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# Physical Interaction-Process driven Investigation of Quantum Mechanics

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# Physical Interaction-Process driven Investigation of Quantum Mechanics

Michael Ambroselli, Ph.D.  
University of Connecticut, 2016

Quantum Mechanics (QM) represents the currently best mathematical theory and computational tool available to quantitatively model the outcomes of measurements in the microscopic world. However, the various interpretations of the theory, including the currently dominant Copenhagen Interpretation, remain actively debated and contested in mainstream scientific journals. This puts us in the curious position of having a theory which evidently maps a significant portion of physical reality, but we still do not fully understand the relationship between its mathematical constituents and physical reality.

This thesis develops a more intuitive understanding of QM based on the physical interaction processes occurring within our detectors. Using a semi-classical point of view, I first present a process-based derivation of the black-body spectrum, the mathematically accurate description of which, as first accomplished by Planck, led to the development of modern QM. I further demonstrate that

ignoring interpretations like the Copenhagen Interpretation and focusing on measurable light-matter interaction processes provides an alternative productive way forward. I focus specifically on superposition effects created by multiple electromagnetic waves simultaneously stimulating appropriate detectors, by mapping the interaction processes that result in the reported data. As a result I present a simple model of the detector signal that incorporates some of the important physical processes which do date have suffered from a lack of attention.

One key observation of this discussion corresponds to the fact that in the linear domain wave amplitudes by themselves do not interact to generate observable data - the response to the joint amplitude stimulation of a detector and the ensuing energy absorption on the other hand do represent observables. Much of the confusion as well as many of QM's paradoxes and contentious issues inherent in various interpretations of the theory resolve automatically when we remain focused on the interaction processes that give rise to the measurable *superposition effect*, rather than the unobservable *superposition principle* that the standard interpretations struggle to give meaning to.

Demanding, as I do in this thesis, a close correspondence between our mathematical symbols and physical quantities, as well as between mathematical operations and physical processes demonstrates that QM maps more of physical reality than what the Copenhagen Interpretation allows us to extract from the QM formalism.

# Physical Interaction-Process driven Investigation of Quantum Mechanics

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A Dissertation  
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Requirements for the Degree of  
Doctor of Philosophy  
at the  
University of Connecticut

2016

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2016

# APPROVAL PAGE

Doctor of Philosophy Dissertation

## Physical Interaction-Process driven Investigation of Quantum Mechanics

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2016

*τη καλλίστη*

(E.N.S.)

und für meine Eltern

(Werner und Manuela)

and

Kay

## ACKNOWLEDGEMENTS

I just recently remembered that a fellow primary school student used to tell a friend and me that we would become scientists and go to the moon. While I have not made it to the moon quite yet, the first part of this “prophecy” appears to have indeed come true, at least for one of us. No one, however, could have predicted the particular circumstances that paved the path I have traveled. Had events taken happier turns at certain points, I definitely would not have gotten here. But then again, this holds for everything and everyone - things happen, and if they didn’t, others things would, and we would not know the difference. Given the chance, I would not want to go back in time to fix things. If time travel movies teach us anything, then that fixing problems in the past will only create more in the future. And after all, things have turned out pretty well and I have many people to thank for that.

First and foremost, of course, I would like to thank my family for their support and everything. No names. You know if that includes you.

Throughout my career as a professional physics student I have had the good fortune to meet some truly unique and interesting individuals, a subset of which became my advisory committee here at UConn. Again, I can only call myself fortunate to have met Chandra Roychoudhuri - my advisor, friend and I dare say



the smartest and wisest individual I have ever met. I have my doubts that I would have made it through graduate school with anyone else. Even though my thesis topic presented somewhat of a moving target, this freedom allowed me to explore many different ideas and issues of which only some made it into the final version this dissertation.

After accepting UConn's offer I heard an NPR interview with Ron Mallett, and I knew I made the right decision. His research and story sounded fascinating and not only did I end up having the opportunity to take his relativity course, but he also has become one of my associate advisors. He has helped me immensely with getting this thesis into its final, coherent form, and provided encouragement when I needed it, for which I can not thank him enough. Juha Javanainen has also helped me enormously with his critical input, comments and review of my draft. Furthermore, I thoroughly enjoyed the classes I took with him years ago - his unique personality made StatMech almost a joy.

I'd like to thank Win Smith for providing critical feedback on various paper drafts that all ended up in this document. Philip Mannheim has also provided significant help throughout the years. Furthermore, I blame him for my decision to consider UConn for graduate school in the first place - a seminar he gave at KU made me want to learn more from this man. I have learned a lot from him and thoroughly enjoyed all of his classes and other lectures.

From my years of taking courses thanks go to those with whom I got to

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Finally, I must not fail to acknowledge John Ralston from the University of Kansas. His brilliance and pragmatic understanding of physics has influenced my thinking significantly, and I find myself regularly going back to his papers and unpublished textbooks, both for entertainment and for continued insights. Along with Chandra and a certain former associate editor of Playboy (whom I've never had the fortune to meet), Ralston probably had the biggest impact on the current state of my reality tunnel.

Lastly, my thanks go to all of those whom I forgot to mention here. As you probably know, my mind consists of mostly holes, held together by more holes. Rest assured that, as soon as I printed and submitted the final version of this, I started kicking myself for all the things and people I forgot to include.

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## Introduction

*It is the theory that decides what  
we can observe.*

---

Albert Einstein

## Background

During the second half of the 19th century, it became apparent that conventional, Newtonian mechanics could not account for some experimentally observed processes and phenomena related to emission of light and light-matter interactions. Most notably, these include black-body radiation, the emission and absorption spectra of various atomic and molecular substances, and the photoelectric effect [1].

In 1905, Einstein explained the photoelectric effect [2] by suggesting that light continues to propagate as discrete energy packets after emission, and a theory based on ad-hoc hypotheses, now called Bohr's "Old Quantum Mechanics", emerged in the early 20th century in order to explain the above and other phenomena. While this old quantum mechanics *could* reproduce and predict *some* of these observations, the theory had no strong physical basis for the assumptions

that made it work. Quantization of atomic energy levels, for example, did not emerge naturally from the theory but required imposition of the Bohr-Sommerfeld quantization condition  $\oint p_i dq_i = n_i h$  to select allowed electron orbits.

In the middle of the 1920s, modern Quantum Mechanics emerged when Heisenberg and Schrödinger developed their mathematically equivalent formulations, namely the Wave Mechanics and Matrix Mechanics, respectively. These formalisms provided a solid mathematical basis and could explain a great number of observed phenomena. Over nine decades later, this new Quantum Mechanics (QM) still represents the best mathematical theory currently available to quantitatively model the outcomes of measurements in the microscopic world. The QM formalism itself makes good predictions, validated by data captured in a large variety of experiments.

However, the dominant [3] Interpretation of the theory, the Copenhagen interpretation (CI), remains actively debated and contested in mainstream scientific journals. The theory hides many details of the microscopic processes due to the statistical nature of the measurement processes considered. This puts us in the curious position of having a theory which evidently maps a significant portion of physical reality, but we still do not fully understand the relationship between its mathematical constituents and physical reality.

In this thesis I aim to develop a more intuitive, physical-process based approach to and understanding of QM. We take on this challenge by consistently

applying an old methodology of thinking. Essentially, we will force ourselves to *visualize* the invisible physical processes<sup>1</sup> happening in nature and ensure their faithful mapping by our mathematics. The core guiding principle here corresponds to the following two demands: 1) mathematical symbols should as much as possible correspond to primary physical parameters and 2) mathematical operations should map physical processes. This *process-based* thinking model has consequences far beyond QM and finds applications in all areas of physics and science in general.

I discuss the importance of identifying the mathematical parameters that most directly participate in the processes effecting the measurable physical transformations within our detectors. Distinguishing and keeping track of quantities we identify as primary, vs. secondary or tertiary etc., determines, in part, how we interpret the theory and how and what experiments we design. Doing this will also help us to better visualize and iteratively refine our mathematical models of nature's processes.

I also find it crucially important to realize and keep in mind that all mathematical theories have limitations. Quantum Mechanics, for example, describes a great many physical phenomena remarkably well, quantitatively. Physicists have used and studied this theory for almost a century, and derived an enormous amount of information from it. This means that QM alone must have captured

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<sup>1</sup>Physicists have done this for ages, but it appears to have fallen somewhat out of favor after the development of QM and the CI.



a considerable part of the rules or logics that govern our universe. However, as Stephen Hawking recognized in a 2002 speech [4], mathematical/formal logic suggests, in the form of Gödel's incompleteness theorem, that we may not consider any existing or future theory as final.

Even from an intuitive standpoint we can see that this must hold true: we humans invented mathematics in order to enable us to quantify and *map* observations made of the world<sup>2</sup> into a set of logical statements that make identifying trends and relationships easier for us. We can not consider the universe a gigantic computer that uses our mathematics in order to function - nature can do “whatever it wants”, it does not have to conform to any of our equations. Our equations merely represent or describe the approximate logical connections between sets of observations that we have found so far.

Since measurements, and thus the interaction processes between measurable quantities and our detectors, represent the *only* way we know *anything* about the universe, it appears absolutely vital for us to take a process-visualization approach. Doing this we quickly realize that any measurement automatically and invariably results in a “loss of information” with respect to the measured quantity - a fact that often goes under-appreciated and misunderstood. Indeed, modern treatments often mix up this “fact of nature” with the “uncertainty principle” - a

---

<sup>2</sup>A world we *only* perceive through various detectors - this includes biological detectors, such as our senses and the associated interpretative neural network, as well as artificial sensors connected to interpretative meters, that we build in order to extend or supplement our senses.

fact of mathematics<sup>3</sup>.

I focus specifically on bridging the differences between the classical and quantum mechanical Superposition Principle (SP) by mapping the interaction processes that appropriate detectors experience and report as data when multiple electromagnetic waves simultaneously stimulate them. One key observation of this discussion corresponds to the fact that, in the linear domain, wave amplitudes by themselves do not interact to generate observable data<sup>4</sup> - the response to the joint amplitude stimulation of a detector and the ensuing energy absorption on the other hand do represent observables<sup>5</sup>. Much of the confusion as well as many of QM's paradoxes and contentious issues inherent in various interpretations of the theory resolve automatically when we remain focused on the interaction processes that give rise to the measurable *superposition effect*, rather than the unobservable *superposition principle* that the standard interpretations struggle to give meaning to.

Understanding and trying to visualize the invisible<sup>6</sup> interaction processes between the measurable quantities and the detecting entities will help us interpret the signals reported by the various detectors we use. Here, I apply this idea directly to the interaction between electromagnetic radiation (light) and material particles (detectors) via a semi-classical approach similar to that advocated by

---

<sup>3</sup>See Appendix E.1

<sup>4</sup>contrary to what the SP seems to suggest and calls "interference"

<sup>5</sup>we suggest using "superposition effect" as a term superior to "interference" for the observed result

<sup>6</sup>i.e. possibly not directly observable by us

Jaynes [5], Lamb [6], Scully [7] and others.

## Overview

In Chapter 1 I review the main historical treatments on black-body radiation. The black-body spectrum represents one of the core problems that eventually led to the development of modern quantum mechanics, and I present an alternative derivation, following in Planck’s footsteps, that does not require counting quantized light (photons), but rather has its basis in the experimentally confirmed process of quantized energy *transfer* between the electromagnetic field and material particles within the black-body’s walls. In addition, I also implement the demand for a closer mapping between mathematical representation and physical reality.

In Chapter 2 I briefly review various “derivations” of the Schrödinger equation, some of which appear to lend themselves to a more useful interpretations of the model than others. Along with our process-visualization approach, these will allow for much more physically insightful interpretations of quantities such as Schrödinger’s  $\psi$ ,  $\psi^*\psi$ , and QM as a whole, than generally considered. I also discuss the Copenhagen Interpretation of the theory and give a detailed account of the difficulties it poses for a process-based understanding of nature.

Chapter 3 reviews the definition, justification and use of the Superposition Principle (SP) in both, Classical Mechanics (CM) and Quantum Mechanics (QM).

I then apply the SP to two superposed beams of electromagnetic radiation in the usual way, without reference to the detector used. An overview and discussion of various detectors for the electromagnetic spectrum in the three major frequency regimes: radio, optical and gamma follows. Each of these require a distinctly different detection mechanism. Then I re-examine the two-beam superposition setup by analyzing the superposition effects as reported by various detectors. Understanding the emergence of superposition fringes by dissecting the physics and engineering behind the detector responses to various physical stimuli represents a major step in the direction of a better understanding of QM and physical phenomena in general. I present a process-based model for the detector signal. Finally we will see that a process-driven interpretation of the superposition effects detected, along with this simple detector model, will, for example, allow us to differentiate between the two experimental techniques of Fourier transform spectroscopy and heterodyne spectroscopy simply on the basis of detector properties and design.

In Chapter 4 I briefly outline the future I see for the process-centered approach presented here, including its application to the other major theories. I also describe an experiment that we hope to carry out in the near future and which may strengthen our theory of a complex tension field (CTF) that we have put forth elsewhere.

## Further Context

Quantum mechanics, and particularly the Copenhagen Interpretation of it, has left scientists with an uneasy feeling since the beginnings. Considering its computational utility most end up getting over the philosophical issues of QM mostly associated with its interpretation, and use it as a useful tool to calculate things. Some of us, however, continue to feel the urge to gain a deeper understanding of QM. Schrödinger, one of the primary creators of modern QM himself struggled with some of the implication of the Copenhagen Interpretation. This originated the three papers [8–10] introducing the now famous and frequently abused illustrative example of what we now call “Schrödinger’s cat”. The main themes he addressed included the superposition principle, the uncertainty principle, as well as entanglement - issues that to this day remain contentious.

Around the same time, just slightly earlier, the well known paper by Einstein, Podolsky and Rosen (EPR) [11] appeared and actually gave rise to Schrödinger’s aforementioned series of papers [10]. EPR showed via a simple example, which also implies a contradiction of the uncertainty principle, “that the wave function does not provide a complete description of the physical reality” [11]. Today, over 85 years later, this issue, as well as all the others Schrödinger raised, has still not found a satisfactory resolution. Even prominent contemporary physicists continue to point out the many inconsistencies of QM [12] and state that it has not “yet achieved the necessary coherence as a theory” [13].

One of the main issues of QM originates from the fact that, while it has time and again proven an excellent calculational tool, it does little to help us better understand the interaction processes occurring at the fundamental level. For example, the ubiquitous *uncertainty principle* (UP), has a rather weak physical foundation [14], but plays a prominent role in the theory. Especially the version concerning time and energy finds its use as a convenient catch-all explanation or justification for inconvenient questions.

Despite the fact that even some undergraduate texts caution against the sloppy use of the uncertainty principle [15], doing just that appears to have become standard practice, particularly in the area of Quantum Field Theory (QFT). We see, for example, pair production explained by the UP [16], as well as things like zero point motion/energy [17]. Such “non-explanation explanations” only serve to impede our quest for a better understanding of nature.

### **Where this Thesis fits in**

Since the mathematics of QM already does an adequate job of modeling the dynamics, it appears that what needs further development corresponds to our *interpretation* of it. In 1952 Bohm suggested that the statistical nature of QM comes from averaging over some hidden variables of a more precise theory [18,19]. In his 1957 Ph.D. thesis Everett developed his now famous “many-worlds” interpretation [20]. In later years various others [21–23] have contributed their ideas, and the

issue remains open and active to this day [24–27].

A process-based view forces us to side with Bohm’s assertion of the incompleteness of QM, which does not diminish the utility of the theory itself. It clearly works very well for a large variety of problems. However, it should come as no surprise that our theories do not only determine to a large degree how and what experiments we conduct, but also how we interpret the results. As Einstein put it, “It is the theory that decides what we can observe” [28]. This implies that we must always strive to question and iteratively enhance our original postulates and fundamental assumptions behind any working theories as our state of knowledge advances. Roychoudhuri [29] discusses this important idea extensively. When working *within* a given theory, one can rarely discover an inconsistency or ways to better accommodate observations. Ralston got to the heart of this problem when he wrote “When logic is made the logic is seamless. Its blindness is invisible. This is the problem of a closed logical loop, in which everything is self-consistent” [30].

Following in the footsteps of the above authors, I aim to provide another point of view to the discussion of the interpretation of QM. Again, QM lets us calculate probabilities of experimental outcomes, but says very little about the interactions that govern individual processes. In addition, the Copenhagen Interpretation, still the most widely accepted interpretation of QM, discourages us from even asking questions about such processes. Since *all* our knowledge about the universe derives precisely from such interactions - i.e. the “outside world”

*interacting* with our sensors - I find it of the utmost importance to focus on these processes and will do so in this thesis.



# Chapter 1

## Black-body Radiation

*What becomes of the energy of a  
photon after complete emission?*

---

Max Planck

In this chapter I re-examine black-body radiation, one of the core observations which ultimately resulted in the creation of modern QM, and the derivation of the Einstein-Bose statistics that helped explain it. Following a historical review of the defining contributions to its explanation, I present a slightly different, interaction process based derivation that does not depend on counting indivisible photons<sup>1</sup>. Planck never felt comfortable with the notion of photons remaining and propagating as indivisible packets of energy after emission. This discussion follows and extends his process-rooted way of thinking.

### 1.1 Historical Review

The black-body spectrum has spurred much of the early development of Quantum Mechanics and ultimately resulted in the creation of QM as we know it today.

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<sup>1</sup>This discussion closely follows my corresponding Physics Essays publication [31].

Planck derived a mathematical expression that describes it accurately in 1901 [32]. The successful derivation of the black-body spectrum required Planck to postulate the quantized emission and absorption of radiation energy by the material inside a black-body cavity. Later some, particularly Bose and Dirac, interpreted these as quanta of the electromagnetic field itself<sup>2</sup>, while Planck never intended this and continued to point out the open question of the behavior of photons *after* their emission [33]. Some sources claim that a successful explanation of the black-body spectrum *requires* the quantization of the electromagnetic field [34,7]. My analysis shows that this does *not* represent a physical or a mathematical necessity.

Even though the black-body spectrum played a crucial role in the development of Quantum Mechanics, some books do not even mention the subject [35], while others relegate it to a brief comment at the end of a chapter [36]. Most contemporary (textbook) discussions and derivations of Bose statistics [15], and hence the radiant energy distribution of a black body, as derived by Planck [32,37], and later by Bose [38], focus purely on combinatorics<sup>3</sup>. This mathematical procedure “gets us there”, but it diverts us from exploring the *physical processes* involved in the emission and absorption of radiation. Roychoudhuri’s recent book [29], explicitly dedicated to Planck and his process driven approach to physics, heavily emphasizes the importance of visualizing, understanding and modeling the invisible interaction processes occurring in nature.

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<sup>2</sup>i.e. photons

<sup>3</sup>counting states

One of the important, but rarely mentioned processes responsible for the radiation spectrum emanating from a black body occurs at the physical surface of the object. In a physical black-body cavity, continual energy exchange takes place between the radiation field and the surface molecules in their solid state. These material particles, via stimulated emission and absorption, facilitate the thermodynamical equilibrium necessary to produce the well known steady-state frequency distribution.

As many other models in physics, a theoretical black body represents a platonic [39], non-physical<sup>4</sup> idealization and thus an oversimplification of nature, albeit a very useful one. Clearly, the black-body concept has successfully captured, and mathematically encoded, some part of our physical reality. Thus, I find it important to explore and understand the physical processes that would result in such measurements as successfully modeled by Planck's formula.

Since most modern derivations of Planck's formula seem to prefer the more mathematical and less physical route of plain combinatorics over one rooted in physical considerations, I will go back and re-examine the line of reasoning presented by the original authors. Interpreting their reasoning in light of our *current* knowledge will enhance our understanding of the physical processes that lead to Planck's formula and Bose's statistics.

---

<sup>4</sup>Meaning: no actually 100% black bodies exist in our universe

### 1.1.1 Planck

At the end of the 19<sup>th</sup> century, Planck had set out to find a connection between the second law of thermodynamics and electromagnetic theory [40]. To him, the radiation spectrum emanating from a black body seemed to intimately connect these two disciplines on an experimental basis. He pointed out that the problem of actually doing this theoretically *lies in the inherent absence of irreversible processes within electromagnetics as described by Maxwell's equations* [40]. This, as a result, makes it impossible to define a thermodynamic entropy<sup>5</sup> for such radiation and the associated absorption and emission processes.

Since the emission and absorption processes involve *matter*, and we *can* describe *it* using thermodynamics, Planck argued that we need to introduce a new hypothesis to facilitate the connection. The basis for this he finds in the finite width of spectral lines. He postulates a completely random distribution of the light's energy over the constituting frequency band, coining the term “natural radiation”. By presupposing that all (thermal) radiation has this property, it follows that we *can* now define an electromagnetically based function of state analogous to the thermodynamic entropy.

While it did not come across very clearly in his paper on irreversible radiation processes [40], in a later paper [41] Planck points out that only thermal radiation has his “natural radiation” property, while Hertzian waves most certainly

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<sup>5</sup>I.e. a quantity that will only ever change in *one* direction.

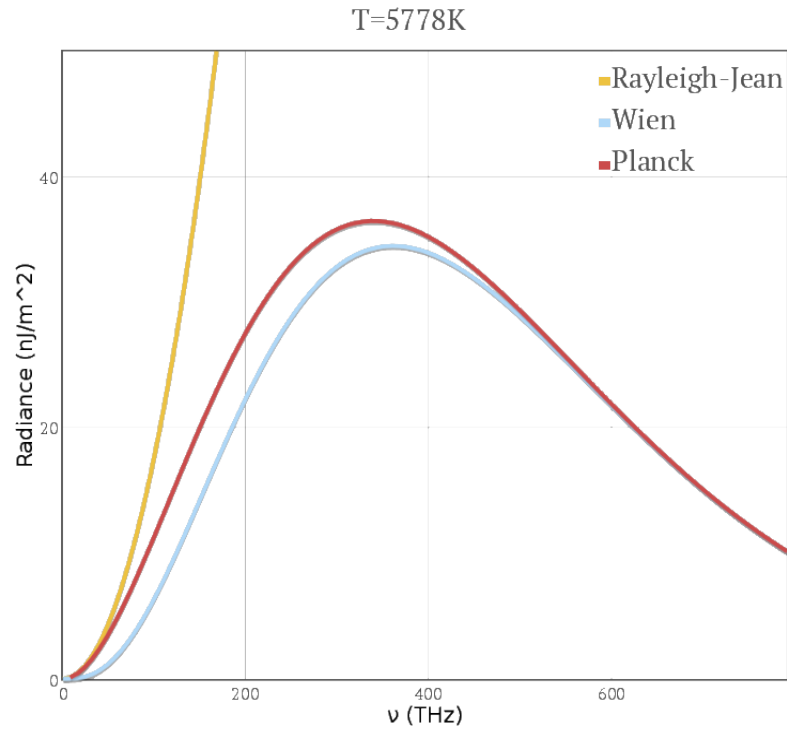
do not.

A Hertzian oscillator functions like a continuous wave radio antenna, producing radiation with well defined frequency and phase. Thermal radiation inside a black body, on the other hand, consists of innumerable wave packets with widely varying frequencies and essentially random phases.

Since the fundamental nature of both “types” of radiation coincides, i.e. it corresponds to that of electromagnetic waves, we see here already the first evidence that the black-body spectrum represents the result of the *interaction processes* between electromagnetic waves and matter, and not a property of the radiation itself. Nor does the black-body spectrum come about as a result of the radiation field’s entropy or provide evidence of it even possessing such a property in the first place.

Planck’s postulate of “natural radiation” allowed him to derive Wien’s law for the energy distribution of the black-body radiation. Soon, new measurements began to indicate disagreements between Wien’s law and the observed energy distribution, and he began to investigate alterations to his derivation necessary to account for the discrepancy [42].

While previously he had, without further explanation, *defined* a relationship between the energy of the radiation and its entropy, he now found it necessary to provide a more physically grounded derivation. The first attempt of this, which occurred only shortly after the first reports of the Wien law’s possible failure,



**Fig. 1.1:** The energy distribution of a black body at the approximate temperature of the sun. Rayleigh-Jean’s law works well for low frequencies but fails catastrophically (“ultra-violet catastrophe”) at higher frequencies. Wien’s approximation works well at high frequencies but only poorly at the lower end of the spectrum. Both represent limiting cases of Planck’s result which works well throughout the entire frequency range.

got him back to Wien’s equation [41]. As evidence of the insufficiency of Wien’s law in the domain of longer wave lengths mounted, Planck successfully continued his quest for the necessary changes to his approach that would provide him with a more complete description of the black-body spectrum [32]. Figure 1.1 shows Planck’s result in comparison to its predecessors.

At first, Planck arrived at an improved and now well-known result through educated guessing of the form of derivatives of the entropy [42]. Following this first success, he worked on actually deriving the entropy expression and thus his

final black-body equation [32]. Instead of trying to find a relationship between the entropy  $\mathcal{S}_N$  of the system and its *total* energy  $U_N$ , Planck realized that he needed to go back and focus on all the *material oscillators*, and their *individual* energies present in the system. Along with the *meaning* of the entropy  $\mathcal{S}$  as a measure of orderliness in a system, this led him to *postulate* that the energy  $U_N$  of all  $N$  oscillators can only take on integer ( $P$ ) multiples of some unspecified energy element  $\varepsilon$ , i.e.

$$U_N = P \cdot \varepsilon \tag{1.1}$$

The necessity of this postulate originates in the need to *count* (energy) states in order to, ultimately, quantify the system's orderliness and thus specify  $\mathcal{S}_N$ . Planck could only do this if each oscillator could support only a finite, albeit possibly very large, number of such states. This allowed him to count the number  $\mathfrak{R} = \frac{(N+P-1)!}{(N-1)!P!}$  of micro states, each of which he calls a “complexion”, that make up a given macro state of total energy  $U_N$ . Now he could write down an expression for the entropy using Boltzmann's  $\mathcal{S}_N = k \log \mathfrak{R}$  [43].

His initial assumption, that

$$U_N = N \cdot U \tag{1.2}$$

i.e. that the total energy of the system distributes itself over  $N$  oscillators that

can only support energies of increments of the single, fixed energy amount  $\varepsilon$ , now got his very novel line of reasoning stuck, although he probably did not realize this at the time. In order to make progress, he had to introduce Wien’s displacement law<sup>6</sup>,  $E \cdot d\lambda = \vartheta^5 \psi(\lambda\vartheta) \cdot d\lambda$ , where  $\vartheta$  stands for the temperature. This ends up leading him to the conclusion that  $\varepsilon = h\nu$ , as well as the final form of the energy distribution formula [32]

$$u = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{\frac{h\nu}{k\vartheta}} - 1} \quad (1.3)$$

Note, however, that Planck rewrites the displacement law in terms of *frequency*, before actually using it in his derivation. Furthermore, in his 1906 book, Planck specifically points out the more fundamental character of the frequency parameter [44] as opposed to the wavelength, which changes from medium to medium, and thus represents a *derived parameter*<sup>7</sup>. I highlight this point to underscore an important physical consideration that to this day remains widely under-appreciated [29].

While the introduction of Wien’s classically derived formula proved success-

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<sup>6</sup>Planck means by this a somewhat generic form of what we now more commonly call “Wien’s approximation”. We will continue to use Planck’s terminology, in line with the other historical figures / papers discussed.

<sup>7</sup>See Appendix A for more on this.



ful, Planck apparently never followed the procedure he had described *verbally* in one of his 1900 papers [37], which would have given him the same result, as I will now demonstrate.

It should make us wonder, that the *constant* amount of energy,  $\varepsilon$ , introduced by Planck out of necessity, does not, in fact, have a constant value at all, but depends on the frequency  $\nu$ . What does this imply for the oscillator picture in the beginning of the discussion?

Evidently, the energy does not get distributed over a large number of oscillators of only *one* type, but over a large number of oscillators of *many* different types. Each of these oscillator types distinguishes itself from the others by the basic quantity of energy  $\varepsilon_i$  that it can absorb, or emit. From this follows, that we need to combine and alter Eqs.(1.1) and (1.2) as follows.

$$U_N = P \cdot \varepsilon \quad \Longrightarrow \quad U_N = \sum P_i \cdot \varepsilon_i \quad (1.4)$$

$$U_N = N \cdot U \quad \Longrightarrow \quad U_N = \sum N_i \cdot U_i \quad (1.5)$$

and

$$P = \sum P_i \quad (1.6)$$

Note that  $U$  and  $U_i$  refer to the *average* energy of the oscillators and the oscillators

of a given type respectively.  $N_i$  now represents the number of oscillators of a given type / frequency ( $\nu_i$ ), and  $P_i$  represents the number of energy packets with energy  $\epsilon_i$ . It also follows, that Planck's expression for the number of micro states ( $\mathfrak{R}$ ) only refers to *one* single type of oscillator, and we need to modify it thusly:

$$\mathfrak{R} = \prod_i \frac{(N_i + P_i - 1)!}{(N_i - 1)!P_i!} \quad (1.7)$$

Now we can write down the entropy  $\mathcal{S} = k_B \ln(\mathfrak{R})$ , which, using Stirling's approximation for  $\ln(n!)$ , becomes

$$\begin{aligned} \mathcal{S} \approx k_B \sum_i (N_i + P_i - 1) \ln(N_i + P_i - 1) \\ - \ln((N_i - 1)!) - P_i \ln(P_i) \end{aligned} \quad (1.8)$$

Varying this expression with respect to  $P_i$  to find the maximum-entropy distribution given the constraints (1.4) and (1.6), i.e.

$$\frac{\partial}{\partial P_i} \left( \mathcal{S} + \alpha(P - \sum P_i) + \beta(U_N - \sum P_i \epsilon_i) \right) = 0$$

results<sup>8</sup> in

$$P_i = \frac{N_i}{e^{\beta \varepsilon_i} - 1} \quad (1.9)$$

where we have set  $\alpha = 0$ , because the total number of “energy chunks”  $P$  does not, even at equilibrium, necessarily represent a conserved quantity. Next, we need to find an expression for  $N_i$ . For this, we use the facts that our radiation exists within a cavity of volume  $V$ , and that our measurement takes place after the black body cavity and the contained radiation have had time to reach thermodynamic equilibrium. Since the result Eq.(1.9) already required this, this does not represent an additional assumption.

At equilibrium, the emission and absorption rates at the boundaries coincide, making the process essentially equivalent to total reflection. Thus, we can use the mathematical eigenmodes of the cavity, i.e. waves with nodes at the cavity boundaries, to calculate the number of modes  $\mathcal{N}_\nu d\nu$  in the frequency interval  $d\nu$ , where  $\mathcal{N}_\nu \equiv N_i$ , since  $i$  really only labels the different frequencies ( $\nu_i$ ). We find

$$\mathcal{N}_\nu = \frac{4\pi\nu^2}{c^3}V \quad (1.10)$$

which we should multiply by 2, to take into account the two orthogonal polariza-

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<sup>8</sup>After dropping the  $-1$  of the first term in the sum, which we can legitimately do when Stirling’s approximation applies.

tion directions [45]. Using this result, we find

$$\rho(\nu) = \frac{8\pi}{c^3} \frac{\nu^2 \varepsilon_i}{e^{\beta \varepsilon_i} - 1} \quad (1.11)$$

where  $\rho(\nu)d\nu \equiv P_i \varepsilon_i d\nu / V$  corresponds to the energy density at frequency  $\nu$ .

It remains to *insert* the fact that the energy emitted from the physical oscillators (atoms / molecules) in the cavity walls increases linearly with the frequency, i.e.  $\varepsilon_i \equiv \varepsilon_\nu = h\nu$ . This represents an experimental result [46], as well as one derivable from quantum mechanics. With that, we finally arrive at Eq.(1.3).

So we see that Planck's original assumption regarding the discreteness of the energy amounts that each oscillator can support, paired with their linear frequency dependence, has exactly the same effect as introducing Wien's displacement law.

A further very important assumption made, that we have to consider when interpreting the result, corresponds to the condition, that *at equilibrium* the total energy tied up in a given frequency does not change. We have used this as a constraint on the entropy of the entire system.

This also implies, that Planck's equation *only* applies to *large systems at equilibrium*. It always requires some amount of time to reach this state, through innumerable emissions and absorptions by the surface molecules of the cavity. The black-body spectrum does not tell us anything about the *processes* involved that brought the system to the state of equilibrium in the first place. We will see that

both, Einstein and Bose, had to make equivalent assumptions.

### 1.1.2 Einstein

Einstein had also, in the following years, written some papers on the subject [2,47–50]. The one explaining the photoelectric effect originated the modern idea of quantized electromagnetic radiation (photons) [2], and won him the Nobel prize. Later he derived Planck’s equation by going a somewhat different route than Planck, introducing the idea of *stimulated emission and absorption* in the context of quantized systems along the way [50], but still using Wien’s classical displacement law to arrive at an equation equivalent to Planck’s.

The processes of stimulated emission and absorption, as Einstein himself mentioned [50], do not correspond to anything particularly quantum mechanical. Classical driven oscillators can, and do, absorb or emit energy in a way that depends on the *phase relationships* between the oscillators themselves and the incoming radiation. Einstein worked out some important implications for the case of *quantized* oscillators, such as atoms and molecules. While some find them surprising [15], they seem a rather natural, but non the less very important.

Einstein’s insightful paper [50] demonstrates, that the assumption of quantized absorbers and emitters (atoms / molecules), along with a rather different introduction of statistics into the problem ( $A$  and  $B$  coefficients), results in his

version of Planck's equation

$$B_m^n \varrho = \frac{A_m^n}{e^{\frac{\varepsilon_m - \varepsilon_n}{kT}} - 1} \quad (1.12)$$

The main physical input to derive this expression came from the requirement of a balance between the processes of spontaneous (proportional to  $A_m^n$ ), and stimulated emission ( $\varrho B_m^n$ ) and absorption ( $\varrho B_n^m$ ) (i.e. thermal equilibrium between the emitters and the radiation), where  $\varrho$  represents the energy density of the radiation. Further, Einstein assumes a statistical distribution of energy states proportional to the corresponding Boltzmann factor ( $e^{-\frac{\varepsilon_n}{kT}}$ ) and statistical weight  $p_n$ . The equilibrium between the probabilities of emission and absorption

$$p_n e^{-\frac{\varepsilon_n}{kT}} B_n^m \varrho = p_m e^{-\frac{\varepsilon_m}{kT}} (B_m^n \varrho + A_m^n) \quad (1.13)$$

then led him to Eq.(1.12).

Again, we must note that Einstein's  $A$  and  $B$  coefficients, as he introduced them, already *presuppose* thermal equilibrium. Due to this, he does not need to use a variational procedure, as we did above and will see that Bose utilized, but still needs Wien's classical displacement law to recover the final form of the energy distribution. The three coefficients ( $A_m^n$ ,  $B_m^n$  and  $B_n^m$ ) thus do not represent individual properties of a given kind of atom or molecule, but rather a *statistical, average* description of a large ensemble of these particles, which hides one or many

individual physical processes that actually cause the behavior they describe.

The famous paper explaining the photo-electric effect and later writings<sup>9</sup> building upon the photon idea, firmly cemented the concept of quantized light into the belief systems of most modern physicists. At the time, as an understanding and experimental observations of quantization had just started to emerge, Einstein's proposal represented an important avenue to explore. Scully and Lamb [7,52], much later, showed that the mathematical formalism of quantum mechanics, which of course had not existed when Einstein wrote his paper, does not require quantized photons to explain the photo-electric effect. This also holds true for the black-body spectrum, as I will discuss further in sections 1.2 and 1.3.

### 1.1.3 Bose

About 23 years after Planck's initial paper, Bose suggested that one did not need, and in fact should not use, the (classical) displacement law to derive the black-body result [38]. He criticized both, Planck's, as well as Einstein's derivations for lacking logical justifications, by which he mainly referred to their reliance on classical assumptions, such as Wien's displacement law. To provide a derivation that removes these apparent shortcomings, he picked as his starting point the *assumption of quantized light* (Einstein's "indivisible quanta"), which together with the formalism of statistical mechanics provided everything he needed.

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<sup>9</sup>mainly by other authors, like Dirac [51]

In order to apply statistical mechanics, Bose had to somehow distribute his light quanta within a given closed volume  $V$  in a well defined way. To this end, he, quite arbitrarily as he himself admitted [38], divided the associated phase space up into cells of size  $h^3$ , where  $h$  corresponds to Planck's constant. However, since the reason that  $h$  occurs in his equations in the first place at most comes down to an arbitrary scale or unit convention for the action, we cannot, and *should* not, interpret this manner of division as in any way physically significant. Ralston gives a very interesting account of the generality of this idea [53–55].

Through this division of the phase space of his light quanta, Bose finds the total number of phase space cells associated with a given “specie”<sup>10</sup>  $s$  of quanta as<sup>11</sup>

$$A^s = V \frac{8\pi\nu^2}{c^3} d\nu^s \quad (1.14)$$

Then, defining  $p_r^s$  as the number of cells of the  $s$ -variety that contain  $r$  quanta, he gets for the total number of all possible permutations of available light quanta<sup>12</sup>

$$\prod_s \frac{A^s!}{\prod_r p_r^{s!}} \quad (1.15)$$

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<sup>10</sup>The members of each specie essentially correspond to the photons of a given frequency.

<sup>11</sup>Neither Einstein's translation [38] to German, nor the back-translations to English [56,57] I found, include the volume  $V$  in this expression.

<sup>12</sup>Bose refers to this expression as the *probability* of the state defined by all  $p_r^s$  (as opposed to the “relative probability” - a common practice as it turns out), while we actually would have to divide this by the total number of states in order to obtain a value smaller than or equal to 1.



From the definition of  $p_r^s$  we also have that

$$N^s = \sum_r r p_r^s \quad ; \quad A^s = \sum_r p_r^s \quad (1.16)$$

The logarithm of Eq.(1.15), under the condition that its variation with respect to  $p_r^s$  vanishes, given the following constraint,

$$E = \sum_s N^s h \nu^s \quad (1.17)$$

and those of Eqs.(1.16), results in an expression for the total energy that corresponds to Planck's formula:

$$E = \sum_s \frac{8\pi h \nu^s{}^3}{c^3} V \left[ \exp \left( \frac{h \nu^s}{kT} \right) - 1 \right]^{-1} d\nu^s \quad (1.18)$$

While the physical interpretation and thus the counting differs quite dramatically, this procedure essentially coincides with that presented at the end of Section 1.1.1.

As a historically interesting side note, I have tried very hard, and failed, to get a hold of the *original* (English) version of Bose's paper. Curiously, Bose himself did not appear to have kept a copy, neither did Einstein, who translated the paper into German to have it published in the *Zeitschrift für Physik* in 1924, nor did the *Philosophical Magazine*, which had rejected to publish it in 1923 [58].

Only Einstein's translation [38] into German and various back translations [56,57] thereof remain.

## 1.2 Alternative Derivation

Planck, Einstein and Bose all made certain assumptions, in addition to physically observed facts, in order to derive their results. Particularly in Bose's case the physical implications of these assumptions seem somewhat opaque and have caused, in my opinion, much confusion.

For a more process-based, semi-classical approach, I start with the experimentally validated assertion, that atoms have sharp energy levels and can only absorb or emit energy in quantities matching the differences between such levels. Molecules and higher order composites of atoms and molecules will have increasingly more complicated level structures that will eventually turn into continua (bands). So what I will count will essentially amount to the number of occupied/excited levels corresponding to a given energy in such a system. The math does, of course, not differ much from that used by the authors discussed. In fact, my approach parallels remarkably with the procedure Planck outlined in words in the paper mentioned earlier [37]. The essential differences lie in the interpretation and visualization of the processes that correspond to the mathematics.

Let us imagine that the material body under consideration has a total of  $N_\nu$

available energy levels<sup>13</sup> corresponding to energy  $\epsilon_\nu$ . Of these,  $n_\nu$  have absorbed a corresponding amount of energy. Thus, we have for the total amount of energy contained

$$E = \sum_{\nu} n_{\nu} \epsilon_{\nu} \quad (1.19)$$

where  $\epsilon_\nu$  represents the energy of a transition corresponding to frequency  $\nu$ .

We now want to find the number of excited levels of a given energy at thermal equilibrium, i.e. in the state when the total energy (supplied by the surrounding radiation) contained in the system (our material object) has reached a constant value. The thermodynamic definition of temperature involves a derivative of the entropy ( $S$ ), so we need an expression for  $S$  of the body, and then find where  $S$  has an extremum. This corresponds to the condition  $\delta S = 0$  and the state of thermal equilibrium.

There exist  $\binom{n_\nu + N_\nu - 1}{n_\nu}$  ways to distribute  $n_\nu$  amounts of energy  $\epsilon_\nu$  among  $N_\nu$  available states. To find the *total* number  $\Omega$  of all possible distributions, we multiply all these together

$$\Omega = \prod_{\nu} \binom{n_{\nu} + N_{\nu} - 1}{n_{\nu}}, \quad (1.20)$$

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<sup>13</sup>In large enough bodies we have bands rather than individual levels, and the  $N_\nu$  correspond the number of possible transitions between these levels given the frequency  $\nu$ .

which results in an entropy<sup>14</sup> of

$$\begin{aligned} \frac{S}{k_B} &= \ln \Omega \\ &\approx \sum_{\nu} (n_{\nu} + N_{\nu}) \ln (n_{\nu} + N_{\nu}) - n_{\nu} \ln n_{\nu} - N_{\nu} \ln N_{\nu} \end{aligned} \quad (1.21)$$

Setting the variation of this with respect to  $n_{\nu}$ <sup>15</sup>, under the constraint Eq.(1.19), equal to zero in order to find the equilibrium condition results in

$$\ln (n_{\nu} + N_{\nu}) - \ln n_{\nu} - \beta \epsilon_{\nu} = 0 \quad (1.22)$$

and we end up with

$$n_{\nu} = \frac{N_{\nu}}{e^{\beta \epsilon_{\nu}} - 1} \quad (1.23)$$

which matches Eq.(1.9). Note, however, that this represents or describes the energy level distribution *within* the material object under consideration, not the radiation within a cavity, and thus demonstrates more clearly that this result may apply to any material object in thermodynamic equilibrium (if it satisfies the physical assumptions made thus far), while leaving some important details

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<sup>14</sup>after using Stirling's approximation for large  $n_{\nu}$  and  $N_{\nu}$ , and dropping the -1

<sup>15</sup>The  $N_{\nu}$  always remain constant for a given object, so we do not need to consider their variation.

unspecified.

If we wanted to actually *build* a black body, we would have to set up a situation in which the material object could reach a steady state between its own internal energy states and impinging radiant energy. A cavity with reflecting walls, as usually assumed in discussions of black-body radiation, represents *one* such modality. Our sun, provides a different example. Its emission spectrum has black body like characteristics without the help of such a cavity. Radiation originating within the sun continually interacts with material particles of its photosphere - thermalizing in the process - until it eventually escapes into free space [59,60].

Realizing that the (ensemble-) *average* energy  $U_\nu$  of an energy level associated with frequency  $\nu$  corresponds to  $U_\nu = n_\nu \epsilon_\nu / N_\nu$  gives us the corresponding version [32] of Planck's formula:

$$U_\nu = \frac{\epsilon_\nu}{e^{\beta \epsilon_\nu} - 1} \quad (1.24)$$

Determination of the parameter  $\beta$  as  $1/k_B T$  follows from the thermodynamic definition of the temperature,  $\frac{1}{T} \equiv \frac{\partial S}{\partial E} = \frac{\partial S}{\partial n_\nu} \frac{dn_\nu}{dE}$ , taken together with all the other assumptions required to arrive at Eq.(1.23). Writing Eq.(1.24) in terms of energy *density* will get us back to Planck's Eq.(1.3).

### 1.3 Discussion

From all three authors we see, that it comes down to two key ingredients that precipitate the successful derivation of Planck’s black-body formula: the use of *statistics* and some form of energy-quantization.

All discussions regarding the radiation from a black body *necessarily* made and make use of statistical mechanics, due to the large quantity of matter that actually produces the described spectrum. We know that single atoms and molecules do not produce anything close to a black-body spectrum. Rather, we see line spectra, which tell us about the quantized nature of the energy levels of *atoms and molecules*. Hence I argue here that we cannot possibly draw any conclusions about the various details of the individual elementary processes involved in the emission or absorption of radiation based on simple measurements of spectra or the photoelectric effect. The relevant equations only model the final measurable energies, not the physical interaction processes that produce them. From this we can also conclude that only macroscopically large systems have the ability to produce anything close to black-body radiation. Sufficiently small systems<sup>16</sup> will not exhibit this phenomenon.

Planck had to introduce his ad-hoc postulate of “natural radiation”, in order to connect electromagnetic radiation to statistical mechanics. This assigns a property to his black-body radiation which makes it essentially different from

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<sup>16</sup>As defined by the inapplicability of the Stirling approximation w.r.t. the number of constituent material particles and their energy levels.

electromagnetic radiation as described by Maxwell's equations, mainly adding phase-randomness to the radiation itself as an intrinsic property. While Planck does not discuss the physical processes responsible for this randomness, we now know that it corresponds to random *spontaneous emission*. This removes an ad-hoc hypothesis imposed on the radiation itself, and identifies a well known physical process, involving matter, as the cause instead. We note, however, that the details of this process still remain an open question, and QM has yet to address this issue directly. It only provides transition *probabilities*, but tells us nothing about the transition *process*.

Now, as reviewed earlier, the successful derivation of Planck's black-body equation also requires the introduction of discrete energies  $E_{mn} = h\nu_{mn}$ . Planck's reason for doing this originated mainly with the need to count energy states in order to define an entropy, without debating the physical significance [32]. Einstein uses thermodynamical arguments to show that low energy density monochromatic radiation behaves as what others have called a photon gas [2]. Bose, on the other hand, takes up Einstein's photon proposal, assigns the quantization directly to the light itself and assumes their separate, particle-like existence [38]. My own derivation in section 1.2 essentially counts net occupation of quantized energy levels of a material object. Evidently, as long as we introduce quantized amounts of energy *somehow*, we will arrive at the black-body equation, regardless of what we propose as the physical reason for quantization.

However, as Keller points out [61], everything we know about light, we only know through its interaction with matter. Thus, it seems more prudent to interpret the energy quantization inherent to the problem as that of the (discrete) energy levels of the material particles making up the black-body cavity, as proposed in Section 1.2, especially since there already exists compelling evidence for the quantized nature of *matter*, rather than taking it as evidence for the quantization of *light* itself.

The problem with assuming, as Bose did, that we have a *photon gas*, i.e. assigning the quantization to the light, emerges when we want to understand the process of how the black-body spectrum comes about. Mathematically maximizing the expression for the entropy, as Bose did, gives us the right mathematical expression for the measurable energy distribution at thermal equilibrium. But what *physical* process does this mathematical step describe? Bose did not discuss this in his paper [38].

Physically the whole system starts out *not* in equilibrium. Any radiation present in the very beginning certainly does not have the characteristics of the black-body spectrum. Only after *thermalization* will we see the characteristic energy distribution over the frequencies. *The radiation has to thermalize through interaction processes* to reach a state of equilibrium. Since light<sup>17</sup> outside of a material medium and at currently attainable energy densities does not appear to

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<sup>17</sup>or *any* waves in the linear domain really. Roychoudhuri has termed this “Non-Interference of Waves”, or NIW [62].



interact with itself [29], this process must *necessarily* involve light-matter interaction.

Planck himself pointed out [40] that in the real world there do not exist any absolutely rigid surfaces, nor ones that reflect 100% of the light impinging upon them. This implies that light within a cavity, even if it has highly reflective walls and does not contain any other material particles, will always interact and exchange energy with the material particles of the walls, to some degree. This interaction suffices to eventually thermalize the radiation within the cavity, producing a black-body like spectrum. In this case, however, we expect this process to take a much longer time.

As mentioned earlier, current treatments of black-body radiation also often approach the derivation of Planck's formula by considering a photon gas [63,45]. While they usually acknowledge that the interaction between these photons happens only through continual emission and absorption at the cavity walls (i.e. through light-matter and not light-light interaction), I believe the whole concept of a photon gas misleads our physical intuition and understanding by implying the existence or necessity of indivisible, particle like light quanta. As discussed previously, I see no evidence, or need, for the light-particle concept when it comes to black-body radiation.

This exposition does not intend to diminish the value of the historic papers discussed, nor do I want to unduly criticize the thought processes and scientific

contributions of their authors. On the contrary, considering the state of knowledge at the time - physicists knew little about the detailed structure of atoms; Thomson had barely discovered the electron [64] when Planck wrote his papers; Rutherford had just recently discovered the proton [64] when Bose wrote his - I find it incredible how much these authors have contributed to the development of Quantum Mechanics and physics as a whole. However, since our horizons have broadened considerably over the course of the following 100 years, we need to re-evaluate and re-interpret everything we have learned in the past in light of our *current* state of knowledge. Furthermore, we should apply this same procedure continually as we move forward, in order to evolve our knowledge about the universe in the most efficient way [29].

## Chapter 2

### Quantum Mechanics

*I think I can safely say that nobody  
understands quantum mechanics.*

---

Richard Feynman

In this chapter I review and critically examine the mathematical formalism and the predominant interpretation of modern QM. I review various “derivations” of the Schrödinger equation and present some interesting perspectives and mathematical facts that, while extremely relevant, the standard literature does not appear to consider very often. I also state and discuss the core ideas of the Copenhagen Interpretation and its implications for physical processes at the micro scale. It turns out that many of the problems associated with understanding QM, as expressed by Feynman in his famous quote [65], originate from the *interpretation* of the formalism, rather than from the formalism itself.

## 2.1 The Schrödinger Equation

Since its conception in 1926 [66–69], the Schrödinger equation<sup>1</sup>

$$i\dot{\psi} = \hat{H}\psi \tag{2.1}$$

has embodied the cornerstone of modern QM.  $\psi \equiv \psi(\vec{x}, t)$  here represents the primary dynamical quantity - the wave function. Eq.(2.1) tells us that the unspecified operator  $\hat{H}$  determines the time evolution of  $\psi$ .

Heisenberg had developed his matrix mechanics [70] almost exactly 6 months before Schrödinger developed his wave mechanics [66]. However, the most popular formulation of QM and the one that we teach to beginning physics students today almost exclusively corresponds to that of Schrödinger [71]. I suspect that this reflects, in a subtle way, the human need for a process-oriented description of nature. Heisenberg’s formulation represents experimental observations very mathematically abstract via time-evolving operators, a notion for which we possess very little intuition. Schrödinger, on the other hand, models physical systems as time-evolving wave functions - something we can immediately picture by thinking about, for example, water waves.

Schrödinger later proved his wave mechanics mathematically identical to

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<sup>1</sup>Often  $\hbar$  appears here - we leave it out at this point. I can give various reasons for doing so:  $\hbar$  represents an artifact of the MKS system [55]; physicists like to use units where  $\hbar \equiv 1$ , which de-clutters expressions; we could also absorb this constant into our definition of the operator  $\hat{H}$  etc.

Heisenberg’s formulation [72]. In the same paper he suggested, that via his undulatory version of QM one can interpret matrix elements and related quantities as amplitude oscillations of the atomic dipole moment [72] - a now largely abandoned notion to which I shall return later.

Various other mathematically equivalent formulations of QM, besides those due to Schrödinger and Heisenberg, exist [71], but we see none of them as widely used as wave mechanics.

### 2.1.1 Various Derivations

While one cannot *derive* QM as a whole<sup>2</sup>, various more or less rigorous procedures exist for ending up with something resembling Eq.(2.1). Including Schrödinger’s original argument, I will present and critique a selection of such “derivations”.

### Operator Substitution

We sometimes see the Schrödinger equation motivated by analogy with classical mechanics. This usually comes in variations of the following procedure. For closed<sup>3</sup> Hamiltonian systems we have the relation

$$H(q, p) = E \tag{2.2}$$

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<sup>2</sup>It consists of more than just the differential equation that models system time evolution, including various axiomatic postulates that one can, by definition, not derive from anything more fundamental.

<sup>3</sup>I.e. systems whose (especially potential) energy does not depend explicitly on time.

which says that the numerical value of the Hamiltonian remains constant. The Hamiltonian resulting in the familiar Newtonian description for a single point particle in a potential corresponds to  $H = p^2/2m + V(q)$ . Multiplying Eq.(2.2) by  $\psi(q, t)$  on both sides and postulating the “quantum replacement rules”  $E \rightarrow i\hbar \frac{\partial}{\partial t}$  and  $p \rightarrow -i\hbar \nabla$  immediately results in

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + V(q)\right) \psi(q, t) = i\hbar \frac{\partial}{\partial t} \psi(q, t) \quad (2.3)$$

The replacement postulates essentially remove any possibility of intuition, so this “derivation” provides little more than a mnemonic device.

A related and not much more illuminating procedure starts with a plane wave solution of the form  $\psi = Ae^{i(\vec{k} \cdot \vec{q} - \omega t)}$  along with the de Broglie relations  $p = \hbar k$  and  $E = \hbar \omega$ , as well as Eq.(2.2). Taking various derivatives of  $\psi$  and using the other relations reconstructs the differential equation, Eq.(2.3).

### Schrödinger’s derivation

In contrast to the above method, Schrödinger’s own approach appears much more process oriented. Geometrical optics<sup>4</sup> represents the high frequency or short wavelength limit of wave optics<sup>5</sup> [73]. In analogy to this, Schrödinger tried to find the wave-analog to classical mechanics. In essence, this corresponds to going back-

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<sup>4</sup>a.k.a. ray optics

<sup>5</sup>a.k.a. physical optics

wards from ray optics to wave optics - i.e. from an approximate description to a more general one - an arguably much more complicated process. He started with the Hamilton-Jacobi equation [67]

$$H + \frac{\partial S}{\partial t} = 0 \quad (2.4)$$

where  $H$  corresponds to the Hamiltonian as a function of the generalized coordinates  $q_k$  and the corresponding momenta  $p_k \equiv \frac{\partial S}{\partial q_k}$ , and  $S$  represents the action. In addition, he wanted to concentrate on a single particle in a time-independent potential. This essentially nailed down the form of his Hamiltonian:

$$H = \frac{p_k^2}{2m} + V(q_k) \quad (2.5)$$

Another assumption he had to make consists of the assertion of additive separability of the action<sup>6</sup>, so that

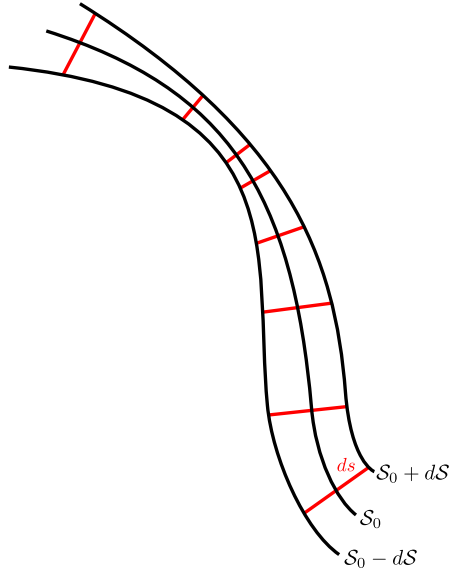
$$S(q_k, t) = W(q_k) - Et \quad (2.6)$$

With this, Eq.(2.4) becomes

$$\left( \frac{\partial S}{\partial q_k} \right)^2 = 2m(E - V) \quad (2.7)$$

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<sup>6</sup>This really does not represent an independent assumption - it merely states that the total energy of the system indeed remains constant over time.



**Fig. 2.1:** Surfaces of constant action.

which looks very much like the eikonal equation of geometrical optics [73]. Defining a line-element  $ds^2 = \frac{2T(q_k, \dot{q}_k)}{m} dt^2$ ,  $T(q_k, \dot{q}_k)$  standing for the kinetic energy as a function of generalized positions and velocities, we get a geometric interpretation of<sup>7</sup>  $ds = \frac{dS}{\sqrt{(2m(E-V))}}$  as the “distance” between surfaces of constant action, for example  $S_0$  and  $S_0 + dS$ , as shown in Fig.2.1.

Schrödinger now had the idea that these surfaces of constant action might correspond to surfaces of constant *phase*  $\phi(x, t) \equiv \frac{2\pi S}{h}$  of some wave  $\psi$ . Well-known relations such as the frequency  $\nu \equiv \frac{\dot{\phi}}{2\pi} = \frac{E}{h}$  follow automatically. If  $S$  evolves into  $S + dS$  in a time  $dt$  we find that  $dS = E dt$  and the phase velocity

$$u = \frac{ds}{dt} = \frac{E}{2m\sqrt{E-V}} \quad (2.8)$$

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<sup>7</sup>  $\left(\frac{\partial S}{\partial q_k}\right)^2 = (\text{grad } S)^2 \equiv \left(\frac{\partial S}{\partial s}\right)^2$



He followed this with the *guess* that  $\psi$  evolves according to the wave equation<sup>8</sup> with characteristic velocity  $c \equiv u$ , so that

$$\nabla^2 \psi + \frac{2m(E - V)}{E^2} \ddot{\psi} = 0 \quad (2.9)$$

Assuming further that  $\psi$  only depends on time through  $e^{2\pi i \nu t}$  results in the customary time-independent Schrödinger equation

$$\left( -\frac{\hbar^2}{2m} \nabla^2 + V \right) \psi = E\psi \quad (2.10)$$

and re-instating one time derivative on the right gets us Eq.(2.1).

Evidently some amount of trickery and educated guesswork went even into the original derivation. In fact, Schrödinger's strategy appears to change from his first [66] to his second [67] communication. Just as in Planck's derivation of the black-body spectrum, some non-trivial amount of reverse engineering may have taken place in that month's time to come up with a better procedure, after seeing that the end result<sup>9</sup> seemed to work. Indeed, it took all together about 6 months until Eq.(2.1) finally appeared.

The Schrödinger equation of form Eq.(2.1) couples the real and imaginary parts of  $\psi$  due to the explicit appearance of  $i$ . For most common forms<sup>10</sup> of  $\hat{H}$ ,

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<sup>8</sup> $\nabla^2 \psi - \frac{1}{c^2} \ddot{\psi} = 0$

<sup>9</sup>finding a new explanation for energy quantization within the Hydrogen atom

<sup>10</sup>mainly ones with time-independent potentials

including Schrödinger's, we can convert this differential equation for a *complex* function  $\psi(x, t)$  into one for a *real* function  $\eta(x, t)$ . In the process, the order of time derivatives increases to two, and that of spatial derivatives to at least<sup>11</sup> 4:

$$i\dot{\psi} = \hat{H}\psi \quad \longrightarrow \quad \ddot{\eta} = -\hat{H}^2\eta \quad (2.11)$$

In the last of his four communications [69] Schrödinger starts out with this fourth order equation and goes the other way, by effectively taking an operator-square root, not unlike Dirac did.

### Classical Mechanics in Schrödinger form

Ralston has demonstrated that one can cast Hamilton's equations (Eqs.(2.12) and (2.13)) into a mathematical form equivalent to Schrödinger's equation [74,54,75]. This may give us another clue that a quantum-classical boundary does not really exist. I will replicate the essential arguments here.

Start with Hamilton's equations:

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \quad (2.12)$$

$$\dot{p}_i = -\frac{\partial H}{\partial q_i} \quad (2.13)$$

---

<sup>11</sup>depending on the actual form of  $\hat{H}$

which we can write more compactly as

$$\dot{\phi} = \mathbf{J} \frac{\partial H}{\partial \phi} \quad (2.14)$$

using the  $2N \times 2N$  skew-symmetric matrix<sup>12</sup>

$$\mathbf{J} = \begin{pmatrix} 0 & I_N \\ -I_N & 0 \end{pmatrix} \quad (2.15)$$

where  $\phi$  represents the list  $(q_1, q_2, \dots, q_N, p_1, p_2, \dots, p_N)$ . We can now make a coordinate transformation  $\mathcal{U} : \phi \rightarrow \psi$  that diagonalizes  $\mathbf{J}$ , where

$$\mathcal{U} = \frac{1}{\sqrt{2}} \begin{pmatrix} I_N & iI_N \\ I_N & -iI_N \end{pmatrix} \quad (2.16)$$

$$\psi = \mathcal{U}\phi \quad (2.17)$$

With this  $\mathcal{U}\mathbf{J}\mathcal{U}^\dagger = -i \begin{pmatrix} I_N & 0 \\ 0 & -I_N \end{pmatrix}$  and  $\psi = (\psi_1, \psi_2, \dots, \psi_N, \psi_1^*, \psi_2^*, \dots, \psi_N^*)$  Eq.(2.14) becomes

$$i\dot{\psi} = \frac{\partial H}{\partial \psi^*} \quad (2.18)$$

where  $H = H(\psi, \psi^*)$  here. Choosing  $H$  to have the usual bilinear form  $H = \psi^* \hat{H} \psi$  results in Schrödinger's  $i\dot{\psi} = \hat{H}\psi$ . In this sense, the Schrödinger equation

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<sup>12</sup> $I_N$  represents the  $N \times N$  unit matrix.

corresponds to the complex representation of Hamilton's equations<sup>13</sup>.

It almost looks like we have doubled the number of degrees of freedom in the process.  $\psi$  still has  $2N$  entries, but each of them consists of a complex number - all in all we now have  $4N$  real numbers when before we only had  $2N$ . However, the second half of the list  $\psi$  contains only to another copy<sup>14</sup> of the first half, so we do still have the same number of DOF after all.

### Low Frequency effective Field Theory

Some treatments [76,54] also consider the Schrödinger equation as the low-frequency limit of the Klein-Gordon equation<sup>15,16</sup>, or a wave equation with a more general coupling [55]. The simplest version of this that will result in (2.3) corresponds essentially to a Klein-Gordon equation with a space-variable frequency parameter:

$$\frac{\partial^2 \phi}{\partial t^2} - c^2 \nabla^2 \phi + \mathcal{W}(x) \phi = 0 \quad (2.19)$$

We can write  $\mathcal{W}(x)$  as the sum of its constant part<sup>17</sup> and its variable part,  $\mathcal{W}(x) \equiv \omega^2 + W(x)$ . Now write  $\phi = e^{-i\omega t} \psi$  and plug this into Eq.(2.19) to get an equation

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<sup>13</sup>The operator  $\hat{H}$  here corresponds to the “usual” Hamiltonian operator divided by  $\hbar$ .

<sup>14</sup>complex conjugated

<sup>15</sup>a.k.a. the massive wave equation

<sup>16</sup>For the K-G equation, the parameter  $\mathcal{W}$  has a constant value and units of frequency squared - we often see it written as  $m^2 c^4 / \hbar^2$ .

<sup>17</sup>similar to the DC offset of an AC signal

for  $\psi$  only:

$$\frac{\partial^2 \psi}{\partial t^2} - 2i\omega \frac{\partial \psi}{\partial t} - c^2 \nabla^2 \psi + W(x)\psi = 0 \quad (2.20)$$

Now, assuming slowly varying  $\psi$ <sup>18</sup> we get

$$i \frac{\partial \psi}{\partial t} = \left( -\frac{c^2}{2\omega} \nabla^2 + \frac{W(x)}{2\omega} \right) \psi \quad (2.21)$$

Multiplying both sides by  $\hbar$  and making appropriate identifications between the parameters of equations (2.3) and (2.21) gives us the Schrödinger equation with potential  $V(x) \equiv \frac{\hbar W(x)}{2\omega}$ .

### 2.1.2 Solution

We can formally<sup>19</sup> solve Eq.(2.1) by writing  $\psi(q, t) = e^{-i\hat{H}t}\psi(x, 0)$ , i.e. unitary operator  $U(0, t) = e^{-i\hat{H}t}$  will evolve the given initial conditions  $\psi(x, 0)$  from time  $t = 0$  to  $t$ . In practice this does not actually help with finding a solution. A more useful solution strategy<sup>20</sup> involves the following steps<sup>21</sup>:

i) As with all linear theories, normal modes of the system will evolve very

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<sup>18</sup>i.e. assuming that  $\frac{\partial^2 \psi}{\partial t^2} \ll 2i\omega \frac{\partial \psi}{\partial t}$

<sup>19</sup>We assume here and in general that  $\hat{H}$  does not depend on time (i.e. we only consider closed systems and assume that energy conservation holds).

<sup>20</sup>At least for simple (single particle) cases with time-independent Hamiltonians.

<sup>21</sup>Dirac notation will help keep clutter to a minimum.

simply in time. Hence, we expand

$$|\psi(t)\rangle = \sum_n c_n e^{-iE_n t} |n\rangle \quad (2.22)$$

which transforms the time-dependent Schrödinger equation (TDSE) into the time-independent Schrödinger equation (TISE):

$$i|\dot{\psi}(t)\rangle = \hat{H} |\psi(t)\rangle \quad \longrightarrow \quad E_n |n\rangle = \hat{H} |n\rangle \quad (2.23)$$

and allows us to solve an eigenvalue problem to find the normal modes  $|n\rangle$ .

ii) Having found the normal modes we can determine the expansion coefficients  $c_n$  by projecting the initial conditions onto them:

$$c_n = \langle n|\psi(0)\rangle \quad (2.24)$$

iii) Putting these back into our expansion Eq.(2.22) we get

$$|\psi(t)\rangle = \sum_n \langle n|\psi(0)\rangle e^{-iE_n t} |n\rangle \quad (2.25)$$

which we may write in the position basis  $|x\rangle$ , i.e. terms of spatial coordinates<sup>22</sup>,

---

<sup>22</sup> $\psi(0, x) \equiv \langle x|\psi(0)\rangle$ , i.e. the initial conditions in the position basis, and  $\psi_n(x) \equiv \langle x|n\rangle$ , i.e. the normal modes in the position basis.

in order to actually perform calculations

$$\psi(x, t) = \sum_n \int \psi(0, x') \psi_n^*(x') \psi_n(x) e^{-iE_n t} dx' \quad (2.26)$$

Alternatively, we may represent the wave function in any *other* basis if convenient.

## 2.2 Interpretation

Quantum Mechanics has at least two parts to it: 1) a differential equation that models the time evolution of the physical system and 2) a number of postulates and statements regarding measurement and interpretation. Number 1) in the wave-function formulation corresponds to the Schrödinger equation as discussed earlier, and number 2) corresponds to various related mathematical postulates as well as its interpretation, including the statistical interpretation of the wave function and the Copenhagen Interpretation of the whole formalism.

The statistical interpretation of the wave function holds that<sup>23</sup>  $|\psi|^2$  represents a probability density [77]. In particular, in the discrete case  $|\langle a|\psi\rangle|^2$  gives the probability of measuring the eigenvalue of eigenstate  $|a\rangle$ , and in the continuous case for example  $|\langle x|\psi\rangle|^2 dx$  gives the probability of measuring the position eigenvalue between  $x$  and  $x + dx$ . Similarly,  $|\langle \phi|\psi\rangle|^2$  gives the transition probability between states  $|\psi\rangle$  and  $|\phi\rangle$  [78].

As for the Copenhagen Interpretation, Peres suggests that there may exist

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<sup>23</sup>a.k.a. the Born rule

“at least as many different Copenhagen interpretations as people who use the term” [79], and that the majority of modern physicists use the term to express views not actually held by Bohr [79], the primary originator of this interpretation. Others [80,81] have come to similar conclusions. While sources often ascribe the Copenhagen Interpretation to Bohr in particular, according to Peres, Bohr had a much more pragmatic interpretation of QM. He did not believe that the theory described physical reality; it merely provided accurate answers “to meaningful questions about experiments done with physical systems” [79]. This view appears in stark contrast to claims made in some introductory QM books. They claim, that the Copenhagen Interpretation represents the view that the particle described by QM does not have a definite position (or other properties) *pre* measurement. Rather, the measurement process itself forces the particle into whatever state we end up measuring [82].

While little consensus exists with regard to what actually constitutes “*the* Copenhagen Interpretation”, it appears that Cramer [81] has come up with a list of five core elements that most Copenhagen Interpretations have in common. We will adopt these as our working definition of the interpretation here:

- i Heisenberg’s uncertainty principle
- ii Born’s statistical interpretation
- iii Bohr’s principle of complementarity
- iv  $|\psi\rangle$  contains all we can *know* about the system
- v Heisenberg’s positivism



## 2.3 Discussion

Seeing, as in Section 2.1.1, that we can arrive at a given mathematical formulation<sup>24</sup> in a multitude of ways, and considering the abundance of *different* mathematical formulations [71] of QM overall, the mathematical formalism appears extremely general. Consequently, the formalism itself seems much less contested than its *interpretation*, and those critical of QM, not surprisingly, mainly focus their efforts on the latter. There also exists a camp that apparently wants to do away with interpretations entirely as exemplified by a recent heated debate [83–86] that started in response to a paper considering the current predominance of the CI as the result of a historical accident [87].

I do not deny the fact that we *can* use the QM formalism to calculate many predictions for outcomes of various experiments, even without any interpretation at all. However, in order to further our *understanding*, improve upon current theories and discover new things, we need to figure out how, in detail, QM relates to the physical processes occurring in nature. I find rumors that we have created *the final theory* vastly overrated, and others<sup>25</sup> have proclaimed similar things in the past, just to find themselves proven wrong almost immediately.

Let us now look at how, in *my* mind, the Copenhagen Interpretation undermines our ability to gain a better understanding, and why I see a need to shift

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<sup>24</sup>i.e. the Schrödinger equation

<sup>25</sup>Michelson, for example, provided an extensive justification for the statement that “our future discoveries must be looked for in the sixth place of decimals” in his 1903 book [88].

to a more process-oriented interpretation. To this end we shall now examine the five core propositions of the CI listed earlier.

i) *Heisenberg's uncertainty principle*: Textbooks [15] sometimes illustrate the uncertainty principle via results such as described in Appendix E.1. Other sources use commutator relations [36,89] to arrive at the same result. In all discussions, the uncertainty relation expresses information about the standard deviations of a set of observables - a statistical statement. I agree here with claims by Roychoudhuri [14] that, as a consequence, the relations do not refer to single measurements, but at most to statistical scatter within ensembles, and say nothing about individual physical processes or measurements.

ii) *Born's statistical interpretation*: This, formally embodied by the Born rule as described in the previous section, in essence does away with our ability to visualize the wave function as anything physical. From my point of view, since the theory's predictions match a large variety of experimental observations, QM must have captured a significant amount of physical reality. By extension, the constituents of the theory must have *some* relation to the physical processes occurring in nature, and we need to attempt to interpret them accordingly. In the following chapter I will reconsider Schrödinger's proposal of  $\psi$  representing a form of dipolar oscillation [72,68], as opposed to a purely probabilistic quantity. The assertion that nature behaves fundamentally probabilistic famously caused Einstein to state that "[God] is not playing at dice" [90]. Critical of QM until the

end, he probably could also not accept the CI's denial of visualizable interaction processes occurring at a more fundamental level.

iii) *Bohr's principle of complementarity*: According to Cramer [81] this has mainly two parts to it, both of them objectionable from a process-based point of view: wave-particle duality and the assertion that the uncertainty principle constitutes an intrinsic property of nature. Firstly, since both, mathematical points and mathematical waves represent non-physical idealizations, it stands to reason that neither actually exist as physical objects<sup>26</sup>. While not a logically *necessary* corollary, it then makes sense that actual physical objects may exhibit qualities that appear consistent with *both* of these idealized concepts. It follows that the “duality” concept may have little physical significance. Secondly, the uncertainty principle appears more an artifact of our definitions and mathematical representations<sup>27</sup> along with an *inherent* fuzziness of measurements<sup>28</sup> rather than a principle or inherent property of nature itself. Postulating a built-in uncertainty principle presents the danger that we may stop questioning or exploring certain observations because the uncertainty principle conveniently “explains” them away.

iv)  $|\psi\rangle$  *contains all we can know about the system*: This part of the CI appears the least problematic, but also seems almost tautological in nature.  $|\psi\rangle$  encodes our knowledge of the system because we input initial conditions and the Schrödinger equation, via the system's Hamiltonian, tells us how they evolve in

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<sup>26</sup>Mathematical points have *infinitesimal* extent; mathematical waves have infinite extent.

<sup>27</sup>See Appendix E.1

<sup>28</sup>See Appendix E.2

time. This seems reasonable. If we emphasize the “can”, the assertion lacks something in content because until we specify a Hamiltonian and the initial conditions,  $|\psi\rangle$  remains entirely undefined.

v) *Heisenberg’s positivism*: On the face of it, it makes sense to focus our attention *only* on what we can observe, and refrain from discussions of unobservable underlying processes or “reality” in general. Heisenberg later distanced himself from such a view [91]. I have to agree - otherwise we could only talk about effects, not their causes; only about results or events, not the processes that lead to them. Doing the former, leads to descriptions, while doing the latter leads to explanations. In physics we aim to *explain*<sup>29</sup> and consequently need to consider physical processes even if we lack the ability to observe them directly.

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<sup>29</sup>Views on this differ among physicists, and some would argue for the sufficiency of good mathematical models, without the need for deeper explanations.

## Chapter 3

### Superposition Effects

*[T]he mathematical superposition principle can become physical only through the mediation of some interacting detector.*

---

Chandrasekhar Roychoudhuri

This chapter first critically reviews the statement, interpretation and application of superposition principles within both classical and quantum mechanics. Then I review the usual two-beam superposition discussion, only considering the fields, without mention of the detector used. I follow this with an overview of the major detection mechanisms for the electromagnetic spectrum, and a subsequent re-examination of the two-beam superposition, this time explicitly taking into account the detector and the associated interaction mechanism. From this I derive a simple model of the detector signal, which incorporates some of the important but often under-appreciated or ignored physical processes that impact our measurement results. Finally, using this model, I present two examples of different spectroscopic techniques whose main differences we can trace back to

the properties of the detector used.

### 3.1 Superposition Principles

Classical and quantum mechanics liberally utilize a consequence of linear mathematical models commonly referred to as the superposition principle, often in the guise of Fourier analysis and related techniques.

#### 3.1.1 Classical Mechanics

Classical mechanics, mainly in the context of gravitation and electrodynamics, introduces the superposition principle as a helpful tool to calculate the influence of a collection of field sources on a test particle. For example, in electrodynamics [82] we want to know the force exerted by a number of charges on a test charge. As stated, the superposition principle allows us just to consider the individual contributions of each source charge, and add them all up in order to find the resultant force.

Generally, if they discuss it on physical grounds at all [92], authors justify the superposition principle by stating that it constitutes an experimental fact [82], which indeed it does, but provide little discussion beyond that. Jackson [93], on the other hand, devotes a whole section of his first chapter to the topic, citing the linearity of the vacuum Maxwell's equations as one reason for why it works on mathematical grounds. To discuss possible failures, he includes a brief discussion

of QED's prediction of light-by-light scattering. The latter process, on top of having a vanishingly small calculated cross section [94], remains experimentally unobserved to this day, unlike the related process of Delbrück scattering<sup>1</sup>, which occurs only in the presence of material particles.

Superposition also plays a role in the theory of mechanical oscillations, where we often approximate small vibrational motions by those of suitable coupled harmonic oscillators and decompose them into normal or eigenmodes [95]. These eigenmodes form a (finite) orthogonal, or often by definition *orthonormal*, set, and we can describe all possible motions of the system as some linear combination of these. Taking certain limits to make a continuum out of such coupled oscillators results in certain differential equations, such as the Klein-Gordon equation or the wave equation (see Appendix B). It follows from this that for these equations we can also find a set of independent solutions, linear combinations of which can describe all possible solutions.

### 3.1.2 Quantum Mechanics

The superposition principle also resides at the core of QM. Here we routinely decompose the wave function  $\Psi$  into a sum of eigenstates  $\psi_i$  of some operator  $\mathbf{A}$  such that  $\Psi = \sum_i c_i \psi_i$ , where the  $c_i$  correspond to the weights of the eigenstates [36] necessary to represent the wave function with them. The math here actually corresponds exactly to that applicable to case of classical mechanical oscillations.

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<sup>1</sup>this really corresponds to the process Jackson discusses

The linear nature of the Schrödinger equation makes it so that we can model any solution via a suitable linear combination of elements of a complete set of functions (such as eigenmodes).

Particularly in terms of physical implications, Dirac's excellent book [51] gives a comparatively comprehensive discussion of the superposition principle in QM. Written only shortly after the invention of the modern version of QM in 1926<sup>2</sup>, the experiments and interpretation regarding the superposition principle described by Dirac evidently defined and set the stage for the views held by the majority of physicists to this day.

He discusses two experimental setups in some length, describing implications for light quanta (photons). The first experiment concerns a beam of photons passing through a plane-polarizing crystal<sup>3</sup>, which presents no problems in the classical case, while it does for indivisible particles of light. Due to its indivisible nature, a photon with a polarization *not* exactly perpendicular or parallel<sup>4</sup> to the optical axis of the crystal presents us with a problem: will it get absorbed entirely or transmitted entirely, and what determines which of these happens? Dirac solves this by suggesting that each photon always exists in a superposition of all possible outcomes and then collapses into one of these upon detection.

The second experiment considers a beam of light passing through an in-

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<sup>2</sup>1<sup>st</sup> ed.: 1930

<sup>3</sup>tourmaline

<sup>4</sup>In the perpendicular case, the photon would pass through the crystal unhindered, while the crystal would absorb it entirely in the parallel case - both of these special cases cause no problems in the indivisible photon picture.

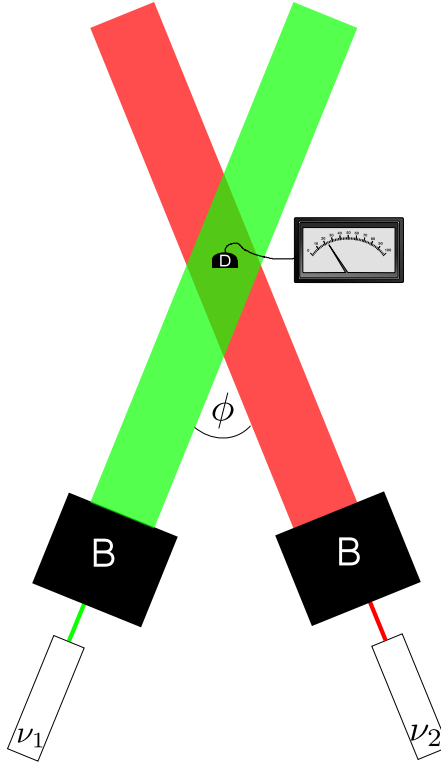


terferometer, the beam getting split in two and each one traversing a separate path of possibly different optical length, before recombining at a detector. Again, classically nothing out of the ordinary happens, but the indivisible photon picture causes us to wonder what happens to a single light particle in such a device. Again, Dirac suggests that the photon entering the interferometer now consists of a combination (superposition) of two states: the photon going one way and the photon going the other way. Only upon detection does it somehow collapse into one definite outcome. Furthermore, he ends up making his famous statement “Each photon then interferes only with itself. Interference between two photons never occurs” [51].

Dirac also suggests that the “main object of physical science is not the provision of pictures” [51] of physical phenomena and processes, a statement with which I have to disagree emphatically. His continual use of the photon picture actually undermines his own point there, and also constitutes the main reason for the difficulty of visualizing the processes involved in the suggested experiments.

### 3.2 Superposition of Fields

Let us consider two collimated, monochromatic laser beams intersecting at an angle  $\phi$ . Figure 3.1 shows the intended experimental setup. Focusing on a small enough, amplitude-flat region near the center of the cross section of the two expanded beams allows us to neglect its Gaussian profile as well as diffrac-



**Fig. 3.1:** The experimental setup: two collimated beams of frequencies  $\nu_1$  and  $\nu_2$  pass through beam expanders B and intersect at an angle  $\phi$ . A detector D placed within the superposition region will register *Interference effects* (fringes). The nature of the fringes will change when the properties of the detector change.

tion effects. In this case, we can mathematically represent the two beams by  $\vec{E}_n(\vec{x}, t) = \vec{E}_0 \cos(2\pi\nu_n t - \vec{k}_n \cdot \vec{x})$ , where  $\vec{E}_0$  represents the direction and magnitude of the electric field of the beam,  $\vec{k}_n$  the wave vector,  $\nu_n$  the frequency, and  $n \in \{1, 2\}$  [96]. I purposely choose to represent all physical quantities and processes using *real* math (as opposed the commonly used complex notation) in order to maintain a close/faithful mapping between our mathematical representation and the physical entities and processes it describes.

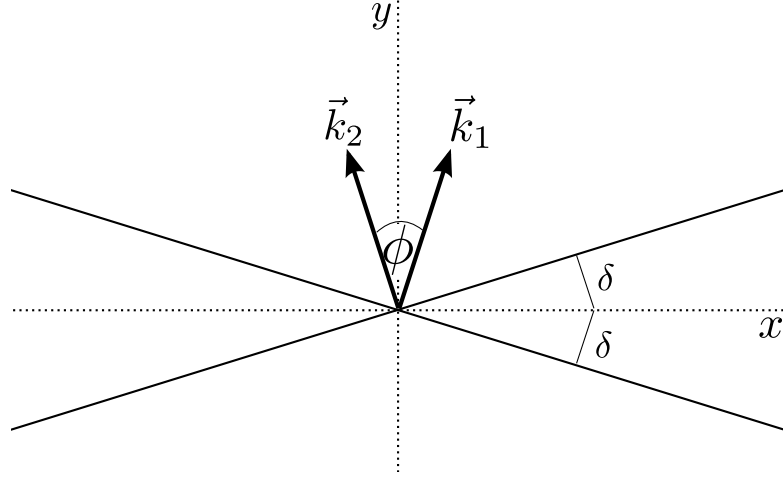
When formulating the superposition effects experienced by detectors, I use

the frequency  $\nu$  rather than the common approach of using the wavelength of the incident radiation. I do this deliberately to underscore that frequency represents the primary, unchanging parameter of an EM wave that remains unaltered as it propagates through different linear media. The wavelength on the other hand represents a “secondary” changeable parameter, derived from the velocity  $c/n$  of the wave in a medium, i.e.  $\lambda = c/n\nu$ . Here  $n$  represents the refractive index of the medium. Planck underscored this point [32] as an important necessity behind deriving his blackbody radiation law. Focusing on the primary physical parameters facilitates construction of potentially causal hypotheses to visualize invisible interaction processes. For example, the oscillatory phase property,  $\exp(i2\pi\nu t)$ , built into Schrödinger’s  $\psi$ , represents the harmonic dipolar oscillation of the quantum mechanical optical detector due to the physical stimulation induced by the incident EM wave(s).

We can simplify the math by focusing our attention on a plane within the central region of the superposition volume (indicated by the detector in Fig.3.1). Furthermore, let us assume, that both beams have the same direction of linear polarization, say vertical. Now,

$$\begin{aligned} E_1(x, t) &= E_0 \cos(2\pi\nu_1 t - k_1\delta) \\ E_2(x, t) &= E_0 \cos(2\pi\nu_2 t + k_2\delta) \end{aligned} \tag{3.1}$$

where  $\delta \equiv x \sin \frac{\phi}{2}$  (see Fig.3.2).  $2\delta$  represents the total relative path difference



**Fig. 3.2:** Wavefronts of the two beams with wave vectors  $\vec{k}_1$  and  $\vec{k}_2$  pass through each other at an angle  $\phi$ .

between the two beams.

According to the mathematical *superposition principle* (SP), we can add the contributions from the two separate beams to get the net field amplitude within the superposition region:

$$\begin{aligned}
 E_{\text{net}} &= E_1 + E_2 \\
 &= E_0 [\cos(2\pi\nu_1 t - k_1\delta) + \cos(2\pi\nu_2 t + k_2\delta)] \\
 &= E_0 [\cos(2\pi\nu_1 t - \pi\nu_1\tau) + \cos(2\pi\nu_2 t + \pi\nu_2\tau)] \quad (3.2)
 \end{aligned}$$

I have defined the total relative *phase delay*  $\tau = 2\delta/c$  here, and used  $k_n = 2\pi\nu_n/c$ , to simplify the notation.

For the superposition of two beams generated by the same single-frequency

source, i.e.  $\nu_1 = \nu_2 \equiv \nu_0$ , we can also write:

$$E_{\text{net}} = 2E_0 \cos(\pi\nu_0\tau) \cos(2\pi\nu_0t) \quad (3.3)$$

Unlike for radio waves, for the very broad band of EM waves from infrared through x-rays, all experiments validate that we can only measure intensities, rather than field amplitudes directly. Thus, we commonly square the above results to get the instantaneous intensity from Eq.(3.2):

$$\begin{aligned} i_{\text{net}} &\equiv E_0^2 \left( \cos(2\pi\nu_1t - \pi\nu_1\tau) + \cos(2\pi\nu_2t + \pi\nu_2\tau) \right)^2 \\ &= E_0^2 + \frac{E_0^2}{2} \cos\left[4\pi\nu_1\left(t - \frac{\tau}{2}\right)\right] + \frac{E_0^2}{2} \cos\left[4\pi\nu_2\left(t + \frac{\tau}{2}\right)\right] \\ &\quad + 2E_0^2 \cos\left[2\pi\nu_1\left(t - \frac{\tau}{2}\right)\right] \cos\left[2\pi\nu_2\left(t + \frac{\tau}{2}\right)\right] \end{aligned} \quad (3.4)$$

or, for the same frequency, the square of equation Eq.(3.3):

$$\begin{aligned} i_{\text{net}} &= 4E_0^2 \cos^2(2\nu_0\tau) \cos^2(2\pi\nu_0t) \\ &= 2E_0^2 \cos^2(\pi\nu_0\tau) + 2E_0^2 \cos^2(\pi\nu_0\tau) \cos(4\pi\nu_0t) \end{aligned} \quad (3.5)$$

Unfortunately, we cannot *directly* observe any of the just mentioned quantities in the lab. An observable *becomes* measurable to us only as some physical transformation experienced by a detector. Some allowed force facilitates this interaction and effects energy absorption from the superposed stimulating entities.

The spatial range of the force of interaction between the interactants dictates the physical range of “locality” [97]. In the optical domain, the released photo electrons, removed from quantum energy levels (or bands), in detectors cannot register the fringes oscillating with the frequency  $\nu_0$  or  $2\nu_0$ . We will see later that for the case of single frequency  $\nu_0$  a quantum detector can only register the time-averaged fringes given by the first term of Eq.(3.5). For the two-frequency case (Eq.(3.4)), the situation becomes more complex, as discussed in Section 3.4. Depending upon the time-constant of the external electrical circuit of the detector, one can observe a steady current (no fringes, hence no “interference”) for a very slow detector, or time-varying heterodyne beat fringes for a fast detector.

The temporal properties of the *detector* determine which superposition term or terms can become registered data. The fields alone do not determine the superposition effect we register. Hence, measurement results, and with that our theory dictated observables, depend crucially on the interaction processes between the light and the detecting dipole as I will show. This makes these processes a critical issue of which we need to stay aware.

### 3.3 Detectors

*I see no nouns, I only see verbs.*

---

Robert Anton Wilson

It turns out that we never actually see or measure *things*; we only see or

measure the results of interaction *processes*. This chapter will give an overview of the types of detectors and detecting mechanisms used when measuring within the electromagnetic spectrum. I will break the discussion up into three major domains: radio, optical and gamma. At this point in time, the detection mechanisms and detector designs we use for each of these types of EM radiation differ distinctly from one another. I will look in detail at each category of radiation and associated detection mechanism, highlighting commonalities and differences.

### 3.3.1 How we see

We experience and learn about our universe by using various detectors - some of them “built in” (our eyes, sense of smell, hearing, touch etc.) and some external (CCDs, microphones etc.). First and foremost, one needs to remember that such detectors represent the *only* way we interact with and gather information about the physical world. Furthermore, we often seem to forget the fact that all detectors have inherent limitations with regard to the information they can convey to us<sup>5</sup>. Different detectors have different limitations. Just as in the old Indian story about the blind men trying to visualize an elephant by using their sense of touch, we have to realize that one and the same thing can appear differently depending on how we “look” at it.

For a long time now have we known that radio waves, optical light and  $x/\gamma$  rays all represent a slightly different manifestation of the same physical phe-

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<sup>5</sup>For more on this see Appendix E.2.

nomenon: electromagnetic (EM) waves. All of the mentioned waves have in common such properties as their characteristic speed of propagation ( $c$ ), diffraction (observation of which we still lack for hard x-rays and  $\gamma$  rays), and we can do spectroscopy due to their oscillatory nature of fixed frequency.

In free space, light travels and behaves to the best of our current knowledge exactly as predicted by the sourceless Maxwell wave equation in vacuum

$$\nabla \times \nabla \times \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} = 0 \quad (3.6)$$

The linearity of this equation encodes the observational fact that different perturbations (waves) can pass through the same spatial location at the same time without influencing one another. In order for one wave to have an effect on another, while occupying the same spatial volume, they both need to interact with some non-linear entity (a material medium) that can facilitate this interaction.

### 3.3.2 Detection of EM-Waves

We call the fact that light does not interact with light<sup>6</sup> the “Non-Interference of Waves principle” (NIW) [62], as it applies to any kind of linear wave. Only material particles can “see” light, and then only through their unique band-limited goggles. The information they can gather about light always lacks completeness<sup>7</sup>.

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<sup>6</sup>When referring to ‘light’ here and in what follows I will usually also mean all (EM) waves in general.

<sup>7</sup>See Appendix E.2.



In addition, humans have not succeeded in inventing any instrument that has 100% fidelity in transferring the information gathered by a detector to the final data recorders. Roychoudhuri calls this the “Perpetual Information Retrieval Problem” [29] nature has imposed on us. This does not correspond to a simple Measurement Problem that we can solve by brilliant mathematical theorems.

Furthermore, we seemingly cannot use the same type of detector, or even the same theoretical detection principle, in order to gather information about the entire spectrum of EM radiation. Different parts of the spectrum appear to require different physical detection processes [96]. In the following sections I will examine the major detection mechanisms for the various spectral regions<sup>8</sup>: Radio Frequency ( $< \sim 10\text{GHz}$ ), Optical ( $\sim 10\text{GHz} - \sim 10^4\text{THz}$ ),  $\gamma/\text{X}$  Rays ( $> \sim 10^4\text{THz}$ ).

## RF Waves

The detection of RF Waves physically corresponds to an energy transformation, typically occurring in LCR “tank”<sup>9</sup> circuits, which we can analyze classically. The LCR circuit may receive multiple radio signals via an antenna, a conductor in which semi-free electrons respond to the electric field of the impinging EM waves.

Upon resonating, this produces an undulating (AC) signal made up of the conduc-

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<sup>8</sup>I consolidated regions like IR, UV, Microwave, etc. into these groups for the present purposes.

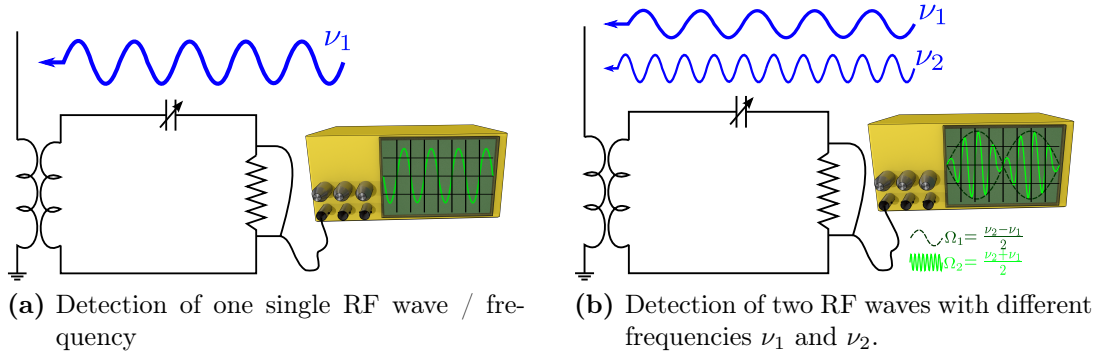
<sup>9</sup>So called because electron “slosh” around in such a circuit agitated by potential differences from an antenna, much like water in a tank agitated by mechanical forces.

tion electrons in the circuit, which oscillates at the same frequency as the incident radio wave, as seen in Fig.3.3.a. If the incident radio wave consists of several distinct frequencies, and the circuit has the necessary broad response, then the AC signal consists of a linear superposition of alternating currents at the incident frequencies (Fig.3.3.b). This makes the signal amenable to analysis using Fourier mathematics. Most importantly, in RF detection the LCR circuit generates “classical” currents which maintain the frequency information of the detected wave(s).

Specifically, a RF detector consists of an antenna connected to a tuned LCR circuit. We can set the circuit’s resonance, via appropriate design decisions, such as to select a potentially very narrow frequency range out of the broad band of radiation received by/at the antenna. The conduction electrons within the circuit respond to an induced EMF at the radio wave frequency, absorbing energy from the EMF to oscillate within the circuit. This corresponds to the electrical current we then measure. This measured AC signal preserves many properties of the radio wave, with the exception of a relative phase shift w.r.t. the original wave.

Mathematically, we can represent the EM-wave itself by  $E = a \cos(2\pi\nu_0 t)$  and the induced AC signal as  $I = \eta a \cos(2\pi\nu_0 t + \phi)$ , where  $\eta$  represents the energy transfer coefficient, and  $\phi$  the phase shift. Again, note that we do not lose the frequency information during the detection process. Also note that, in general,  $\eta$  depends on the frequency, i.e. that  $\eta = \eta(\nu)$ , and the actual design of the circuit

determines its value(s).



**Fig. 3.3:** RF Detection using LRC circuit.

Now, let us consider the behavior of such a circuit with two waves with different frequencies  $\nu_1$  and  $\nu_2$  superposed, i.e. present at the same time in a volume occupied by the detecting antenna. Furthermore we assume that the circuit permits a frequency band wide enough to accommodate both of these frequencies, that the amplitudes coincide and that  $\eta$  does not vary with frequency within the resonance band:

$$E_1 = a \cos(2\pi\nu_1 t)$$

$$E_2 = a \cos(2\pi\nu_2 t)$$

In this special case, the LCR circuit actually performs Fourier synthesis of the two signals, and what we register corresponds to

$$i = \eta E_1 + \eta E_2 = 2\eta a \cos(2\pi\Omega_1 t) \cos(2\pi\Omega_2 t) \quad (3.7)$$

i.e. we see a signal at the *mean* frequency  $\Omega_1 = \frac{\nu_1 + \nu_2}{2}$  with an envelope frequency of  $\Omega_2 = \frac{\nu_1 - \nu_2}{2}$  (see Fig.3.3.b).

## Optical Waves

Unlike in the case of RF waves, detectors for optical (and above) frequencies can only measure energy - a quantity proportional to the square of the electric field. This means that we have a less direct measurement (since we no longer measure the field directly), and we also retrieve less information about the primary quantity (the E-field) as we lose crucial frequency information in the square-law transformation.

One can divide optical detectors into two categories: photon detectors and thermal detectors [98]. I will only consider photon detectors here, since the output of thermal detectors shows very little if any frequency dependence. Furthermore, I could divide photon detectors into a number of different categories and look at each separately. All of them share their most important features, though, and for simplicity's sake I shall instead consider a model that incorporates these features and which I assume to represent the fundamental physical light-matter interaction process.

RF detectors directly measure the dipole-undulation induced by the impinging EM-wave. This represents a continuous process of energy absorption. Absorption and thus detection of *visible light* takes place in quantum mechanical

detectors that exhibit discrete (quantized) energy levels or bands. As an example we can think of a simple atom, but the arguments will apply to any device with quantized energy levels. In these devices, electrons can only occupy certain energy levels determined by the specific physical characteristics of the particular QM entity. In order for these electrons to change to a different energy state, they must either absorb or emit energy, depending on whether the change corresponds to increasing or lowering its energy. If the energy change comes about due to electromagnetic interaction, the emitted or absorbed quantity of energy follows the rule

$$\Delta E = h\nu \tag{3.8}$$

When considering wave packet emission, Eq.(3.8) tells us that the emitted packet will have frequency  $\nu$ . On the other hand, in order to absorb EM energy, the frequency of the impinging wave has to exactly match one of the possible transition frequencies and its associated energy as determined by Eq.(3.8). The transition frequencies in turn depend on the particular quantum system. Detectors made from quantum systems such as atoms or molecules in a sufficiently dilute state, such as a gas, tend to exhibit sharp resonances (discrete energy levels), whereas solid state detectors tend to have energy bands, allowing for broad band

detection [99].

We can map and summarize the behavior of these two types by using the linear (i.e. first order) susceptibility<sup>10</sup>  $\chi(\nu)$  of the atoms. Note that this quantity depends on the (physical) frequency of the EM wave(s). For  $N$  different EM waves impinging on a detector, we can represent the absorbed energy ( $\mathcal{D}$ ) mathematically as<sup>11</sup>

$$\mathcal{D} = \int_0^T |d|^2 dt \quad (3.9)$$

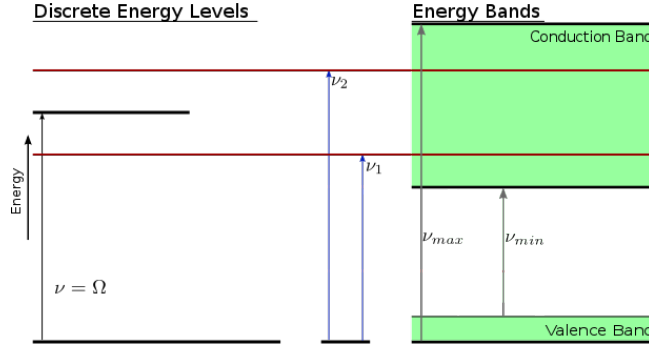
where  $d \equiv \sum_{n=1}^N \chi(\nu_n) E_n$  represents the dipole-stimulation due to all of the EM waves. The restrictions of what does and does not get detected, based on the energy levels or bands actually present, lie encoded within  $\chi(\nu)$ , which goes to zero if  $\nu$  does not correspond to an allowed value. More concretely we can write this function as  $\chi(\nu) \equiv \sum_i \kappa(\nu) \delta_{\nu, \nu_i}$  for the case of discrete levels, and  $\chi(\nu) \equiv \sum_i \kappa(\nu) [\Theta(\nu - \nu_{min,i}) - \Theta(\nu - \nu_{max,i})]$  for energy bands (see Fig.3.4)<sup>12</sup>. Also note the time integration in Eq.(3.9). Each detector has a certain time constant, or “reaction time”, during which it determines its “compatibility” with the frequency of a given wave packet and then absorbs energy accordingly. While this integration

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<sup>10</sup>or polarizability

<sup>11</sup>We will ignore the direction of polarization of the EM waves, as well as that of the material, entirely in order to keep the argument concise. Physically this means essentially that we have aligned everything such as to make the relative angle between EM polarization and polarization of the material vanish.

<sup>12</sup> $\delta_{m,n}$  represents the (slightly generalized) Kronecker delta,  $\Theta(x)$  the Heaviside step function, while  $\kappa(\nu)$  determines the strength of the response. The subscript  $i$  here stands for one possible level or band transition. The sums run over all possible (read: allowed) transitions.



**Fig. 3.4:** A detector with energy bands registers frequencies  $\nu_1$  and  $\nu_2$ , if they fall within the range  $\Delta\nu = \nu_{max} - \nu_{min}$ . Due to the NIW principle, a detector with discrete energy levels 'sees' nothing (as demonstrated experimentally [101]), even though the average frequency  $\frac{\nu_1 + \nu_2}{2}$  corresponds exactly to the resonance frequency  $\Omega$  of the system.

time may vary from detector to detector, when using complex notation and the square-modulus we implicitly *assume* it equals exactly 2 periods of oscillation, as I have shown elsewhere<sup>13</sup> [100].

The two types of detectors considered so far demonstrate how much the information we can get about a physical entity (in this case EM waves) can differ, depending on the detector we use. Let us, for example, consider two superposed plane waves with different frequencies  $\nu_1$  and  $\nu_2$ :

$$\begin{aligned} E_1 &= a \cos 2\pi\nu_1 t \\ E_2 &= a \cos 2\pi\nu_2 t \end{aligned} \tag{3.10}$$

I assume equal amplitudes and polarization directions for simplicity. Now consider a detector with a single energy level (beyond the ground state). If the resonant

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<sup>13</sup>Also see Appendix C.

transition according to Eq.(3.8) corresponds to one of the two frequencies given in Eq.(3.10) the detector's output will report a value proportional to  $\mathcal{D} = \frac{1}{2}\chi^2(\nu_1)a^2$  or  $\mathcal{D} = \frac{1}{2}\chi^2(\nu_2)a^2$  respectively.

What happens if our detector has its resonance at  $\nu = \Omega = \frac{\nu_1 + \nu_2}{2}$  instead? If we blindly assume that Fourier synthesis represents a principle inherent in the EM field, as opposed to a phenomenon whose validity depends on the properties of particular detectors, we would expect to see a response [99]. However, the detector will “see” nothing and thus report nothing back to us, because none of the original frequencies match its resonance frequency, and EM waves do not interact and thus cannot create the required frequency on their own [101].

If, on the other hand, we have a detector with one *band* of allowed energies (see Fig.3.4), where  $\nu_1$  and  $\nu_2$  fall within the allowed band, the detector will respond to both frequencies simultaneously as indicated by Eq.(3.9): adding up the individual dipole stimulations, squaring and time-integrating. This type of detector uses an applied electric field to extract the conduction band electrons in form of a measurable current. This current, though, only exhibits the *difference* frequency ( $\nu_2 - \nu_1$ ); the sum frequency ( $\nu_2 + \nu_1$ ), as present in RF heterodyne<sup>14</sup>, gets filtered out by the time-integration process. Mathematically, the result turns

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<sup>14</sup>which also follows a square-law



out proportional to:<sup>15</sup>

$$\mathcal{D} = \chi^2 a^2 + \chi^2 a^2 \cos 2\pi(\nu_1 - \nu_2)t \quad (3.11)$$

**Direct optical E-Vector detection** Fairly recently Goulielmakis et al. developed a method of directly measuring the electric vector of a few-cycle optical pulse [102]. By sending the pulse through a gas of Ne atoms, along side of an XUV pulse which can photo-ionize these atoms, the group measured the resulting kinetic energy which the optical pulse’s electric field imparted on the now free electrons. Detection by such entirely free electrons likely represents the best method of detecting EM radiation (regardless of type) so far, as these electrons seem to have no frequency band limit (unlike the LCR circuit discussed earlier), and thus should have the ability to perform Fourier Synthesis over an extremely wide range of frequencies. Free electrons add up the superposed fields and accelerate proportional to that sum. Doing this many times with identical pulses and varying phase allows for mapping of the net pulse shape. However, we cannot distinguish between single-carrier-frequency pulses and “chopped” frequency-comb pulses in this manner.

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<sup>15</sup>assuming  $\chi(\nu_1) = \chi(\nu_2)$

## $X/\gamma$ Rays

X-rays and  $\gamma$ -rays represent the high-frequency end of the EM spectrum. We cannot clearly separate the two in terms of their placement in the spectrum as there exists some overlap. The distinction between the two stems rather from their origin, or generation mechanism. While EM pulses due to transitions of inner electrons get labeled “x-rays”, people often call those originating from *nuclear* transitions “ $\gamma$ -rays”, even if some of the latter happen to have lower frequencies than some of the former. With this in mind, I should point out that this section will consider only such radiation that will require detection mechanisms markedly different from those described in previous sections. Detection mechanisms for lower frequencies correspond to those discussed when we looked at the optical regime. For this reason we shall refer to all the frequencies considered here from now on collectively as  $\gamma$ -rays, regardless of their origin.

Due to their high energy, and thus high frequency<sup>16</sup>, the interaction of  $\gamma$ -rays with matter appears very classical. Furthermore, while visible light<sup>17</sup> and other lower frequency radiation exhibits a significant amount of diffractive spreading,  $\gamma$ -rays do not seem to suffer from this effect, which generally behaves inversely proportional to the frequency in the far field.

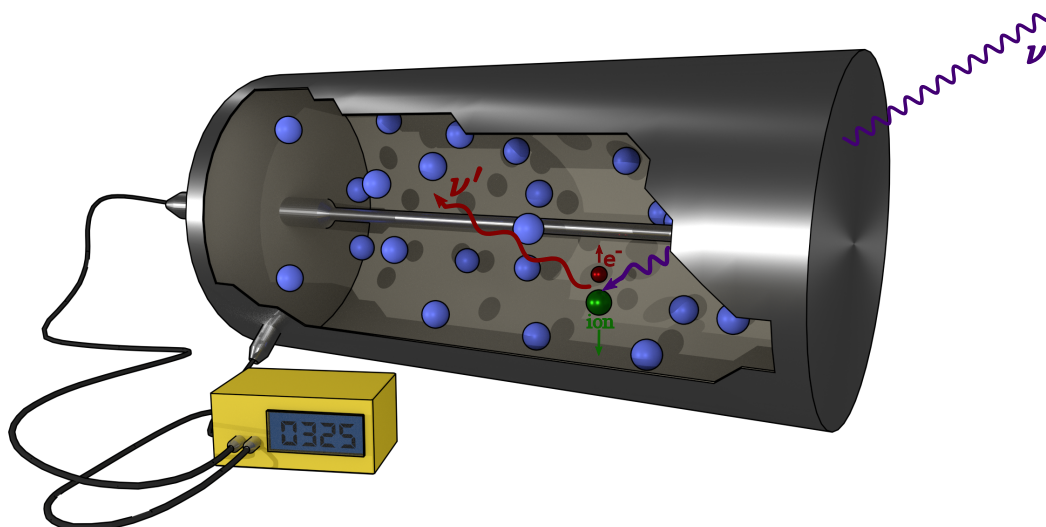
$\gamma$ -radiation seems to mainly effect ionization of the matter it encounters.

Thus most, if not all  $\gamma$ -ray detectors try to measure the results of such ionizations

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<sup>16</sup>On the order of  $10^{20} Hz$ .

<sup>17</sup>On the order of  $10^{14} Hz$



**Fig. 3.5:** The  $\gamma$ -ray enters the proportional counter with frequency  $\nu$ . After ionizing one of the atoms of the gas, thereby losing part of its energy, it continues to propagate, but with reduced frequency  $\nu'$ . The voltage difference between the central wire and the housing of the device accelerates the charged particles in opposite directions as indicated, effectively producing a current which we can detect.

in one form or another. I will mention the three main detector types here very briefly [98]. While the details of the actual detectors vary, they all have the key  $\gamma$ -ray interaction process (ionization), illustrated in Fig.3.5, in common.

**Gas-Filled Detectors** Fig.3.5 shows a detector in which an inert gas provides the active medium with which the radiation will interact. A  $\gamma$ -ray will ionize a gas atom or molecule, thereby losing some of its energy, and move on to ionize another and another until it either gets absorbed entirely, or leaves the detector. The voltage applied between the central wire and the outer cylinder accelerates the thus created positive and negative charges in opposite directions, creating

an electrical current which subsequently gets measured<sup>18</sup>. This behavior closely resembles that of an energetic *particle* entering the medium, knocking electrons out of their bound states and losing kinetic energy in the process. The  $\gamma$ -ray does not lose energy in terms of kinetic energy, however, since it has no mass and never stops moving at the speed of light  $c$ . Rather, its frequency diminishes according to Eq.(3.8).

**Scintillation Detectors** As we get to shorter wavelengths, the efficiency of gas-filled detectors declines and we require a different method of registering such radiation. Instead of a gas, scintillation detectors employ a large crystal<sup>19</sup> (the scintillator), typically thallium doped sodium iodide, as the active medium. Ionization within this medium does not produce free electrons as in the gas, but creates electron-hole pairs that end up recombining at impurity sites, which in turn causes the emission of typically visible scintillation light. This light then propagates through the crystal, transparent at these wavelengths, and ends up getting amplified by photomultiplier tubes (PMTs) and converted to a measurable current using optical detectors. Again, one  $\gamma$ -ray will typically cause multiple such interactions.

**Solid-State Detectors** Much like in the case of optical radiation, the charge produced by ionizing radiation in solid state detectors gets collected directly, using

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<sup>18</sup>We ignore here the more intricate details of gas gain and saturation etc.

<sup>19</sup>or a volume of a suitable liquid

an applied electric field, and converted into a measurable signal. Unlike detection of optical radiation in the corresponding detector, detection of  $\gamma$ -rays does *not* have its basis in a resonance process. As before, the highly energetic radiation directly imparts enough energy upon an electron to transfer it to the conduction band (creating electron-hole pairs), skipping the “usual” determination of “quantum compatibility”. The total energy of the incident  $\gamma$ -ray then gets determined using

$$E = n\epsilon \tag{3.12}$$

where  $n$  represents the number of electrons detected (i.e. the number of ionization events), and  $\epsilon$  corresponds to the energy gap between the valence band and the conduction band of the detector.

### 3.4 Superposition effects as reported by Detectors

*A photon is what a photodetector detects.*

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Roy Glauber

As beautifully captured by Glauber’s remark [103], what we usually call a “photon” actually corresponds to the detector output resulting from a rather complex chain of interaction processes taking place within a photodetector. I will now re-examine the two beam superposition setup discussed earlier from this point

of view.

### 3.4.1 Two beam superposition re-examined

The quantity  $i_{net}$ , or instantaneous intensity, resulting from superposition of multiple beams, exhibits what textbooks commonly call “interference”. However, the waves cannot carry out the mathematical step represented by  $i = E^2$  themselves as they exist as linear excitations of the field. Mathematically, the linearity in the fields of Maxwell’s equations implies that different EM waves can propagate through the same volume of space without perturbing each other’s wave characteristics<sup>20</sup> and do not interact or interfere by themselves [104,29] to reorganize their spatial or temporal energy distributions.

The quadratic process step gets carried out by some optical detectors (atoms, molecules, etc.) that have the intrinsic propensity to do so, as “designed” by nature. When provoked by the resonant stimulation process induced by the fields, such detectors may extract energy from them. Thus, it becomes important to think about what exactly happens in that microscopic world. We need to try to visualize the invisible interaction process between the EM fields and the detector.

The simplest, and in the optical domain most useful detection mechanism corresponds to frequency resonant, but linear dipolar stimulation by the electric vector of the propagating EM waves, followed by energy absorption from the

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<sup>20</sup>I.e. waves in the linear domain simply represent linear excitations facilitated by  $\epsilon_0$  and  $\mu_0$  in the vacuum.

fields resulting in a quantum transition. As this gives the most physical insight and appears to corroborate most of what we observe in experiments, I will assume linear dipoles as our quantum detectors.

Physical dipoles react directly to the field *amplitudes*, and the magnitude and direction of the response depends on both, the impinging field and the properties of the detector. Lamb and Scully have presented such an analysis as early as 1969 [7] in support of a semi-classical model for superposition effects. Jaynes and Cummings have also argued in favor of a semi-classical approach six years earlier [5].

Modeling the interaction between EM waves and matter involves a surprising amount of assumptions and approximations. For example, we often neglect the spatial extent of the elementary particles involved, and assume that linear dipole interactions greatly dominate over those of higher orders. Fortunately, in many cases these approximations give results that agree very well with our observations. For this reason I shall also use these assumptions here. I will neglect the much weaker non-linear light-matter interaction processes, driven by intense fields and higher order dipolar stimulation susceptibilities [105], even though in reality their contributions rarely equal exactly zero.

Very generally, we can mathematically model the dipole response  $\vec{d}$  to  $N$

impinging electromagnetic waves  $\vec{E}^{(n)}$  as<sup>21</sup>

$$\psi_i \equiv d_i = \sum_{n=1}^N \chi_{ij} E_j^{(n)} \quad (3.13)$$

The linear susceptibility matrix  $\chi_{ij}$  depends on the physical properties of the detecting material. Via this suggestive notation I have implemented Schrödinger's proposed interpretation of  $\psi$  [68].<sup>22</sup>  $\chi$  represents the material part of the interaction and encodes the quantum mechanical energy level information [105]. This form does not give us many insights as to the possible physical processes involved in the interaction. Therefore, I propose this perhaps more restrictive but also more illuminating form for the dipole interaction:

$$\psi_i \equiv d_i = \chi_i (E_j \cdot \hat{\chi}_j) \quad (3.14)$$

or in vector form

$$\vec{\psi} \equiv \vec{d} = \vec{\chi} (\vec{E} \cdot \hat{\chi}) \quad (3.15)$$

$\vec{\chi}$  represents the linear dipolar susceptibility *vector* of the dipole, whose magnitude corresponds to the usual linear dipole susceptibility. The material

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<sup>21</sup>using summation convention

<sup>22</sup> $\psi$  here does not necessarily correspond to an actual solution to the Schrödinger equation, but the two quantities (Schrödinger's  $\psi$  and my  $\psi$  here) appear intimately connected, from a process perspective.



properties of the detector determine the direction  $\hat{\chi}$  of  $\vec{\chi}$ . For example, if we have an isotropic detector with freely orientable dipoles, the direction of  $\vec{\chi}$ , and thus the direction of oscillation of the dipoles, would always correspond to that of the impinging field, i.e.  $\hat{\chi} = \hat{E} \equiv \vec{E}/|\vec{E}|$ . If we have a crystalline detector whose dipoles have fixed orientations,  $\hat{\chi}$  would correspond to *that* direction, and we would find the impinging field projected onto, and the dipoles oscillating in the direction determined by the crystal structure. The newly introduced spatial direction of  $\psi$  here corresponds to and merely indicates the direction of undulation of the dipole  $\hat{\chi}$ , i.e. the vector  $\vec{\psi} \equiv \psi \hat{\chi}$ .

The linear susceptibility  $\chi$  does in general exhibit frequency dependence, i.e.  $\chi \equiv \chi(\nu)$ , and we can not treat it as a mere detector *constant*. In the case of low-frequency waves, whose amplitude stimulation we can measure directly using antennae connected to LCR circuits,  $\chi(\nu)$  corresponds to a smooth function of  $\nu$  following the LCR-resonance response curve. However, in the regime of optical frequencies and beyond, detectors operate through quantum mechanical energy level (or band) *transitions*. In these cases we really should denote  $\chi$  as  $\chi_{mn}(\nu)$ , where  $m$  and  $n$  correspond to either the initial and final energy states or bands, respectively. In order to not clutter up the notation too much I will suppress the  $mn$  subscripts, but we must always keep that dependence in mind.

In the most common (isotropic) detectors, the dipoles have the freedom to align themselves in the direction of the stimulating electric field vector, which

reduces equation (3.15) to

$$\vec{\psi} = \chi(\nu)\vec{E} \quad (3.16)$$

When multiple stimulating waves simultaneously impinge upon the same detector, we can represent the net dipole stimulation amplitude by

$$\vec{\psi}_{\text{net}} = \sum_i \chi(\nu_i)\vec{E}_i \quad (3.17)$$

Only under the special condition that  $\chi(\nu)$  does not vary appreciably within the range of frequencies present in the beams can we mathematically factor it out of the sum. This then gives the *appearance* that the fields sum themselves, without the involvement of the material dipoles:

$$\vec{\psi}_{\text{net}} = \chi \sum_i \vec{E}_i = \chi(\vec{E}_1 + \vec{E}_2) = \chi\vec{E}_{\text{net}} \quad (3.18)$$

This equation corresponds to the two-beam case treated in Chapter 3.2. Perhaps the mathematical form of Eq.(3.18) has led us to believe that waves directly interfere (sum themselves) without the active participation of the detectors, while  $\chi$  becomes just a detector constant.

Again, with optical detectors we do not generally observe this amplitude response, but only the energy absorbed by such a dipole through a quantum

mechanical level (or band) transition, in form of the consequent physical transformation within the detector (release of a photo-electron). As in the previous section, the measured result will correspond to a quantity involving the square of Eq.(3.17) or Eq.(3.18). Furthermore, the process of energy absorption does not proceed instantaneously, but rather takes a certain, albeit generally relatively small, amount of time. Mathematically, we will represent this using time averaging and time integrals.

For the moment let us explore the effects of time integration on a measured signal  $\mathcal{D} \equiv \psi^2$ . In the case of two beams with two frequencies, we have (analogous to Eq.(3.4))

$$\begin{aligned} \mathcal{D} = E_0^2 \int_0^T \bigg\{ & \frac{\chi^2(\nu_1) + \chi^2(\nu_2)}{2} + \frac{\chi^2(\nu_1)}{2} \cos [4\pi\nu_1(t - \tau)] \\ & + \frac{\chi^2(\nu_2)}{2} \cos [4\pi\nu_2(t + \tau)] \\ & + 2\chi(\nu_1)\chi(\nu_2) \cos [2\pi\nu_1(t - \tau)] \cos [2\pi\nu_2(t + \tau)] \bigg\} dt \end{aligned} \quad (3.19)$$

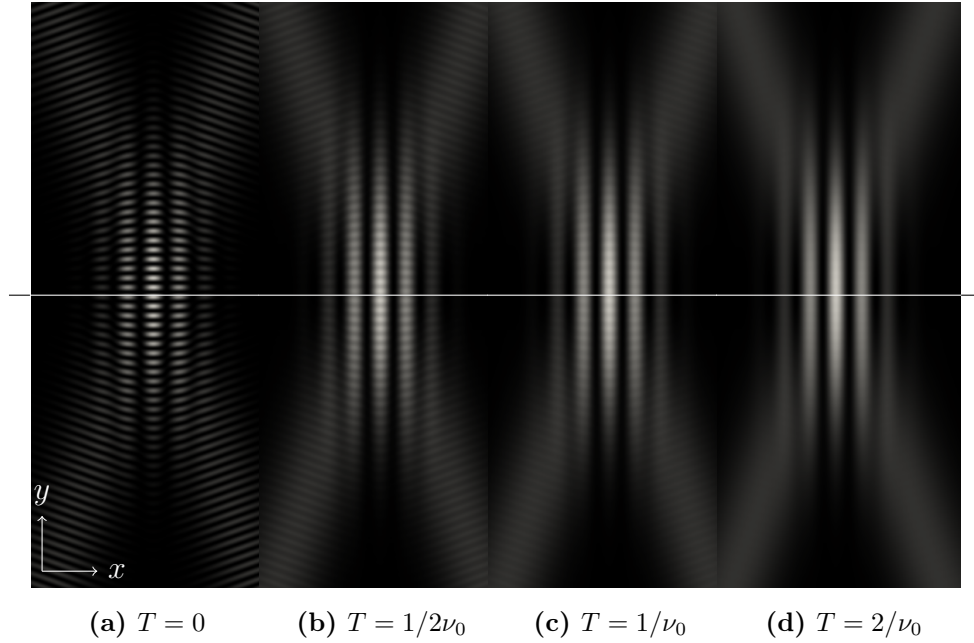
In the case of a detector for which  $\chi(\nu_1) = \chi(\nu_2) \equiv \chi$ , this simplifies to

$$\begin{aligned} \mathcal{D} = E_0^2 \chi^2 \int_0^T & \left\{ 1 + \frac{1}{2} \cos \left[ 4\pi\nu_1 \left( t - \frac{\tau}{2} \right) \right] \right. \\ & + \frac{1}{2} \cos \left[ 4\pi\nu_2 \left( t + \frac{\tau}{2} \right) \right] \\ & \left. + 2 \cos \left[ 2\pi\nu_1 \left( t - \frac{\tau}{2} \right) \right] \cos \left[ 2\pi\nu_2 \left( t + \frac{\tau}{2} \right) \right] \right\} dt \end{aligned} \quad (3.20)$$

FIGs. 3.6 and 3.7 show the result of integrating this over various time spans  $T$  and helps visualize the effect of the integration time.

In the case of equal frequencies we find, as the integration time increases towards about two periods (Fig.3.6b-d), the familiar experimental result of time-steady fringes within the superposition region. When the two frequencies differ (Fig.3.7), we see no fringes at all after integrating over at least one period of the lower-frequency light ( $T = 1/\nu_2$ ).

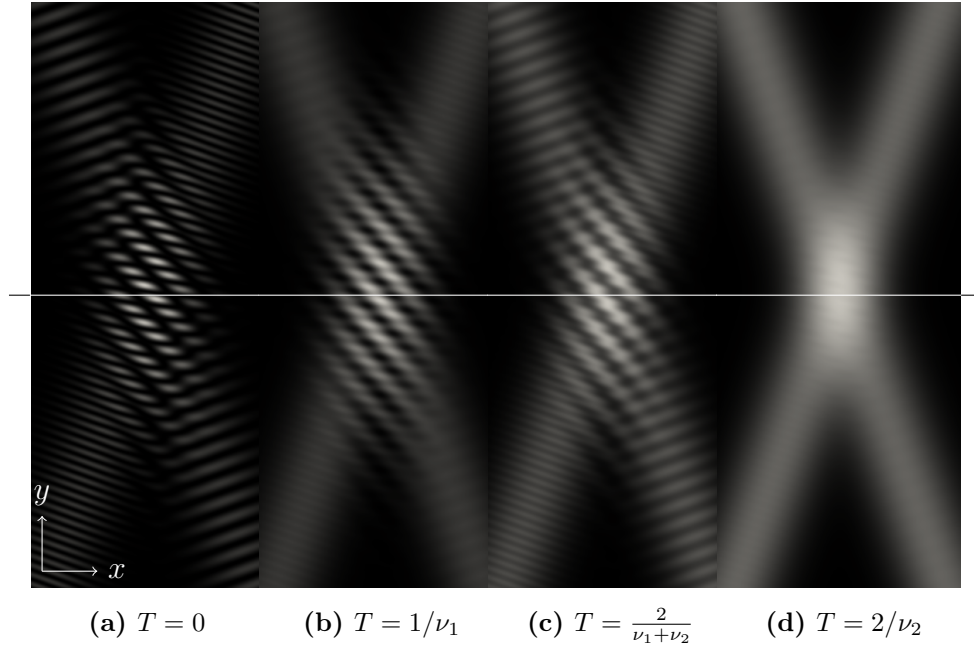
Both of these findings correspond to commonly accepted and observed results. Note, that neither of these results emerged naturally from the analysis in Section 3.2 (using real representation for the EM waves), which neglected to explore the physical interaction processes and the detector's integration time(s). The appearance of static fringes for beams of equal frequencies, and the absence of fringes when the frequencies differ, entirely rely on the properties of the detector, without the ad-hoc hypothesis that “different frequencies do not interfere”, or postulating the “incoherence of different frequencies”, both of which many books



**Fig. 3.6:** Superposition of two beams with equal frequencies. (a) Instantaneous intensity, no time integration: the secondary high-frequency fringes move along the  $y$ -direction with time. (b)-(d) Time integrated energy registered by the detector after integrating for an increasing duration  $T$ . (b) and (c) show steady disappearance (contrast reduction, or time averaging) of the high-frequency fringes. After integration over two cycles, as in (d), the vertical fringes appear as steady  $1 + \cos(2\pi\nu_0\tau)$ . We would normally register this on a photographic plate. (See video of temporal evolution of the fringes online at <http://natureofflight.org/MA/SameVid.mpg>)

commonly employ.

Figure 3.6 allows us to observe another phenomenon: while the bright fringes exhibit time dependence given a sufficiently fast detector, the central portions of the dark fringes remain constantly dark throughout, regardless of how long we time-integrate. The detector never absorbs energy in these areas. This may lead us to conclude that no energy passes through these regions. If we switched one of the

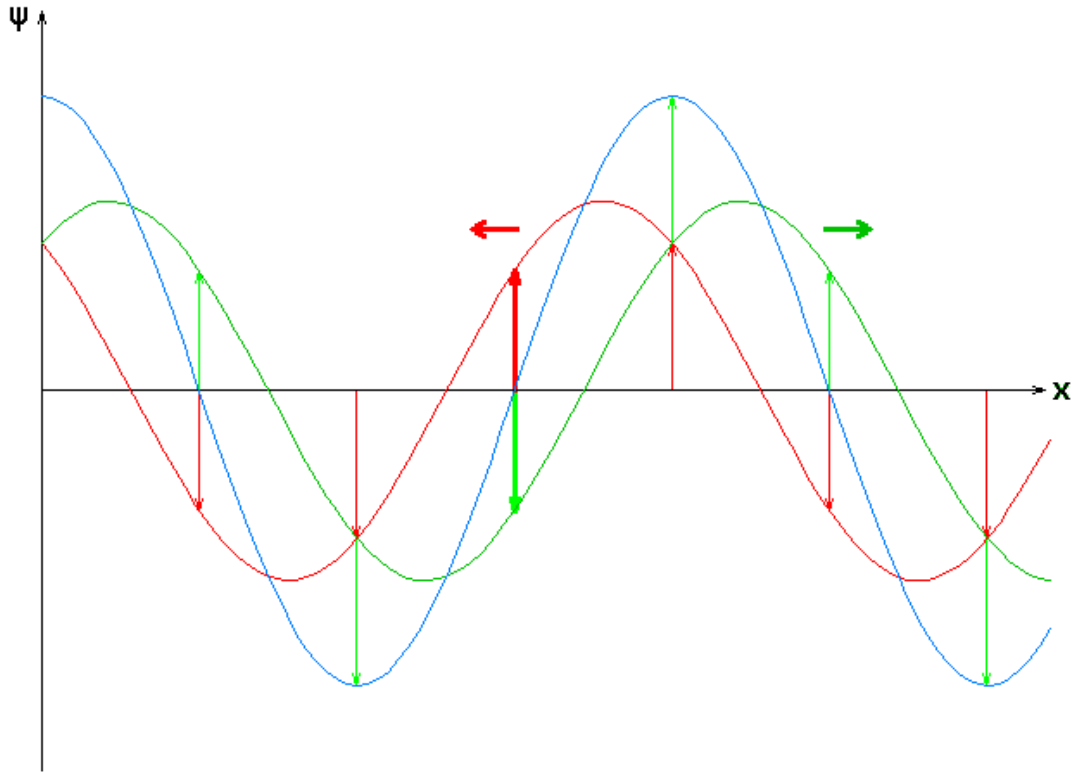


**Fig. 3.7:** Superposition of two beams with different frequencies. (a) Instantaneous intensity, no time integration: the secondary high-frequency fringes move along both the x and y-direction. Along a spatially fixed, 1D detector array (white line), the fringes seem to continuously appear and disappear. (b)-(d) Time integrated energy registered by the detector after integrating for a duration  $T$ . Along a spatially fixed, 1d detector array, the fringes corresponding to figures (b) and (c) would appear as steady amplitude fringes, translating in the horizontal direction (to the right in this case), as time passes. After sufficiently long time integration, about two cycles as in figure (d), the time dependence as well as the fringes disappear completely. This corresponds to the common perception / interpretation that different frequencies do not interfere. (See video of temporal evolution of the fringes online at <http://natureofflight.org/MA/DiffVid.mpg>)

beams off, however, an appropriate detector *would* absorb energy in these places.

Since the two beams when superposed will not influence each other's energy distribution, the absence of a signal in this case must represent a detector-based effect.

Indeed, looking at Fig.3.8 we see that while the individual beams continue to pass



**Fig. 3.8:** In this one dimensional cross section (corresponding to the horizontal line indicated in Fig.3.6.a) the amplitudes corresponding to the red beam appear to travel to the left, while those corresponding to the green beam appear to travel to the right. At each spatial point, a detecting dipole will add these amplitudes, as represented by the blue curve, while absorbing energy proportional to the square of this sum. At the center of the dark fringes, the amplitudes always sum to zero. (See animation online at <http://natureofflight.org/MA/superposition.html>)

through these spatial regions, a *detector*, such as a dipole, responding linearly to amplitude stimulation finds itself *unable to* oscillate and absorb energy in these locations.

### 3.4.2 Process-based Model for Detector Signal

When studying such measurement processes in detail, we find that we must account for and distinguish between at least *two* different physical processes when it comes to energy absorption by quantum entities. At the fundamental level, we have a quantum mechanical interaction process between the optical beam and the detector which absorbs energy in quantized amounts via level (or band) transitions. As such transitions depend on the frequency, the detecting atom or molecule needs time to determine the beam’s frequency, and thus whether it matches an allowed quantum energy level transition  $\Delta E_{mn} = h\nu_{mn}$ . This immutable, detector-intrinsic time interval,  $T_Q$ , depends on the quantum mechanical properties of the detecting dipole. If the frequency matches an allowed transition, each detecting dipole fills up its “quantum cup” [29] with the necessary amount of energy,  $h\nu_{mn}$ , from the propagating field that surrounds it, and makes a sudden transition releasing one photo electron<sup>23</sup>.

The second process we need to model corresponds to the integration time of the entire detecting system, electronics and all. Photo electrons, rapidly released from the valence band within a solid state detector, start to accumulate in the conduction band. We extract these slowly over a time period into an external circuit to register as photo electric current. Depending on the system’s design, the signal integration time, which I call the detector or *system time constant*  $T_d$ ,

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<sup>23</sup>For some subtleties involved here see Appendix D.



can vary.  $T_Q$  always remains the same for a given type of detector (i.e. a given type of atom or molecule).

Every physical detector has a detector time constant, effectively defining the instrumental integration time. Figure 3.6 indicates that as  $T_d$  gets smaller than about two periods of oscillation, the familiar static spatial fringes become time dependent: if the horizontal line represents our linear detector array, the intensity of the bright fringes will oscillate in time with frequency  $\nu_0$ , while the dark fringes remain dark. For radio frequencies, when detected by classical LCR circuit of fast response, one does observe such oscillatory currents (temporal “fringes”) [106].

Similarly, when the two frequencies differ, the instantaneous intensity fringes in Fig.3.7a will appear to move both vertically<sup>24</sup> (as in Fig.3.6a) and horizontally<sup>25</sup>, indicating time varying relative phase. We will see no time dependence for  $T_d$  larger than about twice the longest period of oscillation, as seen in Fig.3.7d. For smaller  $T_d$ , the time dependence will become evident. For a range of integration times (Figures 3.7b-c), the spatial fringes will appear to translate in the  $x$ -direction, along our detector array. In a single wide area and slow detector, the external photo current will appear as temporal “fringe” free DC current, even though the internal rate of photo electron emission will oscillate in time.

For this reason, heterodyne sensor technology uses collinearly superposed beams along with fast detectors [107] to avoid spatial fringe formation. Using

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<sup>24</sup>i.e. in time

<sup>25</sup>i.e. in space

physically very small detectors to minimize spatial fringe formation proves counter productive for weak signals, since the detector responsivity reduces with the active physical area of the detector.

Evidently, the signal we receive from a detector represents the result of not just a single, but a complex multi-step process. Remarkably, the current QM formalism appears to already encode much of this multi-step process, which produces the final observed signal in our detectors, as we will see now.

The first step, embodied by Schrödinger's  $\psi$  function, represents the real *physical operation* of dipolar amplitude stimulation of the detecting molecule (or an assembly thereof) induced by all the simultaneously superposed EM waves on it (Eq.(3.17)). If the frequency of some of the incident fields resonate with the detecting dipoles, this induces the second step consisting of the *physical operation* of energy transfer from the superposed fields into the detecting entity. This process, represented by the  $\psi^*\psi$  function in the formalism, triggers the quantum mechanical transition to generate the measurable data (such as the release of a photo-electron). The final steps consist of our *data acquisition process*. We accumulate the data from a similarly prepared ensemble of interactants, the average of which we accept as the measured data. The mathematical operation represented by  $\langle\psi^*\psi\rangle$  incorporates this process. Additionally, constraints on, for example, the processing speed of the electronics involved may result in an overall time integration over a time-interval that depends on the details of the particular detecting

device used.

Given these considerations, we expect a detector signal proportional to

$$\begin{aligned}\mathcal{D} &\equiv \int_t^{t+T_d} \langle \psi_{\text{net}}^2 \rangle dt' \\ &= \int_t^{t+T_d} \left( \frac{1}{T_Q} \int_{t'}^{t'+T_Q} \psi_{\text{net}}^2 dt'' \right) dt'\end{aligned}\tag{3.21}$$

Note that  $T_Q$  may also depend on the frequency  $\nu$ , but we will consider it constant for the present purposes. Expression (3.21) explicitly incorporates and maps the entire multi-step detection process.

### 3.5 Example: Heterodyne and FT Spectroscopy

Applying all these observations, we find that we can easily implement or differentiate between Fourier Transform Spectroscopy and Heterodyne Spectroscopy via the use of slow (for the former) or fast (for the latter) detectors, i.e. by changing the properties of the detector only. Specifically, the essential difference between FT spectroscopy and heterodyne spectroscopy lies in the time constants of the detectors used.

#### 3.5.1 Heterodyne spectroscopy

We start with my proposed generic expression for a signal measured by a detector, Eq.(3.21), and the same setup as in Fig.3.1. Heterodyne spectroscopy requires

collinear beams, so we set  $\tau = 0$  in Eq.(3.21).

$$\begin{aligned} \mathcal{D} = E_0^2 \chi^2 \int_t^{t+T_d} \frac{1}{T_Q} \int_{t'}^{t'+T_Q} & \left( 1 + \frac{1}{2} \cos(4\pi\nu_1 t'') \right. \\ & + \frac{1}{2} \cos(4\pi\nu_2 t'') + \cos[2\pi(\nu_2 - \nu_1)t''] \\ & \left. + \cos[2\pi(\nu_2 + \nu_1)t''] \right) dt'' dt' \end{aligned} \quad (3.22)$$

At this point, in order to make headway, we need to introduce some assumptions about the particular detector. First, we assume that the quantum mechanical time constant  $T_Q$  of the detecting molecules has a value of at least one oscillatory period of the lowest of the individual frequencies.  $T_Q$  represents an *intrinsic* property of the detecting quantum-entity (e.g. an atom), which we commonly think of as the reciprocal of the natural line width of the associated quantum transition. While, as far as I know,  $T_Q$  does not depend on the frequency of the impinging radiation, we impose the above condition in order to model currently known optical detectors. The literature [108,93] commonly does this using a sentence or two, without formally mapping the associated physical process mathematically, as I have done here.

This averages all the high frequency terms to zero, and the measured signal

$\mathcal{D}$  simplifies to

$$\mathcal{D} \approx E_0^2 \chi^2 \int_t^{t+T_d} \left( 1 + \cos [2\pi(\nu_2 - \nu_1)t'] \right) dt' \quad (3.23)$$

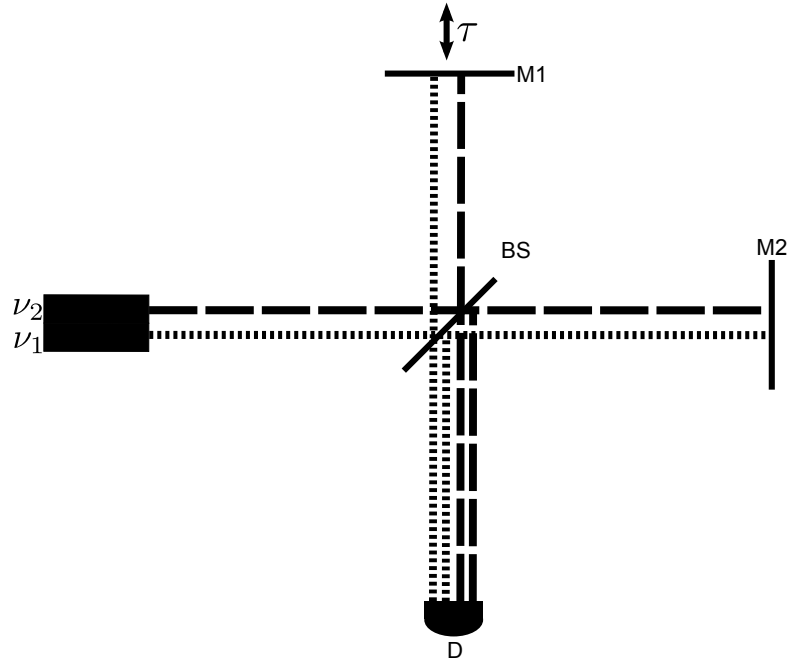
The remaining integration time  $T_d$  depends on the electronics involved, but in heterodyne spectroscopy we require this time constant to have a sufficiently small value. “Sufficiently small” in this example means  $T_d \ll 1/(\nu_2 - \nu_1)$ . As  $T_d$  approaches the value of  $T_Q$ , we can neglect the integral and find the measured signal as

$$\mathcal{D} \approx E_0^2 \chi^2 T_d (1 + \cos [2\pi(\nu_2 - \nu_1)t]) \quad (3.24)$$

Removing the DC offset, one obtains an oscillatory cosine signal at the difference frequency. Fourier transforming the result yields the spectrum - the difference frequency  $f = \nu_2 - \nu_1$  in this particular case. Using this technique we can determine an unknown frequency, given we know the other one with sufficient accuracy.

### 3.5.2 Fourier transform spectroscopy

Now, let us send our collinear beams through a Michelson interferometer, introducing a time delay  $\tau$  between the beams, before detecting the result, as shown in



**Fig. 3.9:** Typical Michelson interferometer as used in Fourier Transform Spectroscopy. This diagram only shows two distinct frequencies, while generally we have a continuum. Part of the beam gets diverted to mirror M1 at beam splitter BS, while the remainder of the beam continues to mirror M2. The movable mirror M1 introduces a time delay  $\tau$  with respect to the beam traveling to the fixed mirror M2. The two beams recombine again at BS and then travel to the detector D. In this diagram, we depicted the light beams spatially separated to underscore the optical paths, while in reality they propagate collinearly.

Fig.3.9. We assume here that the phase of the E-vector in the light beam remains steady over a long time compared to  $\tau$ , which usually holds true for properly selected lasers<sup>26</sup>. The introduction of the time delay modifies our expression for the

<sup>26</sup>Otherwise, rapid phase fluctuations will generate a rapidly fluctuating detector signal. This still represents the superposition effect, but we can no longer model the signal by a simple cosine or a sum of two cosines. The output signal from the detector has become random. While we could define this to represent a property of the EM radiation itself and say that the “light has become incoherent”, the real physical process corresponds to the randomization of the release of photo-electrons in the detector.

net dipole stimulation Eq.(3.17) to

$$\vec{\psi}_{\text{net}} = \chi \left( \vec{E}_1(t) + \vec{E}_1(t + \tau) + \vec{E}_2(t) + \vec{E}_2(t + \tau) \right) \quad (3.25)$$

Note the implicit assumption, that  $\chi$  remains constant for the narrow band of frequencies considered here. We cannot do this in general, for example when considering a very wide band quantum detector receiving EM waves of a very wide range of frequencies.

Plugging this all into Eq.(3.21) we end up with

$$\mathcal{D} = E_0^2 \chi^2 T_d \left( 2 + \cos(2\pi\nu_1\tau) + \cos(2\pi\nu_2\tau) \right) \quad (3.26)$$

$$+ E_0^2 \chi^2 \int_t^{t+T_d} \left( \frac{1}{T_Q} \int_{t'-T_Q}^{t'} f(t'') dt'' \right) dt' \quad (3.27)$$

where  $f(t)$  contains only time dependent (cosine) terms of frequencies greater than  $\nu_1 - \nu_2$ . Thus, if  $T_d > 1/\nu_1 - \nu_2$ , this part of the integral vanishes, and only

$$\mathcal{D} = E_0^2 \chi^2 T_d \left( 2 + \cos(2\pi\nu_1\tau) + \cos(2\pi\nu_2\tau) \right) \quad (3.28)$$

remains. Removing the  $\tau$  independent DC offset as before,

$$\mathcal{D}_{osc.} = E_0^2 \chi^2 T_d \left( \cos(2\pi\nu_1\tau) + \cos(2\pi\nu_2\tau) \right) \quad (3.29)$$

The Fourier transform of this expression then gives us the full spectrum

$$S(\nu) = \delta(\nu - \nu_1) + \delta(\nu - \nu_2) \quad (3.30)$$

of the incident beams under analysis [109]. We see, Fourier transform spectroscopy can provide us with all of the frequencies in a spectrum, without having to know any of them beforehand, unlike in heterodyne spectroscopy. However, in heterodyne spectroscopy we usually only care about the difference frequencies anyway.

### 3.6 Conclusion

I have demonstrated that we can get a more intuitive understanding of QM by considering the physical interaction processes occurring within our detectors. I have presented a process-based derivation of the black-body spectrum using the semi-classical view of a continuous electromagnetic field and quantized material oscillators (atoms, molecules etc.). The variational procedure represents the physical process of continual interaction (thermalization) between matter and radiation, *eventually* leading to equilibrium and the associated spectrum.

I have also demonstrated the surprising generality of the QM formalism in form of the Schrödinger equation, which can emerge from various quite different considerations. Further, I have identified the various interpretations of QM, and the Copenhagen interpretation in particular, as the seat and origin of many trou-



bling aspects associated with the theory. Focusing instead on the detector and detection processes brings more clarity into QM.

I have shown that different parts of the electromagnetic spectrum require distinctly different physical mechanisms for detection. Since, in the linear domain, waves do not interact on their own, we cannot blindly apply the mathematical superposition principle without considering the properties of the particular detector used. While the superposition principle does not represent an observable process, we do observe superposition *effects* as manifestations of light-matter<sup>27</sup> interaction processes.

I have pointed out the important fact that *all* detectors have limitations and only respond to a rather narrow subset of the aspects associated with a physical object<sup>28</sup>. One universal feature of interaction or detection processes includes time integration or time averaging. I have argued that the mathematical formalism of QM includes a short time averaging by virtue of the complex representation used. I presented a generic model of the detected signal, that accounts for some of the important physical properties and processes of the detector, and demonstrated its utility via the example of distinguishing between two different spectroscopic techniques on process-based grounds.

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<sup>27</sup>or wave-detector

<sup>28</sup>The glaring ambiguity of the word “object” here represents evidence of this very fact. Without following the temptation of getting into a lengthy tangent on the subject, I will merely state that I mean for it to include things like pulses of “light” etc..

## Chapter 4

### Future Work

In physics, as in everything else, progress comes about through iterative improvement. Often we can incrementally *evolve* the current state of knowledge by improving upon *it*. Sometimes however, we find that our current model has reached the boundaries of its usefulness and we need to start over anew to make progress. And make progress we must - *ceaseless change corresponds to the only constant thing in the universe*, to paraphrase the ancient saying. This applies in particular to our mathematical theories - very powerful, but nonetheless imperfect constructs of the human imagination.

I have shown that we can make some headway with QM by focusing on detector-detectee interaction processes, as well as demanding a close correspondence between our mathematics and physical reality. Again, mathematical symbols should correspond as much as possible to primary physical parameters, and mathematical operations should map physical processes. We saw that this allows us to extract some more explanatory power from the formalism. In this thesis I have only applied this to a very limited range of physical situations. A detailed

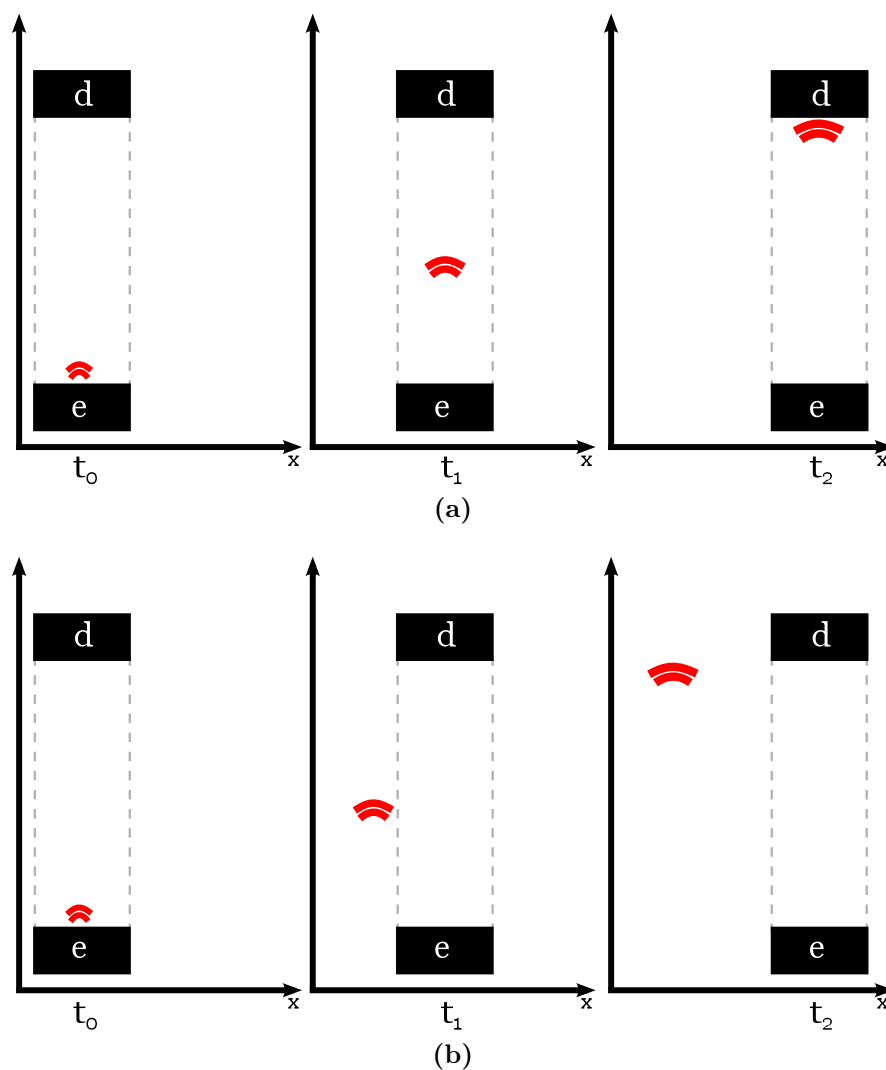
re-analysis of the “canonical” set of controversial experiments and effects, some of which I briefly discuss in Appendix E, represents the next logical step. This might have dramatic consequences for branches of various sciences that rely rather heavily on the Copenhagen way of interpreting the math. The formalism itself also has some subtle problems, an example of which I have described in Appendix E.3.

Like QM, the other major physical theories appear to also work reasonably well for computational purposes, but *explain* relatively little when it comes to very fundamental questions. For example, Special Relativity (SR) postulates the constancy of the speed of light within inertial reference frames (IRFs), but gives no hint as to the *physical* reasons for this condition or the dynamics involved. Furthermore, in SR and General Relativity (GR) time has practically the same status as, and gets treated like the three dimensions of space<sup>1</sup> - again, the theories do not provide any *physical* motivation or explanation for this. An unresolved problem related to this corresponds to the apparent unidirectionality of time. The discussion in Appendix A hints at the possibility of an insurmountable boundary of the theory in this regard.

We have already proposed a possible route that addresses this space-time issue rather directly via the proposition of a complex tension field (CTF) [104,110]. In connection with this, I propose an experiment that essentially corresponds to

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<sup>1</sup>with minor differences in the mathematical representation



**Fig. 4.1:** (a) Emitter  $e$  sends out a short pulse of light at time  $t_0$  which detector  $d$  registers at time  $t_2$ . (b) Emitter  $e$  sends out a short pulse of light at time  $t_0$ . The pulse never reaches the detector  $d$ , due to the relative velocity of the device with respect to the stationary CTF, which determines the propagation of light after emission.

physically implementing the oft-used “light clock” of SR. Treatments on the subject often postulate the behavior of a thought-experimental setup as shown in Fig.4.1.a in order to derive the mathematics of the formalism. A light pulse gets emitted at the bottom of an apparatus moving with velocity  $v$ , and gets detected some time

later by a detector opposite the emitter. Postulating the constancy of the speed of the light pulse regardless of the reference frame results in the mathematics and interpretations characteristic of SR. Again, the theory does not appear to explain the mechanism of this effect in terms of physical processes.

To my knowledge a faithful realization of this experiment does not yet exist. CTF suggests the possible outcome depicted in Fig.4.1.b: given a sufficient horizontal velocity of the device, the light pulse might miss the detector. This outcome would suggest the stationarity of CTF. I have to emphasize the importance of carrying out this experiment in as perfect a vacuum as possible. At least the volume through which the light pulse travels must fulfill this condition. The presence of any material medium<sup>2</sup> would introduce a drag effect in the direction of motion of the device, similar to the effect demonstrated by the Fizeau experiment [111].

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<sup>2</sup>such as air or any other gas

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## Appendix A

### Primary vs. Derived parameters

From a process stand point it proves advantageous to distinguish between “primary” and “derived” parameters. Planck already hinted at the importance of this idea in his 1906 book [44], and Roychoudhuri independently introduced and developed it much more explicitly many years later [29]. While a distinction appears largely arbitrary, I will give here some justification for the particular choices we made in the following examples.

Examples of *primary* parameters include i) distance  $\Delta x$ , ii) frequency  $\nu$ , iii) vacuum permittivity  $\epsilon_0$ , iv) vacuum permeability  $\mu_0$ . Examples of *derived* parameters include i) the speed of light  $c$ , ii) duration  $\Delta t$ , iii) wave length  $\lambda$ .

While admittedly arbitrary, we make the distinction based on the following process-based considerations: “ease” of “measurability” of the parameter, and whether it represents a composite of more fundamental parameters / properties. This leads to the separation into the two categories of above mentioned parameters as follows.

*Primary:* We can measure macroscopic<sup>1</sup> distances rather simply - just use a ruler<sup>2</sup> and hold it up to another object or the separation between two objects.  $\varepsilon_0$  and  $\mu_0$  represent properties of space.

*Derived:* The speed of light represents a composite, or consequence of the properties of space  $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$ . Duration measurements, in comparison to distances, turn out to require quite sophisticated measurement devices (clocks). None of them measure time directly. What they *do* measure corresponds to frequency ( $\nu$ ): all clocks, no matter how modern or primitive, rely on some periodic event<sup>3</sup>, whose frequency they use<sup>4</sup> to “measure” and indicate time. Wavelength depends on the properties of the propagation medium. If the properties of the medium change,  $\lambda$  changes while frequency remains the same  $\lambda = \frac{c_{\text{medium}}}{\nu}$ .

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<sup>1</sup>At a microscopic level we may run into all kinds of complications which we will not and need not get into here.

<sup>2</sup>or any other object of a given spatial extent

<sup>3</sup>I can think of a couple of *apparent* exceptions to this assertion, two examples of which correspond to the height of a burning candle, and the sand trickling in an hourglass. On the face of it they measure time using *rates* which, granted, generically have the same units as frequency and people often use the two synonymously. Thinking a little longer about it, one finds that eventually we need to replace the candle with a fresh one, or rotate the hourglass. Using similar candles, or for a given hour glass, we see that we actually do measure frequencies in these cases as well, albeit *very* low ones. Nevertheless, one might still make the argument for the (more) fundamental nature of rates...

<sup>4</sup>by defining a period  $T \equiv \frac{1}{\nu}$



## Appendix B

### Mechanical Model of the Klein-Gordon and Wave Equations

We start with an (infinite) number of masses  $m$ , each with one degree of freedom (DOF)  $q$  and separated from one another by a distance  $\Delta X$ . We allow each of these masses to slide without friction along a vertical rod<sup>1</sup>, and attach the mass to a point in space ( $\vec{x}$ ) via a spring of spring constant  $k$ . We can write the Hamiltonian for a single such mass as

$$H = \frac{p^2}{2m} + \frac{1}{2}kq^2 \tag{B.1}$$

where by  $q$  we mean the distance of the mass from its equilibrium position, i.e. with an unstretched spring;  $p$  stands for the corresponding momentum.

Since we have one of these oscillators attached at every point in space<sup>2</sup>, the

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<sup>1</sup>to prevent any sideways motion

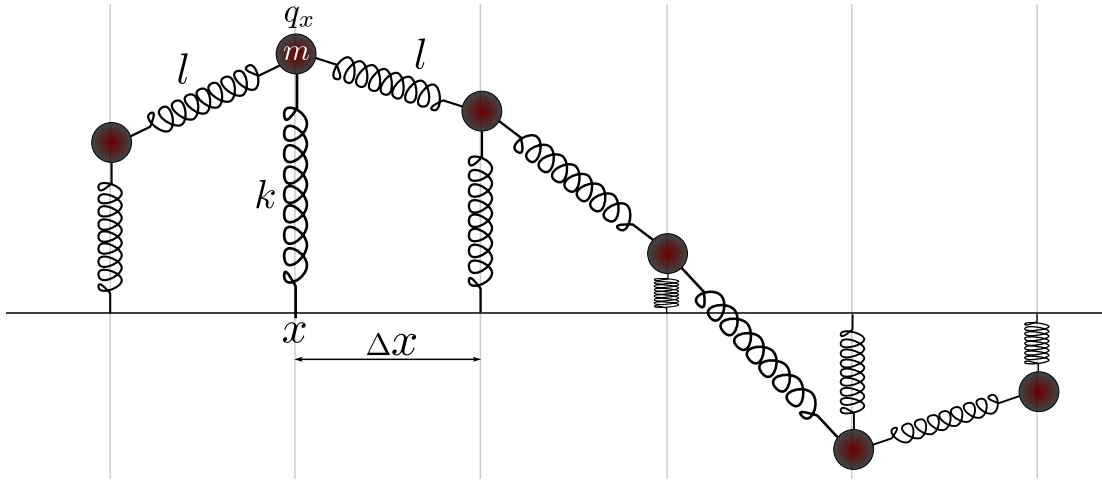
<sup>2</sup>By “space” we mean a regular 3D grid with spacing  $\Delta X$  between points.

full Hamiltonian will become a sum of all of these:

$$H = \sum_{\vec{x}} \frac{p_{\vec{x}}^2}{2m} + \frac{1}{2} k q_{\vec{x}}^2 \quad (\text{B.2})$$

The subscript  $\vec{x}$  now labels the individual HOs via their spatial position. Nothing really interesting can happen in this model - all the oscillators will merely bounce around individually, depending on their initial conditions.

To make things more interesting we need to introduce some kind of interaction between the oscillators. The simplest (interesting) thing we can think of here corresponds to connecting each mass to all of its immediate neighbors via springs with spring constant  $l$ . Figure B.1 shows the situation for the *one dimensional* case.



**Fig. B.1:** 1D version of spring-interconnected masses as represented by Eq.(B.7)

Mathematically, we need to include another term in our Hamiltonian. This

interaction Hamiltonian<sup>3</sup>  $H_{\text{int}}$  will look similar to the other spring constant term already present, but depends on both, the oscillator at  $\vec{x}$  and the ones a distance  $\Delta\vec{x} \equiv \Delta X^i \hat{e}_i$  away from it:

$$H_{\text{int}} = \frac{l}{2} Q_{\vec{x}}^2 \quad (\text{B.3})$$

where

$$Q_{\vec{x}} = \sqrt{\Delta q_{\vec{x}}^2 + \alpha^2 \Delta x^2} \quad (\text{B.4})$$

$$\Delta x^2 = \Delta X^i \Delta X_i \quad (\text{B.5})$$

$$\Delta q_{\vec{x}} = q_{\vec{x}+\Delta\vec{x}} - q_{\vec{x}} \quad (\text{B.6})$$

the  $\Delta X^i$  represent the spatial separation between oscillators in the  $i$ th direction and the  $\hat{e}_i$  stand for the corresponding unit vectors.<sup>4</sup>  $\alpha$ , for our purposes, corresponds to some constant with units suitable to allow us to add a spatial distance to one of the “ $q$ ” variety. In the 1-D and 2-D cases, we can imagine  $q$  to represent an actual spatial extent in the  $z$  direction, and we can set  $\alpha = 1$ . In the 3-D case however, we cannot do this.

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<sup>3</sup>i.e. the part of the total Hamiltonian that facilitates the interaction

<sup>4</sup>We have employed the summation convention for repeated indices here and will continue doing so unless otherwise stated.

The full Hamiltonian now reads

$$H = \sum_{\vec{x}} \frac{p_{\vec{x}}^2}{2m} + \frac{1}{2} k q_{\vec{x}}^2 + \frac{1}{2} l \Delta q_{\vec{x}}^2 + \frac{1}{2} l \alpha^2 \Delta x^2 \quad (\text{B.7})$$

To have an oscillator at *every* point in 3-space, we must shrink the spatial separation  $\Delta X^i$  between our masses to zero. We do this by taking the limit

$$\lim_{\Delta \vec{x} \rightarrow 0} H \quad (\text{B.8})$$

In order to simplify this procedure, we make the following definitions:

- $p \equiv \pi \Delta V$  ;  $\pi$  represents the momentum density
- $m \equiv \mu \Delta V$  ;  $\mu$  represents the mass density
- $k \equiv \kappa \Delta V$  ;  $\kappa$  represents the spring constant density
- $\Delta V \equiv \prod_i^3 \Delta X^i$

Substituting these into Eq.(B.8) results in

$$H = \lim_{\Delta \vec{x} \rightarrow 0} \sum_{\vec{x}} \left[ \frac{\pi_{\vec{x}}^2}{2\mu} + \frac{1}{2} \kappa q_{\vec{x}}^2 + \frac{l}{2} \frac{\Delta q_{\vec{x}}^2}{\Delta V} + \frac{l \alpha^2 \Delta x^2}{2 \Delta V} \right] \Delta V \quad (\text{B.9})$$

The last two terms in the brackets require some more work, after which the Hamil-

tonian becomes

$$H = \lim_{\Delta \vec{x} \rightarrow 0} \sum_{\vec{x}} \left[ \frac{\pi_{\vec{x}}^2}{2\mu} + \frac{1}{2} \kappa q_{\vec{x}}^2 + \frac{\mathcal{Y}}{2} (\nabla q_{\vec{x}})^2 + \frac{3}{2} \alpha^2 \mathcal{Y} \right] \Delta V \quad (\text{B.10})$$

where  $\mathcal{Y} \equiv \frac{l\Delta z}{\Delta x \Delta y} = \frac{l\Delta y}{\Delta x \Delta z} = \frac{l\Delta x}{\Delta y \Delta z}$ . Taking the limit turns the summation into an integral over all space and all the  $\Delta$ s into differentials:

$$H = \int_{\vec{x}} \left[ \frac{\pi_{\vec{x}}^2}{2\mu} + \frac{1}{2} \kappa q_{\vec{x}}^2 + \frac{\mathcal{Y}}{2} (\nabla q_{\vec{x}})^2 + \frac{3}{2} \alpha^2 \mathcal{Y} \right] dV \quad (\text{B.11})$$

By setting all DOF to their equilibrium position ( $q = 0$ ) and giving them zero momentum ( $\pi = 0$ ) we can find out the meaning of the constant term:

$$H|_{\pi, q=0} = \int_{\vec{x}} \frac{3}{2} \alpha^2 \mathcal{Y} dV \quad (\text{B.12})$$

This corresponds to the energy stored within the springs with spring constant  $l$  at equilibrium, i.e. the “ground state” of the system. However, adding a constant to the Hamiltonian does not change the dynamics and we may as well drop it.

Thus, we finally have

$$\boxed{H = \int_{\vec{x}} \left[ \frac{1}{2\mu} \pi_{\vec{x}}^2 + \frac{\kappa}{2} q_{\vec{x}}^2 + \frac{\mathcal{Y}}{2} (\nabla q_{\vec{x}})^2 \right] dV} \quad (\text{B.13})$$

as our model Hamiltonian.

Using Hamilton's equations, we can now obtain the equations of motion, i.e. calculate the time derivatives of the  $qs$  and  $\pi s$ :

$$\dot{q} = \frac{\partial H}{\partial(\pi_{\vec{x}'})} = \frac{\pi}{\mu} \quad (\text{B.14})$$

$$\dot{\pi} = -\frac{\partial H}{\partial(q_{\vec{x}'})} = -\kappa q + \mathcal{Y}\nabla^2 q \quad (\text{B.15})$$

Taking a time derivative of Eq.(B.14) and substituting Eq.(B.15) into the result we find:

$$\ddot{q} - \frac{\mathcal{Y}}{\mu}\nabla^2 q + \frac{\kappa}{\mu}q = 0 \quad (\text{B.16})$$

This almost looks like the K-G equation, and we can make the resemblance even clearer by comparing the coefficients with the usual form of the K-G equation

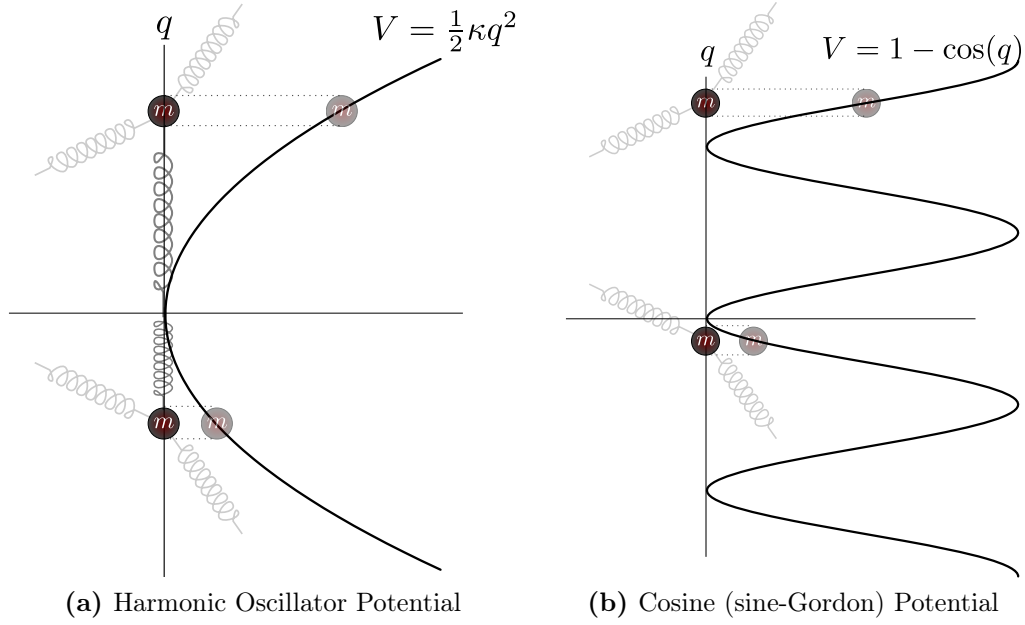
$$\frac{\partial^2 \psi}{\partial t^2} - c^2 \nabla^2 \psi + \Omega^2 \psi = 0 \quad (\text{B.17})$$

and defining

$$\psi \equiv q$$

$$c^2 \equiv \frac{\mathcal{Y}}{\mu}$$

$$\Omega^2 \equiv \frac{\kappa}{\mu}$$



**Fig. B.2:** Two possible potentials for the generalized K-G equation

However, the K-G equation has *partial* time derivatives, while our equation contains *total* time derivatives. The two only coincide under certain circumstances. In that special case we finally find

$$\boxed{\frac{\partial^2 \psi}{\partial t^2} - \frac{\mathcal{Y}}{\mu} \nabla^2 \psi + \frac{\kappa}{\mu} \psi = 0} \quad (\text{B.18})$$

We can also easily extend the equation to potentials more general than that of the simple HO, by rewriting Eq.(B.13) as

$$H = \int_{\vec{x}} \left[ \frac{1}{2\mu} \pi^2 + V(q^2) + \frac{\mathcal{Y}}{2} (\nabla q)^2 \right] dV \quad (\text{B.19})$$

where for our original model  $V(q^2) = \frac{1}{2} \kappa q^2$  (see Fig.B.2a).

Changing the potential to  $V(q^2) = 1 - \cos(q)$  will result in the sine-Gordon Equation (Fig.B.2b) for which Scott [112] has shown classical applications. This non-linear equation exhibits very interesting features, such as solitonic solutions, that appear to become increasingly important in areas such as molecular and cellular biology [113].



## Appendix C

### QM Formalism built-in Time Integration

By virtue of the complex nature of the wave function and the mod-squared rule, QM appears to have a built-in time averaging over high frequencies. We can demonstrate this using the following simple example. Ignoring polarization, we can represent some signal  $E(t)$  using the complex representation

$$\tilde{E} = \sum_n a_n e^{2\pi i \nu_n t} \quad (\text{C.1})$$

where  $E = \text{Re}\{\tilde{E}\}$ . Taking the mod-squared of this, which incidentally uses both - the real and the complex - parts, results in an output varying in time at all the difference frequencies.

$$|\tilde{E}|^2 = \sum_n a_n^2 + 2 \sum_{n < m} a_m a_n \cos 2\pi(\nu_m - \nu_n)t \quad (\text{C.2})$$

Particularly in the optical domain, detector signals often resemble this, which suggests that the mod-squared operation encodes *some* sort of physical process.

We may analyze the same situation using real math only, representing the amplitude and square of the signal as

$$E = \sum_n a_n \cos 2\pi\nu_n t \quad (\text{C.3})$$

$$\begin{aligned} E^2 = & \sum_n a_n^2 (1 + \cos 4\pi\nu_n t) \\ & + \sum_{n < m} a_m a_n [\cos 2\pi(\nu_m + \nu_n)t + \cos 2\pi(\nu_m - \nu_n)t] \end{aligned} \quad (\text{C.4})$$

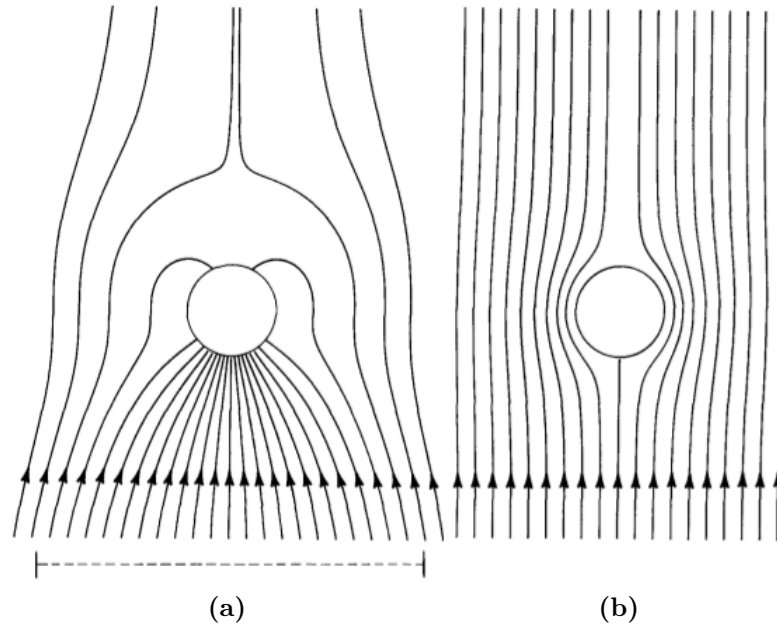
Comparing Eqs.(C.2) and (C.4) we find that they essentially coincide if we time average Eq.(C.4) over the high frequency terms. We conclude that the mod-squared operation encodes a short time averaging.

## Appendix D

### Absorption Cross Section

Absorption experiments done at low energy density indicate a *suction* (by the particle), or *pushing* (by the field) effect: the amount of energy absorbed from the propagating EM waves appears to exceed that intercepted by the geometric cross section of the absorbing atom or molecule (Fig.D.1). Classical and semi-classical theoretical investigations find this effect due to the detector's interaction with the superposition of the fields incident on and those (re-)emitted by the detector [114,115]. The semi-classical treatment additionally finds the intensity dependence of this behavior: with increasing energy density, the effective cross section decreases [114], due to saturation.

A quick back-of-the-envelope calculation for free atoms in a gaseous state will illustrate this enhanced absorption effect. For instance, electronic transitions happen nearly instantaneously, and for a resonant atom to absorb a red photon from a  $1mW$ ,  $1mm$  diameter laser beam [101], it would have to have a radius of about  $55\mu m$ . For comparison, an excited hydrogen atom in its  $n = 137$  state has an atomic radius of about  $1\mu m$ , while 'regular' atoms have radii on the order



**Fig. D.1:** Pointing vector flow (field lines) in the vicinity of a particle at resonance (a), and not at resonance (b) with the impinging EM field. The particle at resonance has an effective cross section 18 times larger than its geometric cross section. Source: Bohren, 1982 [115]

of angstroms. This implies that, frequency-resonant atoms have the ability to increase their interactive volume by at least 17 orders of magnitude!

Hence, a resonantly excited (free) atom may present a very large effective cross-section for harnessing EM energy from its surroundings. This can probably not happen when the atom finds itself confined within the restricted solid state environment. Thus, optical solid state detectors need higher EM energy flux as they become smaller, while *nuclei* in such detectors can offer large cross sections to resonant  $\gamma$ -rays, due to the enormous amount of space between nuclei even in such environments.

## Appendix E

### Some problematic aspects of current interpretations of QM

#### E.1 The mathematical uncertainty Principle

Begin with some function  $f(x)$  of position  $x$  that has a value appreciably different from zero concentrated within a region  $x \in (-a, a)$ . For definiteness we will take  $f(x) = \frac{1}{c\sqrt{\pi}}e^{-x^2/c^2}$  which has a peak at  $x = 0$  and does not differ much from zero<sup>1</sup> for  $|x| > a \approx 2c$ . Since this represents a localized “bump” we may *define* its position as a weighted average

$$\text{position} \equiv \int_{-\infty}^{\infty} xf(x - x_0)dx = x_0 \quad (\text{E.1})$$

So we see that this corresponds to an OK definition of “position”, because this average shifts by  $x_0$  if we translate  $f(x)$  by  $x_0$ . We could now decide to expand  $f(x)$  as an infinite sum of infinitely extended oscillatory functions  $e^{-ikx}$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} c(k)e^{-ikx} dk \quad (\text{E.2})$$

---

<sup>1</sup>For  $|x| > 2c$ ,  $f < 4\%$

where we determine<sup>2</sup>  $c(k)$  as follows

$$c(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x') e^{ikx'} dx' \quad (\text{E.3})$$

$$= \frac{1}{\sqrt{2\pi}} e^{-k^2/d^2} \quad (\text{E.4})$$

where  $d \equiv \frac{2}{c}$ . We find that the coefficients needed to represent our original function in terms of these basis functions appear also localized around the associated  $k$  parameter. In fact, comparing the standard deviations ( $b$  and  $c$ ), it turns out that their product always remains constant:

$$cb = 2 \quad (\text{E.5})$$

By changing definitions suitably, we can make the constant on the right anything we want. As mentioned earlier,  $c$  gives a measure of the spatial localization of  $f(x)$ : the greater  $c$ , the more spread out  $f(x)$ . Similarly,  $b$  measures how much  $c(k)$  spreads. Call  $c \equiv \Delta x$  and  $b \equiv \Delta k$ . We often call this phenomenon the “uncertainty principle”: if you make  $f(x)$  less spread out, you need a wider  $k$  range to represent it in the  $e^{-ikx}$  basis. However, we needed no physical (experimental) input to derive this, thus it represents a fact of mathematics.

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<sup>2</sup>the Fourier transform of  $f(x)$

## E.2 Measurement Uncertainty

Philosophical debates aside, I take the view that the universe and everything in it exists even when we do not look at or measure it; not only in the case of macroscopic objects like the moon [116], but also microscopic and submicroscopic ones, like elementary and other particles. It follows that everything has a well-defined existence, regardless of the presence of conscious observers or measurement apparatuses.

At a very basic level, the universe and all its constituents “just exist” and interact with one another in ways that we as scientists try to understand and quantify. However, it often appears difficult to remember that we constitute a part of the system we seek to understand, and not some external objective observer. As such, we have some inherent and severe limitations that we should acknowledge, as well as realize that in all likelihood there may exist no ways around them.

By the term “measurement” we generally refer to an *effect* created in one object as a result of an interaction between it and something else. We focus on one limited set of properties of our reference object (the detector), and note how they change due to the interaction. Furthermore, only a limited set of aspects of the entity we set out to measure actually influence the properties we decided to monitor. This consideration allows for the following two conclusions: i) detectors do not measure physical objects or their states - they only measure a *subset* of the properties of said object; ii) no inherent difference exists between the detector

and the detectee - changes take place in both during the interaction.

In turn, these two realizations alone showcase two different types of uncertainty inherent in all measurements, no matter how carefully executed: i) with every measurement we remain completely uncertain with respect to the parts of the object that our detector does not interact with, or interacts only in a way that we did not think of monitoring; ii) even if we knew how the measurement process influenced the aspects of the object that we did set out to measure, we do not know whether and how it influenced the other properties.

I may now comment on the century old question of the “measurement problem” and the “collapse of the wave function” rather pragmatically. We can only acquire limited knowledge about the world around us and use the tool of the wave function and the QM formalism to *model* and *represent* this knowledge and the associated processes. While it appears often tempting, we have to make sure not to equate the mathematical tool, for example that of the wave function, with “the real thing”<sup>3</sup>, and conclusions drawn from measurements and represented using our mathematical models need to take this fact into account very carefully.

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<sup>3</sup>at the risk of sounding a bit Kantian



### E.3 The Free Particle

Early on, introductory texts [15] on QM often discuss the free particle<sup>4</sup>, its normal mode solutions and the problem they pose. As Schrödinger points out [69], the associated 4th order equation corresponds to that which also models the vibration of plates [117]. In the case of infinite spatial extent, the eigenvalue solutions correspond to sines and/or cosines<sup>5</sup>. Obviously no *one* of these solutions can represent a free particle, mainly due to the lack of localization.

The course of action to rescue the free-particle interpretation of the proposed free particle Hamiltonian: initial conditions. Essentially, the proposal corresponds to *making* a localized initial condition by hand, calling *it* “the particle”, and then using  $\hat{H}$  to time evolve. Paired with a Fourier transform based procedure<sup>6</sup> to construct the initial conditions in position space from other initial conditions in Fourier-conjugate space<sup>7</sup> and many words [15], one almost has the feeling that this could work.

However, this approach explains very little in terms of physics: we can put together *any* initial conditions we like this way. Do they all correspond to particles? We find it almost impossible to try and visualize the physical process behind all this. Perhaps Feynman also tried to picture the possible processes, and

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<sup>4</sup>importing Hamiltonians from CM via operator substitution, as usually done, gives us  $\hat{H} \equiv -\frac{\hbar^2}{2m}\nabla^2$

<sup>5</sup>often, but somewhat misleadingly, referred to as “plane waves”

<sup>6</sup>As shown in Appendix E.1.

<sup>7</sup>“momentum space”

consequently felt prompted to state that “I think I can safely say that nobody understands quantum mechanics” [65]. It appears that this issue may lie at the very heart of our difficulties with QM.

#### E.4 Locality and Causality

In physics we hold the well-established belief<sup>8</sup> that everything<sup>9</sup> in the universe moves with a finite<sup>10</sup> velocity. Hence, any influence one entity may exert on another must travel from the former to the latter (locality) in a finite time-interval (causality). Claims of non-local interactions<sup>11</sup> remain heavily debated.

In addition, I claim that if non-local or non-causal interactions indeed occurred on the micro scale, it would have consequences for our “every day” observations and corresponding results would have not gone unnoticed. Schrödinger came up with his much abused cat story [9] to exemplify the absurdity of the implied consequences. Despite much propaganda to the contrary, owners of dead-and-alive cats remain unobserved. Non-causality and non-locality generally enter our *equations* as a consequence of certain *mathematical* techniques<sup>12</sup>. Upon inspection we find that these techniques feature mathematical parameters and operations that quite obviously do not match quantities or processes observed in nature, as

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<sup>8</sup>Founded upon and firmly grounded in experimental evidence!

<sup>9</sup>such as material particles and field perturbations

<sup>10</sup>The generally accepted and well known speed limit corresponds to that of the speed of light,  $c = (\mu_0 \epsilon_0)^{-\frac{1}{2}}$

<sup>11</sup>such as certain entanglement-related experimental conclusions

<sup>12</sup>such as the use of Fourier transforms

demonstrated in Appendix E.1.

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