at 13.72%.

The next three columns estimate the CCAPM at annual frequency using the  $Q_X$ - $Q_X$  convention to better account for time aggregation problems. None of the zero beta rates are statistically different than zero. Measured in quarters one, two and three, most of the other failings of the CCAPM are evident. The risk aversion coefficient is not significant in the first three quarters and the R-squareds remain low, peaking at 24% using  $Q_3$ - $Q_3$  and bottoming at 8% using  $Q_1$ - $Q_1$ . The last column replicates the central result of Jagannathan and Wang (2007). Measured from  $Q_4$ - $Q_4$ , consumption is significantly priced at the 10% level, the zero beta rate is not significantly different than zero and the R-squared is considerably higher. The estimated absolute alphas are also lower. In light of the  $Q_X$ - $Q_X$  evidence, Jagannathan and Wang (2007) conclude that quarter four is a special time when agents make consumption and investment decision concurrently.

Figure 3.1 shows the quarterly and annual regressions from Table 3.1, Panel A, graphically. Each portfolio is labeled with two numbers, the first for its size ranking and the second for its book

to market ranking. Portfolio “11”, consist of stocks in the smallest quintile and the lowest book to market (small growth) quintile, whereas Portfolio “55” has stocks in the largest 20% that are in the highest book to market quintile (large, value). Not that the small, growth portfolio is a large outlier in the quarterly model. The high consumption beta corresponds to very low returns, opposite the prediction of the CCAPM.

In panel B of Table 3.1, I present the same two-stage regressions for ten portfolios formed jointly on Value, Profitability and Momentum using rank sorts. Looking across the first three rows, none of the zero beta rates are significantly different than zero. Looking across the next three rows, the risk aversion coefficient (and equivalently the consumption risk premium) is significantly different than zero. The R-squareds are all very large peaking at 77% in the annual specification and only falling as low as 54% in the Q3-Q3 specification. Q4-Q4 is not particularly special on any of the metrics. The R-squared is higher than the other quarter to quarter specifications and the absolute alpha is lower, but the magnitudes are much less than when size and book to market portfolios are test assets. In all specifications, the main implications of the CCAPM are born out, consumption risk is priced, the zero beta rate is near zero, and the predicted returns capture a large spread in average returns leading to high R-squareds. An investor trying to transform these three well studied anomalies into a trading strategy finds that to profit in the form of high average returns requires exposure to considerably more consumption risk. The high returns of combining these strategies, explored in detail by Novy-Marx (2013b), can largely be explained by the CCAPM as compensation for consumption risk.

Figure 3.2 shows the results in Table 3.1, Panel B, graphically. Note the extension of the Y-axis, comparing Figure 3.2 with Figure 3.1. The spread in average returns between the highest and lowest portfolio in Figure 3.1 is 2.5% at a quarterly horizon and 11% at an annual horizon, while in Figure 3.2, the spread is 4% at a quarterly horizon and 19% at an annual horizon. Larger spreads in average returns create more for the model to price. Larger spreads in average returns are harder to price purely by chance. Larger spreads increase test power. The overall picture is that the CCAPM performs better as the test gets *harder*. The more powerful, more informative tests generate overall results corresponding to the CCAPM predictions. The large spread in average returns is accompanied by large spreads in betas. That is the central prediction of the CCAPM. This pattern continues through Table 3.1.

The third panel extends this result to one dimensional sorts on the Dissecting Anomalies portfolios, combining seven anomalies, size, book to market, momentum, profitability, investment, net stock issues and accruals. I weight each anomaly by their contribution in a predictive regression on next month's returns. Again the CCAPM performs extremely well on these one dimensionally sorted portfolios. In the first three columns, the zero beta rate is not significantly different than zero. While the 8.3% zero beta rate is economically quite large, it is potentially due to sampling error. The zero beta rates are considerably smaller and slightly negative in the other specifications.

The next three rows shows that the relative risk aversion coefficient is significantly different than zero in all specifications. This is equivalent to showing a positive and significant consumption risk premium. The R-squareds are considerably higher than in the size and book to market portfolios. There is a large spread in average returns created by the portfolios accompanied by a large spread in predicted returns. Again, the  $Q_4$ - $Q_4$  results are hardly extraordinary. The  $Q_1$ - $Q_1$  results have as high an R-squared with quarters two and three not far behind. All of the  $Q_X$ - $Q_X$  results are broadly similar. The absolute alphas are lowest for the  $Q_4$ - $Q_4$  results, but slightly lower for the plain annual results.

The last panel shows the results for the Dissecting Anomaly, No Peeking portfolios. This combines the seven anomalies, but with only out of sample regressions. The first three rows show that five out of six of the specifications have zero beta rates that are not significantly different than zero. In this specification, the economically large zero beta rate of 11.71% on annual consumption measured over the entirety of the year is significantly different than zero. The next three rows show that in all specifications the coefficient of relative risk aversion is significantly different than zero. Again, the R-squared is highest and the absolute alpha lowest in  $Q_4$ - $Q_4$  relative to the other three quarters, but the fourth quarter is considerably less special.

Figures 3.3 and 3.4 show the results for the Dissecting Anomaly portfolios and the Dissecting Anomaly-No Peeking portfolios graphically. Again the pattern persists that the CCAPM performs better when there are larger spreads in average returns. The high and low average return portfolios have over 8% spreads at quarterly horizons and 30% spreads at annual horizons. Higher average return portfolios tend to have higher covariance with consumption risk. As the spread in average returns widens the fit tightens as the pictures display the relatively high R-squareds graphically.

Taken as a whole, the results indicate that the one dimensionally sorted portfolios that utilize

more return predictors (so called anomalies) are priced much more successfully by the CCAPM. Figures I, II, III and IV show the regression results graphically. The figures display the characteristic low R-squared in the literature when testing the CCAPM on the Fama and French 25 portfolios and the much higher R-squareds from the one dimensionally sorted portfolios. Examining the Y-axes of the figures show that the spread in average returns created by the one dimensionally sorted portfolios are larger than the book to market portfolios. With the annual construction shown on the right hand side of each figure, the largest spread in average returns between portfolios is 11%. The high minus low spread on value, profitability and momentum sorted portfolios is 18%, for Dissecting Anomaly portfolios it is 37% and for their expanding window counterpart it is 31%.

Tables 2 through 6 examine the betas and expected returns across the portfolio sorts. Table 3.2 shows the average returns, consumption betas, and the t-statistics of the betas for the twenty-five portfolios sorted on size and book to market. The first panel shows the well-known result that value stocks earn higher average returns than growth stocks (the vertical trend), and except for growth stocks, small stocks earn higher average returns than large stocks (the horizontal trend). The vertical pattern in consumption betas generally matches the value trend, with higher consumption betas on value stocks, but consumption betas don't present a clear pattern in the horizontal direction. The last panel shows that the betas are measured with considerable error. None of the betas is significantly different than zero.

The bottom of the table presents a chi-squared test of the null hypothesis that all of the twenty-five betas are equal and all of the patterns are due to chance. This hypothesis is rejected with a high degree of confidence. I test three additional hypotheses: whether the betas on the small growth portfolio are equal to the large value portfolio, whether the betas on the small growth portfolio are equal to the small value portfolio, and whether the betas from the large growth are equal to the large value portfolio. None of these hypotheses are rejected. There is little spread in CCAPM betas in the twenty-five portfolios. The consequence is that asking if consumption risk is associated with a risk premium will be a weak test. The considerable uncertainty with which the betas are measured means asking whether the spread in average returns is explained by consumption betas is also a relatively weak test.

Table 3.3 shows the average returns and consumption betas for the quarterly returns. The same patterns emerge. Small stocks have higher returns than large stocks except in the extreme growth

portfolios and value stocks have generally higher returns than growth stocks. The betas show very little pattern across portfolios. The last panel shows that while the betas are significantly different than zero, they aren't significantly different than each other at the 5% level of significance.

One way to summarize the results in Tables I through III is that, while there is little evidence for the CCAPM, there is also little evidence *against* the CCAPM. Even though in most specifications consumption risk is not significantly different than zero, the confidence intervals around the consumption risk premium or equivalently the risk aversion coefficients are often consistent with the other panels in which the CCAPM works well. This is characteristic of a weak test of the CCAPM.

Table 3.4 shows the average returns, consumption betas and t-statistics for both annual and quarterly returns. We see a strong pattern in average returns from the first portfolio to the tenth. The spread in annual returns of 19% is almost 70% larger than the largest spread in the size and book to market sorted portfolios. There is also a clear pattern in the consumption betas with larger betas appearing on the higher average return stocks, but lower betas appearing on the low average return stocks. The first chi-squared test shows the results that such a dispersion in betas is due to chance under the null hypothesis that all the betas are equal. The p-value of 0.11 shows that the hypothesis cannot be rejected, but it's important to note that nothing about the test accounts for the strong increasing pattern in the betas. Alternatively, the second test asks if the consumption beta on portfolio 10 is equal to the consumption beta on portfolio 1. This null hypothesis is rejected at the 1% significance level with a p-value of .005. The second three columns shows that an increasing and large spread in average returns is generally associated with increasing betas, but the pattern is too noisy to reject either the null hypothesis that all the betas are equal or that betas on portfolios 10 and 1 are equal. The results in Table 3.4 suggest a large spread in average returns is associated with a large spread in consumption betas. Overall, the value, profitability, and momentum portfolios seem to generate a stronger test of the CCAPM.

Table 3.5 displays the average returns and consumption betas on the twenty-five Dissecting Anomalies portfolios. The table shows a very large dispersion in average returns with a strong increasing pattern from the low average return portfolio to the high average return portfolio. The 35% spread is over three times as large as the spread created by the size and book to market portfolios. The difference in the annual beta from the first portfolio to the last portfolio is 11, over

three times larger than the largest spread created by the size and book to market portfolios. The chi-squared tests show that the dispersion in betas is too large to be caused by chance alone. Both the null hypotheses that all of the betas are equal and the null hypothesis that the betas on the first and twenty-fifth portfolios are equal are rejected. On the right side of the table, we see an increasing but somewhat noisy pattern in the betas accompanying a strong increasing pattern in the average returns. The null hypothesis that all the betas are equal cannot be rejected, but the null hypothesis that the beta on the first portfolio and last portfolio are equal can be rejected at the 10% level of significance.

Table 3.6 shows the average returns and consumption betas for twenty-five Dissecting Anomaly portfolios with the expanding window (no peeking) methodology. Again, we see large spreads in average returns from portfolio 1 to portfolio 25. The spread from the lowest return portfolio, to the highest return portfolio, is 31%. The annual betas show a similarly increasing pattern. The lowest beta is -8.38 on portfolio 1, while the highest beta is 5.27 on portfolio twenty-five. The Chi-squared test rejects the null hypothesis that all the betas are equal and the null hypothesis that the beta on portfolio 25 is equal to the beta on portfolio 1. A similar pattern in average returns and betas occurs in the quarterly portfolios. The spreads in the no peeking betas are even larger than the spreads in the full sample betas presented in Table 3.5. The Chi-squared test for the quarterly consumption betas rejects the null hypothesis that all the betas are equal and the null hypothesis that the betas on the extreme portfolios are equal.

Taken together, the Dissecting Anomaly portfolios form much stronger tests of the CCAPM than the size and book to market portfolios. The portfolios generate a very strong spread in average returns. There is a lot of spread to price. This creates a powerful test. That the large spread in average returns is accompanied by a large spread in consumption betas is the central prediction of the CCAPM. Of course, we can also read the evidence backwards. A powerful test of the CCAPM requires a large spread in consumption betas. The CCAPM predicts that a large spread in consumption betas should be associated with a large spread in average returns. There is nothing circular about the reasoning. It is mandated by the equality sign.

Next, I extend the reevaluation of the CCAPM by extending the cross-section of test assets. Rather than sort on many anomalies in a one dimensional sort, I test the CCAPM on seventy portfolios, each a decile sort on an anomaly variable, including size, book to market, momentum,



net stock issues, profitability, investment, and accruals. Table 3.7 shows the results of the CCAPM on the seventy anomaly portfolios. Five out of six of the specifications have zero beta rates insignificantly different than zero. Only the annual consumption beta has a zero beta rate significantly different than zero at the 10% level of significance. All of the risk aversion coefficients are significantly different than zero at either the 5% or 1% level of significance. Again,  $Q_4$ - $Q_4$  isn't that special. The R-squared is highest at 48%, but not much higher than  $Q_1$ - $Q_1$  or  $Q_3$ - $Q_3$ , which both have R-squared of 40%. Figure 3.5 shows this result graphically. The left side panel shows the quarterly results. There is a clear trend evident in the graph that higher covariance with consumption tends to correspond to higher returns. In the lower left corner, the first decile sorted on momentum and the first decile sorted on net stock issues jump out as outliers with especially large CCAPM alphas. The tenth decile of momentum appears in top right of the graph, suggesting that momentum risk creates a large spread in consumption betas. Both have low consumption betas, but even lower returns. This pattern is even stronger in the annual returns. Again momentum portfolios appear evident at the extremes, but not as particularly large outliers.

The bottom panel of Table 3.7 looks more closely at momentum alone. Since momentum has increased data availability, I extend the time period back to 1952. The CCAPM again performs well. In four of the five size specifications, zero beta rates are not significantly different than zero. Only momentum at the annual frequency has a zero beta rate significantly different than zero. The coefficient of risk aversion is significantly different than zero for all specifications. Again the R-squared is highest at 66% for  $Q_4$ - $Q_4$ , but not conspicuously higher than  $Q_1$ - $Q_1$  at 53% or  $Q_3$ - $Q_3$  at 52%. Figure 3.6 shows these results graphically. The left panel shows the quarterly regression. Momentum portfolios three through eight line up nicely with the CCAPM predictions, while portfolios one and two stand out as outliers. The low average return portfolios have low betas, but not low enough to justify their low returns. The annual returns again line up better with all portfolios fitting tightly around the line. Again in both graphs there is a clear association of high consumption betas and high average returns.

Table 3.8 explores this association with average returns and consumption betas on momentum portfolios. There is again a clear increasing pattern at both the annual and quarterly horizons. The Chi-squared test that the portfolios have equal betas is rejected at the 10% level, but the null hypothesis that the consumption beta on extreme winner decile is different than the extreme loser

decile is only rejected at the 11% significance level. The quarterly betas show a less consistent pattern. Betas increase with average returns from portfolios seven to 10, but are noisy in portfolios six through one. The chi-squared test of the null hypothesis that all the betas are equal is not rejected nor is the null hypothesis that betas ten and one are significantly different.

### **3.5 Conclusion**

This paper reevaluates the Consumption Capital Asset Pricing Model's ability to price the cross-section of stocks. With a few adjustments the generate more informative tests by increasing test power, I find that the simple linearized CCAPM often excels relative to its previously documented performance. A key stylized fact emerges that many interesting "anomalies" share the characteristic that high expected return portfolios tend to have higher covariance with consumption.

Table 3.1: Linearized CCAPM on Sets of Test Portfolios

<b>FF25</b>						
	Quarterly	Annual	Q1-Q1	Q2-Q2	Q3-Q3	Q4-Q4
Cons	1.42	13.72	4.04	5.05	3.78	4.61
t-FM	(2.25)	(4.47)	(1.53)	(1.76)	(1.33)	(1.32)
t-Sh	(2.07)	(3.33)	(0.99)	(1.50)	(0.93)	(0.71)
$\gamma$	95.69	67.28	83.40	43.88	73.80	108.76
t-FM	(1.03)	(1.74)	(2.13)	(1.18)	(2.28)	(3.13)
t-Sh	(0.95)	(1.32)	(1.40)	(1.02)	(1.62)	(1.73)
$R^2$	5	20	18	8	24	66
Avg $ \alpha $	0.52	2.07	2.04	2.16	1.72	1.30
<b>VPM10</b>						
	Quarterly	Annual	Q1-Q1	Q2-Q2	Q3-Q3	Q4-Q4
Cons	-2.42	6.25	-2.13	-2.99	-2.67	-5.40
t-FM	(-2.81)	(2.41)	(-0.72)	(-1.09)	(-0.89)	(-1.63)
t-Sh	(-1.00)	(0.89)	(-0.29)	(-0.54)	(-0.37)	(-0.61)
$\gamma$	573.45	188.31	159.38	122.17	155.18	170.55
t-FM	(6.67)	(5.65)	(5.93)	(5.65)	(3.40)	(6.65)
t-Sh	(2.40)	(2.69)	(2.53)	(3.11)	(2.67)	(2.65)
$R^2$	66	77	61	58	54	74
Avg $ \alpha $	0.55	2.11	2.46	2.67	2.71	2.18
<b>DA25</b>						
	Quarterly	Annual	Q1-Q1	Q2-Q2	Q3-Q3	Q4-Q4
Cons	-1.10	8.30	-1.19	-2.35	-1.69	-2.15
t-FM	(-1.50)	(2.82)	(-0.40)	(-0.90)	(-0.57)	(-0.67)
t-Sh	(-0.75)	(1.22)	(-0.15)	(-0.43)	(-0.23)	(-0.30)
$\gamma$	381.94	155.78	172.15	130.54	161.36	137.36
t-FM	(6.62)	(6.35)	(6.86)	(5.97)	(5.86)	(5.94)
t-Sh	(3.41)	(3.00)	(2.78)	(3.15)	(2.51)	(2.88)
$R^2$	60	85	79	67	70	79
Avg $ \alpha $	0.72	2.51	2.79	3.07	2.98	2.52
<b>DA-NP 25</b>						
	Quarterly	Annual	Q1-Q1	Q2-Q2	Q3-Q3	Q4-Q4
Cons	0.82	11.71	4.06	1.26	4.19	5.29
t-FM	(1.11)	(3.84)	(1.18)	(0.45)	(1.36)	(1.66)
t-Sh	(0.77)	(2.19)	(0.65)	(0.30)	(0.83)	(0.97)
$\gamma$	244.78	114.1	117.67	84.63	97.87	104.79
t-FM	(4.38)	(4.41)	(4.46)	(3.69)	(3.59)	(4.26)
t-Sh	(3.11)	(2.75)	(2.67)	(2.65)	(2.32)	(2.70)
$R^2$	52	70	57	44	51	67
Avg $ \alpha $	0.64	2.64	3.14	3.22	3.31	2.79

Table 3.1: Linearized CCAPM on Sets of Test Portfolios

The table shows the Linearized CCAPM estimated at quarterly, annual and quarter over quarter frequencies on four sets of test portfolios: twenty-five portfolios sorted by size and book to market (FF25), 10 portfolios sorted by value, profitability and momentum (VPM10), twenty-five Dissecting Anomalies portfolios, and twenty-five Dissecting Anomalies No Peeking portfolios. Each portfolio is regressed on the change in consumption growth measured as non-durable consumption plus services per capita to estimate consumption covariance. Then portfolio returns are regressed on consumption covariances to estimate the risk aversion parameter of the linearized CCAPM. Standard errors are computed in the fashion of Fama-MacBeth with and without the Shanken corrections for generated regressors.

Table 3.2: Annual Average Returns and Betas on Size and Book to Market Portfolios

The table shows average returns, consumption betas, and the t-statistics of consumption betas estimated from time series regressions of returns of twenty-five portfolios sorted by size and book to market on consumption growth measured at an annual frequency.

Average Returns					
	Small		Size		Large
Growth	4.48	5.88	6.88	7.88	7.02
	10.66	9.33	9.73	7.81	6.99
B/M	10.37	11.74	10.03	10.30	7.94
	12.88	12.09	11.81	10.78	8.22
Value	15.32	13.50	13.52	11.75	9.38
Betas					
	Small		Size		Large
Growth	-1.54	-2.07	-1.76	-1.86	-0.59
	-0.67	-1.40	-0.44	-0.71	-0.12
B/M	-0.28	0.30	0.09	-0.15	0.88
	0.82	1.13	0.24	1.41	1.49
Value	0.36	1.11	0.50	0.70	1.24
T-Statistics of Betas					
	Small		Size		Large
Growth	-0.45	-0.76	-0.77	-0.87	-0.32
	-0.23	-0.63	-0.22	-0.38	-0.07
B/M	-0.11	0.13	0.05	-0.08	0.51
	0.34	0.52	0.11	0.68	0.80
Value	0.13	0.46	0.21	0.29	0.58
$\chi^2\beta_1 = \dots = \beta_{25}$	60.44				
p-val	[0.00]				
$\chi^2\beta_1 = \beta_{25}$	1.48				
p-val	[0.22]				
$\chi^2\beta_1 = \beta_5$	1.37				
p-val	[0.24]				
$\chi^2\beta_{21} = \beta_{25}$	1.67				
p-val	[0.19]				

Table 3.3: Quarterly Average Returns and Betas on Size and Book to Market Portfolios

The table shows average returns, consumption betas, and the t-statistics of consumption betas estimated from time series regressions of returns of twenty-five portfolios sorted by size and book to market on consumption growth measured at a quarterly frequency.

<b>Average Returns</b>					
	Small		Size		Large
Growth	1.00	1.59	1.63	1.90	1.32
	2.60	2.27	2.37	1.74	1.49
B/M	2.59	2.81	2.35	2.07	1.34
	3.15	2.87	2.62	2.44	1.53
Value	3.57	2.98	3.20	2.53	1.76
<b>Betas</b>					
	Small		Size		Large
Growth	6.54	4.51	4.19	3.83	3.29
	6.29	3.95	3.79	3.49	2.38
B/M	5.12	4.01	3.21	3.40	3.35
	5.25	4.35	3.40	3.45	2.71
Value	5.64	4.66	3.63	4.38	3.50
<b>T-Statistics of Betas</b>					
	Small		Size		Large
Growth	2.59	2.03	2.08	2.11	2.34
	2.96	2.11	2.23	2.21	1.84
B/M	2.70	2.38	2.10	2.28	2.79
	2.91	2.65	2.19	2.35	2.19
Value	2.78	2.53	2.16	2.61	2.51
$\chi^2 \beta_1 = \dots = \beta_{25}$	35.86				
p-val	[0.06]				
$\chi^2 \beta_1 = \beta_{25}$	2.23				
p-val	[0.14]				
$\chi^2 \beta_1 = \beta_5$	0.41				
p-val	[0.52]				
$\chi^2 \beta_{21} = \beta_{25}$	0.03				
p-val	[0.86]				

Table 3.4: Value, Profitability and Momentum Betas Annual and Quarterly

The table shows average returns, consumption betas, and the t-statistics of consumption betas estimated from time series regressions of returns of ten portfolios sorted by value, profitability and momentum on consumption growth measured at annual frequency and quarterly frequency.

Portfolios	Annual			Quarterly		
	DA	Beta	t	VPM	Beta	t
1	-2.65	-2.17	-0.84	-0.36	2.78	1.54
2	1.97	-0.66	-0.31	0.70	2.96	1.99
3	4.63	-1.55	-0.75	1.30	2.58	1.91
4	5.68	0.06	0.03	1.53	3.00	2.40
5	4.24	-0.10	-0.06	1.24	2.71	2.15
6	6.94	-0.37	-0.19	1.89	2.92	2.37
7	7.54	1.31	0.67	1.94	3.84	3.14
8	8.88	1.21	0.60	2.31	4.39	3.17
9	13.38	1.32	0.58	3.22	4.55	3.09
10	16.01	2.16	1.00	3.91	4.66	3.03
$\chi^2\beta_1 = \dots = \beta_{10}$		2.53			13.64	
p-val		[0.11]			[0.14]	
$\chi^2\beta_1 = \beta_{10}$		23.64			1.24	
p-val		[0.00]			[0.26]	

Table 3.5: Annual and Quarterly Dissecting Anomaly Portfolios

The table shows average returns, consumption betas, and the t-statistics of consumption betas estimated from time series regressions of returns of twenty-five Dissecting Anomaly portfolios on consumption growth measured at annual frequency and quarterly frequency.

Portfolios	Annual			Quarterly		
	DA	Beta	t	DA	Beta	t
1	-12.74	-6.35	-1.74	-3.06	2.60	0.96
2	-3.49	-2.45	-0.92	-0.56	3.21	1.59
3	1.70	-3.27	-1.22	0.72	2.67	1.38
4	2.24	-2.83	-1.09	0.71	2.35	1.41
5	5.22	-2.64	-1.15	1.38	1.84	1.24
6	6.64	-0.23	-0.11	1.65	3.63	2.49
7	5.64	-0.03	-0.02	1.48	3.11	2.35
8	9.17	-0.71	-0.32	2.26	2.79	1.97
9	8.29	0.33	0.17	2.11	3.14	2.24
10	10.39	0.11	0.05	2.57	2.98	2.18
11	10.64	0.60	0.27	2.60	4.36	3.21
12	10.26	0.82	0.38	2.49	3.66	2.67
13	10.04	1.28	0.66	2.55	3.94	2.85
14	11.61	1.63	0.73	2.87	4.90	3.29
15	13.20	1.80	0.79	3.20	4.46	3.01
16	10.59	2.72	1.18	2.67	5.61	3.55
17	11.57	2.75	1.15	2.87	6.04	3.54
18	13.15	0.81	0.31	3.20	5.83	3.42
19	17.56	2.47	0.97	4.18	6.31	3.65
20	15.71	3.52	1.27	3.70	5.80	3.34
21	13.98	3.00	1.33	3.51	6.21	3.53
22	16.93	2.87	1.01	3.99	7.07	3.80
23	16.81	1.49	0.56	4.00	4.98	2.73
24	22.75	3.32	0.82	5.19	8.23	3.44
25	23.57	4.60	1.11	5.33	7.80	3.29
$\chi^2\beta_1 = \dots = \beta_{25}$	81.33			31.14		
p-val	[0.00]			[0.15]		
$\chi^2\beta_1 = \beta_{25}$	4.36			3.75		
p-val	[0.04]			[0.053]		



Table 3.6: Annual and Quarterly No Peeking Dissecting Anomaly Portfolios

The table shows average returns, consumption betas, and the t-statistics of consumption betas estimated from time series regressions of returns of twenty-five Dissecting Anomaly - No Peeking portfolios on consumption growth measured at annual frequency and quarterly frequency.

Portfolios	Annual			Quarterly		
	DA-NP	Beta	t	DA-NP	Beta	t
1	-8.66	-8.38	-1.90	-2.05	1.82	0.57
2	-0.35	-2.51	-0.84	0.44	2.91	1.16
3	6.15	-2.72	-0.84	1.64	3.29	1.54
4	6.18	-4.22	-1.48	1.65	1.64	0.86
5	7.19	-4.72	-1.47	1.77	1.16	0.63
6	7.72	-0.18	-0.07	2.08	3.37	1.92
7	7.93	-0.53	-0.22	2.06	2.63	1.63
8	10.15	-1.71	-0.74	2.57	2.38	1.51
9	10.90	-2.15	-0.93	2.80	1.16	0.70
10	12.01	-0.81	-0.34	2.97	2.70	1.73
11	12.07	0.01	0.01	3.04	3.64	2.25
12	13.88	0.08	0.03	3.38	3.30	2.08
13	10.20	-0.84	-0.33	2.52	4.30	2.59
14	11.19	2.23	0.94	2.83	4.75	3.01
15	15.38	1.86	0.74	3.77	4.58	2.66
16	12.09	-1.04	-0.44	3.05	4.34	2.64
17	12.20	1.12	0.49	3.14	4.79	2.76
18	14.99	1.53	0.52	3.60	7.42	3.87
19	15.52	2.69	0.93	3.79	6.46	3.33
20	13.88	2.83	1.03	3.49	6.07	3.07
21	17.49	2.42	0.75	4.21	8.25	3.83
22	14.90	3.47	1.37	3.79	6.24	2.97
23	17.28	0.23	0.08	4.38	6.78	2.82
24	19.56	0.25	0.07	4.60	7.05	3.10
25	22.18	5.27	1.08	5.33	10.88	3.42
<hr/>						
$\chi^2\beta_1 = \dots = \beta_{25}$	170.97			48.72		
p-val	[0.00]			[0.00]		
$\chi^2\beta_1 = \beta_{25}$	4.03			3.95		
p-val	[0.04]			[0.047]		

Table 3.7: Linearized CCAPM on Sets of Test Portfolios

The table shows the Linearized CCAPM estimated at quarterly, annual and quarter over quarter frequencies on two sets of test portfolios, seventy portfolios of stocks sorted into deciles by size, book to market, momentum, asset growth, profitability, accruals, and net stock issues, and ten portfolios sorted on momentum. Each portfolio is regressed on the change in consumption growth measured as non-durable consumption plus services per capita to estimate consumption covariance. Then portfolio returns are regressed on consumption covariances to estimate the risk aversion parameter of the linearized CCAPM. Standard errors are computed in the fashion of Fama-MacBeth with and without the Shanken corrections for generated regressors.

**RNM 70**

	Quarterly	Annual	Q <sub>1</sub> -Q <sub>1</sub>	Q <sub>2</sub> -Q <sub>2</sub>	Q <sub>3</sub> -Q <sub>3</sub>	Q <sub>4</sub> -Q <sub>4</sub>
Cons	0.43	6.03	0.97	1.31	0.91	0.01
t-FM	(0.73)	(2.16)	(0.32)	(0.54)	(0.30)	(0.22)
t-Sh	(0.59)	(1.72)	(0.20)	(0.42)	(0.20)	(0.14)
$\gamma$	165.52	58.25	93.44	59.01	82.05	79.53
t-FM	(3.31)	(2.74)	(5.23)	(2.90)	(4.40)	(4.07)
t-Sh	(2.72)	(2.30)	(3.65)	(2.40)	(3.19)	(2.94)
$R^2$	0.23	0.36	0.40	0.20	0.40	0.48

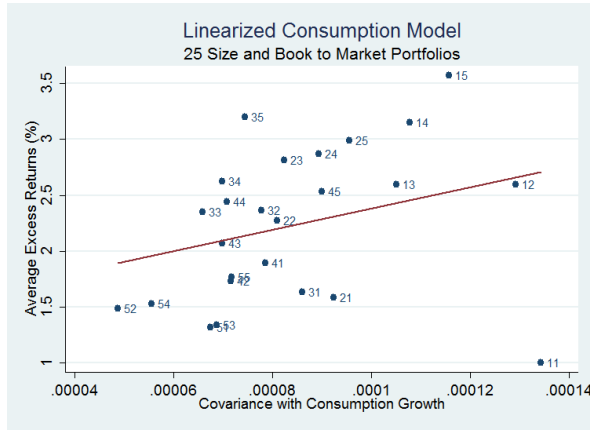
**Mom 10**

	Quarterly	Annual	Q <sub>1</sub> -Q <sub>1</sub>	Q <sub>2</sub> -Q <sub>2</sub>	Q <sub>3</sub> -Q <sub>3</sub>	Q <sub>4</sub> -Q <sub>4</sub>
Cons	0.35	9.37	-1.72	1.52	-0.52	-3.59
t-FM	(0.61)	(4.27)	(-0.57)	(0.67)	(-0.21)	(-1.05)
t-Sh	(0.39)	(2.20)	(-0.23)	(0.40)	(-0.08)	(-0.42)
$\gamma$	238.72	131.57	160.66	97.33	168.68	161.69
t-FM	(3.80)	(4.52)	(4.88)	(4.26)	(5.35)	(5.14)
t-Sh	(2.44)	(2.43)	(2.00)	(2.69)	(2.21)	(2.12)
$R^2$	0.14	0.89	0.52	0.36	0.53	0.66

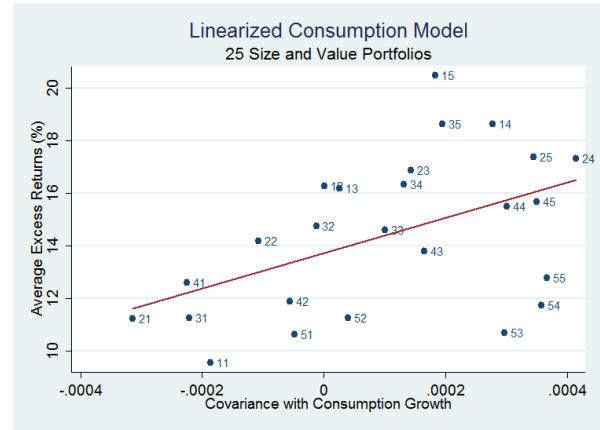
Table 3.8: Annual and Quarterly Momentum Average Returns and Betas

The table shows average returns, consumption betas, and the t-statistics of consumption betas estimated from time series regressions of returns of ten portfolios sorted by momentum on consumption growth measured at annual frequency and quarterly frequency.

Portfolios	Annual			Quarterly		
	Mom	Beta	t	Mom	Beta	t
1	-0.06	-4.94	-1.58	-0.08	2.55	1.48
2	4.22	-1.45	-0.63	1.10	2.82	2.11
3	6.03	-2.20	-1.07	1.48	1.37	1.22
4	6.58	-0.86	-0.48	1.68	2.10	2.04
5	6.52	-0.46	-0.27	1.69	1.99	2.06
6	7.34	-0.93	-0.52	1.86	1.98	2.01
7	7.69	-0.69	-0.43	1.98	1.70	1.86
8	9.84	-0.06	-0.03	2.42	2.38	2.56
9	10.66	0.46	0.25	2.59	2.80	2.82
10	15.49	2.05	0.89	3.73	4.16	3.24
$\chi^2\beta_1 = \dots = \beta_{10}$		15.73			11.77	
p-val		[0.07]			[0.23]	
$\chi^2\beta_1 = \beta_{10}$		2.65			0.96	
p-val		[0.10]			[0.33]	



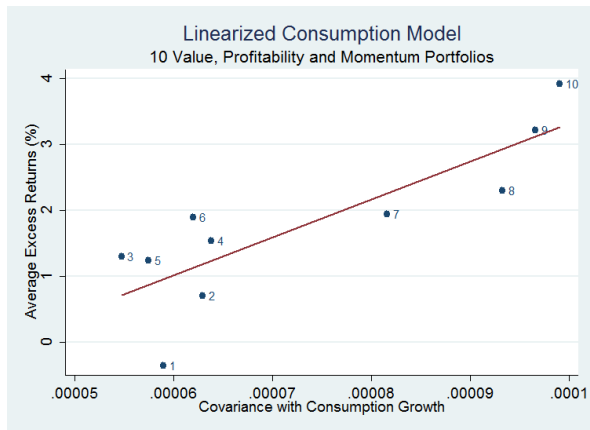
(a) Quarterly



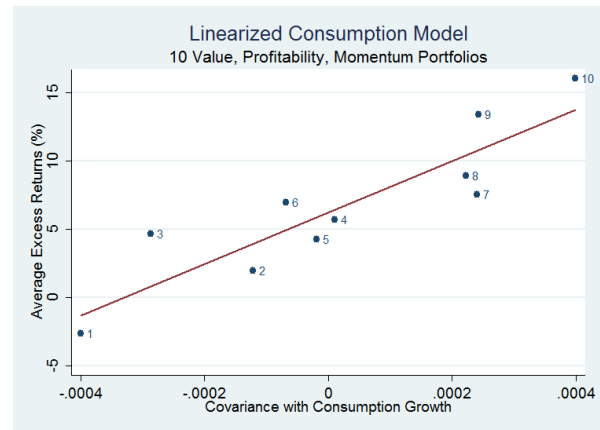
(b) Annual

Figure 3.1: CCAPM vs 25 Portfolios Formed on Size and Book to Market.

The figure shows the results of the cross-sectional regressions of consumption growth on ten portfolios formed by size and book to market. The X axis is the covariance with consumption growth. The Y axis is the average return in excess of the risk free rate, the return on either a three month or 1 year treasury security.



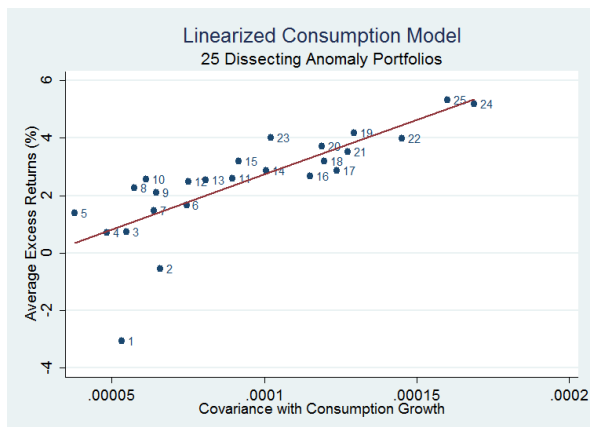
(a) Quarterly



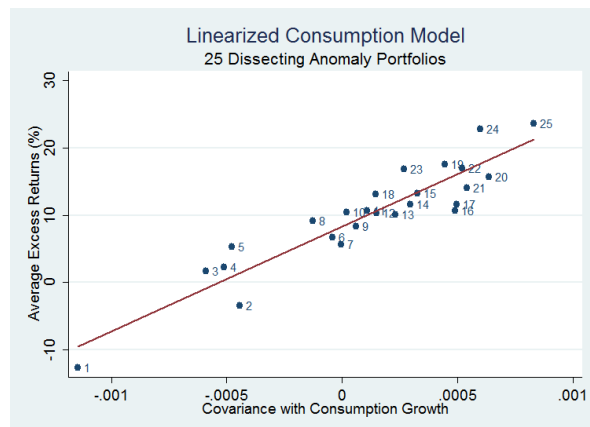
(b) Annual

Figure 3.2: CCAPM vs 10 Portfolios Formed on Value, Profitability, Momentum

The figure shows the results of the cross-sectional regressions of consumption growth on ten portfolios formed on value (book to market), profitability (gross profit), and momentum. The X axis is the covariance with consumption growth. The Y axis is the average return in excess of the risk free rate, the return on either a three month or 1 year treasury security.



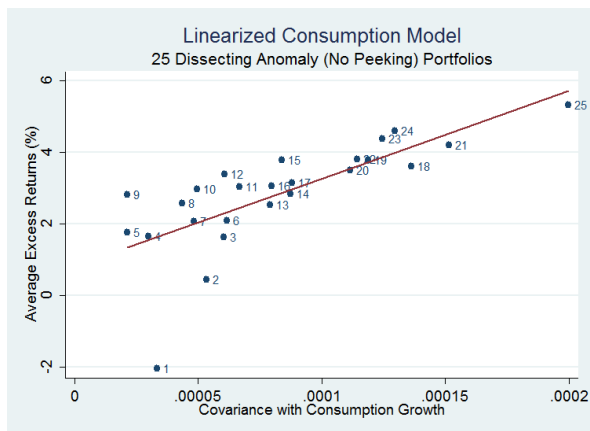
(a) Quarterly



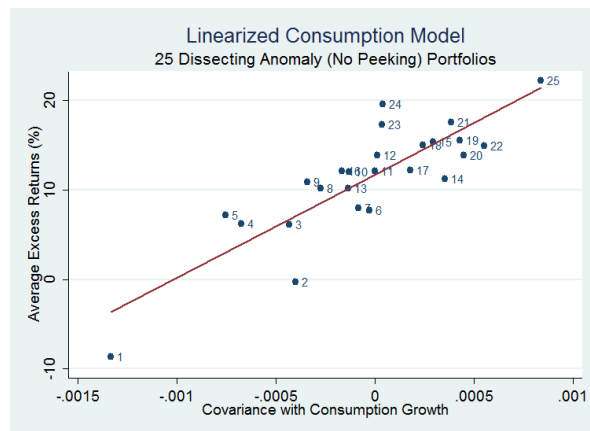
(b) Annual

Figure 3.3: CCAPM vs 25 Portfolios Formed in the Style of Dissecting Anomalies

The figure shows the results of the cross-sectional regressions of consumption growth on ten portfolios formed by regressions on seven anomaly variables, size, book to market, momentum, investment, profitability, net stock issues and accruals. The X axis is the covariance with consumption growth. The Y axis is the average return in excess of the risk free rate, the return on either a three month or 1 year treasury security.



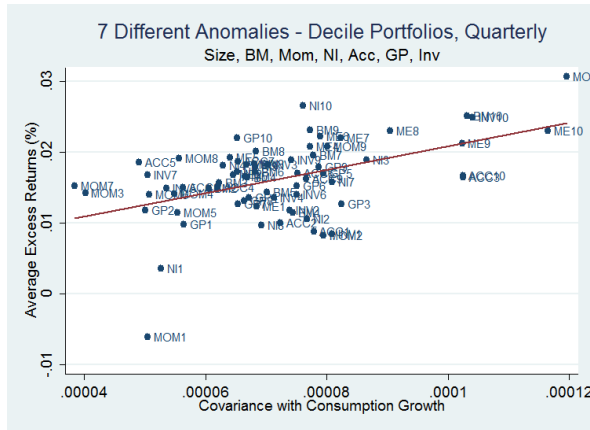
(a) Quarterly



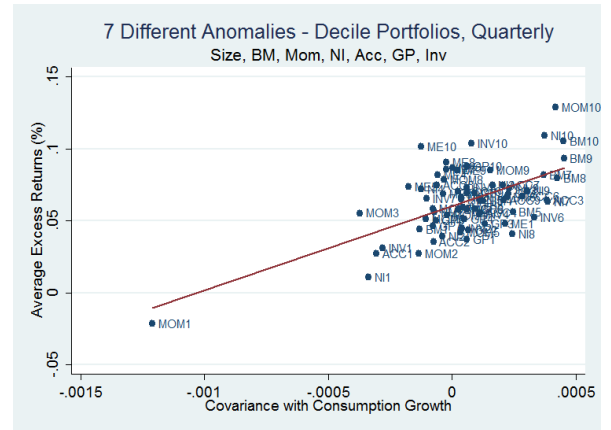
(b) Annual

Figure 3.4: CCAPM vs 25 Portfolios Formed in the Style of Dissecting Anomalies (No Peeking)

The figure shows the results of the cross-sectional regressions of consumption growth on ten portfolios formed by regressions on seven anomaly variables, size, book to market, momentum, investment, profitability, net stock issues and accruals. The portfolios are formed with completely out of sample coefficients formed using an expanding window of only data available to the investor at the time of investment. The X axis is the covariance with consumption growth. The Y axis is the average return in excess of the risk free rate, the return on either a three month or 1 year treasury security.



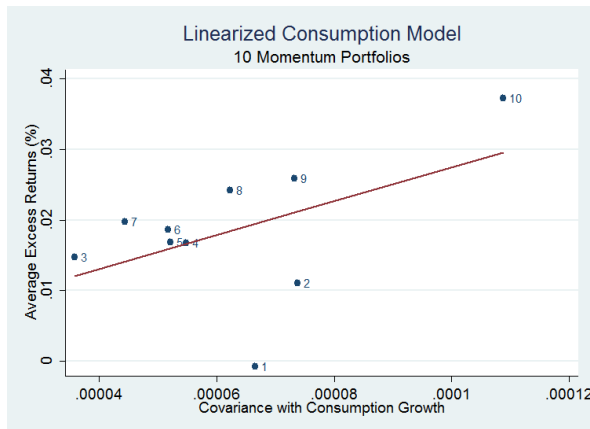
(a) Quarterly



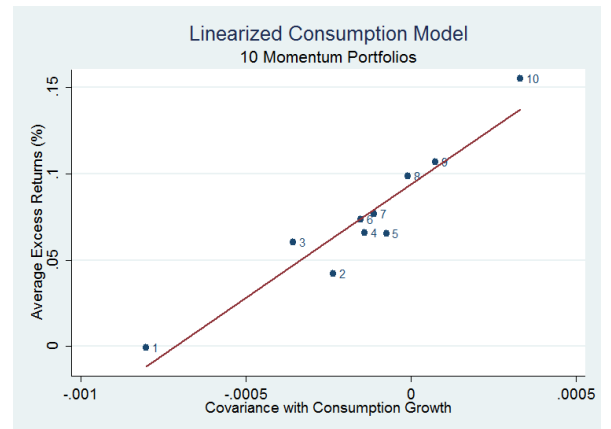
(b) Annual

Figure 3.5: CCAPM vs 70 Portofflios Formed on 7 Anomaly Deciles

The figure shows the results of the cross-sectional regressions of consumption growth on seventy portfolios formed by regressions on decile sorts on seven anomaly variables, size, book to market, momentum, investment, profitability, net stock issues and accruals. The X axis is the covariance with consumption growth. The Y axis is the average return in excess of the risk free rate, the return on either a three month or 1 year treasury security.



(a) Quarterly



(b) Annual

Figure 3.6: CCAPM vs 10 Portfolios Formed on Momentum

The figure shows the results of the cross-sectional regressions of consumption growth on ten portfolios formed by regressions on decile sorts on momentum, size, book to market, momentum, investment, profitability, net stock issues and accruals. The X axis is the covariance with consumption growth. The Y axis is the average return in excess of the risk free rate, the return on either a three month or 1 year treasury security.

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