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Money Within a Direct Search Framework

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Ping M. Ang

University of Connecticut, 2016

ABSTRACT

Chapter 1 shows that total anonymity in matching is not a necessary condition to give rise to the use of money, contrary to the existing literature, which finds that total anonymity is required. The first paper presents a simple model of direct money search in a decentralized environment. This paper presents three interesting findings: (i) partial anonymity is sufficient for money to be useful, (ii) commodity money does not exist in symmetric direct money search at steady state, (iii) bartering and commodity money can co-exist in the asymmetric case. The introduction of fiat money drives out bartering and commodity money, consistent with previous studies.

The second chapter presents two thought-provoking results on money search models. For random money search models, it has been taken for granted that monetary equilibrium unquestionably holds if the number types of good is greater than three. This chapter reports that monetary equilibrium in random money search models fail to exist when the number of types of goods is significantly greater than three. An increasing number of types of goods reduces the matching probability and expected utility, ultimately approaching zero as the number of types of goods approaches infinity. A quasi-direct search model is presented as a solution for this problem. This leads to another more thought-provoking result: a connection between two major macroeconomics models, namely cash-in-advance and money search models.

In Chapter 3, a search-theoretic model is used to show that money is a necessary instrument to clear the market. In decentralized markets with imperfect information, money assumes the role of a perfectly informed Walrasian auctioneer. This chapter presents a novel framework that utilizes money to clear the market in a decentralized environment, in contrast to Lagos and Wright (2005), where a centralized price mechanism clears the market. Without the use of money, markets with imperfect information are almost impossible to clear. Markets can exist without money, and the market is not a substitute for money.

Money within a Direct Search Framework

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APPROVAL PAGE

Doctor of Philosophy Dissertation

Money within a Direct Search Framework

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Contents

Summary	ix
1 : A Simple Model of Direct Money Search	1
1.1 Introduction	1
1.2 Model	4
1.2.1 Baseline	4
1.2.2 Location and Information	5
1.2.3 Search and Match Mechanism	7
1.2.4 Strategies and Equilibrium	10
1.3 Autarkic	12
1.3.1 Symmetric Case	12
1.3.2 Asymmetric Case	14
1.4 Barter and Commodity Money	14
1.4.1 Symmetric Case	14
1.4.2 Asymmetric Case	18
1.5 Fiat Money	20
1.5.1 Symmetric Case	21
1.5.2 Asymmetric Case	22
1.6 Conclusion	23

2 : Stability of Monetary Equilibrium in a Direct Search Model	24
2.1 Introduction	24
2.2 The Model	26
2.2.1 The Environment	27
2.2.2 Search Process / Mechanisms	28
2.2.3 Value Functions and Equilibrium	30
2.3 Non-monetary Equilibrium	32
2.4 Monetary Equilibrium	34
2.5 Conclusion	38
 3 : Money for Market Clearance in Decentralized Markets	 42
3.1 Introduction	42
3.2 Model	44
3.3 Barter and Market	46
3.4 Monetary Equilibrium	50
3.5 Conclusion	57
 Appendices	 59
 Bibliography	 77

List of Figures

1.1	Marketplaces and travel paths among agents in a direct search.	6
1.2	Example of a perfect frictionless exchange where every agent engages in a successful barter and the market clears.	15
1.3	Symmetric bartering exchange.	16
1.4	Asymmetric bartering exchange.	18
2.1	The roles and strategies of agents.	29
2.2	A comparison of utility gained in accepting money between random and di- rected money search frameworks, where k denotes the number of good types.	39
2.3	Expected utility for different searching strategies of a moneyholder at steady state in non-cooperative directed money search, where k denotes the number of good types.	39
2.4	Expected utility at steady state for value functions in non-cooperative directed money search, where k denotes the number of good types.	40
3.1	A diagram showing the role of agents and the strategies for each sub-period within periods $t - 1, t$, and $t + 1$	46
3.2	The effect of k on market clearance in a monetized market and a bartering market, where k denotes the number of good types.	58

List of Tables

1	Comparison of models.	xiii
2.1	A table showing the sensitivity analysis for the expected utility of holding money with different paramater values for r , m , c_1 , c_m and x	41

Summary

These three papers have a unifying theme: they present direct money search models where money matters. This approach differs from the existing literature, which has relied heavily on random search models of the role of money. The definition of direct search here means that an agent directly approaches another agent who has the good he wants. The existing literature commonly uses random search or partially directed search to show how money matters in the economy. Fully direct search has not been presented in the monetary economics literature.

The first paper explicitly shows how a person would approach a specific supplier to acquire a desired good. Like Kiyotaki and Wright (1989), henceforth KW, and Goldberg (2007), I allow three types of goods in the simple model. Some of the results in this first paper mirror their earlier results, but some differ. For example, I find that no commodity money equilibrium exists in a symmetric direct search environment, unlike when search is non-direct. This is because the non-direct search models assume that each agent cannot consume his own production, where the type of consumption good and the type of production good are fixed for each agent. In that environment, *quid pro quo* trade may emerge, which means that goods or services are exchanged with one transfer contingent on the other. Non-random models can lead to a commodity being used as a medium of exchange, because a person might be better off if he temporarily sacrifices an immediate gain and accepts an unwanted good that later can be exchanged for a good he wants. That scenario is not possible with direct search, because an agent would have the same chance of getting his desired good by

holding his own good or any other good, without having to rely on a *quid pro quo* exchange. The primary purpose of this first paper is to clearly show the direct search mechanism in a simple model that focuses on both symmetric and asymmetric steady-state equilibria. The major finding is that commodity money does not exist in the symmetric direct money search model at steady state, but bartering exists. The role of fiat money can still be explicitly modeled when direct money search is introduced, but it drives out bartering and commodity money. Fiat monetary equilibrium exists in both symmetric and asymmetric cases.

In the second paper, the importance of the variety of goods in establishing the role of money is investigated. It has been taken for granted that a fiat money equilibrium always exists, provided that the number of types of goods exceeds three. This paper shows this is not necessarily true. In addition, agents in the first paper cannot hold a unit of a good and a unit of money at the same time, but this second paper relaxes that restriction. This structure is emphasized because holding money should not prevent or hinder production. Money is allowed to be a true medium of exchange, free from the production constraint. Even if a person does not accept money, he may be able to obtain his desired good by bartering. In KW, if a money-holder cannot meet someone else who will accept his money, there is no other way to obtain his desired good, such as bartering with his own good, because each agent can only hold either money or a good in each period. The second paper also extends the number of goods from three to any arbitrarily large number. While the first paper draws on the work of Kiyotaki and Wright (1989), this paper is best compared to a later paper by Kiyotaki and Wright (1993). This chapter finds that a consistency problem arises with random search when a large number of goods is assumed. When the types of goods increase in a random search environment, the willingness to accept money decreases, but I show that direct search may solve this problem. When the types of goods increase in direct search, the willingness to accept money increases, which is more consistent with our intuition and real world observations. The second paper also presents an interesting technical finding that has not been shown in existing money search models: a link between cash-in-

advance models and money search models. This integration of two major macroeconomic models is incomplete, since I assume that both money and goods are indivisible, while the cash-in-advance model assumes divisibility in both goods and money. Nevertheless, this link may serve as a beginning for a proper integration. Such integration is potentially important because the underlying foundation of cash-in-advance is the concept of general equilibrium, where all markets potentially clear with a perfect relative price mechanism and the equilibrium has other desirable attributes. But the conventional general equilibrium model is a poor one for understanding the fundamental nature and role of money, while previous money search models have lacked a mechanism for determining relative prices and clearing markets. Since market clearance is one of the important reasons to integrate the cash-in-advance and money search models, my third paper suggests a way to achieve market clearance in a money search model. This chapter is short but perhaps important. If a market potentially clears within one period, this feature can be extended to multiple periods for sequential equilibria, permitting analysis of macroeconomic and monetary policies within a dynamic setting. Currently, there are attempts to achieve market clearance within one period using a money search model. For example, Lagos and Wright (2006) have modeled market clearance within a period by introducing two sub-periods: one is a centralized market with a general good and the other is a decentralized market with specific goods. It is a move in the direction of merging monetary theory and general equilibrium analysis.

An attempt is made in this third paper to achieve market clearance within a single period, in the sense that everyone may obtain the good they desire, but this process is made price-independent by assuming the price of each good is fixed (or that there is no price). It differs from Lagos and Wright's (2006) model by only having decentralized markets, and it captures the notion of market clearance without having to compromise on its flexibility or the notion that money matters and is used to facilitate trade among agents. Departing from the traditional models, where a Walrasian auctioneer clears the market given perfect information, money is a necessary instrument to coordinate market clearing in a decentralized

market system with imperfect information.

	Number of good types	Sub-period	Searcher and Stayer	Asymmetric case	Commodity Money	Market clears within One period
Chapter 1	3	No	Not Fixed	Yes	Yes	No
Chapter 2	$\{2, 3, \dots, \infty\}$	No	Not Fixed	No	No	No
Chapter 3	$\{2, 3, \dots, \infty\}$	Yes	Fixed	No	No	Yes

Table 1: Comparison of models.

Chapter 1 :

A Simple Model of Direct Money Search

1.1 Introduction

Money search theory has progressed significantly from the seminal random search paper of Kiyotaki and Wright (1989), henceforth KW, to the cooperative directed search approach of Corbae, Temzelides and Wright (2003), to the partial directed search model of Goldberg (2007). For several reasons, monetary theorists have sought to develop a fully direct search model of money.^{1.1} First, with random matching, who would lend money to a borrower if he does not even know if he will ever meet that agent again? Second, it is difficult to imagine that our daily transactions could be conducted in a random way to obtain the things we want. Finally, Howitt (2003) has simply noted that random search is unusual in the real world. The challenge, one that is addressed in this paper, is to model credit where the role of money is also explicit in the model.

Total anonymity (or randomness) as a prerequisite for establishing the role of money in search, as suggested by Korchelakota (1999), appears to be unnecessary in the following model. In this direct search approach, only some degree of anonymity is needed to ensure a useful role for money. This anonymity or randomness arises from imperfect information

^{1.1}This desire to formulate a direct search model of money has been mentioned by KW, Wallace (2007), Goldberg (2007), Williamson and Wright (2013), Wallace (2013) and others.

about the preferences of other agents. Unlike the previous money search literature, the idea of *quid pro quo* in exchange does not arise in a symmetric case in this direct search model. This feature distinguishes this paper from earlier money search papers.

Moving toward direct search in a monetary model is important because of the inability to explicitly incorporate money in existing economic models. One of the solutions stemming from the Lucas critique is to inject micro-foundations into macroeconomic models, however money is not explicit in those models, as noted by Wallace (2008). Yet, money is important to the economy and should not be overlooked or omitted. Several ideas have been put forth to address this issue. Patinkin (1950) suggested including money in the utility function, Clower (1967) introduced the cash-in-advance approach, and both Townsend's (1980) turnpike model and Wallace's (1978) overlapping-generations model also address this problem. Yet none of these earlier models explicitly incorporated money as a medium of exchange.

Using search theory, Kiyotaki and Wright (1989, 1993) made an important breakthrough in the monetary literature by explicitly incorporating money as a medium of exchange, but their models involved random search. The literature has progressed from their fully random approach to Goldberg's (2007) partially directed search model. However, non-direct search (whether random or partially directed) imposes a hurdle to fully integrating money into economic models, because non-direct search models must assume total anonymity for money to be essential. This total anonymity in money search theory leads many to question the validity and significance of this line of research, as noted by Goldberg (2007). One of the strongest criticisms of the total anonymity assumption is that it prevents further development of an explicit model of credit, as noted by Wallace (2012). For example, would you loan money to anyone, not knowing if or when you will meet him again in the future or when you do not know who the next person will be? This is the "credit problem" in the random search literature. But, the problem does not end here, because it also prevents modeling loans and banking, including the role of a central bank. This study starts with simple but essential questions of why money matters, why it also matters in credit, and how

money can be explicitly modeled using direct search.

Another basic criticism of random search, by Howitt (2005), is that people normally do not randomly meet others to obtain the goods they want. They commonly know where to go to get specific goods. If we know a particular seller does not have the good we want, we simply would not visit that seller. Randomness in trade matching only makes sense if individuals meet once and are very unlikely to meet again in the future. Examples of such random matching may exist (ancient Silk Road trading, exchange in Venice or other historically important trading ports, contemporary swap meets, etc.), where traders come and go without knowing if they will ever meet again, but such anonymous trading encounters are not the norm.

In the conventional random search setup, where consumption and production type are individual specific, knowing a person's production immediately reveals his consumption preference. This assumption of fixed consumption and production types hinders the modeling of direct money search, because it eliminates the double coincidence of wants. When you search directly, knowing the seller type also gives you additional information about the type of good he wants. Thus, if you do not hold the good he wants, you would not approach him. You only approach him if you have the good he wants and you also want the good he holds. This ensures the double coincidence of wants and eliminates the need for money. The absence of double coincidence of wants is a necessary condition for money to serve as a medium of exchange. Thus, the random search approach makes it hard to properly explore the emergence of money and its economic role.

This first chapter addresses a fundamental question: Can money be explicitly modeled in a direct search framework? Some simple modifications are made. I maintain the setup of fixed production types, but drop the assumptions of fixed consumption types and random search by generalizing consumption and introducing direct search with only three goods. The restriction to three goods makes it easier to compare results with those of Kiyotaki and Wright (1989) and Goldberg (2007), who also made this assumption.

In Section 1.2 of this chapter, the basic model is described. In Section 1.3, we investigate the condition where an autarkic equilibrium can emerge in asymmetric and symmetric cases. In Section 1.4, we investigate the equilibrium for barter and commodity money under symmetric and asymmetric cases. Fiat money is introduced into the model in Section 1.5 and Section 1.6 concludes.

1.2 Model

This paper is somewhat closer in structure to Goldberg (2007) than KW, but like both of those papers, the variety of goods is limited to three to keep the model simple and to facilitate the comparison of results.

1.2.1 Baseline

A continuum of infinitely lived agents with unit mass face an environment that contains a set of only three goods, $I = \{1, 2, 3\}$. Besides the three goods, there is an intrinsically worthless and inconvertible object called “fiat money”, introduced later in Section 1.5. All three goods and money are indivisible and exist in units of one. Each of the three goods has the same price, or the three goods always trade on a one-for-one basis with no reference to any price. To facilitate the understanding of why we want to hold money in the exchanges of goods, prices are not discussed here. Trade is bilateral, mutual, and unforced.

Each agent has three roles: (i) producer, (ii) consumer, and (iii) holder of a good. Henceforth, for simplicity, a producer of good type i is called an “ i -producer”, a consumer of good type i is called an “ i -consumer”, and a holder of good type i is called an “ i -holder”. In his role as a producer, each agent has a fixed type of production and engages in a specialized production of good type i in each period. His role as a consumer consists of drawing a “taste” or type of good he wants to consume and he engages in a generalized consumption of all type of goods. Each agent initially draws a taste shock from a uniform distribution, and he

repeats this process after consuming a desired good. If he does not acquire his desired good, his taste remains the same until he acquires and consumes his desired good. At that point, a new taste shock is drawn. Once a good is in his possession, he becomes a holder of that good and also can choose to hold that good rather than consuming or trading it away.

Let V_{ijk} be the value function of an agent who produces good i , wants to consume good j , and holds good k , where $i, j, k \in I$. Agents receive utility $u > 0$ from consuming their desired good and disutility $d \geq 0$ from production, as well as incurring a transaction cost $\tau > 0$ when an agent exchanges a good. An agent can only produce after consuming the good he desires. An agent produces another unit of his output only after consuming his desired good. If an agent draws a taste shock to consume the good he produces, he will immediately consume that good and draw another taste shock.^{1,2} So, each agent will end up wanting to consume a good $i^* \neq i$, where $i^* \in \{j, k\}$, which he cannot produce. He must therefore rely on trade to acquire the desired good. Each agent can only hold either one unit of production or one unit of money at a time. Money is loosely conceived in this section because it can be commodity money (which is accepting one of the commodities as money) or fiat money (where a fraction of m agents is endowed with one unit of fiat money).

1.2.2 Location and Information

There are three potential locations to visit and each is called a marketplace. Each agent lives at home and his home is also his “factory” or production site. Each marketplace is

^{1,2}Let V_{iii} be the value function of an agent who draws a desire to consume his own output i , such that

$$V_{iii} = \frac{1}{3}(u - d + V_{iii}) + \frac{2}{3}V_{iji}$$

where $j \neq i$. This means he has a probability of $\frac{1}{3}$ wanting to consume his own output, which provides a net utility of $u - d$ before drawing another taste shock and $\frac{2}{3}$ wanting to consume something other than his own output. Let the expected net utility after acquiring his consumption be $U = u + \frac{1}{6}(u - d) - \tau$, then $V_{iii} = U - u + \tau - V_{iji}$. $V_{iii} = u - d + \frac{1}{3}(V_{iii} + V_{iji} + V_{iki})$, $V_{iii} = \frac{1}{2}(u - d) + \frac{1}{2}(V_{iji} + V_{iki})$. Then the value of acquiring the good he desires $u + \frac{1}{3}(V_{iii} + V_{iji} + V_{iki})$ will be $u + \frac{1}{6}(u - d) + \frac{1}{2}(V_{iji} + V_{iki})$, which can be simplified to:

$$U + \frac{1}{2}(V_{iji} + V_{iki}).$$

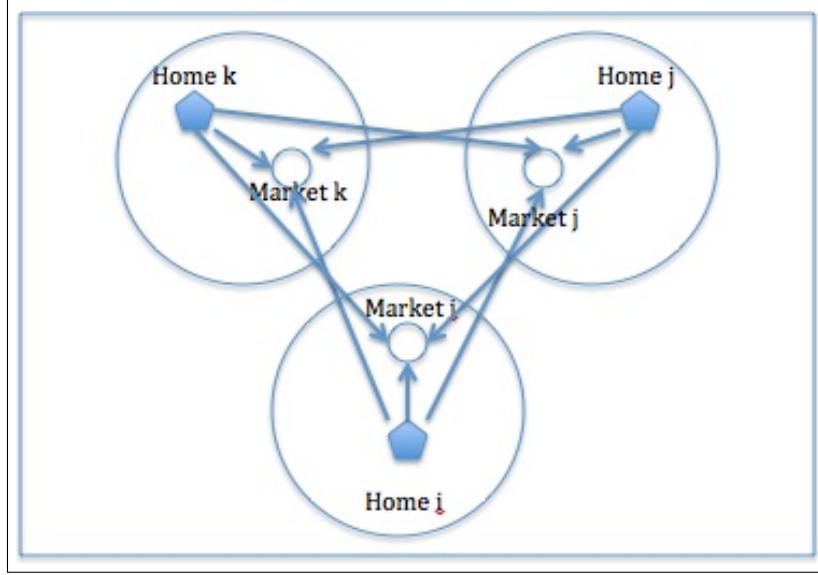


Figure 1.1: Marketplaces and travel paths among agents in a direct search.

a location where a specific type of good is sold, and the marketplace is close to the home, so the cost of taking his production to market is negligible. Unlike in Goldberg (2007), the marketplace is separate from the home. This set-up avoids agents of the same type knowing what good their neighbors hold when they are at home, and it avoids trading among themselves without visiting a marketplace if they hold different goods. An i -producer must go to market i to sell his produced good, which is the marketplace that allows i -producers holding good i to gather and sell. People who visit market i can only trade for good i .

The idea of such a market can be related to having a good-specific place to facilitate search and exchange. If a market were to allow other than its own production good to be sold, it would become a location where a seller could be selling any type of good. People would randomly search to get their good, and if this occurs, the environment would resemble the one in random search models. It is hard to imagine that a fisherman wants a carpenter, cobbler, or mechanic to sell their goods or services in his fish market. This sort of "mixed marketplace" is not allowed in this model; only good i can be sold at market i . A marketplace is a location of exchange for suppliers offering one specific type of good only. If a supplier does not hold the good of his own production type, he can still visit his own marketplace,

but he is not allowed to offer other goods for exchange. Only an i -type is allowed to be a supplier at marketplace i to offer good i for trade. If an i -producer holds good $i^* \neq i$, he cannot offer it for trade at market i .

Everyone knows the location of each market and the type of product being sold there. Everyone also knows his own taste, but not other agents' tastes. This differs significantly from KW and Goldberg (2007). Hence, an agent's current taste is private information, which is only revealed to another agent after they engage in a bilateral meeting (trade need not occur). To ensure the absence of double coincidence of wants, even the type of good his partner wants is revealed to him after they meet and have parted, so he could approach her again in the future when he has her desired good, hoping for a guaranteed barter exchange.^{1.3} However, if she later wants a different good because she already acquired her desired good and drew a new taste shock,^{1.4} a double-coincidence of wants for a barter exchange may not occur. For this reason, there are only double coincidence of wants and single coincidence of wants in this model.

Goldberg (2007) assumes that agents do not know which shops are available and the types of goods being offered. His assumption is crucial for his model because it motivates the lack of double coincidence of wants and gives rise to the need for a medium of exchange. I will investigate dropping this assumption to create a more realistic environment, where agents know which shops and goods are available. Every agent in my model knows this information.

1.2.3 Search and Match Mechanism

After establishing a location, each agent needs to choose which market he wants to visit. If Agent i who produces i , goes to market i to wait for visitors, he is called a "stayer" at his

^{1.3}To facilitate discussion and avoid confusion, I use a masculine pronoun for one agent and a feminine pronoun for his trading partner.

^{1.4}A larger number of good types accentuates the absence of double coincidence of wants. For further discussion of this issue, see chapter 2 of this dissertation. The current chapter only focuses on symmetric equilibria and how the search mechanism works in a simple model, so that results can be compared to those of KW and Goldberg (2007).

own marketplace. If Agent i goes to market j or k to look for goods, he is called a “searcher”. There is no travel cost going to his own market i because the market is very near and travel cost becomes negligible.^{1.5} However, a travel cost, $c_i \geq 0$, is incurred when he travels from his home to markets other than his own. An agent must return to his home at the end of each period, and another travel cost is incurred on the way home. So total travel cost of any visit is $2c_i$. No agent is allowed to consume goods in any market in the model; he must return home to consume.^{1.6}

In each period, a stayer always arrives before any visitors at his market because the market is closer to him, as depicted in Figure 1.1. If agents choose to search, then all searchers will arrive at the same time at the market^{1.7}. Stayers at any particular market are *ex-ante* identical, holding the same type of good. Recall that information about the type of good a stayer wants to consume in that particular period is unknown or uncertain to searchers visiting the market. If searchers know this piece of information, the friction of matching vanishes and it becomes a Walrasian market.

All searchers arrive at a market at the same time and all the stayers are identical to them because all stayers sell identical goods. Searchers would choose randomly which stayer to meet bilaterally because it makes no difference in terms of outcome. He knows that anyone he approaches in that market will provide the particular good he wants in exchange for the good he holds. No one can meet more than once each period and all agents will go home and wait for the next period after each bilateral meeting.

In the bilateral meeting, agents are only allowed to meet one partner. The arrival rate, α_{ii} , is the chance that Agent i stays in his own market i and meets with an arriving visitor for possible trade. The arrival rate, α_{ii^*} , where $i^* \neq i$, is the chance that Agent i searches at a different market i^* to meet a stayer for possible trade. The arrival rates, $\alpha_{ii}, \alpha_{ii^*} =$

^{1.5}Alternatively, we could assume that travel costs of going to other markets are normalized to the travel cost to his own market.

^{1.6}Goldberg (2007) allows consumption on the spot, but that assumption would not affect the equilibrium results that follow.

^{1.7}It would be interesting to explore the effects of different market arrival times by specifying a circular geography with different distances between sites.

$\alpha(n_{12}, n_{13}, n_{21}, n_{23}, n_{31}, n_{32})$, depend on the proportions each type of agent who search (n_{ij} where $i, j = 1, 2, 3$ and $i \neq j$). No more than one searcher will visit a stayer at a time. Since each Agent i will want to consume either good j or k in each period, and the taste shock is drawn from a uniform distribution, half of the i -type agents will always want to consume good j and the other half will want to consume good k in the symmetric case. The arrival rates will depend on the proportions of each type of agent who search, as follows:

$$\alpha_{ii} = \begin{cases} \frac{n_{ji}+n_{ki}}{2-n_{ij}-n_{ik}} & \text{if } \frac{n_{ji}+n_{ki}}{2-n_{ij}-n_{ik}} < 1 \\ 1 & \text{if } \frac{n_{ji}+n_{ki}}{2-n_{ij}-n_{ik}} \geq 1 \end{cases} \quad (1.1)$$

$$\alpha_{ij} = \begin{cases} \frac{2-n_{ji}-n_{jk}}{n_{ij}+n_{kj}} & \text{if } \frac{2-n_{ji}-n_{jk}}{n_{ij}+n_{kj}} < 1 \\ 1 & \text{if } \frac{2-n_{ji}-n_{jk}}{n_{ij}+n_{kj}} \geq 1 \end{cases} \quad (1.2)$$

$$\alpha_{ik} = \begin{cases} \frac{2-n_{ki}-n_{kj}}{n_{ik}+n_{jk}} & \text{if } \frac{2-n_{ki}-n_{kj}}{n_{ik}+n_{jk}} < 1 \\ 1 & \text{if } \frac{2-n_{ki}-n_{kj}}{n_{ik}+n_{jk}} \geq 1 \end{cases} \quad (1.3)$$

In 1.1, if Agent i stays at this marketplace, he will meet a visitor with probability equal to the ratio of the people ($n_{ji} + n_{ki}$) who produce good j and k to the number of stayers ($2 - n_{ij} - n_{ik}$).

When visitors equal or exceed stayers, the arrival rate is one, $\alpha_{ii} = 1$. It means each stayer will definitely meet with one searcher. The stayers will conduct a lottery as to which ones the visitors will meet, for the case where visitors exceed stayers. When there are fewer visitors than stayers, the arrival rate is the ratio of visitors to stayers. The searchers will conduct a lottery as to whom to meet. In (1.2), Agent i meets a stayer in market j with probability equal to the ratio of the proportion of Agent j stayers to the proportion of agents who visit market j . If stayers exceed visitors, then the arrival rate is one for Agent i visiting market j , $\alpha_{ij} = 1$. If visitors exceed stayers, the visitors will conduct a lottery to determine who will have a chance to meet with the stayers. The third equation is similar to the second equation, so the explanation is omitted.

1.2.4 Strategies and Equilibrium

Following Goldberg (2007), agents choose two strategies: (i) location and (ii) trading. As for the location strategy, an agent chooses which market he wants to visit. V_{xyz} is the value function of an agent at the end of a period who produces good $x \in I$, wants to consume good $y \in I$, holds good $z \in I$, and who faces three location strategies: visit market i , market j , or market k , given that β is the discount factor:

$$V_{xyz} = \beta \max \{ \text{visit market } i, \text{visit market } j, \text{visit market } k \}. \quad (1.4)$$

After choosing his location strategy, he chooses his trading strategy if he gets to meet a supplier. He becomes a “seller” when when he exchanges his good for money (or for something he does not consume). He becomes a “buyer” when he exchanges his money (or something his partner does not consume) for goods. Otherwise, he becomes a “barterer” if both parties exchange goods for immediate consumption.

As in KW, with the imposition of a positive transaction cost τ , an agent will not trade if he is indifferent between trading two goods. He always accepts the good he wants to consume.

$$\begin{aligned} V_{iki} = & \max \beta \left\{ \alpha_{ii} \left[\frac{1}{2} (U + \frac{1}{2} (V_{iki} + V_{iji})) + \frac{1}{2} \max \{ V_{iki}, V_{ikj} \} \right] + (1 - \alpha_{ii}) V_{iki}, \right. \\ & - c_i + \alpha_{ij} \left[\frac{1}{2} V_{iki} + \frac{1}{2} \max \{ V_{iki}, V_{ikj} \} \right] + (1 - \alpha_{ij}) V_{iki} - c_i, \\ & - c_i + \alpha_{ik} \left[\frac{1}{2} (U + \frac{1}{2} (V_{iki} + V_{iji})) + \frac{1}{2} (\Pi_1 (U + \frac{1}{2} (V_{iki} + V_{iji}))) + (1 - \Pi_1) V_{iki} \right] \\ & \left. - (1 - \alpha_{ik}) V_{iki} - c \right\} \quad (1.5) \end{aligned}$$

The equation above shows that the expected utility of an agent who produces i , consumes k , and holds i , at the beginning of a period, with a discount factor β when he holds his own production good. The first strategy is to visit his own market i , where he meets a partner

with arrival rate α_{ii} , with probability $\frac{1}{2}$ that he gets the good he wants and with probability $\frac{1}{2}$ that he does not get the good, but has to decide if he wants to accept good j .

The second strategy is to visit market j , incurring cost c_i , where no stayer has the good he wants and he meets a partner with arrival rate α_{ij} , and with probability $\frac{1}{2}$ he meets with a partner that wants his good that he may want to trade, and with probability $\frac{1}{2}$ nothing happens. On his way back he incurs another transport cost c_i .

The third strategy is visiting market k , incurring cost c_i , where every stayer has the good he wants and he meets a partner with an arrival rate α_{ik} , but only with probability $\frac{1}{2}$ he meets a partner who wants to consume his good and will exchange with him; with probability $\frac{1}{2}$ she does not want to consume his good. However, if she is willing to accept his good, then he can obtain the good he wants to consume. On the return, he incurs another transport cost c_i .

$$\begin{aligned}
V_{ikj} = \max \beta \{ & V_{ikj} , \quad -c_i + \alpha_{ij} \left[\frac{1}{2} V_{ikj} + \frac{1}{2} \max \{ V_{iki} , V_{ikj} \} \right] + (1 - \alpha_{ij}) V_{iki} - c_i, \\
& -c_i + \alpha_{ik} \left[\frac{1}{2} (U + \frac{1}{2} (V_{iki} + V_{iji})) + \frac{1}{2} (\Pi_1 (U + \frac{1}{2} (V_{iki} + V_{iji}))) + (1 - \Pi_1) V_{iki} \right] \\
& - (1 - \alpha_{ik}) V_{iki} - c_i \}
\end{aligned} \quad (1.6)$$

Equation (1.6) shows the value function for Agent i at the beginning of a period when he wants to consume good k and holds good j . Market i does not allow any goods other than good i to be ‘sold’, so Agent i holding good j will not be able to trade at all going to market i as shown as the first strategy.

Since market i does not allow any other goods to be ‘sold’ except good i , then Agent i holding good j will not be able to trade at all going to market i , shown as the first strategy. The second strategy is going to market j where no stayer wants the good j brought by him because no stayer will want to trade for the same type of good j . The third strategy is going to market k where the stayer has the good he wants, with a transport cost c_j , and he will

readily exchange because market k only sells his consumption good. With the probability $\frac{1}{2}$ he meets with a partner who wants his good j , otherwise he meets a partner who does not want his good, times the arrival rate α_{ij} . Then he returns home with another transport cost, c_j .

Lemma 1.1. *Value functions are non-negative.*

Definition 1.1. *An equilibrium is a Nash equilibrium with a set of steady-state strategies for trading and location, where expected utility is maximized, given the strategies played by other agents, and its distribution results from the chosen strategies.*

There are two cases here: (i) symmetric and (ii) asymmetric. The symmetric case is considered, such that the transport costs for all goods are equal ($c_1 = c_2 = c_3 = c$). The asymmetric case is one where the transport costs are unequal and $c_1 < c_2 < c_3$.

1.3 Autarkic

There is an autarkic equilibrium where everyone stays at their own market, no one wants to visit a market other than their own. Consider the case where no one searches.

1.3.1 Symmetric Case

In equation [1.7], V_{131} denotes the value function of an agent who produces good 1, wants to consume good 3 and holds good 1. The agent has three strategies: (i) stay at his own market 1, (ii) go to market 2 or (iii) go to market 3. If he stays at his own market 1, no trade happens because no one visits him. If he searches, he is the only searcher in the market (either 1 or 2). If he goes to market 2, he incurs a transport cost. Since he wants to consume good 3 but market 2 only offers good 2, he will not get his desired consumption good. There are two possibilities in market 2: (1) if he meets a producer who wants to consume good 1, he can choose to accept good 2 by giving up his good 1 to become a holder of good 2 in the

next period, or (2) if he meets a producer who wants to consume good 3 and does not accept his good 1, nothing happens and he goes home. His value function would be:

$$V_{131} = \max \beta \{ V_{131}, -c + \frac{1}{2} (V_{131}) + \frac{1}{2} \max \{ V_{131}, V_{132} \} - c, -c + \frac{1}{2} (U + \frac{1}{2} (V_{131} + V_{121})) + \frac{1}{2} V_{131} - c \} \quad (1.7)$$

If he goes to market 3, he incurs the same transport cost as going to market 2, but market 3 only sells his desired consumption good. There are two possibilities in market 3: (1) he meets a producer who consumes good 1, so they will engage in barter exchange, or (2) he meets a producer who consumes good 2 in market 3, so they trade if she is willing to accept his good 1, otherwise no trade happens and he goes home.

In equation [1.8], V_{132} denotes the value function of an agent who produces good 1, desires to consume good 3 and holds good 2. He has three strategies. If he stays at his own market 1, nothing happens because no one visits him. If he goes to market 2 to search, he incurs a transport cost, but since no supplier of good 2 in market 2 will exchange for the good 2 he brought, nothing happens and he goes home. If he goes to market 3, he incurs the same transport cost, but he will readily exchange because market 3 only sells his desired consumption good. There are two possibilities in market 3: (1) he meets a producer who consumes good 2, so they will engage in barter exchange; or (2) he meets a producer who consumes good 1 in market 3, so they both trade if she is willing to accept his good 2, otherwise no trade happens and he goes home. He gets the same payoff outcome as holding good 1 entering market 3, since there will always be half of the people in market 3 who want good 1 and half who want good 2.

$$V_{132} = \max \beta \{ V_{132}, -c + V_{132} - c, -c + \frac{1}{2} (U + \frac{1}{2} (V_{131} + V_{121})) + \frac{1}{2} V_{132} - c \} \quad (1.8)$$

In this case, it is optimal to stay home at his own market i because the expected utility

gained is not greater than the expected transport costs.

Proposition 1.1. *Everyone stays is a symmetric autarkic equilibrium iff $U \leq 4c$.*

The probability of getting the good you want and gaining the utility is half of U and the search cost of moving back and forth of a market not of his own is $2c$, hence the condition $U \leq 4c$. In the symmetric case, no one searches because the expected utility is no greater than the expected cost of searching.

1.3.2 Asymmetric Case

In the asymmetric case, $c_1 < c_2 < c_3$. The expected benefit at steady state is less than the expected transport costs when he makes trips back and forth to visit a market twice. From the assumption that $c_1 < c_2 < c_3$, we have the following proposition:

Proposition 1.2. *Everyone stays is an asymmetric autarkic equilibrium iff $U \leq 4c_1$.*

The proof is straightforward for Proposition 1.2 and similar to Proposition 1.1, thus the proof is omitted. In both the asymmetric and symmetric cases, autarkic equilibrium may exist. A high enough search cost c deters people from searching and, hence, trading.

1.4 Barter and Commodity Money

1.4.1 Symmetric Case

A perfect frictionless and complete barter economy with market clearance might occur in a symmetric case, but it is very unlikely. There are two possible cases. To help explain, $I(i, j, k)$ denote agents who produce i , want to consume j , and hold k . In Figure 1.2, below, the left, middle and right circles represent the markets of producers of types 1, 2 and 3, respectively. Half of Type 1 producers want to consume good 2, shown by $I(1, 2, 1)$, and the other half want to consume good 3, shown by $I(1, 3, 1)$, in the left circle in Figure 1.2. The

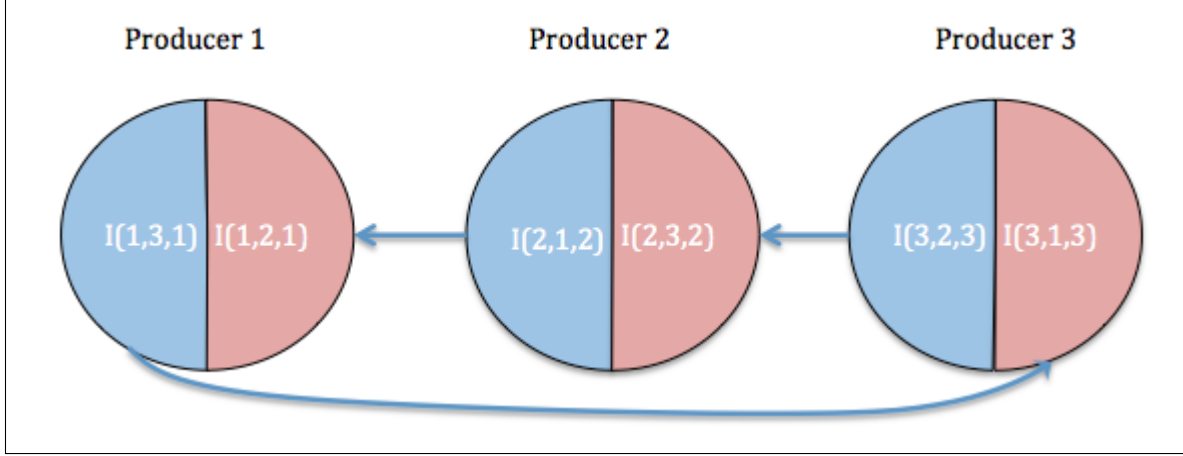


Figure 1.2: Example of a perfect frictionless exchange where every agent engages in a successful barter and the market clears.

same holds for Type 2 producers – half are $I(2, 1, 2)$ and half are $I(2, 3, 2)$, as shown in the middle circle – as well as for Type 3 producers, as shown by the equal groups $I(3, 1, 3)$ and $I(3, 2, 3)$ in the right circle.

Case 1: Suppose that all $I(1, 3, 1)$ agents visit market 3 and all $I(3, 1, 3)$ agents stay, all $I(3, 2, 3)$ agents visit market 2 and all $I(2, 3, 2)$ agents stay, and all $I(2, 1, 2)$ agents visit market 1 and all $I(1, 2, 1)$ agents stay. Then all agents would be perfectly matched to barter in a frictionless environment and still achieve market clearance, as illustrated in Figure 1.2.

Case 2: Suppose that all $I(1, 3, 1)$ agents stay and all $I(3, 1, 3)$ agents visit market 1, all $I(3, 2, 3)$ stay and all $I(2, 3, 2)$ agents visit market 3, and all $I(2, 1, 2)$ agents stay and all $I(1, 2, 1)$ agents visit market 2. This is another possible case, but it is basically the opposite direction of Case 1. Both cases may be possible if there is perfect coordination. With a simultaneous stage game with “complete but imperfect” information, having perfect communication is a necessary condition for perfect frictionless exchange. Complete information means everyone knows the payoff and all the possible strategy profiles. Imperfect information means no one knows the new draw for type from the taste shock.

Even with perfect information, there is a search cost to visit a market. The question is: Which group would sacrifice to search to earn a lower payoff, and why? Since no agent

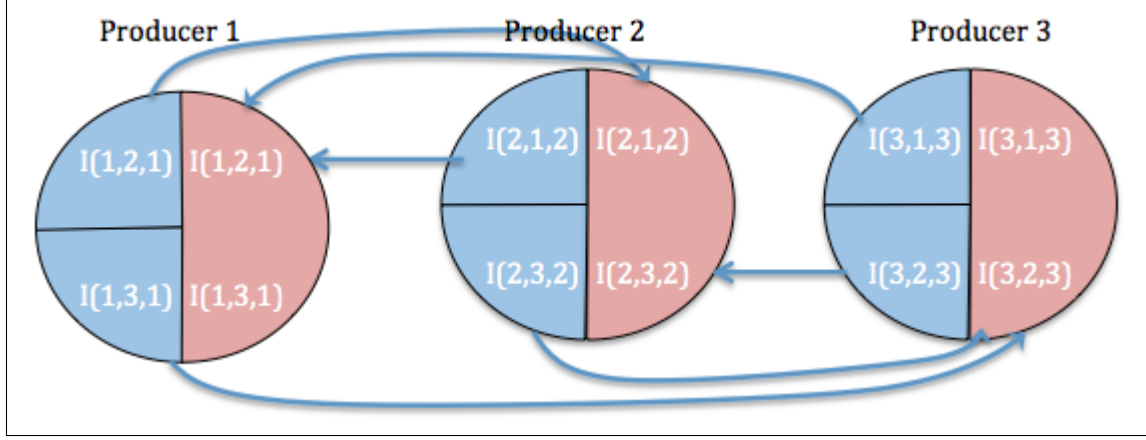


Figure 1.3: Symmetric bartering exchange.

would sacrifice to have a lower payoff, then all agents prefer to stay.

The aforementioned frictionless exchange is a very restrictive case and is not a possible equilibrium, given that no communication is assumed in this model. Some agents of the group may choose to stay and search, this would lead to a non-cooperative environment as described below.

A steady-state non-autarkic equilibrium exists, where all agents engage in trade and no agent wants to deviate from it. My result, that the proportion of agents who search is slightly less than half, differs from Goldberg's (2007) finding of exactly half because of the transport cost.

Agents have to be compensated for transport costs in the non-autarkic case. This is why the proportion of agents who search will never be half in a steady-state equilibrium.

Proposition 1.3. *Given $n = \frac{u-4c}{2u-4c}$, there exists a bartering mixed strategy equilibrium iff $U > 4c$.*

We can see from the result that if $u = 4c$, then $n = 0$. That is the corresponding autarkic equilibrium where no one searches. If $c = 0$, then $n = \frac{1}{2}$, as shown in Figure 1.3 Half of the population of each type searches and stays when there is no transport cost. Otherwise, the proportion of agents who search is less than half to compensate for the lower payoff to searching when transport costs exist. When agents coordinate in such a way that slightly less

than half of each of them searches, and the rest stay in their own market, this is a barter equilibrium. Every searcher will be matched with at least one stayer, and it is optimal for them to barter in the bilateral meetings. Contrary to Goldberg's (2007) and KW's results, where the *quid pro quo* condition allows any commodity to become money in the symmetric case, an equilibrium does not exist in the symmetric case for the direct search model. There is no need to hold a certain commodity as a medium of exchange. In symmetric direct search, agents are not better off holding goods (other than your own production), because the chance you will encounter someone who wants to trade is the same. When an agent is indifferent, nothing happens. Then no one would accept a good that is not his consumption good. Direct money search for the symmetric case does not give rise to the *quid pro quo* structure; it is not profitable to do so. Hence, commodity money does not arise in the symmetric case.

Proposition 1.4. *A steady-state commodity monetary equilibrium does not exist in the symmetric case.*

Recall that there are two types of consumers in each market. For instance, half of the suppliers (producer and holder) of good 1 in market 1 are Type 2 consumers, and the other half are Type 3 consumers. If a Type 1 agent who holds good 2 wants to visit market 1, he will have a one-half chance to be matched with someone in market 1 who wants to consume good 2. If a Type 1 agent who holds good 3 wants to visit market 1, he will also have a one-half chance to be matched with someone in market 1 who wants to consume good 3. Regardless if a Type 1 agent holds good 2 or 3 when he enters market 1, the chance of matching with someone who wants the good he brought is the same. Then, Type 1 who holds good 3 will never trade with anyone for good 2 because it does not improve the chance for a successful trade. Since this is a symmetric case, the payoff would be the same for a Type 2 producer holding good 2 and a Type 3 producer holding good 3. Then no one would accept another commodity as a medium of exchange. So, in the symmetric case, there is no commodity monetary equilibrium.

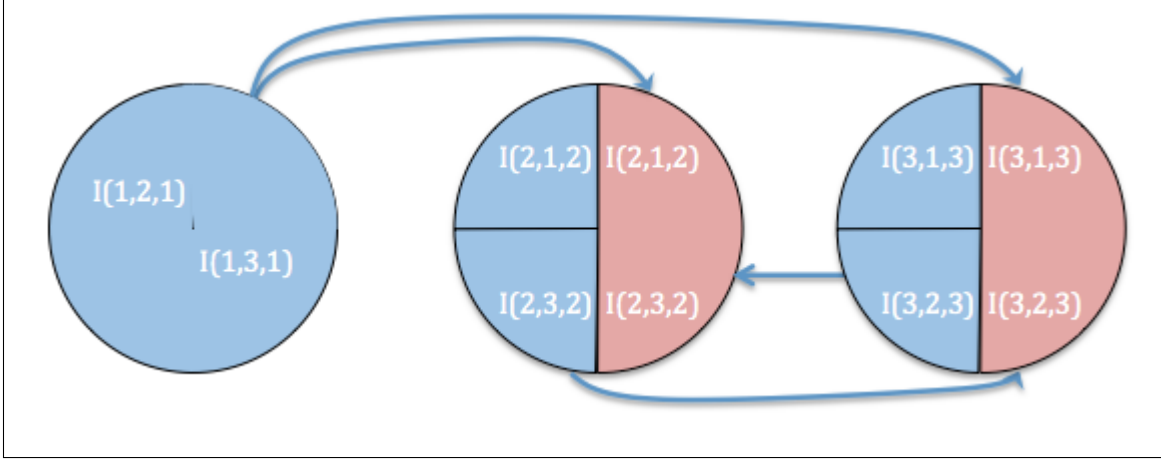


Figure 1.4: Asymmetric bartering exchange.

1.4.2 Asymmetric Case

In the asymmetric case, as predicted in KW and Goldberg (2007), it appears that the commodity with the lowest transport cost will emerge as the medium of exchange. Initially, imagine that exactly half of each type of agent search for a good. The producer of good 1 has the initial advantage to establish his good as a medium of exchange because c_1 is the lowest transport cost. Later, in a steady state, when good 1 has been circulated as the medium of exchange (until half of Type 2 and 3 producers hold good 1), Type 1 producers no longer have the advantage of getting the good they want, relative to other producers. At steady state, as shown in Figure 1.4, all Type 1 producers become searchers because good 1 is readily accepted in transactions, even if the trading partner does not consume good 1. Half of Type 1 producers desire good 2 and will visit market 2 and the other half desire good 3 and will visit market 3. In the steady state, half of the Type 2 producers and half of the Type 3 producers hold good 1 as a medium of exchange.

For Type 3 producers, half the agents hold good 3; a quarter stay and desire good 1 and quarter stay and desire good 2, as shown by the two blue segments in Figure 1.4. They are visited by twice as many visitors as themselves, so they conduct a lottery to see who gets to meet a visitor. Since all visitors hold good 1, a quarter of Type 3 stayers who hold good

3 and desire good 2 will trade for good 1 and become good 1 holders in the next period. A quarter of Type 3 stayers who desire good 1 will trade, consume it immediately, and draw a new taste shock.

Half of Type 3 agents hold good 1 and desire good 2. They visit market 2 because suppliers in market 2 will accept good 1. However, only half of Type 2 agents stay at market 2, and have twice as many visitors (half of Type 1 and Type 3 visitors holding good 1). Type 2 producers conduct a lottery to determine with whom they will trade, which means that both Type 3 and Type 1 agents have half a chance to get their good. At the end, a quarter of Type 3 searchers will get good 2, and nothing happens to the other quarter. At the end of the period, half of Type 3 agents will hold good 1 and desire good 2. Another half of Type 3 agents will hold good 3; half of Type 3 agents who hold good 3 desire good 1 and the other half desire good 2. The case is similar for Type 2 agents, so explanation is omitted.

Half of Type 1 searchers in market 2 must compete with Type 3 searchers. Type 1 searchers have half a chance of getting good 2 in market 2 at steady state. It is the same case for Type 1 searchers in market 3. At the end of the period, only half of Type 1 agents acquire their consumption good. This analysis shows that commodity money will arise in an asymmetric case, but it neither improves nor worsens the matching probability. However, Type 2 and Type 3 agents are better off because they are able to trade for a commodity with a lower transport cost. Type 1 agents have the initial advantage of getting their desired good. However, the advantage fades as the steady state is approached. Type 1 agents are not better off than Type 2 and Type 3 agents in successful matching at steady state. Half of each type will acquire their desired consumption in each period at steady state and no one would deviate from this monetary equilibrium.

Proposition 1.5. *A steady state commodity monetary equilibrium exists in the asymmetric case.*

Goldberg (2007) shows that money drives out bartering. However, this paper shows that bartering can co-exist with commodity money. At steady state in each period, a quarter of

Type 1 agents and one-eighth of Type 2 and Type 3 agents are barterers, while the remaining agents are either a seller or buyer. This interesting result coincides with the intuition that if a miner has gold and uses the gold to get the good he wants, and if his trading partner wants to use the gold, they become barterers. The supplier of the commodity money will have the chance to barter because he may meet someone who wants to trade for his *good*.

Proposition 1.6. *The commodity with the lowest transport cost emerges as the commodity money.*

Hence a commodity with the lowest transport cost will be the preferred choice for commodity money.

Proposition 1.7. *Bartering and commodity money coexist in equilibrium in the asymmetric case.*

Bartering and commodity money may co-exist in equilibrium in an environment without fiat money. This result is interesting because it has not been shown in previous studies that bartering can actually coexist with commodity money. The intuition is that there must be a group of agents who produce a commodity that can be used as money. The producer of this commodity money will have half a chance to meet with another agent who wants to “literally” consume his good, so they would barter. He also has half a chance to meet with another agent who does not want to consume his good but would accept it as a medium of exchange because it has a lower transport cost. That is when the commodity becomes a commodity money.

1.5 Fiat Money

In this section, fiat money is introduced into the model. It is indivisible (comes in units of one) and searching with it incurs a transport cost $c_0 \geq 0$. Initially, a fraction $m \in (0, 1)$ of agents are endowed with money, not real goods. This paper only focuses on the analysis

where fiat money crowds out other means of exchange. However the condition for fiat money to be accepted as a medium of exchange in equilibrium is different for asymmetric and symmetric cases.^{1.8}

Definition 1.2. *In a fiat monetary equilibrium, fiat money is accepted as a medium of exchange and agents never accept other goods except the good they want to consume.*

This definition follows Goldberg (2007). The analysis here considers both asymmetric and symmetric types of goods. Since direct search is used, there is no shopping or door-to-door equilibrium like those discussed in Goldberg (2007). The fiat money works in both asymmetric and symmetric cases.

1.5.1 Symmetric Case

Consider a symmetric fiat money equilibrium and a Type i agent who expects the following: all agents who hold their production goods go to their markets and agree to accept only their desired consumption goods. In the symmetric case, fiat money is used to improve trade because agents are willing to accept the fiat money, believing that the next agent he meets will accept money to get the good he wants. The belief that the next agent he meets will accept the money is due to the lower transport cost of fiat money. If the belief is true, then the payoff is greater than the payoff of trying to barter with only half the chance of getting the achieving double coincidence of wants. So the likelihood of the next agent being willing to accept fiat money must exceed $\frac{1}{2}$. The expected utility would exceed the expected utility from pure bartering. Since no one would accept any good as commodity money, the introduction of fiat money could encourage trade due to its relatively lower transport cost, and it would become a medium of exchange in the steady state. In the symmetric case, the use of fiat money improves the chance of trade.

^{1.8}There is no market to "sell" the money; basically you cannot be a stayer and a buyer at the same time. Markets like "pawn shops", where searchers can bring a commodity to a market to exchange for money, do not exist in this model. The purpose of having a marketplace in the model is to dedicate a place for selling specific products.

Proposition 1.8. *A symmetric fiat monetary equilibrium exists iff $c_0 < c_i$ where $i = 1, 2, 3$.*

Since the use of commodity money does not exist in equilibrium in the symmetric case, fiat money fills the void, serving as a medium of exchange to improve the chances of successful trade. The introduction of fiat money crowds out pure bartering. This result corresponds to Goldberg (2007) and KW.

1.5.2 Asymmetric Case

However, in the asymmetric case, the introduction of money does not improve the *chance* of trade when compared to the use of commodity money. In the asymmetric case, fiat money improves the payoff because it has a *lower transport cost* than goods 1, 2, and 3. Fiat money does not improve the chance of trade, but it emerges as the medium of exchange because it has a lower transport cost and this enables agents to achieve higher utility.

Proposition 1.9. *An asymmetric fiat monetary equilibrium exists iff $c_0 < \min\{c_1, c_2, c_3\}$.*

With the introduction of fiat money, agents are not able to search for the good they want with their own production commodity good. In asymmetric equilibrium, fiat money and commodity money have slightly different implications. There is a tradeoff between the lower chance of matching and the gain from lower transport costs. The chance of matching to get the desired good is not higher for any agent when fiat money is introduced.

Without fiat money, all Type 1 agents are searchers in a commodity money environment. However, only half of Type 1 agents are searchers with fiat money and they stop searching with their production good once fiat money is introduced. This is because Type 2 and 3 agents are more willing to accept fiat money for the lower transport cost of fiat money, than good 1 as commodity money. Due to the lower “saleability” of good 1 among Type 2 and 3 agents, Type 1 agents lose their “privilege” as a commodity money producer and have to acquire fiat money as a medium of exchange in order to get the good they want.

In a fiat money steady state, all types of agents who search will be matched and obtain the good they want to consume. The fiat money equilibrium exists and there exists a unique fraction of agents endowed with money m such that the expected utility of each agent is maximized.

The results in this paper show that fiat money drives out both bartering and commodity money, at the same time. This outcome has not been shown in previous papers. Goldberg (2007) shows that fiat money drives out commodity money, and KW show that fiat money drives out bartering. Over time, fiat money does not promote more successful trading among agents, on average, but fiat money exhibits second-order stochastic dominance over commodity money in the asymmetric case or over pure bartering in the symmetric case.

1.6 Conclusion

The *quid pro quo* in non-direct money search models enables a commodity to become a medium of exchange in the symmetric case. The equilibrium for commodity money in the symmetric case does not exist in direct search. There is a steady state for people to search for bartering. The introduction of fiat money deters people from bartering and there exists a monetary equilibrium for fiat money. However, consideration of the symmetric case is very specific. It would be interesting for future research to analyze the asymmetric case. It is interesting that the introduction of fiat money does not improve the chance of trading in the asymmetric case, because there could exist a commodity money. But this perhaps depends the three-good case considered here. It would be interesting to see the results when we extend the number of goods beyond three. This is an interesting historical issue because the high transport cost of holding gold or silver coins prompted the early use of fiat money, easily transported notes called "money" that were backed in gold.

Chapter 2 :

Stability of Monetary Equilibrium in a Direct Search Model

2.1 Introduction

Random-matching models, spawned from two seminal papers by Kiyotaki and Wright (1989, 1993), have been common in the money search theory literature. Technical restrictions in those models have drawn criticism. One criticism is the randomness in matching. As noted by Howitt (2003), most people do not conduct their daily economic transactions in a random-matching setting. Randomness in matching is unrealistic but has been maintained in the models for tractability. Further developments in money search models have moved away from random-matching towards more directed-matching. For example, Corbae, Temzelides and Wright (2003) present a directed search model with a cooperative mechanism in an exchange market, while Goldberg (2007) introduces partial directed money search in a fully decentralized market.

For the analysis of monetary equilibrium, non-cooperative directed search is a more realistic feature than partial directed or cooperative directed search. People normally visit stores to get the goods they want without a third party coordinator. The entire search

process is self-directed and each agent can generally reach the sellers of their desired good. The non-cooperative directed search model of money introduced in this paper is similar to the random money search model of Burdett, Coles, Kiyotaki and Wright (1995), henceforth BCKW, except that people directly search and know which available goods other agents sell. Ex-ante, there are many identical sellers of a good, so the searcher can either randomize the choice of seller or revisit a known seller to obtain the desired good. Unlike in partial directed search, a seller in non-cooperative directed search is always able to provide the good whenever approached by a buyer.

The random-matching models generated from Kiyotaki and Wright's (1993) framework pose a technical challenge. An increasing number of good types, denoted k , will reduce the expected utility with or without money in a random search model^{2.1}. When k is sufficiently large in a random search model, it will deter people from trade because the expected utility diminishes. Even with the presence of money, people will not choose to trade when there is a sufficiently large variety of goods because the expected utility diminishes as k increases. In the random meeting money search model, the role of trade and money would disappear and the expected utility approaches zero as $k \rightarrow \infty$. In other random meeting papers (e.g. BCKW), a higher k increases the likelihood of an autarkic equilibrium, even when money is introduced, given that search cost is imposed. This means the variety of goods is not always positively correlated to trade or to the role of money in the random-matching setting. However, it conflicts with the general perception that people are not less willing to hold money to trade when the variety of goods increases in the market. This paper shows that this is not the case in a directed money search model: when $k \rightarrow \infty$, the role of money becomes more apparent. This occurs because k is not in the matching probability function

^{2.1}In Kiyotaki and Wright (1993), $\lim_{k \rightarrow \infty} rV_1 = \frac{\beta(1-M)}{k(k-1)}(U - \epsilon + V_n - V_1) + \frac{\beta M}{k}\pi(V_m - V_n - 1) = 0$ and $\lim_{k \rightarrow \infty} rV_m = \frac{\beta(1-M)}{k}\Pi(U - \epsilon + V_n - V_m) = 0$ where V_1 , V_m and V_n are the value functions for not holding money, holding money, and drawing a new preference shock. In the probability measure of k , the number of desired good types being searched has to be infinitely many. Otherwise, if k is finite, this gives a zero measure. In Kiyotaki and Wright (1993), $\frac{\partial V_1}{\partial k} < 0$ and $\frac{\partial V_m}{\partial k} < 0$.

with money holdings in the present direct search model.

When the information of the seller's desire is absent, and the probability of a single coincidence of want matching is low because k is sufficiently large, money plays a role to facilitate exchange. This would be sufficient for money to appear as a medium of exchange.

When $k \rightarrow \infty$, the cash-in-advance constraint framework is shown to be a very special case of the money search framework. When there is a sufficiently large number of good types, trading without money is not profitable because the possibility of barter is near zero, so agents will only search for a good when they have money. The variety of goods is positively correlated with trade with money, but negatively correlated with trade without money. No agent will find it profitable to barter unless they have the cash in advance before searching for a good. The cash-in-advance framework also proves to be a special case of the direct money search framework, where k only needs to be sufficiently large that there exists a transaction cost or search cost.

The model setup is described in the next section. In Section 3, I describe the decision problem faced by an agent. The autarkic equilibrium is also described in Section 3. In Section 4, a monetary equilibrium with directed search is presented. A comparison between the random-matching model and directed search, in terms of the effect of k , is shown in Section 5. A welfare comparison between random-matching and directed search is shown in Section 6. In Section 7, I conclude and discuss further lines of research.

2.2 The Model

The environment used in this paper is a modified version of BCKW. They use random matching, whereas a non-cooperative directed search framework is used in this paper.

2.2.1 The Environment

A continuum of agents, $i \in [0, 1]$, live in discrete time with an infinite horizon and a finite number of good types, $k \geq 3$, where k is a finite integer. Each good is indivisible, uniform in size and perishable at the end of each period. Each agent continues to specialize in producing one unit of an agent-specific commodity good at the beginning of each period^{2.2} with a disutility (cost) of production, x . There are no production shocks, and each agent can use his own production for consumption or possible bilateral exchange.

Each agent is a generalist in consumption and receives positive utility, u , by consuming the type of good desired in that period, where $u > x$. Each agent is randomly assigned a preference for the type of good he initially desires. If he happens to desire his own production type, the agent will immediately consume it and draw a new preference. A new random preference will be drawn at the end of the period only after the agent acquires the desired good. Otherwise the agent will continue to desire the same type of good. If the agent consumes a good not desired, his utility equals the disutility (cost) of producing a unit of the good and it gives zero net utility^{2.3}. An agent always consumes his own production when there is no trade by the end of each period. The preference shocks are assumed to be identically and independently drawn from a known uniform distribution of good types. The preference drawn is private information known only to the agent, but his production type is public information.

Money is indivisible, not perishable, cannot be produced and is randomly assigned initially to a fraction of agents, m . Each agent can only hold one unit of money balances. A “non-moneyholder” possesses only one unit of a self-produced commodity good; a “moneyholder” possesses one unit of money in addition to one unit of a commodity good^{2.4}. A

^{2.2}This feature of producing a good in every period is a closer fit to the non-monetary general equilibrium model.

^{2.3}This simplifies the model by ensuring that an agent will not receive any utility from consuming his own production when he does not desire it.

^{2.4}The setup in the conventional money search framework differs from this paper, where an agent expects to produce in every period and is able to possess a unit of a good and money at the same time.

non-moneyholder can only bring one unit of a commodity into the marketplace to trade; whereas a moneyholder can bring one unit of money or one unit of a commodity, or both. For each successful trade, both agents incur a transaction cost, ϵ .

There are two decisions to make in every period: (i) search or stay, and (ii) reject or accept trade. Both moneyholder and non-moneyholder can choose to search or stay. Each agent chooses to search and trade only once and meets only one agent in a given period. Otherwise, an agent has to wait for the next period to decide to search and trade again. The trading decision is straightforward in the sense that an agent will only trade when he finds the good he desires or when he is willing to exchange his production good for money. Besides that, no agent would want to trade for a good he does not desire because each good is perishable within the period and all agents exhibit self-interest; he would end up with negative net utility from the trade due to the transaction cost. So, we focus more on the decision to search or stay.

2.2.2 Search Process / Mechanisms

A moneyholder becomes a *buyer* when he exchanges his money for a good. A non-moneyholder becomes a *seller* when he trades his good for money. Anyone becomes a *barterer* whenever he swaps his good for another agent's good. If an agent gets his desired good, he consumes it immediately. All agents who search will return to their home at the end of the period.

All sellers of the same type of production are located in a specific marketplace known to all agents. The matching is endogenous such that the searcher self-directs exactly whom to meet. However, the searcher does not know the seller's desire. The information of the seller's desired good is revealed only after meeting. If an agent decides to search, he is assumed to approach only a person who holds and owns the good he desires. So he know with certainty who holds which goods, but he doesn't know which good(s) those persons desire. When a searcher arrives at a marketplace, the assumption of a continuum of agents means there are

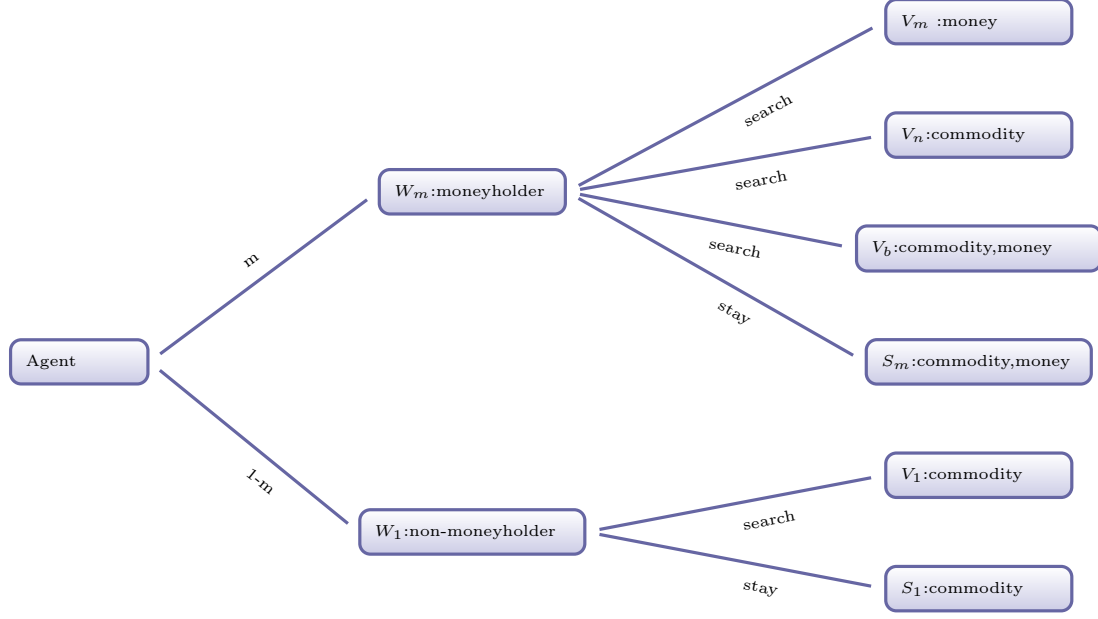


Figure 2.1: The roles and strategies of agents.

infinitely many identical sellers of the desired good type for potential trade^{2.5}. A searcher is indifferent about the identical sellers at the marketplace. Hence, a searcher is always matched with a seller of the searcher's desired good. The searcher knows his desired good is *available*, but he does not know if one of the sellers will *accept* his offer^{2.6}. This feature is more realistic than the partial directed search: we normally have prior knowledge about a seller's available stock of the desired good when we want to make a purchase, but we don't always know what the seller desires in return.

Every meeting between agents in this exchange economy will produce either a single coincidence of wants or double coincidence of wants. No coincidence of wants does not happen because an agent always directs himself to the seller of his desired good in this non-cooperative directed search^{2.7}. Revisitation is permissible in this model, but has no

^{2.5}This is a mathematical convenience to assume away the number of agents in matching as long as not all agents are searching. The assumption of infinite agents is less realistic, but makes the model more tractable.

^{2.6}Goldberg (2007) motivates the frictions in partially directed search with a probabilistic measure that a searcher will acquire a good from a seller of his desired good. Unlike in Goldberg's (2007) model, searchers here also know which seller's shop is open and has the capacity to supply the desired good.

^{2.7}The absence of no coincidence of wants distinguishes this model from random-search or partially directed-search models.

measurable effect on payoffs. A searcher is indifferent between randomly picking or revisiting a seller because there is no advantage to knowing the seller's past desire from a bartering exchange in the previous period^{2.8}. A searcher without money can only barter and a searcher with money can either barter or exchange with money if the seller is willing to accept.

On the other hand, the stayer cannot dictate who will visit him for an exchange. As a stayer, you remain at your store to welcome traders at no cost. A stayer who does not hold money can either barter or be a seller, and a stayer who holds money can only barter because of the assumed degenerate distribution of money holdings. The stayer has the opportunity to meet with those who are searching only for that specific good type. This is symmetric for all other agents.

2.2.3 Value Functions and Equilibrium

The probability you will meet an agent who wants your good is $\frac{1}{k-1}$, and the probability that you would want his good is always 1 because you would only look for the good you desire in this self-directed mechanism. The probability that a searcher would have the good you desire is $\frac{1}{k-1}$. Let m be the probability that you would meet with someone with money and $1 - m$ without money.

Let c_1 and c_m be the cost of searching and transport with commodity and with money, where $c_1 > c_m > 0$. Assume that the cost of transport with money is zero, so c_m is simply the search cost^{2.9}. The rate of time preference is $r > 0$, and π and Π denote the best response of an agent to accept money and the likelihood of acceptance of money by the other agent. Let n_1 , n_m , n_b and n_s be the proportions of non-moneyholders who search with a good, moneyholders who prefer to search with money, moneyholders who search with both money

^{2.8}Let a searcher who draws the same preference choose to revisit the same seller who previously bartered with him. He may not be able to barter with the seller because the seller may desire a good other than the searcher's production good due to the seller's preference shock. When the searcher draws the same preference again and intends to barter with the same trader, the probability of a successful barter by revisiting a previously traded seller is the same as a random pick for any other identical seller.

^{2.9}Because the transport cost with money is zero, c_m is the search cost, and so $c_1 - c_m$ is the transport cost of a commodity.

and commodity, and moneyholders who prefer to search with a good.

Let W_j denote the value function for an agent, where $j \in \{1, m\}$; subscript 1 denotes a non-moneyholder and subscript m denotes a moneyholder who holds a unit of money and a unit of commodity. V_j and S_j denote the sub-value functions of a searcher and a stayer, respectively. V_c and V_b are special cases for a moneyholder who prefers to search with commodity only and with both money and commodity.

If the agent is a non-moneyholder, he chooses to search with commodity or stay at steady-state, so as to maximize:

$$\begin{aligned}
W_1 = \max \{ & \underbrace{\frac{U}{r(k-1)} - \frac{c_1}{r}}_{V_1 : \text{search with commodity}}, \\
& \underbrace{\frac{n_1(1-m)}{r(k-1)}U + \frac{mn_c}{r(k-1)}U + \frac{mn_b}{r(k-1)}U + \max_{\pi} [\pi(\frac{mn_m}{r} + \frac{mn_b}{r} \frac{k-2}{k-1})(-x - \epsilon + W_m - W_1)]}_{S_1 : \text{stay with commodity}} \}
\end{aligned} \tag{2.1}$$

where^{2.10} $U = \frac{(u-x)k}{k-1} - \epsilon$.

If the agent is a moneyholder, he chooses to search with money at steady-state, with commodity, with money and commodity, or stay with money and commodity, so as to max-

^{2.10}Similar to BCKW, if you draw a preference desiring your own good, you would consume it immediately and then draw a new preference. The value function of drawing a preference desiring your own production type, V_0 , is:

$$\begin{aligned}
V_0 &= (u-x) + \frac{1}{k}V_0 + \frac{k-1}{k} \max\{V_1, S_1\} \\
&= \frac{k}{k-1}(u-x) + \max\{V_1, S_1\}
\end{aligned}$$

u denotes the utility in consuming one unit of desired good, and, U denotes the expected utility gained from acquiring and consuming his desired good, including a draw to desire his own production type from a taste shock.

imize:

$$\begin{aligned}
W_m = \max \{ & \underbrace{\frac{\Pi(1-m)}{r}[U+x+W_1-W_m] - \frac{c_m}{r}}_{V_m : \text{search with money}}, \underbrace{\frac{U}{r(k-1)} - \frac{c_1}{r}}_{V_c : \text{search with commodity}}, \\
& \underbrace{\frac{\Pi(1-m)}{r}(1 - \frac{1}{k-1})[U+x+W_1-W_m] + \frac{U}{r(k-1)} - \frac{c_1}{r} - \frac{c_m}{r}}_{V_b : \text{search with money and commodity}}, \underbrace{\frac{n_1(1-m) + m(n_c + n_b)}{r(k-1)}U}_{S_m : \text{stay with money and commodity}} \}
\end{aligned} \tag{2.2}$$

An equilibrium is defined to be a Nash equilibrium such that each agent chooses the pure and stationary strategy which maximizes his expected utility, contingent upon other agents' strategies and the distributions of n_1, n_m, n_b, n_s resulting from the strategies chosen by other agents.

Assumption 2.1. *A continuum of agents means there are infinitely many identical sellers of the desired good type for potential trade as long as the fraction of agents who search is not equal to 1.*

Assumption 2.2. $u \geq x > c_1 > c_m > \epsilon > 0$.

Lemma 2.1. $V_i, S_i \geq 0$ for $i \in \{1, m, c, b\}$.

Any strategy used must have a value greater than or equal to zero because an agent could choose the strategy to consume his own production forever to have a zero expected utility.

2.3 Non-monetary Equilibrium

In this model, the friction of matching is due to incomplete information: not knowing the desire of the seller. If knowledge of the seller's desire can be easily acquired, money is not

needed for the agents to perfectly coordinate. Hence, a non-monetary Walrasian equilibrium could occur when everyone knows each seller's desire.

Lacking information of the seller's desire, there can be two possible non-monetary equilibria in a directed search money model. First, a non-monetary autarkic equilibrium could exist when the searching strategies imply that no one is willing to search. Everyone stays, and no trade occurs (similar to the results of Goldberg (2007) and BCKW).

Proposition 2.1. *There exists a non-monetary autarkic equilibrium iff $c_1 > \frac{U}{(k-1)}$ and $c_m > \Pi(1 - m)[U + x]$.*

This result appears to be consistent with the standard money search models: (i) a non-monetary autarkic equilibrium exists when the cost of searching with a commodity and money is higher than the possible gain from trade, (ii) no agent would want to search with commodity when the variety of goods is sufficiently large which diminishes the utility gained from trade, and (iii) no agent wanting to search with money depends on the probability of money being accepted and the fraction of money supplied to agents in the market. This makes the probability of meeting another agent willing to accept money low, making it unprofitable to search with money and a commodity. When each agent believes that all agents follow these strategies, his optimal strategy is to stay. Since no agent chooses to search, no trade occurs. This is an interesting result, in that the threshold of searching with money is independent of the number of good types.

Proposition 2.2. *There exists a non-monetary barter equilibrium iff $c_1 < \frac{U}{k-1}$, with the resulting distribution of $n_1 = 1 - \frac{c_1(k-1)}{u}$ and $c_m > \Pi(1 - m)[U + x]$.*

When the number of good types is very low and the value of having money as a medium of exchange is very small, a barter economy can emerge. This is consistent with a number of observable cases. In ancient or primitive economies, where the variety of goods is limited

and the number of traded goods is low, bartering tends to prevail. Similarly, even in more modern war-torn economies, survival depends on fewer types of goods, primarily food and shelter, increasing the reliance on barter transactions. Indeed, bouts of hyperinflation during such periods might be viewed as efforts to flee from the use of money in favor of the direct exchange of goods.

2.4 Monetary Equilibrium

A monetary equilibrium is a Nash equilibrium with a list (W_1, W_m) that satisfies the incentive condition to hold money, $-x - \epsilon + W_m - W_1 > 0$, such that the gain in accepting money exceeds zero, given a stationary distribution of n_1, n_m, n_s . Only the equilibria where people accept money with probability $\Pi = \pi = 1$ are considered.

There would exist a monetary equilibrium when agents get higher value by choosing the strategy to accept money when the cost of searching is lower than the benefit of being matched with the desired good in exchanging with money. The cost of searching with a commodity must exceed the benefit of being matched in a bartering exchange, given the utility in successfully acquiring the desired good and the probability of getting matched $\frac{1}{k-1}$.

Proposition 2.3. *There exists a monetary equilibrium if a moneyholder would not carry only a commodity to search.*

A moneyholder would not carry only a commodity to the market to search because this implies a moneyholder chooses a strategy for which the payoff is the same as for a non-moneyholder. No one would accept money if a non-moneyholder can be equally as well-off as a moneyholder. If that is the case, monetary equilibrium does not exist. A moneyholder who carries only a commodity to search implies that search with money is not profitable;

this strategy is not possible in monetary equilibrium. It must be the case that searching with money is profitable for a moneyholder; so this strategy of carrying only a commodity to the marketplace is Pareto dominated in a monetary equilibrium.

Lemma 2.2. *When an agent searches with money and a commodity, he will choose the strategy to barter if possible; if not, then he would offer money in exchange for a good.*

Proposition 2.4. *In monetary equilibrium, a moneyholder would carry both money and a commodity to search iff $\frac{U}{k-1} + c_m < c_1 < \frac{1}{k-1}[mU + (1-m)x] + c_m$, or there exists a k such that $\frac{U}{c_1 - c_m} + 1 < k \leq \frac{mU + (1-m)x}{c_1 - c_m} + 1$.*

It would not be profitable for the moneyholder to search with money and a commodity if the cost of carrying the commodity, $c_1 - c_m$, exceeds the gain of engaging in barter with a probability of $\frac{1}{k-1}$. Searching with both will save on the search cost, since search cost is incurred only once for each search, regardless if the agent carries money, a good, or both.

Proposition 2.5. *In monetary equilibrium, a moneyholder would carry only money to search iff $c_1 > \frac{1}{k-1}[mU + (1-m)x] + c_m$ or there exists a sufficiently large k such that $k > \frac{mU + (1-m)x}{c_1 - c_m} + 1$.*

Figures 2.2, 2.3, and 2.5 show some numerical results for this model. For all numerical results, we set $r = 0.001$, $m = 0.5$, $c = 0.01$, $x = 0.1$ and $U = 1$. Molico (2006), who provides a good example of numerical analysis of a money search model, finds that $r = 0.001$ yields an approximately normal distribution for money holdings and other key variables. In my model, welfare is maximized when m is set to approximately 0.5, and the decision to set $U = 1$ follows BCKW, mostly for reasons of tractability. The value $c = 0.01$ and $x = 0.1$ are used to introduce a simple form of transaction and production costs to the model.

Figure 2.2 shows that the gain for accepting money in the random money search model diminishes in k , as in BCKW. Note, however, that direct money search shows that the gain

for accepting money monotonically increases in k . This is particularly important, in that it conforms with economic intuition about the gain in accepting money being positively related to the number of good types, k . The gain from searching with a commodity diminishes when there are more type of goods in the market.

In a random search framework, the expected utility of a moneyholder and a barterer in monetary and non-monetary equilibria approaches 0 when k approaches infinity. With random money search, the likelihood of searching with money decreases because the expected utility of accepting money decreases with an increasing number of good types, k . However, in reality, the opposite seems to be more plausible: having more types of good to choose from should not decrease the value of holding money. In the random search framework, whether money is introduced or absent, the expected utility converges to zero as the number of good types increases, implying that no one would trade and the system would converge to an autarkic equilibrium.

Even given some search cost in random search^{2.11}, there exists a sufficiently large k such that the expected utility of a moneyholder and barterer will be zero. Hence, it would result in autarkic equilibrium for any given value of m, c_1, c_m, r and x . It seems unrealistic that, when the number of goods is large enough, no one will trade with or without money. For example, we have a healthy monetary exchange economy with millions of types of good in the world today.

In comparison, non-cooperative directed search presents a result closer to the general economic intuition that the benefit of accepting money increases or at least does not diminish with k .

Proposition 2.6. *When $k \rightarrow \infty$, monetary equilibrium still exists and a moneyholder would carry only money to search for any $c_1, c_m, r, m > 0$ and $m, r \in (0, 1)$.*

^{2.11}The traditional KW random search framework contains storage costs rather than search costs, while BCKW use search cost in their money search model.

Figure 2.3 shows different value functions for the directed money search model. The value function for holding money and a commodity to search converges to a positive value as k approaches infinity. The value function of searching with a commodity only quickly goes to zero as k increases. This result has an interesting implication: agents will be unlikely to search without money unless there are few good types in the market. The more good types in the market the more likely the agents will search with money and commodities or money alone. These intuitive results follow from the present direct search framework, but contradict the earlier random search models.

Corollary 2.1. *The cash-in-advance constraint framework is a (strict) special case of the money search model with conditions that satisfy Proposition 2.5 (Proposition 2.6).*

People will only search and buy when they have money in hand. This phenomenon is equivalent to the cash-in-advance constraint framework. That being said, the cash-in-advance framework can be regarded as a special case of the direct money search model, where the agent would search only when they have the money (or cash) in hand when the search cost is positive and k is large enough to deter search with a commodity, which appears to be unprofitable. This implies that for any given parameter values, there exists a unique solution to the equation for monetary equilibrium.

2.5 Conclusion

People have different desires and needs at different times. Without perfect knowledge of the seller's desire, there exists a role for money in exchange, given a sufficiently large number of good types. Compared to the random search framework, directed money search conforms better with basic economic intuition. The idea that random search becomes infeasible with a sufficiently large number of goods is particularly troubling. The inefficiency of bartering due to low probability of matching depends on the large number of good types.

In the directed money search model presented here, monetary equilibrium can be satisfied with increasingly large k . In addition, the value of accepting money is at least non-decreasing when the number of good types increases. Generally, the variety of goods is positively correlated with trade with money, however, random matching in money search models leads to a *negative* correlation. Given these technical challenges, there must exist other reasons to proceed with the equivalent class of random money search models. The use of random money search may be applicable to specific cases, but non-cooperative directed money search appears to be a useful alternative.

By focusing on the number of types of goods in a directed money search model, this paper offers a novel connection between two well-known frameworks, the cash-in-advance framework and the money search framework. This connection may serve as a starting point for intuitively understanding both frameworks and combining their features to improve the tools of monetary policy analysis. In this regard, future research should extend the integration of cash-in-advance models with money search models.

In closing, I would like to highlight a point that has not been discussed so far. There could be more than one variable which affects the exchangeability between money and good. The number of good types (k) is just one of the possible variables that influences the use of money. However, finding a set of necessary and sufficient conditions for the demand for money when there exists an alternative medium of exchange, other than bartering, remains unresolved in this paper.

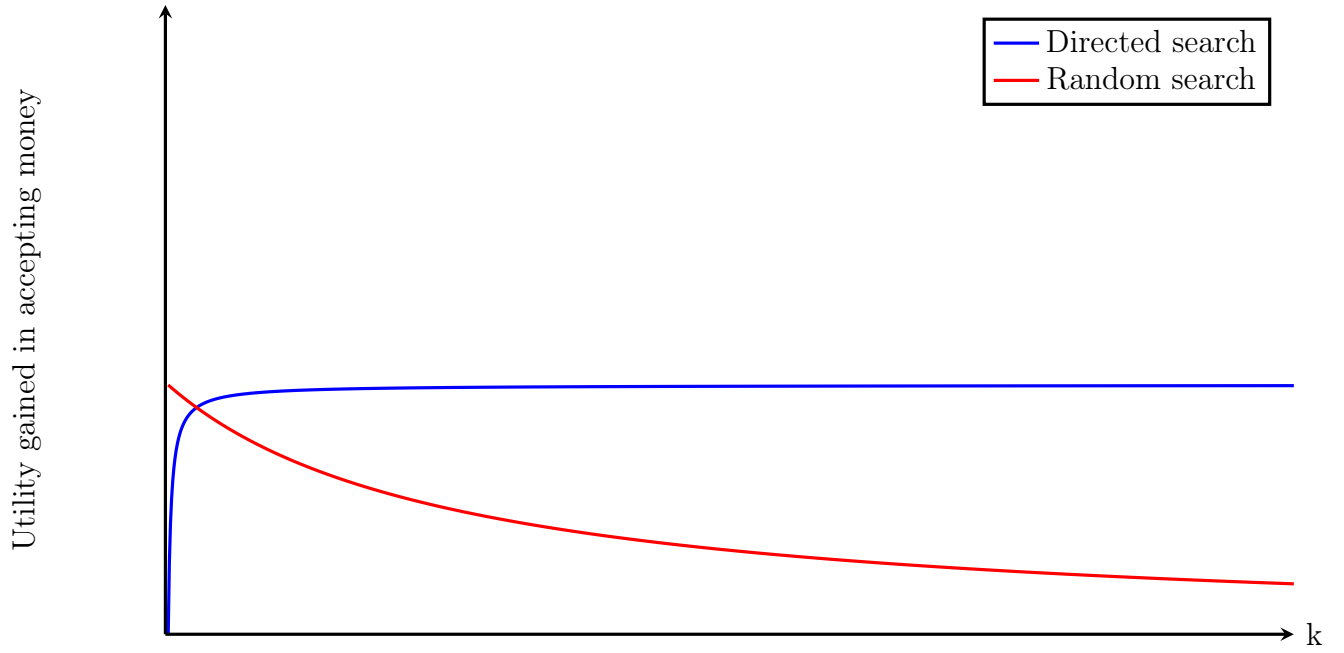


Figure 2.2: A comparison of utility gained in accepting money between random and directed money search frameworks, where k denotes the number of good types.

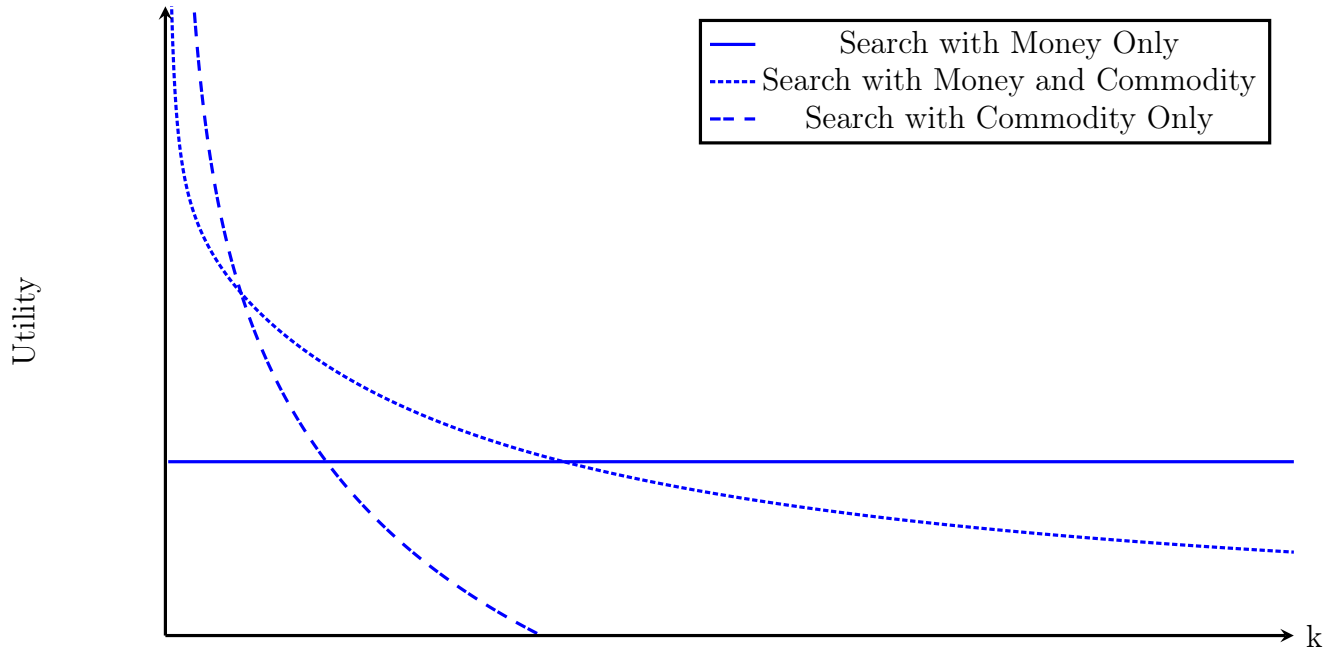


Figure 2.3: Expected utility for different searching strategies of a moneyholder at steady state in non-cooperative directed money search, where k denotes the number of good types.

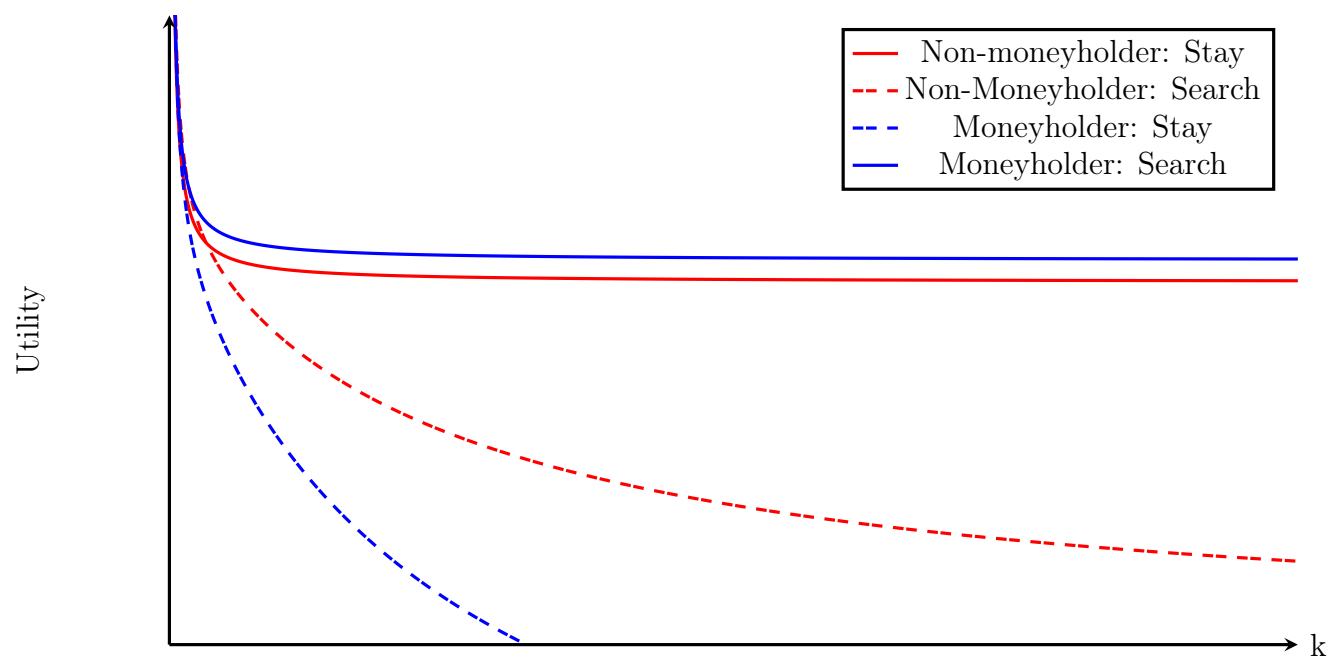


Figure 2.4: Expected utility at steady state for value functions in non-cooperative directed money search, where k denotes the number of good types.

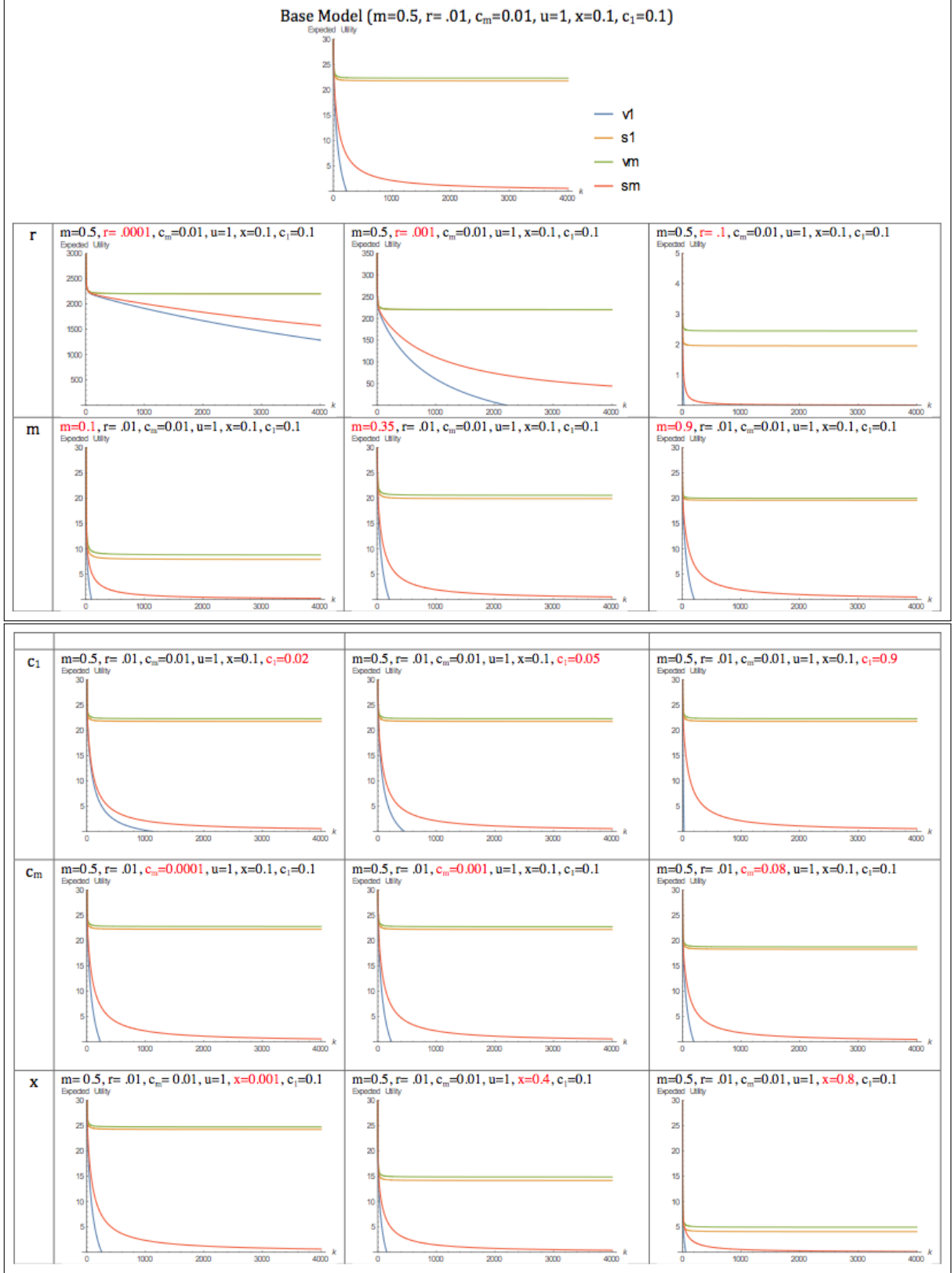


Table 2.1: A table showing the sensitivity analysis for the expected utility of holding money with different parameter values for r , m , c_1 , c_m and x .

Chapter 3 :

Money for Market Clearance in Decentralized Markets

3.1 Introduction

General equilibrium models typically involve decentralized decision-making with a centralized clearing mechanism, employing an imaginary “Walrasian auctioneer” to do complex computations for market clearing. With imperfect information and no transaction costs, money assumes a role equivalent to the imaginary Walrasian auctioneer, so that each market attains zero excess demand within one period, even when friction arises.

In this chapter, I introduce a model with fully decentralized markets that can potentially clear within each period, an alternative to Lagos and Wright’s (2005) model. The desirable feature of market clearing within a period also ensures that markets clear in subsequent periods. The results correspond to Kiyotaki and Wright’s (1993) model, except agents are more likely to accept money when the number of types of goods is infinite. Not only does money, as a medium of exchange, facilitate successful transactions among individuals, but it also serves as an important and necessary instrument to assist market clearance in decentralized markets when frictions arise. Without money, when the number of types of good exceeds

two, the market is impossible to clear when there is imperfect information among agents. Even though introduction of the market improves on successful trade, it cannot completely overcome the search friction.

Since the seminal work of Kiyotaki and Wright (1989, 1993), there have been successful efforts to explicitly incorporate money as a medium of exchange in economic models using search theory. However, most of the first and second-generation money search models do not achieve market clearing within a period. The idea that the market clears within one period is important because it ensures that markets can potentially clear in any subsequent periods. This is also true for the Arrow-Debreu model of market clearance within one period. It later extends to sequential equilibrium in a dynamic model, where each market clears with a perfect relative price mechanism. From sequential equilibrium, many policy issues can be analyzed and explored. However, if a money search model can be developed with similar properties, we can potentially have an integrative model where good markets clear within a competitive decentralized market mechanism and money serves as the medium of exchange that facilitates market clearance.

Third generation money search models begin to feature market clearance within one period, starting with a paper by Lagos and Wright (2005)^{3.1}. In their paper, each period is cleverly divided into two sub-periods: one is decentralized and the other is centralized. The search mechanism is incorporated in the decentralized market and a market-clearing mechanism is used in the centralized market. This approach has some weaknesses. For example, in their model, market clearance is not in the market where money matters. Money is used to overcome the search frictions in the decentralized market, but money is pre-conditioned for acceptance in the centralized market for clearance purposes. This downplays another important role of money as an instrument to clear the market.

This paper investigates two aspects of money. *First*, can fully decentralized good markets

^{3.1}Lagos and Wright (2005) also contribute by introducing divisibility for goods and money, and introducing sub-periods, where one of the markets in a sub-period is decentralized and exchange involves Nash bargaining as a solution.

with frictions use money to achieve market clearance? *Second*, this paper investigates if money plays an instrumental role in clearing the market within a period. *Third*, do markets need money to function, or is money a necessary condition for the existence of markets? Kultti (1994) shows that agents are able to achieve economic efficiency with money as a pre-condition for the existence of markets. I show here that markets can exist before money; we called those markets barter markets. I also show that money is a necessary tool to achieve market clearance when the number good types is greater than 2. Without money, market clearance is hard to achieve, due to imperfect information.

In this model, we are not concerned about whether agents want to search or stay, or their choice of which market to visit. For that, see the second chapter of this dissertation and BCKW. In this chapter, we only investigate monetary equilibrium, where money is accepted as a medium of exchange, and how money is crucial in clearing decentralized markets for goods.

3.2 Model

This model consists of N infinitely-lived agents with specialized production and generalized consumption, where N is an infinitely-large number. The goods in production and consumption are indivisible and called *real commodities*, as in KW. The set $K = \{k \in K : k \geq 2 \text{ and } k \in N\}$ represents the types of goods and $\frac{1}{k}$ denotes the equal proportion of agents who will consume that type of good^{3.2}.

Market clearance in this paper requires that each agent gets his desired good in exchange for one unit of his own production. Each period has two sub-periods, as in Lagos and Wright (2004). The critical difference between our models is that markets in *both* sub-periods are decentralized in my model. Initially, regardless of type, agents are randomly assigned and grouped in equal proportion into two categories: (i) searchers and (ii) stayers. An agent

^{3.2}This notation $\frac{1}{k}$ corresponds to x in KW's paper.

whose turn it is to search is called a *searcher*, and an agent whose turn it is to stay is called a *stayer*. As depicted in Figure 3.1, a searcher in the first sub-period becomes a stayer in the second sub-period, and a stayer in the first sub-period becomes a searcher in the second sub-period. Subsequently, stayers and searchers will alternate their roles in each sub-period, so there are always fixed and equal shares of searchers and stayers in every sub-period.

Consider a market where people coordinate such that there are equal proportions of searchers and stayers at steady state.^{3.3} Imagine a world where people work on a given schedule to indirectly coordinate the search process, such that whoever stays to offer a good or service to trade gets visited by a searcher. Each sub-period is a decentralized market where agents can directly search for the good they want.^{3.4} The commodity goods being produced by everyone are agent-specific and fixed, and this is public information. Everyone knows the quantity of money in the economy. Everyone can produce only once in each period and must consume a good in order to produce. Whenever an agent does not hold his production good and has acquired the good he wants, he will exit the market immediately and rejoin the market in the next period.

In the model, there are organized markets for each type of good. The type of consumption good that an agent wants is private information, only known by him and revealed to those who have visited him or to the person he visits. After acquiring the good he wants to consume, he will consume it immediately and draw a new taste shock for the next sub-period. Each searcher can visit the exact market that sells the good he wants to consume. After meeting with a stayer and trading with the stayer, he knows the good the stayer wants to consume in that period. He can choose to visit him again in the next sub-period if he happens to draw a taste shock for the same type of good again. However, the stayer may have drawn a different type of good that he wants to consume in the next sub-period. Hence,

^{3.3}This is a pre-condition which ensures that we are not addressing who is searching or staying and the matches between searchers and stayers are always complete.

^{3.4}Unlike Lagos and Wright's (2003) model, where the first sub-period is decentralized but the second is decentralized, this paper presents a model with market clearing in a decentralized search market within a period.

the absence of double coincidence of wants may occur because the stayer may not want to consume the same good from the previous sub-period, and thus may not trade with the same good that he brought. The only way the same stayer wants the good that the searcher brought in the next sub-period is if and only if the stayer failed to get and consume the good he wants.

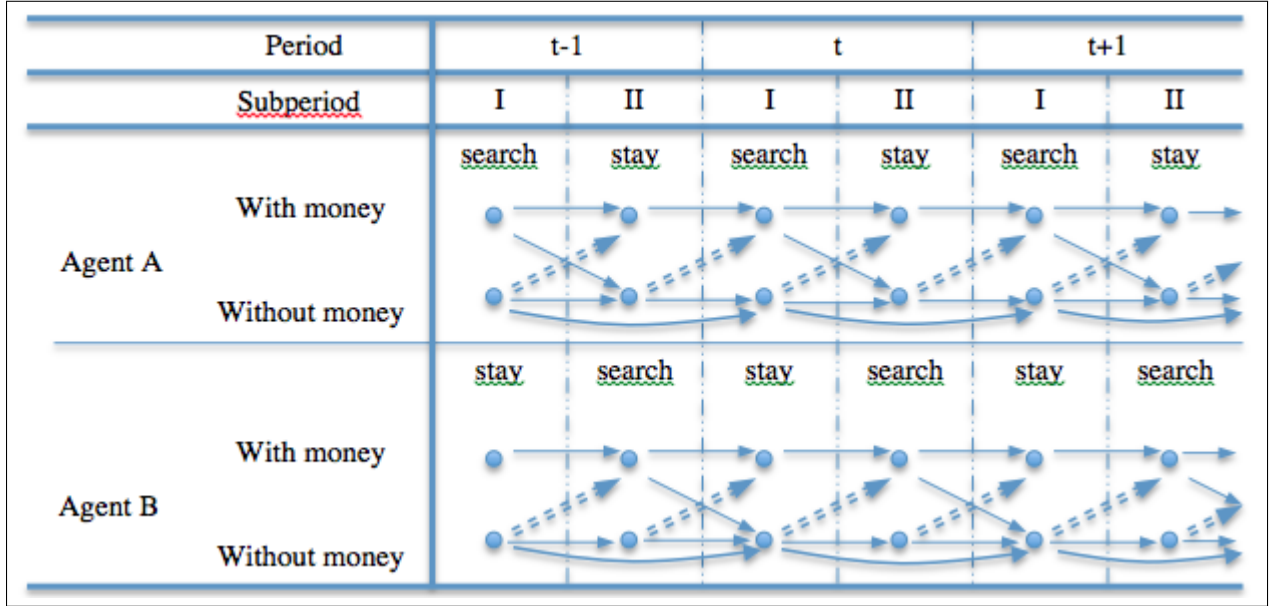


Figure 3.1: A diagram showing the role of agents and the strategies for each sub-period within periods $t - 1$, t , and $t + 1$.

3.3 Barter and Market

In this section, each agent produces one unit of a type-specific commodity good. Initially, each agent only holds one unit of a commodity good they produce. There are two assigned groups: searchers and stayers. Trade between any two agents is mutual. If an agent is offered the good he wants to consume, he will be ready to accept it. They trade if and only if the outcome benefits both; or in this case, if both want the good offered they barter. Otherwise, they part. When they barter, the stayer and searcher each get the good they

want to consume. After a successful bartering, both agents will exit the market and return in the next period to the market.

In this section, each agent only has one unit of a commodity good. They produce one unit of a type-specific commodity good. There are two assigned groups: searchers and stayers. Trade between two agents is mutual when they meet and they trade if and only if the outcome benefits both; otherwise, they part. If an agent is offered the good he wants to consume, he will definitely agree to exchange, or in this case, if both want the good offered they barter. After a successful bartering, both agents will exit the markets and will come back in the next period. When they barter, the stayer and searcher get the good they want to consume.

The value functions for the searcher (V_1) and stayer (S_1) are as follows:

$$V_1 = \beta \left[\frac{1}{k-1} (u + V_1 - S_1) + S_1 \right] \quad (3.1)$$

$$S_1 = \frac{u}{k-1} + V_1 \quad (3.2)$$

V_1 is the value function for a searcher with a commodity in the first sub-period where he gains utility u when matched with someone who holds the good he wants with probability $\frac{1}{k-1}$ and will exit the market to become a searcher in the next period, V_1 with discount rate, β . If the searcher barter and get his good, he will not participate in the next sub-period and will wait until the end of a period to consume his good, and will then become a searcher again. If the searcher does not find the good he wants to consume, he will engage as a stayer (S_1) in a new search in the second sub-period.

S_1 is the value function for a stayer with a commodity in the second sub-period where he gains utility u when matched with probability $\frac{1}{k-1}$ and he becomes a searcher with a

commodity in the next first sub-period. Manipulating the Equations [3.1]-[3.2] gives

$$rV_1 = \frac{2k-3}{(k-1)^2}u \quad (3.3)$$

The equation above denotes the expected utility for an agent initially assigned to be searcher in a non-monetary environment, where r is the time preference rate. Now consider the other group of agents who initially are assigned to be stayers.

$$X_1 = \beta \left[\frac{1}{k-1} (u + X_1 - W_1) + W_1 \right] \quad (3.4)$$

$$W_1 = \frac{u}{k-1} + X_1 \quad (3.5)$$

X_1 is the value function for a stayer with a commodity in the first sub-period, where he gains utility u when matched with someone who holds the good he wants with probability $\frac{1}{k-1}$ and becomes a stayer with a commodity in the next period with discount rate β . If a stayer barter and gets his good, he would not participate in a search in the next sub-period, but waits until the end of the period to consume his good, and then becomes a stayer again. If the stayer does not find the good he wants to consume, he will engage in a new search in the second sub-period. W_1 is the value function for a searcher with a commodity in the second sub-period, where he gains utility u when matched with probability $\frac{1}{k-1}$ and becomes a stayer with commodity in the next first sub-period.

Solving Equations [3.4] and [3.5] gives the following expression:

$$rX_1 = \frac{(2k-3)}{(k-1)^2}u \quad (3.6)$$

The equation above denotes the expected utility for agents initially assigned to be a stayer in a non-monetary environment, where r is the time preference rate. Money is not essential for a market to exist. Unlike Kultti (1995), who shows that introducing money enables the

formation of a market, implying that markets can only function when money exists, I show this is not necessarily the case. The preceding equation shows that a market can form and operate without the need for money. Without money, barter markets can exist and trade can still be conducted. There are examples of barter markets in the real world, even in modern economies, as described by Dalton (1982) and Argumedo and Pimbert (2010).

In this paper, I show that a barter market can exist without money, and money does not replace the role of the market. The question of which good becomes commodity money and the adoption of fiat money are not issues to be addressed here.^{3.5}

Although people can achieve better trade outcomes with a market, even without the use of money, the good market gets harder to clear when market friction arises as the number of types of goods increases. Without the use of money, people will only find a successful match to barter, with probability $\frac{2k-3}{(k-1)^2}$ within a period. without the use of money. Let's say there are N agents in the market where $N > nk$ such that $n > n_0$ and n_0 is some large integer, then:

$$\sum_{i=1}^N \frac{(2k-3)}{(k-1)^2} 1_i \leq \sum_{y=1}^N 1_y \quad (3.7)$$

In the above inequality, the term on the left denotes the quantity demanded that is acquired from successful bartering, and the term on the right is the total quantity supplied within a period.^{3.6} In a non-monetary environment, the market cannot clear at all when $k > 2$. Let $B = \frac{(2k-3)}{(k-1)^2} \sum_{i=1}^N 1_i$, then we have $\frac{\partial B}{\partial k} < 0$ and $\lim_{k \rightarrow \infty} B = 0$. The market gets more difficult to clear as the number of goods increases. In fact, the market can only fully clear when $k = 2$. Without the use of money, decentralized markets make it impossible to

^{3.5}Search costs are used in the money search model to give rise to the choice of money. In this model, I focus on the question of whether a barter market can exist with or without money, so search costs are omitted for parsimony without much affecting the results. Search costs are more important when we address questions about the emergence of commodity money or location strategies.

^{3.6}In this model, we only compare quantity demanded with quantity supplied, rather than with the quantity produced in a period, because Walrasian market clearance requires that the quantity demanded equals the quantity supplied. However, the set-up can be modified to also account for quantity produced without affecting the major findings.

achieve Walrasian market clearance for any $k > 2$. In the next section, money is a necessary tool to achieve market clearance within one-period.

3.4 Monetary Equilibrium

We focus only on monetary equilibrium at steady state, where agents accept money as a medium of exchange. Money is introduced into the model in this section as a token that is indivisible, intrinsically worthless and inconvertible. Each monetary token is tradable for a one-to-one swap with any real commodity. Each agent can only hold at most one unit of money or commodity at a time. Initially, only a portion m of the agents are endowed with one unit of money, where $0 < m < 1$.

As in Figure 3.1, a searcher and stayer can either be a money-holder or a non-money-holder. Every agent maximizes his expected utility by choosing whether or not to accept money as a medium of exchange to achieve a trade. If a stayer does not have money, he can choose to accept money as a medium of exchange in return for his commodity good, or he can reject and nothing happens. If a stayer holds money, he cannot sell it, because the market is only meant for selling goods; he can only wait for the next sub-period to search with money.

A money-holder is someone who holds one unit of money, and a commodity holder is someone who holds one unit of a good. Every agent holds either one unit of a good or money at any time. Townsend's (1998) turnpike model restricts money to be the sole medium to achieve successful trade. His concept of alternating the fixed role of agents is similar to this model, except that the present model allows another alternative: a decision to barter to achieve successful trade. Initially, a fraction $1 - m$ of the agents are endowed with a unit of money. The arrival rate is normalized to 1 to simplify the exposition without affecting the equilibrium outcomes.

The setup remains the same, where the population is divided into two equal groups: one

is a group of searchers and the other is a group of stayers in the first sub-period. There are four value functions for agents from the two groups. The first two value functions are for first sub-period *searchers* who are money-holders (V_m) or commodity-holders (V_1):

$$V_m = \beta[(1 - m_x)\Pi(u + S_1 - S_m) + S_m] \quad (3.8)$$

$$V_1 = \beta[(1 - m_x)\frac{1}{k-1}(u + V_1 - S_1) + S_1] \quad (3.9)$$

V_m denotes the value function for a searcher who holds money in the first sub-period, with a discount rate β , times the probability $(1 - m_x)$ of meeting a stayer who is not a money holder and the probability (Π) that the stayer accepts money, after which the searcher gains utility u from consumption. After consuming the good, he produces a unit of good and will join the second sub-period as a stayer holding a commodity that he hopes to exchange for money from a searcher, so that he can become a searcher holding money in the next sub-period. If he does not exchange to get the good he wants, he will become a stayer with money (S_m) in the second sub-period.

V_1 denotes the value function for a searcher who holds one unit of commodity in the first sub-period with a discount rate β and the probability $(1 - m_x)$ of meeting a stayer who is not a money holder and the probability $\frac{1}{k-1}$ of meeting a stayer who is not a money holder and the utility u from consumption. He will also exit the market after consumption to wait until the end of the period to produce a unit of good to become a stayer with commodity (S_1) in the second sub-period.^{3.7}

$$S_m = V_m \quad (3.10)$$

^{3.7}Note: There is no discount rate β for Equations 3.10 and 3.11 because both value functions denote the second sub-periods. The discount rate is only applied to inter-period, not across subperiods within a period.

$$S_1 = (1 - m_w) \left[\frac{u}{k-1} + V_1 - V_m \right] + V_m \quad (3.11)$$

S_m denotes the value function for a stayer holding money in the second sub-period who becomes a searcher with money (W_m) in the next first period. A stayer holding money in the second sub-period will not be able to trade with anyone because he cannot go to his good market to sell his money. Each good market is restricted to selling a specific type of good.

S_1 denotes the value function for a stayer holding a commodity in the second sub-period, with probability $(1 - m_w)$ of meeting a searcher who is not a money holder and probability $\frac{1}{k-1}$ that he would want the good brought by the searcher, after which he gains utility u from consuming it. If the stayer fails to meet a searcher who has the good he wants, they part and the stayer becomes a searcher holding commodity (V_1) in the next first sub-period. With probability m_w , he meets a searcher with money who wants to buy his good. If he accepts the money in exchange for his good, he will become a searcher holding money (V_m) in the next first sub-period.

$$X_m = \beta W_m \quad (3.12)$$

$$X_1 = \beta \left[(1 - m_v) \frac{1}{k-1} (u + X_1 - W_1) + m_v (W_m - W_1) + W_m \right] \quad (3.13)$$

X_m denotes the value function for a stayer holding money in the first sub-period, given by the discount rate β times W_m , the value of becoming a searcher with money in the second subperiod. As a stayer holding money in the first sub-period, he will not be able to trade with anyone, because he cannot go to his good market to "sell" his money (same as trading his money for a good at his own market), as in 3.10. Each good market is restricted to selling a specific type of good.

X_1 denotes the value function for a stayer holding a commodity in the first sub-period, given by the discount rate β times the probability $(1 - m_v)$ of meeting a searcher who is not a

money holder and the probability $\frac{1}{k-1}$ that he would want the good brought by the searcher, after which he gains utility u from consuming it. If he does not meet a searcher who has the good he wants, they part and he becomes a searcher W_1 holding a commodity, similar to S_1 . With probability m_v , he meets a searcher with money who wants to buy his good. If he accepts the money in exchange for his good, he will become a searcher W_m holding money in the second sub-period.

$$W_m = (1 - m_s)\Pi(u + X_1 - X_m) + X_m \quad (3.14)$$

$$W_1 = (1 - m_s)\frac{u}{k-1} + X_1 \quad (3.15)$$

W_m denotes the value function for a searcher holding money in the second sub-period with probability $(1 - m_s)$ of meeting a stayer who is not a money holder and probability Π that the stayer accepts money, after which the searcher gains utility u from consumption. After consuming the good, he produces a unit of good and will join the next first sub-period as a stayer X_1 holding a commodity that he hopes to exchange for money from a searcher, allowing him to become a searcher holding money in the next second sub-period. If he cannot exchange to get the good he wants, he will remain a stayer X_m holding money in the next first sub-period.

W_1 denotes the value function for a searcher holding money in the second sub-period with probability $(1 - m_s)$ of meeting a stayer who is not a money holder and with probability $\frac{1}{k-1}$ that the stayer wants the good he brought, after which they engage in a barter and he gains utility u from consumption. After consuming the good, he produces a unit of good and will join the next first sub-period as a stayer holding a commodity. There is no market for money, so he can only barter in this situation. If he did not exchange to get the good he wants, he will continue to be a stayer holding a commodity in the next first sub-period.

The definition of monetary equilibrium in this paper is a steady-state search equilibrium

denoted by the value functions $(V_m, S_m, W_m, X_m, V_1, S_1, W_1, X_1)$, money supply m and a distribution of agents (m_w, m_x, m_s, m_v) in the market that satisfies the incentive to hold money: $\max\{V_m, S_m\} > \max\{V_1, S_1\}$ and $\max\{W_m, X_m\} > \max\{W_1, X_1\}$.

To ensure that monetary equilibrium exists, the best response for a non-money holder stayer to accept money as a medium of exchange is when a searcher finds that it is more profitable holding money than holding a commodity in the next sub-period, $(V_m - V_1 > 0)$. Solving the incentive constraint with Equations [3.8]- [3.11], gives:

$$\Pi > \frac{(k - 2 + m_w) + (k - 1)(1 - m_w)}{(k - 1)(k - m_w)} \quad (3.16)$$

At steady-state, with optimal money supply $m = .5$ and the distribution $m_w = 1$ and $m_x = 0$, the best response for a monetary equilibrium to accept money as a medium of exchange can be simplified at steady-state to:

$$\Pi > \frac{1}{k - 1} \quad (3.17)$$

This result is the same as in BCKW. It means that the acceptability of money as a best response strategy holds when the probability of finding someone who will accept money as a medium of payment exceeds the probability of finding someone who wants your good to engage in a barter. Now, we look at the group who is assigned as stayers in the first sub-period.

To show the existence of monetary equilibrium, there must exist be incentive for an agent to accept money because it gives a higher utility. Using equations (3.12)-(3.15) in the inequality $W_m > W_1$ and solving for Π gives:

$$\Pi > \frac{\beta(1 - m_v)(k - 2 - m_s) + (k - 1)(1 - m_s)}{(1 - m_s)[(k - 1)^2 + \beta(1 - m_v)(k - 2 - m_s)]} \quad (3.18)$$

At steady state, with optimal money supply $m = .5$ and the distribution $m_s = 0$ and $m_v = 1$, a stayer will choose to accept money as a best response strategy in monetary

equilibrium if:

$$\Pi > \frac{1}{k-1} \quad (3.19)$$

Intuitively, a stayer will accept money as a medium of exchange when the expected value of meeting a non-moneyholder stayer who is willing to accept money is higher than the expected value of meeting someone who would want to consume the good he brought. The results here correspond to KW in that Π is dependent on k . Both searcher and stayer have the same result in monetary equilibrium at steady state. However, when the set of k goods becomes infinitely large in KW or BCKW, the monetary equilibrium does not hold because it does not satisfy the condition to be a medium of exchange, as shown in the second chapter of my dissertation.

In monetary equilibrium with $m = .5$, which results in $(m_s, m_v, m_w, m_x) = (0, 1, 1, 0)$ at steady state, the value function for the agents of both groups can be simplified to:

$$rV_m = u \quad (3.20)$$

$$rX_1 = u \quad (3.21)$$

This interesting outcome has not been shown in previous studies. It says that if you are a searcher or stayer, or if you are a buyer or seller, the expected utility within a period is symmetric. Essentially this shows that any agent is indifferent in choosing to become a searcher or seller in a symmetric setup.

$$\sum_{i=1}^N 1_i = \sum_{y=1}^N 1_y \quad (3.22)$$

In equation [3.22], the left-hand side denotes the quantity of acquired good that they want (or acquired quantity demanded) from successful trade with the use of money, and the right-

hand side denotes the total quantity supplied within a period. In this period, if everyone chooses to accept money as the medium of exchange, it ensures that everyone is able to obtain the good they want to consume in every period in steady state. Everyone produces one unit of output and is able to exchange for one unit of consumption good in a totally decentralized market. Since this model only focuses on good frictions and the price is assumed to be fixed, market clearance means that every unit of production is being consumed at steady state, given optimal money supply. An optimal money supply means the fraction of m that gives the first-best expected utility. It also means each agent is being matched to get the good they want to consume within a period. This would satisfy Walrasian equilibrium in that the market potentially clears with the use of money. Since the market clears in one-period, it can be easily shown that the market potentially clears in all subsequent periods with the introduction of money.

Proposition 3.1. *Given imperfect information, money is necessary for market clearing to achieve Walrasian equilibrium when $k > 2$.*

Without money, an agent matching to achieve a successful barter is inversely proportional to the variety of goods. The larger the variety of goods, the less likely an agent will be matched with the good they want, as depicted in Figure 3.2. With the introduction of money, an agent chooses the strategy to accept money to improve the chance of acquiring the desired good; otherwise they would have a lower probability of getting the good. Any deviation from the equilibrium path results in a lower chance for an agent to get the good they want and, hence, lower expected utility. In each sub-period, there are agents who stay and choose to accept money in exchange for their production good and agents, or buyers, who trust and hold money as an instrument of exchange to get the good they want. Otherwise, in a non-monetary economy, people will have difficulty finding a match to exchange for the good they want. Thus, markets will be very difficult to clear in a non-monetary economy. Money is crucial in assisting markets to clear.

3.5 Conclusion

Moving away from the traditional setup with perfect information, where a Walrasian auctioneer clears the market, money is a necessary instrument to coordinate market clearing in a decentralized market system with imperfect information. The analysis also shows that markets can function without money where people can barter when they meet at a market. Nonetheless, money helps to improve successful trade with markets.

However, if the number of types of good exceeds two, the use of money is necessary for markets to clear within one period and in all subsequent periods. The proposed model shows that in a simple world with one fixed price, money is essential to achieve Walrasian equilibrium. Inclusion of an aggregate price mechanism in this model could potentially represent significant progress toward an integration of money search and general equilibrium models.

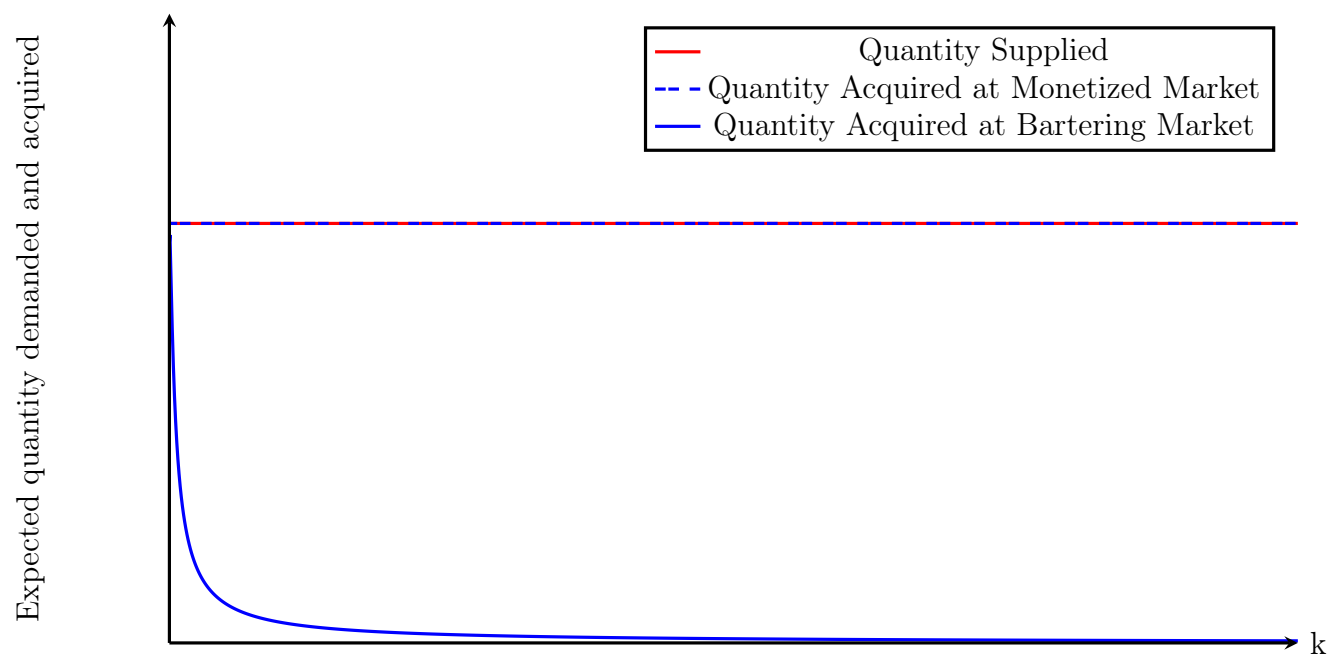


Figure 3.2: The effect of k on market clearance in a monetized market and a bartering market, where k denotes the number of good types.

Appendices

Appendix A: Proofs for Chapter 1

Proof of Lemma 1.1. There is no search or storage cost to stay at his own market (or stay home). He can always choose the strategy to stay home which has a lower bound of zero utility if no one visits his market. This lemma is the same as in Goldberg's (2007) Lemma 1. If visiting other markets will result in negative outcomes, he can always choose not to visit other markets and get zero utility. \square

Proof of Proposition 1.1. We solve the equilibrium by tracing backward. Between choosing V_{131} and V_{132} in market 2, he would not choose to trade for good 2 with his good 1 when he is indifferent, since he does not gain in expected utility for trading in good 1. Evaluating agent 3 in market 3, agent 3 who consumes good 2 will not want to exchange with agent 1 because $V_{323} \geq V_{321}$ in a symmetric case. This makes $\Pi_3 = 0$. This simplifies the equation to:

$$V_{131} = \max \beta \{0, -c + \frac{1}{2}(V_{131}) + \frac{1}{2}V_{131} - c, -c + \frac{1}{2}(U + \frac{1}{2}(V_{131} + V_{121})) + \frac{1}{2}V_{131} - c\} \quad (\text{A.1})$$

$$V_{131} = \max \beta \{0, -2c, -2c + \frac{1}{2}(U + \frac{1}{2}(V_{131} + V_{121})) + \frac{1}{2}V_{131}\} \quad (\text{A.2})$$

$$V_{132} = \max \beta \{V_{132}, -c + V_{132} - c, -c + \frac{1}{2}(U + \frac{1}{2}(V_{131} + V_{121})) + \frac{1}{2}V_{132} - c\} \quad (\text{A.3})$$

From the equations above, $V_{132} - V_{131} = 0$, then $V_{132} = V_{131}$. Hence the agent will not accept the trade of commodity 1 for commodity 2.

$$V_{121} = \max \beta \{V_{121}, -c + \frac{1}{2}(V_{121}) + \frac{1}{2} \max\{V_{121}, V_{123}\} - c, -c + \frac{1}{2}(U + \frac{1}{2}(V_{131} + V_{121})) + \frac{1}{2}V_{121} - c\} \quad (\text{A.4})$$

$$V_{121} = \max \beta \{0, -2c, -2c + \frac{1}{2}(U + \frac{1}{2}(V_{131} + V_{121})) + \frac{1}{2}V_{121}\} \quad (\text{A.5})$$

$$V_{121} = \max \beta \{0, -2c, -2c + \frac{1}{2}(U + V_{121})\} \quad (\text{A.6})$$

It is easy to show that $V_{131} = V_{121}$. So, solving these equations and simplifying them gives:

$$V_{131} = V_{121} = \max \beta \{0, -2c, -2c + \frac{1}{2}U\} \quad (\text{A.7})$$

He chooses to stay at his own market if and only if $U \leq 4c$. It is optimal to stay home/stay at his own market i because the expected utility gained is not greater than the expected transport costs. □

Proof of Proposition 1.2. Refer to the text. □

Proof of Proposition 1.3. In the symmetric case, we know that neither all agents search or

all agents stay. Some agents will search, and some agents will stay. So,

$$V_{iji} = r \max[\alpha_{ii}(\frac{1}{2}(u + \frac{1}{2}(V_{iji} + V_{iki}) - V_{iji})), -2c_i + \alpha_{ij}(\frac{1}{2}(u + \frac{1}{2}(V_{iji} + V_{iki}) - V_{iji})), -2c_i + \alpha_{ik} \max\{V_{iji}, V_{ijk}\} - V_{iji}] \quad (\text{A.8})$$

$$V_{ijk} = r \max[V_{ijk}, -2c_i + \alpha_{ij} \max\{V_{iji}, V_{ijk}\} - V_{ijk}, -2c_i + \alpha_{ik}(\frac{1}{2}(u + \frac{1}{2}(V_{iji} + V_{iki}) - V_{ijk}))] \quad (\text{A.9})$$

$$V_{iki} = r \max[\alpha_{ii}(\frac{1}{2}(u + \frac{1}{2}(V_{iji} + V_{iki}) - V_{iji})), -2c_i + \alpha_{ij} \max\{V_{iji}, V_{ijk}\} - V_{iki}, -2c_i + \alpha_{ik}(\frac{1}{2}(u + \frac{1}{2}(V_{iji} + V_{iki}) - V_{iki}))] \quad (\text{A.10})$$

From $V_{iji} - V_{iki} = 0$, then $V_{iji} = V_{iki}$. This simplifies the above equations to:

$$V_{iji} = r \max[\alpha_{ii}\frac{u}{2}, -2c_i + \alpha_{ij}\frac{u}{2}, -2c_i + \alpha_{ik} \max\{V_{iji}, V_{ijk}\} - V_{iji}] \quad (\text{A.11})$$

$$V_{ijk} = r \max[0, -2c_i + \alpha_{ij}\frac{u}{2}, -2c_i + \alpha_{ik} \max\{V_{iji}, V_{ijk}\} - V_{ijk}] \quad (\text{A.12})$$

$$V_{iki} = r \max[\alpha_{ii}\frac{u}{2}, -2c_i + \alpha_{ij} \max\{V_{iji}, V_{ijk}\} - V_{iki}, -2c_i + \alpha_{ik}\frac{u}{2}] \quad (\text{A.13})$$

If an agent wants to exchange for commodity k with commodity i , he would not exchange for commodity i with commodity k when he visits market k . For an agent who produces i and desires j , but holds k , it must be the case that $V_{ijk} > V_{iji}$. Otherwise he would not

exchange them in the first place if we assume that agent is rational. This implies that:

$$V_{ijk} = r \max[0, -2c_i + \alpha_{ij} \frac{u}{2}, -2c_i] \quad (\text{A.14})$$

Since $V_{ijk} \leq V_{iji}$, an agent does not trade commodity i for k since we assume no trade if agents are indifferent due to transaction cost. This further simplifies Equation A.11 to:

$$V_{iji} = r \max[\alpha_{ii} \frac{u}{2}, -2c_i + \alpha_{ij} \frac{u}{2}, -2c_i] \quad (\text{A.15})$$

An agent who produces i , desires j , and holds i will not visit market k because of a lower payoff. To show that there is a bartering equilibrium, it must be the case that some agents would search and some would stay. So he would either stay or visit market j . In steady state, $\alpha_{ii} = \frac{n_j + n_k}{2(1-n_i)}$ and $\alpha_{ij} = 1$.

$$\alpha_{ii} \frac{u}{2} = -2c_i + \alpha_{ij} \frac{u}{2} \implies \alpha_{ii} - \alpha_{ij} = -\frac{4c_i}{u} \implies \frac{n_j + n_k}{2(1-n_i)} - 1 = -\frac{4c_i}{u}$$

In the symmetric case, all agents will have the same payoff and choose the strategy identically and independently. So, $n_1 = n_2 = n_3 = n$.

$$\frac{2n}{2(1-n)} - 1 = -\frac{4c_i}{u} \implies \frac{4n-2}{2(1-n)} = -\frac{4c_i}{u} \implies n = \frac{1-\frac{4c_i}{u}}{2-\frac{4c_i}{u}} \implies n = \frac{u-4c_i}{2u-4c_i}$$

Since this is a symmetric case, all other agents' strategies will be the same since they are all identical. Thus there is an equilibrium when they are indifferent between staying or visiting market j , given that $n = \frac{u-4c_i}{2u-4c_i}$, such that n fraction of agents search for each type of agents.

□

Proof of Proposition 1.4. From Proposition 1.3, we know that $V_{ijk} \leq V_{iji}$, an agent does not trade commodity i for k since we assume no trade if agents are indifferent due to the transaction cost. An agent will not trade for a commodity he does not want to consume. This is because holding commodity k is as likely to get the good he desires as with commodity j . There is no improvement in the chance of getting the good he desires. He does not trade when he is indifferent due to the transaction cost. So he would not accept a good he does

not desire, hence the role of a commodity as a medium of exchange does not arise. \square

Proof of Proposition 1.5. In the asymmetric case with common knowledge and complete information, first, we check at the initial state on who would accept a good he does not consume. Suppose initially no one would accept any good other than the good they want to consume, then we would have $\Pi_{ij} = 0$ for all i, j . There would be some agents from each type who search when $U > 4c$, where $c = \max\{c_1, c_2, c_3\}$. First, we check with agent 2 holding good 2,

$$rV_{233} = -2c_2 + \alpha_{23} \frac{1 - n_{23}}{2 - n_{23} - n_{13}} \left(u + c_2 + \frac{1}{2} (V_{232} + V_{212}) - V_{232} \right) \quad (\text{A.16})$$

$$rV_{231} = -2c_1 + \alpha_{23} \frac{1 - n_{13}}{2 - n_{23} - n_{13}} \left(u + c_1 + \frac{1}{2} (V_{232} + V_{212}) - V_{231} \right) \quad (\text{A.17})$$

We can safely assume that $n_{ij} = n$ for all i, j because they all choose the same strategy not to accept a good they do not want to consume. Let $n = n_{23}, n_{13}$,

$$rV_{232} = -2c_2 + \alpha_{23} \frac{1}{2} \left(u + c_2 + \frac{1}{2} (V_{232} + V_{212}) - V_{232} \right) \quad (\text{A.18})$$

$$rV_{231} = -2c_1 + \alpha_{23} \frac{1}{2} \left(u + c_1 + \frac{1}{2} (V_{232} + V_{212}) - V_{231} \right) \quad (\text{A.19})$$

Then $V_{231} \leq V_{232}$ must be true for agent 2 not to accept good 1 when they do not want to consume, or $V_{231} > V_{232}$ must be false. From the equations above, we get

$$V_{231} - V_{232} > 0 \implies -2c_1 + 2c_2 + \frac{\alpha_{23}}{2} (c_1 - c_2) > 0$$

$(c_2 - c_1) \left[2 - \frac{\alpha_{23}}{2} \right] > 0$. We get $c_2 > c_1$ and $4 > \alpha_{23}$. $V_{231} > V_{232}$ is true, so $V_{231} \leq V_{232}$ cannot be true. The result shows that $V_{231} > V_{232}$ when $c_2 > c_1$. It must be the case that exchanging good 2 for good 1 yields higher expected utility. Commodity 1 serves as a medium of exchange for agent 2.

Similarly we check with agent 3 holding good 3,

$$rV_{323} = -2c_3 + \alpha_{32} \frac{1}{2} \left(u + c_3 + \frac{1}{2} (V_{323} + V_{313}) - V_{323} \right) \quad (\text{A.20})$$

$$rV_{321} = -2c_1 + \alpha_{32} \frac{1}{2} \left(u + c_1 + \frac{1}{2} (V_{323} + V_{313}) - V_{321} \right) \quad (\text{A.21})$$

Then $V_{321} \leq V_{323}$ must be true for agent 3 not to accept good 1 when they do not want to consume. Or $V_{321} > V_{323}$ must be false. From the equations above, we get

$$V_{321} - V_{323} > 0 \implies -2c_1 + 2c_3 + \frac{\alpha_{32}}{2}(c_1 - c_3) > 0$$

$(c_3 - c_1) \left[2 - \frac{\alpha_{32}}{2} \right] > 0$. We get $c_3 > c_1$ and $4 > \alpha_{32}$, given $\alpha_{23} \in [0, 1]$. $V_{321} > V_{323}$ is true, so $V_{321} \leq V_{323}$ cannot be true. The result shows that $V_{321} > V_{323}$ when $c_3 > c_1$ and $4 > \alpha_{32}$. It must be the case that exchanging good 3 for good 1 yields higher expected utility. Commodity 1 serves as a medium of exchange for agent 3.

However, for agent 1 is the opposite case, such that $V_{121} \geq V_{123}$ and $V_{131} \geq V_{132}$. Agent 1 will not accept good 2 or 3 if he does not want to consume them.

The type 1 agent knows that type 2 and 3 agents would accept commodity 1 regardless if they desire to consume it or not, thus all type 1 agents will search because other agents will accept commodity 1 regardless. Since all type 1 agents search, then commodity 2 cannot be a commodity money. It turns out that type 1 agents will not accept any good except than his own consumption, but type 3 and 2 agent would accept good 1 even if they don't consume it. Then, we have a location strategy equilibrium for the asymmetric case, as shown in Figure 3.

Let $I(1, 2, 1)$ denotes agents who produce good 1, desire good 2, hold good 3. At steady state, all Type 1 agents search. Half Type 1 agents desire commodity 2, $I(1, 2, 1)$, and another half desire commodity 3, $I(1, 3, 1)$; the former visits market 2 and the latter visits market 3.

Examining Type 2 agents, there are a quarter of Type 2 agents who are $I(2, 1, 2)$ and

$I(2, 3, 2)$ and half of $I(2, 3, 1)$ in the beginning of a period at steady state. $I(2, 1, 2)$ and $I(2, 3, 2)$ will stay and $I(2, 3, 1)$ will visit market 3 by holding commodity 1. Since half of Type 2 stays in the market 2 and is visited by half of Type 3 and Type 1, each stayer of Type 2 will meet with two visitors. With the bilateral meetings assumption, Type 2 stayers can choose to meet with only one visitor. Since he knows that the visitors all are holding good 1, stayers are indifferent on whom he choose to trade with. The stayer does not know the visitors' desired type of consumption. So, the stayer conducts a lottery to select which two visitors he wants to meet.

After acquiring from the match and consuming the good he wants, the stayers $I(2, 1, 2)$ draw a new taste shock. The draw results half of the stayers $I(2, 1, 2)$ to become $I(2, 1, 2)$ again and another half to become $I(2, 3, 2)$ in the next period. As for stayers $I(2, 3, 2)$ after matching, they didn't acquire the good they want but accepted commodity 1 as medium of exchange and become a searcher $I(2, 3, 1)$ in the next period.

Searchers of Type 2, $I(2, 3, 1)$, 'compete' with $I(1, 3, 1)$ to be chosen by type 3 stayers to trade. All visitors at market 3 bring commodity 1; stayers are indifferent and conduct a lottery. Since there are twice as many visitors as stayers at market 3, $I(2, 3, 1)$ have only half a chance to be chosen to trade in market 3. So only half of $I(2, 3, 1)$ consume their desired good and draw a new taste shock, either becoming $I(2, 1, 2)$ or $I(2, 3, 2)$. Another half of searchers $I(2, 3, 1)$ remains to be $I(2, 3, 1)$ in the next period.

At the end of the period, there are a quarter of Type 2 who are $I(2, 1, 2)$ and $I(2, 3, 2)$ and half who are $I(2, 3, 1)$. Examining Type 3 will be the same as Type 2. There are a quarter of Type 3 agents who are $I(3, 1, 3)$ and $I(3, 2, 3)$ and half who are $I(3, 2, 1)$ in the beginning of a period at steady state. Given a similar explanation as for Type 2, we get the same result that there are a quarter of Type 3 agents who are $I(3, 1, 3)$ and $I(3, 2, 3)$ and half who are $I(3, 2, 1)$ at the beginning of the period. Half of $I(1, 2, 1)$ and $I(1, 3, 1)$ consume their desired good and draw a new taste shock, so the resulting distribution is half of Type 1 will be $I(1, 2, 1)$ and half will be $I(1, 3, 1)$ at the end of the period.

At steady state, $I(3, 2, 3)$ and $I(2, 3, 2)$ will exchange their produced commodity for commodity 1, as a medium of exchange. Any deviation from the steady state will make them worse off.

□

Proof of Proposition 1.6. Besides commodity 1 being accepted as commodity money, suppose commodity 2 is also accepted as a medium of exchange such that $\Pi_{32} = 1$. First, we look at the strategy for agent 2:

$$(r + \alpha_{23})V_{232} = -2c_2 + \alpha_{23}(u + c_2 + \frac{1}{2}(V_{232} + V_{212})) \quad (\text{A.22})$$

$$(r + \alpha_{23})V_{231} = -2c_1 + \alpha_{23}(u + c_1 + \frac{1}{2}(V_{232} + V_{212})) \quad (\text{A.23})$$

We have $V_{231} - V_{232} > 0$ such that $-2c_1 + \alpha_{23}c_1 > -2c_2 + \alpha_{23}c_2$, this gives us $c_1 > c_2$. This shows that an agent holding commodity 2 will always trade for commodity 1 even if he does not consume it, because the lighter commodity will improve his expected utility when he searches.

Second, we look at the strategy for agent 3 :

$$(r + \alpha_{32})V_{323} = -2c_3 + \alpha_{32}(u + c_3 + \frac{1}{2}(V_{313} + V_{323})) \quad (\text{A.24})$$

$$(r + \alpha_{32})V_{321} = -2c_1 + \alpha_{32}(u + c_1 + \frac{1}{2}(V_{313} + V_{323})) \quad (\text{A.25})$$

Then $V_{321} - V_{323} > 0$ gives us $c_1 > c_3$. This also shows that an agent holding commodity 3 will always trade for commodity 1, even if he does not consume it due to it being a lighter commodity, and for $\Pi_{23} \in [0, 1]$ still holds, regardless if holder 2 would trade for commodity 3.

Since both agent Type 2 and agent Type 3 accept commodity 1 as a medium of exchange,

agent 1 will always search since he knows that everyone accepts his commodity 1. This means all Type 1 agents will always search. There is no reason for agent 1 to accept other commodities he does not want to consume because his production is widely accepted.

Since we know that agent 1 will always search, there is no one supplying good 1 in market 1. Then we check if agent 3 would accept commodity 2 as a medium of exchange. The only Type 3 agents who would accept commodity 2 as money are those who desire good 1. So, Type 3 agents who desire good 1 are offered good 2 when they meet with a good 2 holder when Type 3 agent was a stayer. However, he will not accept commodity 2, even if the transport cost is lower than commodity for 3, because no one is in market 1. Plus, agent 3 desiring good 1 is better off to stay, knowing that everyone holding good 1 will search at equilibrium. This is the same case for agent 2 accepting commodity 3 to search for his desired commodity 1.

Suppose the a commodity with higher transport cost is used as the money. Then a Type 1 agent will not want to exchange his good for a commodity with a higher transport cost because he can search with commodity 1 with a lower cost. Then when a Type 1 agent visits a market, Type 2 or 3 would accept commodity 1, even they do not consume it because they can search in the next period, which saves them on transport cost and yields higher utility. Hence, a commodity with the lowest transport cost will always be preferred as the commodity money. Thus, only commodity 1 is the medium of exchange at steady state because it has the least transport cost. \square

Proof of Proposition 1.7. Agent 1 produces commodity 1 and he brings his own produced commodity to visit a market to search for the good he desires. When agent 1 holds 1 to visit a market to get the good he desires, he will meet with someone who either desires to consume commodity 1 or not. Regardless of the acceptability of commodity money, if he meets someone who desires to consume commodity 1, both will barter. Hence, there is always a bartering in an asymmetric case, together with commodity money. Bartering and commodity money can co-exist. The agents who produce the commodity could be matched

with someone who desires to consume his good (a commodity money), then bartering happens because the commodity money is also a consumption good. He could be matched someone who don't want to consume the commodity, but want to use it as a medium of exchange. \square

Proof of Proposition 1.8. With the introduction of fiat money into the model, the fiat money is equally and identically distributed across different types of agents. From Proposition 1.3, we know that no agent will accept any commodity as money in a symmetric environment because there is no gain from trade. So everyone just barter if they wish to trade. With introduction of money where the transport cost for holding money is lower than holding their own commodity $c_0 < c_i$, a holder of commodity will trade for money to become a moneyholder in the next period. At steady-state, everyone accepts fiat money as a medium of exchange; only holders of money will search and holders of commodity will stay. \square

Proof of Proposition 1.9. Given that $c_0 < c_i \forall i$, everyone accepts fiat money as a medium of exchange at steady state because of lower transport cost. Every agent will trade for money because it has the lowest transport cost. At the optimal money supply $m = \frac{1}{2}$, everyone will only accept fiat money as the medium of exchange. Money is equally distributed across agents. The cost of fiat money is lower than the cost of commodity money $c_0 < c_1$. Agents who produce the type 1 good will trade for fiat money with a lower transport cost. Then a quarter of type 1 agent with fiat money will be searching for good 2 and 3, respectively. The same for type 2 agent with fiat money searching for 1 and 3, as well as, for type 3 agent with fiat money searching for 1 and 2. Only fiat money holders will search, and thus will be called buyers and those who do not hold fiat money will stay to be sellers. Each seller in this case will be approached by exactly one buyer with fiat money who wants the good sold. Then each seller will accept the fiat money in return. Every buyer will consume the good, then produce and draw a new taste shock. In the next period, the buyer will become seller and the seller will become a buyer. Commodity 1 no longer is the commodity money because agent 2 and 3 would accept fiat money due to the lower transport cost. Even agent 1 would exchange commodity 1 with fiat money because it has a lower transport cost.

No buyer will want to deviate from searching because he could not trade for his desired good if he stays with fiat money. So a money holder is better off to search and be a buyer.

No seller will want to deviate from staying because he knows there is an exact match with a buyer to get a unit of fiat money in each period, given that β is large enough. Deviating to search instead of stay, means the deviator will encounter half the chance of being matched with a double coincidence of wants. So he is better off to stay and match with a buyer to get a unit of fiat money, then search for his consumption good with the probability 1. Hence the asymmetric fiat equilibrium has the same outcome, where only money holders search and non-money holders stay. When the money supply is optimal, it creates a perfect coordination environment where everyone gets the good they want over two-periods. The optimality of money supply is defined as fraction of agents who hold money that gives the highest attainable expected utility among agents.

□

Appendix B: Proofs for Chapter 2

Proof of Lemma 2.1. An agent can always choose to stay at home or at his market, which gives him a lower bound of zero utility. It costs him nothing to stay at home or his market. If other strategies give value lower than zero, he will always choose to stay home. So, he can always choose the strategy to stay home or at his market. Hence the value function cannot be lower than zero. This lemma is also established by Goldberg (2007). \square

Proof of Lemma 2.2. If an agent carries both money and a good into the marketplace for trade, he would barter to get his desired good if he finds a seller who likes his good. Otherwise he would exchange with money, unless the seller is already a moneyholder.

Suppose that exchanging with money is preferred to bartering when a searcher brings both money and a good to the market.

$$\begin{aligned}
 & \underbrace{\frac{\Pi(1-m)}{r}[U+x+W_1-W_m] + \frac{mU}{r(k-1)} - \frac{c_1}{r}}_{\text{where the offer of money is preferred for trade}} \\
 & > \underbrace{\frac{\Pi(1-m)}{r}\left(1 - \frac{1}{k-1}\right)[U+x+W_1-W_m] + \frac{U}{r(k-1)} - \frac{c_1}{r}}_{\text{where the bartering is preferred for trade}}
 \end{aligned} \tag{B.1}$$

The RHS denotes the value function for a searcher searching with a good and money who prefers to barter, whereas the LHS denotes the value function for a searcher searching with a good and money who prefers to offer money in exchange for a good. The inequality becomes:

$$\frac{mU}{r(k-1)} > \frac{-\Pi(1-m)}{r(k-1)}[U+x+W_1-W_m] + \frac{U}{r(k-1)} \tag{B.2}$$

Rearranging this inequality gives

$$U\left(1 - \frac{1}{\Pi}\right) + x > W_m - W_1 \tag{B.3}$$

The incentive condition for holding money is

$$-x - \epsilon + W_m - W_1 > 0 \text{ and } \Pi = 1, \tag{B.4}$$

hence, $W_m - W_1 > x + \epsilon > x$ for $\epsilon > 0$.

But the inequality from the incentive condition for holding money and $\Pi = 1$ imply:

$$x > W_m - W_1, \quad (\text{B.5})$$

which contradicts. □

Proof of Proposition 2.1. Assume that an agent will choose not to search if the alternative strategies give the same utility. We have to show that no one would accept money in an exchange and bringing money to market are redundant, such that,

$$\max\{V_1, S_1\} \geq -x - \epsilon + \max\{V_m, V_b\} \quad (\text{B.6})$$

and no one would choose to search. To show that no one would choose to search, it also means that all agents would prefer to stay, such that $V_1 \leq S_1 = 0, \max\{V_m, V_b\} \leq S_m = 0$. This implies that $n_1, n_c, n_m = 0$. This causes the Equations [2.1] and [2.2] to become:

$$W_1 = \max\left\{ \underbrace{\frac{U}{r(k-1)} - \frac{c_1}{r}}_{V_1 : \text{search with commodity}}, \underbrace{0}_{S_1 : \text{stay with commodity}} \right\} \quad (\text{B.7})$$

$$W_m = \max\left\{ \underbrace{\frac{\Pi(1-m)}{r}[U+x] - \frac{c_m}{r}}_{V_m : \text{search with money}}, \underbrace{\frac{U}{r(k-1)} - \frac{c_1}{r}}_{V_c : \text{search with commodity}}, \underbrace{0}_{S_m : \text{stay with money and commodity}} \right\} \quad (\text{B.8})$$

In a non-monetary autarkic equilibrium, $S_1 > V_1$, therefore:

$$c_1 \geq \frac{u}{(k-1)} \quad (\text{B.9})$$

and, $S_m \geq V_m$ gives:

$$c_m \geq (1-m)[U+x] \quad (\text{B.10})$$

Since no one chooses the strategy to search, no matching will occur and no exchange can take place. This is an equilibrium because the cost of searching exceeds the expected gain in utility from matching.

□

Proof of Proposition 2.2. For a non-monetary bartering equilibrium, agents are indifferent between searching with a commodity and staying, such that $V_1 = S_1$, $V_c = S_m$, and $\max\{V_c, S_m\} > V_m$. This implies that $n_m = n_b = 0$, so the Equations [2.1] and [2.2] become:

$$W_1 = \max\left\{\frac{U}{r(k-1)} - \frac{c_1}{r}, \frac{n_1 U(1-m)}{r(k-1)} + \frac{mn_c U}{r(k-1)}\right\} \quad (\text{B.11})$$

$$W_m = \max\left\{\underbrace{\frac{\Pi(1-m)}{r}[U+x] - \frac{c_m}{r}}_{V_m : \text{search with money}}, \frac{U}{r(k-1)} - \frac{c_1}{r}, \frac{n_1 U(1-m)}{r(k-1)} + \frac{mn_c U}{r(k-1)}\right\} \quad (\text{B.12})$$

When a moneyholder abandons searching with money because $\max\{V_c, S_m\} > V_m$, his payoff for W_1 is the same as for a non-moneyholder. This means their strategic behavior is the same, so the distribution of agents with or without money will be the same, such that $n_1 = n_c = \theta$.

Letting $n_c = n_1 = \theta$ gives:

$$\frac{u}{r(k-1)} - \frac{c_1}{r} = \frac{\theta u(1-m)}{r(k-1)} + \frac{\theta u m}{r(k-1)} \quad (\text{B.13})$$

and solving for θ gives:

$$\theta = 1 - \frac{c_1(k-1)}{u} \quad (\text{B.14})$$

and, $0 < \theta < 1$ since $0 < n_1, n_c < 1$, then

$$\frac{u}{c_1} + 1 > k > 1 \quad (\text{B.15})$$

It must be the case that the value of searching with a commodity is greater than 0, satisfying Lemma 2.1.

$$c_1 < \frac{u}{k-1} \quad (\text{B.16})$$

$$k < \frac{u}{c_1} + 1. \quad (\text{B.17})$$

□

Proof of Proposition 2.3. To search with money, the condition for a moneyholder is:

$$\max\{V_m, V_b\} > \max\{V_c, S_m\} \quad (\text{B.18})$$

To accept money in exchange for a good, the condition for the non-moneyholder is

$$-x - \epsilon + \max\{V_m, V_b, V_c, S_m\} > \max\{V_1, S_1\} \quad (\text{B.19})$$

From the condition to search with money, the equation above can be simplified to

$$-x - \epsilon + \max\{V_m, V_b\} > \max\{V_1, S_1\} \quad (\text{B.20})$$

It would be sufficient to show that $W_m > W_1$ to ensure the monetary equilibrium condition is satisfied. Thus, V_c is redundant to check in the sense that $W_m > W_1$ equals $W_m > \max\{V_1, S_1\}$ where $W_1 = \max\{V_1, S_1\}$. Given that the value function of a moneyholder searching with commodity only is V_c , $V_c = V_1$ means $W_m > \max\{V_1, S_1\} \geq V_c$. As long as the condition for monetary equilibrium is satisfied, search with a commodity is not

profitable for a moneyholder.

□

Proof of Proposition 2.4. From Proposition 2.2, we know that an agent will only search when $c_1 > \frac{u}{k-1}$, hence the LHS of the inequality is satisfied such that $k > \frac{u}{c_1} + 1$.

As for the RHS of the inequality, from Proposition 2.3, we know that the inequality must satisfy:

$$\begin{aligned} & \underbrace{\frac{\Pi(1-m)}{r}[U+x+W_1-W_m] - \frac{c_m}{r}}_{\text{searching only with money}} \\ & \leq \underbrace{\frac{\Pi(1-m)}{r}\left(1 - \frac{1}{k-1}\right)[U+x+W_1-W_m] + \frac{U}{r(k-1)} - \frac{c_1}{r}}_{\text{searching with money and a commodity}} \end{aligned} \quad (\text{B.21})$$

$$c_1 \leq \frac{1}{k-1}[mU + (1-m)[x+W_1-W_m]] + c_m \quad (\text{B.22})$$

Hence, there exists a k such that $k \leq \frac{mU+(1-m)[x+W_1-W_m]}{c_1-c_m} + 1$.

□

Proof of Proposition 2.5. For an agent to choose a pure strategy of searching only with money, the value function of searching with only money must exceed the value of holding both money and a commodity, as follows:

$$\begin{aligned} & \underbrace{\frac{\Pi(1-m)}{r}[U+x+W_1-W_m] - \frac{c_m}{r}}_{\text{searching only with money}} \\ & > \underbrace{\frac{\Pi(1-m)}{r}\left(1 - \frac{1}{k-1}\right)[U+x+W_1-W_m] + \frac{U}{r(k-1)} - \frac{c_1}{r}}_{\text{searching with money and a commodity}} \end{aligned} \quad (\text{B.23})$$

$$c_1 - c_m > \frac{-\Pi(1-m)}{k-1}[U+x+W_1-W_m] + \frac{U}{(k-1)} \quad (\text{B.24})$$

$$c_1 - c_m > \frac{1}{k-1}[U(1 - \Pi(1 - m)) + \Pi(1 - m)[x + W_1 - W_m]] \quad (\text{B.25})$$

For a monetary equilibrium, Π is assumed to be 1. Hence,

$$c_1 > \frac{1}{k-1}[mU + (1 - m)[x + W_1 - W_m]] + c_m \quad (\text{B.26})$$

The above inequality is satisfied when k is sufficiently large. \square

Proof of Proposition 2.6. For any r, m, c_1, c_m :

$$\lim_{k \rightarrow \infty} V_1 = \lim_{k \rightarrow \infty} \frac{U}{r(k-1)} - \frac{c_1}{r} = -\frac{c_1}{r} < 0 \quad (\text{B.27})$$

$$\lim_{k \rightarrow \infty} S_m \leq 0 \quad (\text{B.28})$$

$$\lim_{k \rightarrow \infty} S_1 \geq 0 \quad (\text{B.29})$$

$$\lim_{k \rightarrow \infty} (V_m - V_b) \geq 0 \quad (\text{B.30})$$

$$\lim_{k \rightarrow \infty} (V_m - S_1) \geq 0 \quad (\text{B.31})$$

This satisfies the incentive constraint for holding money such that $V_m = \max\{V_m, V_b\} > S_1 \geq 0$ and Lemma 2.1 for the exchange participating constraint. \square

Appendix C: Proofs for Chapter 3

Proof of Proposition 3.1. It follows from a comparison of equations [3.7] and [3.22]. Let

$$B = \frac{(2k-3)}{(k-1)^2} \sum_{n=1}^N 1_n \quad (\text{C.1})$$

Then we have $\frac{\partial B}{\partial k} < 0$ and $\lim_{k \rightarrow \infty} B = 0$ for any $k > 2$. The market gets more difficult to clear as the number of types of good increases. With algebra manipulation of B , the market can only clear without the use of money when $k = 2$. When $k = 1$, it is not meaningful because everyone is self-sufficient in production and there is no need for money and market to trade. □

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