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An Examination of the Factors and Characteristics that Contribute to the Success of Putnam Fellows

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University of Connecticut, 2015

The William Lowell Putnam Mathematical Competition is an intercollegiate mathematics competition for undergraduate college and university students in the United States and Canada and is regarded as the most prestigious and challenging mathematics competition in North America (Alexanderson, 2004; Grossman, 2002; Reznick, 1994; Schoenfeld, 1985). Students who earn the five highest scores on the examination are named Putnam Fellows and to date, despite the thousands of students who have taken the Putnam Examination, only 280 individuals have won the Putnam Competition. Clearly, based on their performance being named a Putnam Fellow is a remarkable achievement. In addition, Putnam Fellows go on to graduate school and have extraordinary careers in mathematics or mathematics-related fields. Therefore, understanding the factors and characteristics that contribute to their success is important for students interested in STEM-related fields.

The participants were 25 males who attended eight different colleges and universities in North America at the time they were named Putnam Fellows and won the Putnam Competition four, three, or two times. An 18-item questionnaire, adapted form the Walberg Educational Productivity Model, was used as a

framework to investigate the personal and the formal educational experiences as well as the role the affective domain played in their development as Putnam Fellows. Further, research (DeFranco, 1996; Schoenfeld, 1992) on the characteristics of expert problem solvers was used to understand those elements of the cognitive domain that contributed to their success.

Data was collected through audio-recorded interviews conducted over the telephone or through Skype, and through written e-mail responses. The interview data was coded according to the coding category it represented and then sorted to identify existing themes and patterns for within-case and cross-case analyses.

The results indicated that four subcategories of personal experiences, four subcategories of formal educational experiences, seven subcategories involving the affective domain, and three subcategories of the cognitive domain all played an important role in the development of Putnam Fellows. Future research should include a thorough examination of female Putnam winners as well as the problem-solving strategies used by Putnam Fellows.

An Examination of the Factors and Characteristics

that Contribute to the Success of Putnam Fellows

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that Contribute to the Success of Putnam Fellows

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DEDICATION

For my first teachers,

Robert R. Stroud and Joan Ferguson Stroud

with love and appreciation

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"And whatsoever ye do in word or deed, *do* all in the name of the Lord Jesus, giving thanks to God and the Father by him" (Colossians 3:17 King James Version).

First and foremost, I am thankful to my advisor, Dr. Thomas C. DeFranco, who has been my mentor throughout this journey, which first began with my favorite class EDCI 369, *The Teaching and Learning of Mathematical Problem Solving*. Dr. DeFranco's endless passion and enthusiasm for the teaching and learning of mathematics has inspired me throughout the years and I will forever be grateful for his wisdom and guidance. I would also like to thank the members of my advisory committee, Dr. Stuart J. Sidney, Dr. Hariharan Swaminathan, Dr. Mary P. Truxaw, and Dr. Charles I. Vinsonhaler, for their guidance and feedback on my research the past few years.

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Finally, I would like to thank the two most important people in my life, my parents. Without the loving memory of my dad and the love and support of my mom, none of this would have been possible. Although my dad did not live to see me complete my doctorate, I know the Fire Chief would be proud. And thank you to my mom for giving me the two greatest gifts of all – her love and her time. ¥

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Chapter I

Overview of the Study

> The William Lowell Putnam Mathematical Competition, since its inception in 1938, has had a substantial impact on the field of mathematics in the United States and Canada. It rivals in this respect the classic Tripos in Cambridge and the influential Eötvös competition in Hungary. While there have been many different reasons for the remarkable expansion of mathematics during the past forty years, we believe that the challenge provided by the Putnam Competition has led many gifted college students into serious involvement with mathematics, and our profession is the richer for it (as cited in Mathematical Association of America, 1980, p. vii).
>
> ~ A. M. Gleason, R. E. Greenwood, & L. M. Kelly, 1978

Introduction

The William Lowell Putnam Mathematical Competition is an intercollegiate mathematics competition, administered by the Mathematical Association of America (MAA), for undergraduate college and university students in the United States and Canada and is regarded as the most prestigious and challenging mathematics competition in North America (Alexanderson, 2004; Grossman, 2002; Reznick, 1994; Schoenfeld, 1985).

William Lowell Putnam II first proposed the notion of an academic competition between colleges in an article he authored for the December 1921 issue of the *Harvard Graduates' Magazine* (Birkhoff, 1965; MAA, 2008; Santa Clara University (SCU), 2013). Putnam believed that intellectual competitions would help promote students' interest in academics, just as athletic competitions, debating teams, and chess teams inspire students (Putnam, as cited in Birkhoff, 1965). The first Putnam Competition was held on April 16, 1938, and its participants consisted of 163 students from 67 institutions (Bush, 1965; Cairns, 1938b). In subsequent years, the number of students competing in the Putnam has increased substantially – for example, during the most recent competition held on December 6, 2014, there were 4,230 contestants from 577 colleges and universities (Klosinski, Alexanderson, & Krusemeyer, 2015). Students are eligible to compete in the Putnam Competition a maximum of four times and throughout the history of the 75 competitions students have participated in the Putnam a total of 140,314 times (see Appendix A). The students who earn the five highest scores on the examination are named Putnam Fellows, and these contestants are not ranked by their scores, but are named alphabetically. Furthermore, because it is possible for two or more individuals to receive a tie score on the examination, there have been 15 competitions in which six Putnam Fellows were named (Klosinski et al., 2015) and one competition, in 1959, when eight Putnam Fellows were named (Bush, 1960; Gallian, 2004, 2014; MAA, 1980).

Over the course of the 75 Putnam Competitions, there have been 280 Putnam Fellows, and counting repeated winners these individuals have received this award

a total of 393 times (see Appendix B). Among the 280 Putnam Fellows, eight students have earned this distinction four times; 21 individuals have won this award three times; 47 students have received this title two times; and 204 individuals have earned this accolade one time (see Appendix C). Therefore, given the number of students who compete in the Putnam Competition each year, to be named a Putnam Fellow is a remarkable accomplishment while being named a Putnam Fellow multiple times is an extraordinary achievement.

After finishing their undergraduate studies, most Putnam Fellows further their education by completing graduate school, and then go on to make significant contributions to academia and industry. For example, many Putnam Fellows have become college and university professors of mathematics, physics, and computer science, while others have worked in industry as mathematicians, physicists, chemists, and engineers, as well as for the United States federal government in the Department of Energy, the Department of Transportation, and the Census Bureau. In addition, Putnam Fellows have also received some of the most prestigious awards in their respective fields including: the Nobel Prize in Physics, the National Medal of Science, the International Medal for Outstanding Discoveries in Mathematics (the Fields Medal), the Abel Prize, and the Albert Einstein Award in theoretical physics (Alexanderson, 2004; Gallian, 2004, 2014; Grossman, 2002; MAA, 2008). Furthermore, some Putnam Fellows have served as presidents of the American Mathematical Society or the Mathematical Association of America (Gallian, 2004, 2014), as well as members of the National Academy of Sciences, the American

Academy of Arts and Sciences, and the National Academy of Engineering (Alexanderson, 2004; Gallian, 2004, 2014).

As noted above, the Putnam Competition has played a central role among mathematical competitions in the United States and Canada, as well as having an impact in the field of mathematics over the past several decades. Therefore, the purpose of this study is to understand the characteristics that contributed to the success of Putnam Fellows and to see if these Putnam Fellows share some of the same characteristics of "expert" problem solvers as defined in the literature (DeFranco, 1996; Schoenfeld, 1992; Schoenfeld, as cited in DeFranco, 1996).

Background of the Study

William Lowell Putnam II, a member of the Harvard class of 1882, had a deep conviction in the merits of academic intercollegiate competition (Birkhoff, 1965; Gallian, 2004, 2014; SCU, 2013). After his death, Elizabeth Lowell Putnam created the William Lowell Putnam Intercollegiate Memorial Fund, in honor of her husband, for the purpose of establishing an intercollegiate competition (Arney & Rosenstein, 2001; Birkhoff, 1965; Bush, 1965; Gallian, 2004, 2014; MAA, 2008; SCU, 2013). In 1928, this memorial fund provided support for an academic competition in the field of English between 10 seniors at Harvard University and 10 seniors at Yale University (Arney & Rosenstein, 2001; Birkhoff, 1965; MAA, 2008; SCU, 2013). The team from Harvard University won the competition, but the contest was never repeated (Birkhoff, 1965).

In 1933, a second intercollegiate competition was held between 10 students from Harvard University and 10 cadets from the United States Military Academy at West Point (Arney, 1994; Arney & Rosenstein, 2001; Birkhoff, 1965; Gallian, 2004, 2014; MAA, 1985, 2008; Page & Robbins, 1984). The examination questions consisted of problems taken from calculus, analytic geometry, and elementary differential equations (Birkhoff, 1965). The team of West Point cadets won the competition, but again the contest was not repeated (Arney, 1994; Arney & Rosenstein, 2001; Birkhoff, 1965).

After the death of Elizabeth Putnam in 1935, the Putnam children consulted with Harvard mathematician George Birkhoff and decided to hold an annual academic competition that was open to all undergraduate students in the United States and Canada (Birkhoff, 1965; MAA, 2008). Because several members of the Putnam family had a long history of majoring in mathematics and working in the field of mathematics the decision was made to hold a mathematics competition (Birkhoff, 1965; MAA, 1985).

In devising the competition Birkhoff created four principles that would eventually become the governing regulations of the Putnam Competition and suggested that the mathematics competition be open to both teams and individuals (Birkhoff, 1965). Birkhoff also believed that the Mathematical Association of America should administer the competition (Birkhoff, 1965; Bush 1965; SCU, 2013), that prizes be awarded to teams and individuals (Birkhoff, 1965), and that one of the top five competitors be awarded a graduate fellowship at Harvard University or

Radcliffe College in recognition of his or her creativity and originality on the examination (Birkhoff, 1965; MAA, 1938).

Birkhoff and members of the mathematics department at Harvard University were asked to create the first Putnam Examination (Birkhoff, 1965; Bush, 1965) and as a result it was agreed that Harvard students would not participate in the first year of the competition (Bush, 1965). The examination questions were taken from calculus, higher algebra, elementary differential equations, and analytic geometry (MAA, 1938).

The University of Toronto won the first Putnam Competition and consequently, members of its mathematics department were asked to construct the examination questions for the second competition, and then disqualify their students from competing the following year (Birkhoff, 1965; Bush, 1965). This pattern of a school winning the competition, preparing the questions for the subsequent examination, and then disqualifying itself the following year was repeated (Birkhoff, 1965; Bush, 1965) until 1942 when faculty members from Dartmouth College and Mount Holyoke College constructed the examination (Bush, 1965). In 1942, the Mathematical Association of America announced that the Putnam Competition would be postponed, “mainly by the preoccupation of both teachers and students with war courses in mathematics” (October 1942, p. 552).

Following World War II, modifications were being considered to the Putnam Examination and mathematicians Pólya, Radó, and Kaplansky, were asked to construct the examination questions (Birkhoff, 1965). The goal of the new committee members was to design mathematics problems that tested a student’s

ingenuity in devising and using algorithms, as well as performing logical analysis (Birkhoff, 1965). Furthermore, it was decided that colleges and universities that won the Putnam Competition in any given year would be eligible to compete in subsequent years (Birkhoff, 1965).

To enter the competition it was decided that any college or university within North America would submit the names of three students, who will constitute a team to represent that institution, to the Secretary of the Mathematical Association of America (Cairns, 1938a; MAA, 2008). In the event that less than or more than three students from a college or university wished to participate in the examination, then they would compete as individuals (MAA, 1938).

Beginning with the first Putnam Competition, the examination has consisted of two, three-hour test periods administered during a morning and an afternoon session (MAA, 1938). The number of test questions varied from 11 to 14 during the competitions held between 1938 and 1961 (Gallian, 2004, 2014; MAA, 1980) and from 1962 to the present, the examination has consisted of six questions in both the morning and afternoon sessions (Gallian, 2004, 2014; MAA 1985, 2008). Today, the Putnam Examination consists of 12 problems, each worth 10 points, for a maximum score of 120 points (Bush, 1965) and during a four-day period, 30 graders from colleges and universities throughout the country convene at Santa Clara University to score the Putnam Examination questions (Beezer, 2004). After the examinations have been graded, the names of the colleges and universities with winning teams are published in *The American Mathematical Monthly* (see Appendix D). From 1938 through 1941, the colleges and universities with the top three winning teams were

named in order of their rankings (Cairns, 1938b; MAA, 1938) and in 1942, and continuing to today prizes were awarded to the institutions with the top winning teams (Bush, 1959, 1965; Cairns, 1942). The practice of naming the schools with the top five winning teams continues to this day.

Over the years the number of students and colleges and universities entering the competition has grown. For example, in 1938, 42 colleges and universities entered teams in the competition (Cairns, 1938b) and in 2014, teams from 431 institutions in the United States and Canada participated in the competition (Klosinski et al., 2015). Although there have been an increasing number of teams participating in the competition, the 355 top five winning teams have come from 43 institutions. Furthermore, 166 of these winning teams, which represent more than 46% of the combined wins, come from four of these 43 colleges and universities – Harvard University has compiled the best record with 60 top five winning teams, the Massachusetts Institute of Technology has had 45 top five winnings teams, the California Institute of Technology has had 33 top five winnings teams, and Princeton University has had 28 top five winning teams.

In addition to printing the names of the institutions that have winning teams, the Mathematical Association of America also publishes the names of the five highest-ranking contestants in alphabetical order (Cairns, 1938b; MAA, 1938). The contestants with the five highest scores, whether competing as part of a team of three people or as individuals, are named Putnam Fellows. The 280 Putnam Fellows, who have collectively won the Putnam Competition 393 times, represent 56 different colleges and universities (see Appendix E).

In recent years the number of women participating in the competition has increased and in 1992 the Mathematical Association of America awarded the Elizabeth Lowell Putnam Prize "to a woman whose performance on the Competition has been deemed particularly meritorious" (Klosinski, Alexanderson, & Larson, 1993, p. 757). During the last 23 years, 11 women have been awarded this honor and counting repeated winners, these individuals have received this award a total of 17 times (see Appendix F). Among the 11 recipients, two students have earned this distinction three times and two individuals have won this award twice. Furthermore, the women who were awarded the Elizabeth Lowell Putnam Prize in 1996, 2002, 2003, and 2004 were also named Putnam Fellows those same years.

Becoming a Putnam Fellow requires strong problem-solving skills and the ability to quickly analyze and understand the structural relationships in the problems. Therefore, because the Putnam Fellows earn the highest scores on the examination and the Putnam Competition is regarded as the most prestigious and challenging mathematics competition in North America (Alexanderson, 2004; Grossman, 2002; Reznick, 1994; Schoenfeld, 1985), these individulas can be classified as exceptional problem solvers.

Over the years, there have been a number of mathematical competitions that have served to discover mathematically talented students in our country and serve as valuable experiences for future Putnam winners. For example, the New York Metropolitan Section of the Mathematical Association of America sponsored its first competition known as the "Mathematical Contest" on May 11, 1950 (MAA, 1950, 2015a; Turner, 1978). Eight years later, as interest and participation in the

Mathematical Contest continued to expand throughout the nation, the Mathematical Association of America and the Society of Actuaries sponsored the first national competition named the "Annual High School Mathematics Contest" (Turner, 1978). As interest in the International Mathematical Olympiad (IMO) began to grow throughout the mathematics and mathematics education communities, Turner advocated for the creation of the United States of America Mathematical Olympiad (USAMO), while considering the benefits to our nation (Turner, 1971).

As noted by Turner,

> It [USAMO] certainly would represent a step higher in secondary school competition in mathematics in our country. As a subjective type examination, the type used for the individual Mathematical Olympiads that are now being held in Eastern bloc countries and England, it would provide the challenging experience needed by students in our country to think a problem through, to organize a proof, and to express that organization in the written word. It could act as the 'go between' between the Annual High School Mathematics Competition and possible participation in an IMO (February 1971, p. 193).

As a result, in 1971 the Mathematical Association of America agreed to sponsor the first United States of America Mathematical Olympiad, which was held in 1972 (Greitzer, 1973; Turner, 1978). The purpose of the Olympiad was to identify mathematically talented and computationally fluent students who possessed mathematical creativity and inventiveness (Greitzer, 1973). The Mathematical Association of America invited 106 students to participate in the first

Olympiad and these individuals included the top scorers on the "Annual High School Mathematics Competition", as well as a select number of students who possessed superior mathematical talent but resided in states which did not take part in the Annual High School Mathematics Competition (Greitzer, 1973). Today, the United States of America Mathematical Olympiad is held in late April and is open to United States citizens and students who have qualifying scores and live in North America (MAA, 2015b).

Campbell and Wu (1996) have conducted a number of research studies that examined the Mathematics Olympiad programs in the United States as well as in other countries. Through a series of studies, Campbell (1996a) designed a questionnaire using other instruments as sources such as: the Johns Hopkins University Study of Mathematically Precocious Youth (Follow-up Questionnaires); the Westinghouse Talent Search winners (After High School Follow-up Questionnaire); the Longitudinal Study of American Youth; and Campbell's international instruments (Campbell, 1996a; Campbell and Connolly, 1987; Campbell and Connolly, as cited in Campbell, 1996a). As part of his investigation of mathematics achievement in pre-collegiate students, Campbell and Wu (1996) adapted the Walberg Educational Productivity Model as the theoretical framework for their Mathematics Olympiad studies (Walberg, 1984a, 1984b, 1986; Walberg, as cited in Campbell & Wu, 1996).

Walberg has synthesized thousands of empirical studies in the construction of his nine-factor educational productivity model (see Figure 1) that is an outgrowth of more than three decades of development (Campbell & Wu, 1996). The Walberg

Educational Productivity Model depicts aptitude, instruction, and the environment as the major causal influences to learning (Campbell & Wu, 1996; Iverson & Walberg, 1982; Walberg, 1984a, 1984b, 1986; Walberg, as cited in Campbell & Wu, 1996; Walberg & Marjoribanks, 1976). In their research, Campbell and Wu (1996) adapted the Walberg Educational Productivity Model (see Figure 2), by subsuming five of the global Walberg factors and expanding the number of variables within the home factor to include: family processes, socio-economic status, and the number of parents living at home. Additionally, Campbell and Wu (1996) expanded the motivation factor to include: mathematics and science self-concepts, general self-concept, and two attribution factors (ability and effort).

As part of a research study of the American Mathematics Olympians, Campbell (1996a, 1996b) examined the factors that contribute to or impede the development of the Olympians' talent in mathematics and investigated the contributions Olympians make to the fields of mathematics and science. The Mathematics Olympiad study revealed that the four most important factors that contribute to the Olympians' mathematical talent include: the home, the school, the Olympiad Program, and mentoring (Campbell, 1996b).

As noted earlier by virtue of the Putnam Fellows' success on the Putnam Competition, individuals who have been named Putnam Fellows multiple times would be categorized as "expert" problem solvers. What are some of the characteristics of "expert" problem solvers? To answer that question, the mathematics research community has examined the differences between the problem-solving behavior of expert and novice problem solvers for over four

decades. Through his research studies, Schoenfeld (as cited in DeFranco, 1996) revised the definition of expert as it pertains to the problem-solving domain, and identified the attributes that constitute problem-solving expertise. The characteristics of expert problem solvers include: domain knowledge (the knowledge base a problem solver brings to a problem), problem-solving strategies (heuristic techniques), metacognitive skills (the ability to comprehend, control, and monitor one's own thought processes, the ability to allocate one's resources, etc.), and a set of beliefs (one's worldview) about mathematics as a domain (DeFranco, 1996; Schoenfeld, 1992; Schoenfeld, as cited in DeFranco, 1996).

Over the past few decades problem solving has occupied a substantial part of the K-12 mathematics curriculum (Cai & Lester, 2010; MAA, 1983; National Council of Teachers of Mathematics, 1980a, 1980b, 1989, 1991, 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; National Research Council, 1989, 2001). Therefore, it would be beneficial to the fields of mathematics and mathematics education to examine the factors that have contributed to the Putnam Fellows' abilities as "expert" problem solvers, and in turn use these findings to improve the problem-solving performance of K-12 students.

The purpose of this research is twofold. First, because Putnam Fellows are expert problem solvers and go on to have extraordinary careers in mathematics or mathematics-related fields, using the Walberg Educational Productivity Model this study will examine the characteristics that led these individuals to becoming Putnam Fellows. Second, to see if these Putnam Fellows share some of the same

characteristics of "expert" problem solvers as defined in the literature (DeFranco, 1996; Schoenfeld, 1992; Schoenfeld, as cited in DeFranco, 1996).

Figure 1

Walberg Educational Productivity Model Depicting the Causal Influences on Student Learning (Campbell & Wu, 1996; Walberg, 1984b)

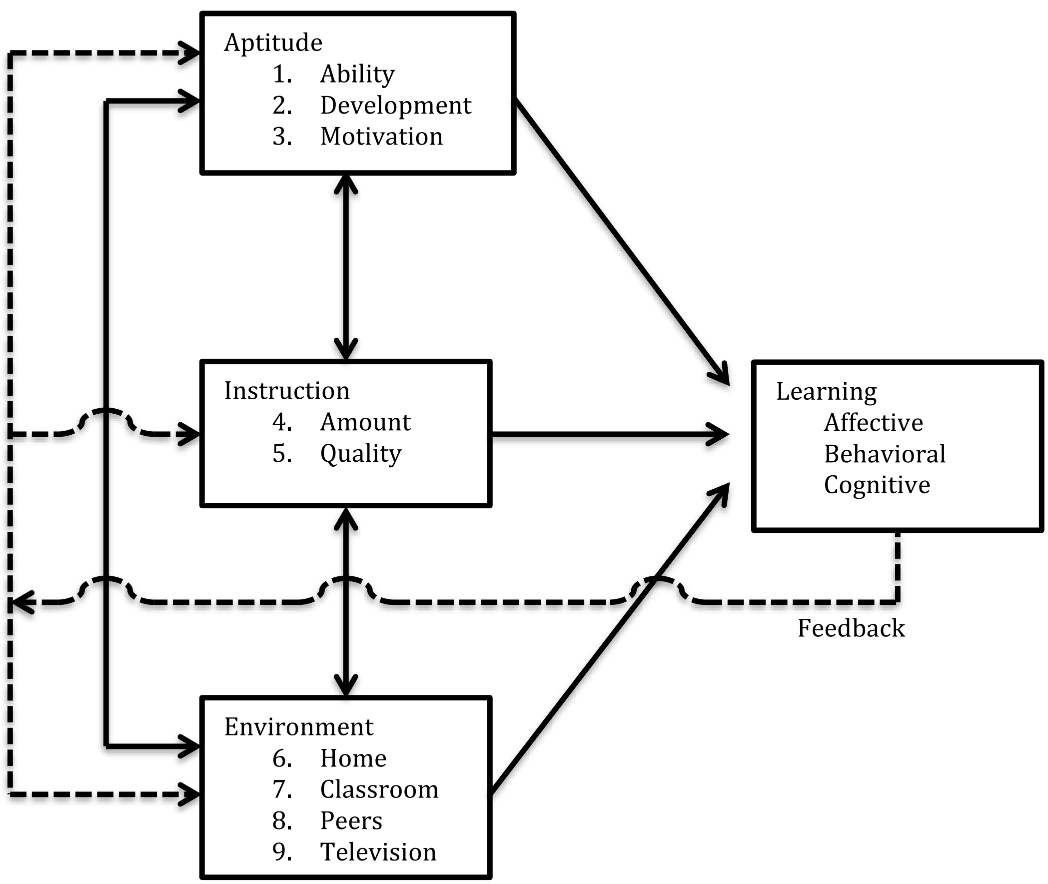
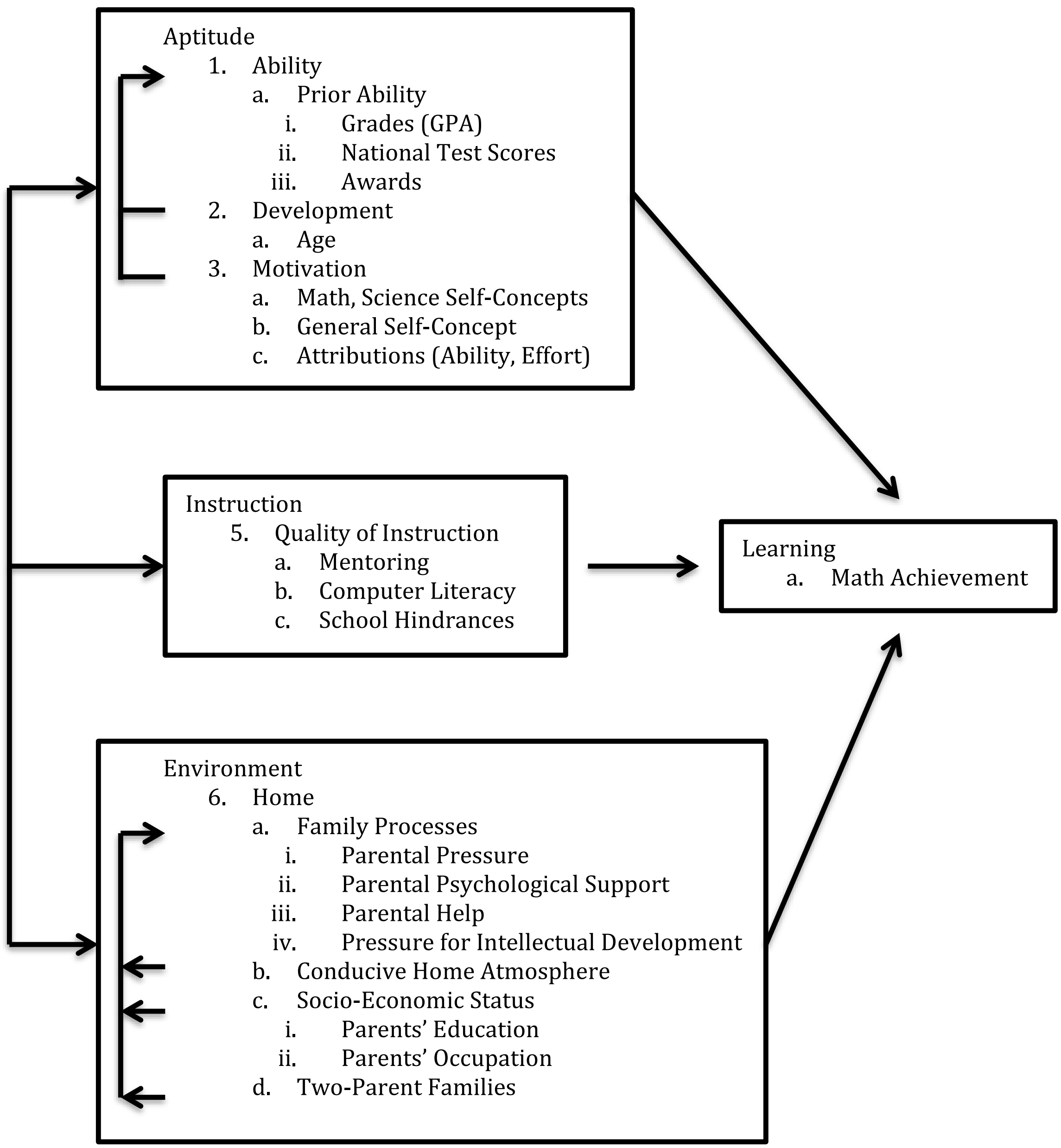


Figure 2

Interconnections Analyzed Within Campbell and Wu's (1996) Adaptation of the Walberg Educational Productivity Model



Research Questions

This study investigated the factors and characteristics that have contributed to the development of the Putnam Fellows' exceptional abilities as problem solvers. In particular, this study proposed to answer the following questions:

RQ1: What personal experiences do Putnam Fellows identify as influential in their success on the Putnam Examination?

RQ2: What formal educational experiences do Putnam Fellows identify as influential in their success on the Putnam Examination?

RQ3: What role does the affective domain play in the development of a Putnam Fellow?

RQ4: What role does the cognitive domain play in the development of a Putnam Fellow?

Methods and Procedures

A description of the participant selection process, the procedures and instrument used for data collection, and the methods employed for data analysis, follows in this section.

Participants

The study took place during the spring, summer, and fall months of 2014. The researcher used the names of the Putnam Competition winners, published annually in *The American Mathematical Monthly* by the Mathematical Association of

America, and the Internet to compile contact information for the 74 individuals who have been named Putnam Fellows four, three, or two times. Among the 74 people who were multiple Putnam Competition winners between 1939 and 2013 inclusive, 25 individuals consented to participate in the study and constituted the sample; five people declined to participate; 27 individuals did not respond to e-mail, mail, and/or telephone invitations; five people could not be located; and 12 individuals are deceased.

Of the 25 individuals who participated in the study, five have been named Putnam Fellows four times; seven have earned this distinction three times, and 13 have won this award two times. For more detailed information about the subjects (i.e., awards, honors, and professional appointments) see Appendix G.

Data Collection

A phenomenological study was conducted whereby in-depth, intensive, and iterative interviews were used to investigate the lived experiences (Rossman & Rallis, 2003) of the Putnam Fellows. Each subject was sent a questionnaire (see Appendix H) via e-mail or through the mail, which allowed the participant the opportunity to review and reflect on the questions prior to the interview.

Data collection began in May 2014 and ended in November 2014. During this period, each Putnam Fellow was asked to participate in an in-depth interview that took the form of a purposeful dialogue, as part of the phenomenological data-gathering process (Erlandson, Harris, Skipper, & Allen, 1993; Rossman & Rallis, 2003; Seidman, as cited in Rossman & Rallis, 2003). The researcher conducted

semi-structured interviews to elicit the Putnam Fellows' views on specific topics and to allow the participants to elaborate on their responses as their stories unfolded (Rossman & Rallis, 2003). Because of the participants' physical locations, summer travel schedules, and time constraints, data was collected through audio-recorded interviews conducted over the telephone or through Skype, and through written e-mail responses. The length of the interviews varied for each participant ranging from 45 to 90 minutes each. The interviews were audio-recorded using Piezo software, transferred to audio compact discs, played back using an Apple QuickTime Player 7 application, and then transcribed for data analysis.

The 18-item questionnaire (Appendix H), which was designed using other instruments as sources (Campbell 1996a, 1996b; DeFranco 1996), was used to uncover information about the factors and characteristics (i.e., personal experiences, formal educational experiences, and beliefs about mathematics and mathematical problem solving) that have contributed to the development of these individuals as Putnam Fellows.

Data Analysis

To answer the research questions, responses from the interviews were transcribed (a sample interview transcription can be found in Appendix I) and each statement was printed in a text matrix. The statements within the text matrices were color-coded according to the personal experience, formal educational experience, or learning domain the response represented. This required interpretation on the part of the researcher to bring meaning and insight into the

words of the participants in the study to determine the type of experience or learning domain a participant's response represented (Marshall & Rossman, 1995). Since the personal and educational experiences, as well as the learning domains, are interrelated, many responses coded into more than one category or more than one subcategory within a category. The statements within the text matrices were then sorted into a subcategory or subcategories within each category. At this point, the researcher read and reread the text matrices to become connected to the data. The data was reduced as the major recurrent ideas within each level of each category were identified. As themes emerged they were organized into summary theme tables.

The researcher worked with a peer debriefer to help build credibility for the study. The peer debriefer is a professor of mathematics education with expertise in qualitative analysis. The researcher and the peer debriefer scheduled sessions to develop techniques for coding the data, examine the coded data, provide opportunities for them to ask probing questions, and discuss different explanations or alternative coding of the data (Erlandson et al., 1993).

Limitations

The trustworthiness of research guarantees some measure of credibility of the research findings and provides the basis for further application (Erlandson et al., 1993). In order to ensure credibility and rigor this study employed data triangulation via the audio compact disc recordings, written e-mail responses, and

follow-up interviews in order to check the consistency of the participants' responses. Regular meetings with the peer debriefer who was trained in the procedures for coding and analyzing the data, offered opportunities to discuss the findings and identify possible researcher bias.

With respect to external validity there is the possibility that the results may only be generalized to individuals who have participated in the Putnam Competition and been named a Putnam Fellow. Nevertheless, this research may provide important insight into the factors that contribute to the Putnam Fellows' problem-solving abilities and in turn could be used to improve the problem-solving performance of K-12 students.

Chapter II

Review of Related Literature

> There is one matter that I have had in my mind ever since my visit and that is the mathematical contest your sister contemplated. I would very much like to test our method of teaching mathematics against that of your institution. I, frankly, think our method is superior to yours, and would like to try it out (as cited in Arney, 1994, p. 14).

~ Major General W. D. Connor, Superintendent of the United States Military Academy at West Point, 1932

> Your challenge is a very interesting one, which we will be glad to accept (as cited in Arney, 1994, p. 14).

~ A. L. Lowell, President of Harvard University, 1932

Introduction

In this chapter, literature related to this study will be reviewed. This includes a summary of the research related to the Putnam Competition. More specifically, a history of the Putnam Competition, mathematical contests and competitions, the role of affect and cognition in problem solving, and the Walberg Educational Productivity Model.

A History of the Putnam Competition

The William Lowell Putnam Mathematical Competition is an intercollegiate mathematics competition, administered annually on the first Saturday in December by the Mathematical Association of America (MAA) for undergraduate college and university students in the United States and Canada and is regarded as the most prestigious and challenging mathematics competition in North America (Alexanderson, 2004; Grossman, 2002; Reznick, 1994; Schoenfeld, 1985).

William Lowell Putnam II first proposed the notion of an academic competition between colleges in an article he authored for the December 1921 issue of the *Harvard Graduates' Magazine* (Birkhoff, 1965; MAA, 2008; Santa Clara University (SCU), 2013). While colleges and universities have long competed in athletic events, Putnam considered it, "a curious fact that no effort has ever been made to organize contesting teams in regular college studies" (as cited in Birkhoff, 1965, p. 469, p. 472). Putnam believed that intellectual competitions between teams of undergraduates would help promote students' interest in academics, just as athletic competitions, debating teams, and chess teams inspire students (Putnam, as cited in Birkhoff, 1965). "It seems probable that the competition which has inspired young men to undertake and undergo so much for the sake of athletic victories might accomplish some result in academic fields" (Birkhoff, 1965, p. 473; Putnam, as cited in Arney & Rosenstein, 2001, p. 1).

The first Putnam Competition was held on April 16, 1938 and its participants consisted of 42 teams of three people and 37 individual contestants for a total of

163 students from 67 institutions (Bush, 1965; Cairns, 1938b). In subsequent years, the number of students competing in the Putnam has increased substantially. For example, during the most recent competition in 2014 there were 431 three-person teams and 2,937 individual participants for a total of 4,230 contestants from 577 colleges and universities (Klosinski, Alexanderson, & Krusemeyer, 2015). Students are eligible to compete in the Putnam Competition a maximum of four times and throughout the history of the 75 competitions students have participated in the Putnam a total of 140,314 times (see Appendix A). The individual students who earn the five highest scores on the examination are named Putnam Fellows and because it is possible for two or more individuals to receive a tie score on the examination there have been 15 competitions in which six Putnam Fellows were named (Klosinski et al., 2015) and one competition when eight Putnam Fellows were named (Bush, 1960; Gallian, 2004, 2014; MAA, 1980).

Over the course of the Putnam Competitions there have been 280 Putnam Fellows, with 8, 21, 47, and 204 students earning this distinction four times, three times, two times, and one time respectively (see Appendix C). Therefore, given the increasing number of students who compete in the Putnam Competition each year, being named a Putnam Fellow is an extraordinary achievement.

Most Putnam Fellows further their education by completing graduate school and go on to make significant contributions to academia and industry. For example, many Putnam Fellows have become college and university professors of computer science, mathematics, and physics, while others have worked in industry as chemists, engineers, mathematicians, and physicists, as well as for the United States

federal government in the Census Bureau, the Department of Energy, and the Department of Transportation. In addition, Putnam Fellows have also received some of the most prestigious awards in their respective fields including: the Abel Prize, the Albert Einstein Award in theoretical physics, the International Medal for Outstanding Discoveries in Mathematics (the Fields Medal), the National Medal of Science, and the Nobel Prize in Physics (Alexanderson, 2004; Gallian, 2004, 2014; Grossman, 2002; MAA, 2008). Furthermore, some Putnam Fellows have served as presidents of the American Mathematical Society or the Mathematical Association of America (Gallian, 2004, 2014), as well as members of the American Academy of Arts and Sciences, the National Academy of Engineering, and the National Academy of Sciences (Alexanderson, 2004; Gallian, 2004, 2014).

As a member of the Harvard class of 1882, William Lowell Putnam II had a deep conviction in the merits of academic intercollegiate competition (Birkhoff, 1965; Gallian, 2004, 2014; SCU, 2013). Elizabeth Lowell Putnam and her brother Abbott Lawrence Lowell, who served as President of Harvard University, also shared William Lowell Putnam's conviction (Arney & Rosenstein, 2001; Birkhoff, 1965) and four years after William Lowell Putnam's death, Elizabeth Lowell Putnam created the William Lowell Putnam Intercollegiate Memorial Fund, for the purpose of honoring her husband and establishing an intercollegiate competition (Arney & Rosenstein, 2001; Birkhoff, 1965; Bush, 1965; Gallian, 2004, 2014; MAA, 2008; SCU, 2013). In 1928, this memorial fund provided support for an academic competition in the field of English between 10 seniors at Harvard University and 10 seniors at Yale University (Arney & Rosenstein, 2001; Birkhoff, 1965; MAA, 2008; SCU, 2013)

in which the team from Harvard University won the competition, however, the contest was never repeated (Birkhoff, 1965).

In the fall of 1932, Army defeated Harvard in a college football game held at Soldier's Field in Cambridge, Massachusetts (Arney, 1994; Arney & Rosenstein, 2001). After the football game, Abbott Lowell held a luncheon at his home during which time he commented to his guests that while Army "could trounce Harvard in football, Harvard could just as easily win any contest of a more academic nature" (Lowell, as cited in Arney & Rosenstein, 2001, p. 1). It was during this luncheon that Mrs. Putnam and Major General W. D. Connor, Superintendent of the United States Military Academy at West Point (USMA), discussed the notion of a mathematics contest between Harvard and West Point (Arney, 1994; Arney & Rosenstein, 2001). As a result, in 1933 a second intercollegiate competition was held between 10 students from Harvard University and 10 cadets from West Point (Arney, 1994; Arney & Rosenstein, 2001; Birkhoff, 1965; Gallian, 2004, 2014; MAA, 1985, 2008; Page & Robbins, 1984). Both schools agreed that the mathematics contest would be held at West Point and the chairmen of the two mathematics departments, Lieutenant Colonel Harris Jones of West Point and Professor William Graustein of Harvard, agreed that Professor Arnold Dresden, the President of the Mathematical Association of America, would write and grade the examination (Arney, 1994). The examination questions consisted of problems taken from calculus, analytic geometry, and elementary differential equations (Birkhoff, 1965) and the team of West Point cadets won the competition (Arney, 1994; Arney & Rosenstein, 2001; Birkhoff, 1965).

After the death of Elizabeth Putnam in 1935, the Putnam children in consultation with Harvard mathematician George Birkhoff decided to hold an annual academic competition that would be open to all undergraduate students in the United States and Canada (Birkhoff, 1965; MAA, 2008). Because several members of the Putnam family had a long history of majoring in mathematics and working in the field of mathematics, the decision was made to hold a mathematics competition (Birkhoff, 1965; MAA, 1985).

When William Lowell Putnam first proposed the idea of an academic competition between colleges and universities, he emphasized, “that the competition to be valuable should be between teams and not individuals” (as cited in Birkhoff, 1965, p. 473). However, when Professor Birkhoff put forth the four principles that would eventually become the governing regulations of the Putnam Competition he suggested that the mathematics competition be open to both teams and individuals (Birkhoff, 1965). Birkhoff believed that individuals should be allowed to compete since colleges with lower enrollments might not have a sufficient number of mathematically talented students required to assemble a team of three (Birkhoff, 1965).

Birkhoff also believed that: (a) the Mathematical Association of America should administer the mathematics competition (Birkhoff, 1965; Bush 1965; SCU, 2013), (b) prizes should be awarded to teams and individuals in order to recognize distinguished performance (Birkhoff, 1965), and (c) one of the top five competitors should be awarded a graduate fellowship at Harvard University or Radcliffe College (Birkhoff, 1965; MAA, 1938). As a result the first Putnam Examination was created

with problems from calculus, higher algebra, elementary differential equations, and analytic geometry (MAA, 1938).

The University of Toronto won the first competition and consequently, members of its mathematics department were asked to construct the examination questions for the second competition, and then disqualify their students from competing the following year (Birkhoff, 1965; Bush, 1965). This pattern of a school winning the competition, preparing the questions for the subsequent examination, and then disqualifying itself the following year was repeated until 1942 when faculty members from Dartmouth College and Mount Holyoke College constructed the examination (Birkhoff, 1965; Bush, 1965). In 1942, the Mathematical Association of America announced that the Putnam Competition would be postponed due to the war (MAA, 1942).

Following World War II, modifications were made to the Putnam Examination and mathematicians Pólya, Radó, and Irving Kaplansky were appointed to a committee that was responsible for constructing the examination questions (Birkhoff, 1965). Their goal was to design mathematics problems that tested a student's ingenuity in devising and using algorithms as well as performing logical analysis (Birkhoff, 1965). Furthermore, it was decided that colleges and universities that won the Putnam Competition in any given year would be eligible to compete in subsequent years (Birkhoff, 1965).

Beginning with the first Putnam Competition, the examination has consisted of two, three-hour test periods and is administered during a morning and an afternoon test session (MAA, 1938). The number of test questions varied from 11 to

14 during the competitions held between 1938 and 1961 (Gallian, 2004, 2014; MAA, 1980) and from 1962 to the present, the examination has consisted of six questions in both the morning and afternoon sessions (Gallian, 2004, 2014; MAA, 1985, 2008).

To enter the competition it was decided that any college or university within North America would submit the names of three students, who will constitute a team to represent that institution, to the Secretary of the Mathematical Association of America (Cairns, 1938a; MAA, 2008). In the event that less than or more than three students from a college or university wished to participate in the examination, then they would compete as individuals (MAA, 1938).

Currently, the Putnam Examination consists of 12 problems with each problem worth 10 points for a maximum score of 120 points (Bush, 1965). During a four-day period, 30 graders from colleges and universities throughout the country convene at Santa Clara University to score the Putnam Examination questions (Beezer, 2004). Prior to grading, each examination is assigned a numerical identification number, which prevents the graders from knowing anything about the test taker or the test taker's college or university (J. A. Gallian, personal communication, July 2, 2015). The six problems from the morning test session are scored by a group of 15 graders during the first two days, while the six problems from the afternoon test session are scored by a group of 15 different graders during the last two days (Beezer, 2004). Initially, two or three grading partners are assigned to score a specific problem guaranteeing consistent grading on problems as well as criteria for a complete proof or solution to a problem (Beezer, 2004). In the event a group of graders finishes scoring their Putnam problem, they may be

assigned to help score a second Putnam problem if needed, in which case one of the initial graders for the problem will work as a consultant to the group of graders now scoring that problem (Beezer, 2004). Lastly, the Putnam Examinations from the top schools and the top individuals are scored a second time by one grader for the purposes of awarding partial credit consistently, if necessary, and to make certain that the numerical scores on the Putnam Examinations are checked against the spreadsheet, which contains all the test scores (Beezer, 2004).

After the examinations have been graded, the names of the colleges and universities with winning teams are published in *The American Mathematical Monthly* (see Appendix D). From 1938 through 1941, the colleges and universities with the top three winning teams were named in order of their rankings (Cairns, 1938b; MAA, 1938), with the ranks of the teams determined by the sum of the rankings of the three individual team members (Gallian, 2004, 2014; MAA, 1938, 2008). Thus, the team with the lowest score when its team members' examination rankings are summed up is awarded first place. From 1942 through 1958, prizes were awarded to the institutions with the top four winning teams (Bush, 1965; Cairns, 1942); and in the fall of 1958, a fifth winning team was added (Bush, 1959, 1965). The practice of naming the schools with the top five winning teams continues to this day.

In 1938, 42 colleges and universities entered teams in the competition (Cairns, 1938b) and in 2014 teams from 431 institutions in North America participated in the competition (Klosinski et al., 2015). Although an increasing number of teams from North American colleges and universities participate in the

Putnam Competition, the 355 top five winning teams have come from 43 institutions. Furthermore, 166 (46.76%) of these winning teams come from four of these 43 colleges and universities – Harvard University, Massachusetts Institute of Technology, California Institute of Technology, and Princeton University.

In addition to printing the names of the institutions that have winning teams, the Mathematical Association of America also publishes the names of the five highest-ranking contestants in alphabetical order (Cairns, 1938b; MAA, 1938). The contestants with the five highest scores are named Putnam Fellows. Today there are 280 Putnam Fellows who have won the Putnam Competition 393 times representing 56 different colleges and universities (see Appendix E).

In 1992, the Mathematical Association of America began awarding the Elizabeth Lowell Putnam Prize, "to a woman whose performance on the Competition has been deemed particularly meritorious" (Klosinski, Alexanderson, & Larson, 1993, p. 757). During the last 23 years, 11 women have been awarded this honor and among the 11 recipients, two have earned this distinction three times and two have won this award twice (see Appendix F). Furthermore, the women who were awarded the Elizabeth Lowell Putnam Prize in 1996, 2002, 2003, and 2004 were also named Putnam Fellows those same years.

The first prizes awarded to the respective departments of mathematics of the top three winning teams were $500, $300, and $200 respectively and each member of the first place team, the second place team, and the third place team received $50, $30, and $20 respectively (Cairns, 1938b). Over the years, the number of winning teams has increased from three to five and the dollar amounts of the prizes have

also increased substantially – for example, in 2015 the departments of mathematics of the top five winning teams received prizes of $25,000, $20,000, $15,000, $10,000, and $5,000 (Klosinski et al., 2015). Furthermore, each member of the first place team through the fifth place team received $1,000, $800, $600, $400, and $200 respectively (Klosinski et al., 2015).

With respect to the Putnam Fellows the first prize awarded was $50 (Cairns, 1938b) and one Putnam Fellow was awarded a one-year, $1,000 scholarship for graduate study at Harvard University (MAA, 1938). Today, each Putnam Fellow receives a prize of $2,500 and one Putnam Fellow is awarded a scholarship of $12,000 plus tuition for graduate study at Harvard University (SCU, 2013).

When the Elizabeth Lowell Putnam Prize was first awarded in 1992, the woman received a prize of $500 (Klosinski et al., 1993). In 2013, the winner was awarded a prize in the amount of $1,000 (Klosinski, Alexanderson, & Krusemeyer, 2014) and the four women who were awarded the Elizabeth Lowell Putnam Prize were also named Putnam Fellows and received both monetary prizes (Klosinski, Alexanderson, & Larson, 1997; SCU, 2013).

Mathematics Contests and Competitions

During the early half of the twentieth century mathematics contests for high school students were sponsored by numerous colleges and universities and held throughout the United States in various cities and geographical regions. In the years immediately following World War II national interest in scholastic contests began to

increase and the president of the National Council of Teachers of Mathematics (NCTM) proposed that a survey be conducted regarding mathematics contests and scholarships that existed throughout the country (Mayor, 1949). Subsequently, the National Council of Teachers of Mathematics mailed a six-question survey to 80 individuals, which included the National Council of Teachers of Mathematics Board of Directors, the National Council of Teachers of Mathematics state representatives, and other leaders in the mathematics education community (Mayor, 1949). Item 4 of the questionnaire asked, "In your opinion, would a plan similar to the Putnam Examination sponsored by the Mathematical Association of America for college undergraduates, be feasible for high school students?" (NCTM, as cited in Mayor, 1949, p. 284). In 1934, the findings from this survey were presented at the annual National Council of Teachers of Mathematics meeting. The report found that 33 of the 54 respondents supported a National Council of Teachers of Mathematics-sponsored national mathematics contest similar to the Putnam Examination (Mayor, 1949).

In 1949 the New York Metropolitan Section of the Mathematical Association of America studied the feasibility of establishing a national high school mathematics competition (Turner, 1978) and in 1956 a nationwide survey was conducted about the potential for a national mathematics competition (Lloyd, 1956). In addition to identifying 43 different mathematics contests throughout North America (Lloyd, 1956), the participants' responses to the 20-item survey prompted the Mathematical Association of America to start making plans for a national mathematics contest which would be an outgrowth of existing Mathematical

Association of America sectional mathematics contests (Lloyd, 1956; Lloyd, as cited in Fagerstrom & Lloyd, 1958).

As a result in 1950 the New York Metropolitan Section of the Mathematical Association of America sponsored its first high school mathematics competition known as the "Mathematical Contest" (MAA, 1950; Turner, 1978). The examination was administered to over 6,000 students from 238 high schools located throughout the state of New York (MAA, 2015a; Turner, 1978) and its questions consisted of material taken entirely from elementary algebra, intermediate algebra, and plane geometry (MAA, 1950). Beginning in 1952, the competition expanded to a number of states including Colorado, Illinois, Nebraska, Oregon, Washington, and Wisconsin (Lloyd, 1956) and by 1956, the competition was renamed the "Annual Mathematical Contest" (Turner, 1978). As interest and participation in the Annual Mathematical Contest continued to expand rapidly throughout the United States, the Mathematical Association of America assumed full responsibility for the competition from the New York Metropolitan Section and together with the Society of Actuaries sponsored the first North American high school mathematics competition. In 1958, the examination was retitled the "Annual High School Mathematics Contest" (Fagerstrom & Lloyd, 1958; Salkind, 1964; Turner, 1978).

During the latter half of the twentieth century, this national high school mathematics contest was renamed four additional times beginning with the "Annual High School Mathematics Competition" in 1969 (Turner, 1978); followed by the "Annual High School Mathematics Examination" in 1977 (Turner, 1978); the "American High School Mathematics Examination" (AHSME) in 1983 (Maurer,

Reiter, & Schneider, 2001); and in 2000, the “American Mathematics Contest 12” (AMC 12), which is a secondary school mathematics contest held every February and open to students in grades 12 and below who are under 19.5 years of age (MAA, 2015b, 2015d, 2015f). Starting in 2002, the AMC 12 has been administered during two different test sessions (A & B) in order to accommodate school districts with different schedules (MAA, 2015b, 2015d).

The purpose of these contests has always been to encourage secondary school students’ interest in mathematics, as well as to help develop their mathematical talent by presenting challenging problems involving topics up to pre-calculus (MAA, 2015b, 2015d). The format of the national high school mathematics contest has changed over the years. For example, in 1958 students were given 80 minutes to solve 50 multiple-choice questions (Fagerstrom & Lloyd, 1958), whereas students who compete in the AMC 12 today have 75 minutes to solve 25 multiple-choice questions (MAA, 2015b, 2015d).

As interest in secondary school mathematics competitions continued to grow throughout the country and within the mathematics and mathematics education communities, Turner (1971) advocated for the creation of the United States of America Mathematical Olympiad (USAMO).

According to Turner (1971),

> It [USAMO] certainly would represent a step higher in secondary school competition in mathematics in our country. As a subjective type examination, the type

> used for the individual Mathematical Olympiads that are now being held in Eastern bloc countries and England, it would provide the challenging experience needed by students in our country to think a problem through, to organize a proof, and to express that organization in the written word. It could act as the 'go between' between the Annual High School Mathematics Competition and possible participation in an IMO [International Mathematical Olympiad] (February 1971, p. 193).

As a result, in 1971 the Mathematical Association of America agreed to sponsor the first USAMO, which was held on May 9, 1972 (Greitzer, 1973; Maurer & Mientka, 1982; Turner, 1978). The purpose of the Olympiad is to identify mathematically talented and computationally fluent students who possess mathematical creativity and inventiveness (Greitzer, 1973). The Mathematical Association of America invited 106 students to participate in the first Olympiad and these individuals included the top scorers on the Annual High School Mathematics Competition, as well as a select number of students who possessed superior mathematical talent but resided in parts of the country, which did not take part in the Annual High School Mathematics Competition (Greitzer, 1973; Turner, 1978). The USAMO is held in late April and is open to United States citizens and students who have qualifying scores and live in North America (MAA, 2015b, 2015e). Beginning with the first USAMO in 1972, students competed in a three-hour examination consisting of five questions, which required written solutions and mathematical proofs (Maurer & Mientka, 1982). The format of the USAMO has been revised through the years and starting in 2002, the six-question, nine-hour

examination has been administered during two consecutive days, where participants have four and one-half hours to write three essays/proofs per day that require the use of pre-calculus methods (MAA, 2015b, 2015e). The students who earn the 12 highest scores on the examination are named USAMO competition winners and these contestants are not ranked by their scores but are named alphabetically (MAA, 2015h).

In 1974, the Mathematical Olympiad Summer Program (MOSP) was created to help prepare students to become potential members of the United States team at the International Mathematical Olympiad (IMO) (MAA, 2015b, 2015e). The purpose of the MOSP is to provide students with intensive training and preparation in areas of mathematics, which have customarily received more emphasis in other nations than in the United States (MAA, 2015b, 2015e). Approximately 28 students who achieve the highest scores at the USAMO are invited to participate each year in the three-to-four week MOSP, which is held at the University of Nebraska-Lincoln (MAA, 2015e, 2015g). At the end of the MOSP training camp six high school students are selected based on their test scores for the team to represent the United States at the IMO (MAA, 2015e, 2015g).

On the international level the Mathematical and Physical Society of Romania invited seven Eastern European countries to participate in the first IMO held in Romania in 1959 (IMO, 2015a; Maurer & Mientka, 1982; Turner, 1971, 1978). Over the next few years, participation in the IMO spread to several Western European countries and by 1974 Professor Greitzer, chairman of the USAMO Committee, had obtained an invitation for the United States to participate in the sixteenth IMO held

in Germany in 1974 (IMO, 2015b; Maurer & Mientka, 1982; Turner, 1978). The IMO is the oldest of the International Science Olympiads and each year students from more than 90 countries take part in this competition (MAA, 2015e, 2015g). The IMO is a six-question, nine-hour examination that is administered during two consecutive days in July where a team of up to six pre-collegiate students under the age of 20, is given four and one-half hours to write three essays/proofs per day (MAA, 2015b, 2015e, 2015g).

In 1983 the members of the Committee on the American Mathematics Competitions (CAMC) introduced a new examination known as the "American Invitational Mathematics Examination" (AIME), which was designed to act as a bridge between the AHSME and the USAMO (MAA, 2015e; Maurer et al., 2001). The purpose of the AIME is to provide high school students who possess exceptional mathematical ability with a greater challenge beyond the level of the AMC 12 (MAA, 2015b, 2015e). When the AIME was first introduced students were given two and one-half hours to solve 15 short-answer questions using pre-calculus methods (MAA, 2015b), while today students are given three hours to solve the 15 short-answer questions in which the solution to each problem is a whole number less than 1000 (MAA, 2015b, 2015e). Starting in 2002, the AIME has been administered during two different test sessions (A & B), to accommodate school districts with different schedules (MAA, 2015b, 2015d).

In 1985 the Committee on the American Mathematics Competitions introduced a new examination known as the "American Junior High School Mathematics Examination" (AJHSME), later renamed the "American Mathematics

Contest 8" (AMC 8), which is a middle school mathematics contest held every November and open to students in grades eight and below who are under 14.5 years of age (MAA, 2015b, 2015c, 2015f). The purpose of the AMC 8 is to help promote the development of middle school children's problem-solving skills across a wide range of applications as well as to stimulate students' interest in their ongoing study of mathematics (MAA, 2015b, 2015c). Children who compete in the AMC 8 are given 40 minutes to solve 25 multiple-choice questions (MAA, 2015b, 2015c).

In 2000 the Committee on the American Mathematics Competitions expanded the American Mathematics Competitions by introducing a new examination known as the "American Mathematics Contest 10" (AMC 10), which is a secondary school mathematics contest held every February and open to students in grades 10 and below who are under 17.5 years of age (MAA, 2015d, 2015f). The purpose of the AMC 10 is to encourage freshman and sophomore students' interest in mathematics as well as to help develop their mathematical talent by presenting challenging problems that can be solved with algebra and geometry concepts (MAA, 2015b, 2015d). Students who compete in the AMC 10 are given 75 minutes to solve 25 multiple-choice questions (MAA, 2015b, 2015d) and starting in 2002, the AMC 10 was administered during two different test sessions (A & B) to accommodate school districts with different schedules (MAA, 2015b, 2015d).

In 2010 the Committee on the American Mathematics Competitions introduced the "United States of America Junior Mathematical Olympiad" (USAJMO) – a six-question, nine-hour examination administered during two consecutive days every April. On this examination participants have four and one-half hours to write

three essays/proofs per day that require the use of pre-calculus methods (MAA, 2015b, 2015e).

The AMC 8, AMC 10, and AMC 12 are classified as mathematics contests (MAA, 2015b) and students who score exceptionally well on either the AMC 10 or the AMC 12 are invited to take part in a series of invitational mathematics competitions beginning with the AIME (MAA, 2015b, 2015e). In order to be invited to participate in the AIME, a student must earn 100 or more points out of a possible 150 points on the AMC 12 or place in the top 5% among all students taking the AMC 12 (MAA, 2015b, 2015e). Similarly, a student taking the AMC 10 will be invited to participate in the AIME if he or she earns 120 or more points out of a possible 150 points on the AMC 10 or places among the top 2.5% students taking the AMC 10 (MAA, 2015b, 2015e). Following the AIME, the second round of invitational competitions includes the USAMO and the USAJMO and over the years the selection process for the participants has been revised. At the present time, a weighted average based on the top AMC 12 and AIME scores is used to invite approximately 270 students to participate in the USAMO (MAA, 2015b, 2015e). Similarly, a weighted average based on the top AMC 10 and AIME scores is used to invite approximately 230 students to participate in the USAJMO (MAA, 2015b, 2015e). Subsequently, approximately 28 students who achieved the highest scores at the USAMO are invited to participate in the MOSP (MAA, 2015e, 2015g). Ultimately, the six high school students who earn the highest scores during the MOSP are designated as the team to represent the United States at the IMO (MAA, 2015e, 2015g).

Problem Solving

According to Pólya (1965),

> In mathematics, know-how is much more important than mere possession of information… What is know-how in mathematics? The ability to solve problems – not merely routine problems but problems requiring some degree of independence, judgment, originality, creativity. Therefore, the first and foremost duty of the high school in teaching mathematics is to emphasize methodological work in problem solving (p. xii).

Since the publication of Pólya's book, *Mathematical Discovery*, a number of national organizations have reiterated his recommendation that problem solving play a central and prominent role in the K-12 school mathematics curriculum (Cai & Lester, 2010; MAA, 1983; NCTM, 1980a, 1980b, 1989, 1991, 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers (NGAC & CCSSO), 2010; National Research Council (NRC), 1989, 2001). Because of the importance of problem solving to both the individual and society, the mathematics research community should investigate the factors that contribute to the development of students with exceptional mathematical talent, and in turn help develop this ability in other individuals.

Affect plays an important role in the teaching and learning of mathematics and reform efforts in the mathematics curriculum have also emphasized the role of affect in student performance (McLeod, 1992). The National Council of Teachers of

Mathematics (1989) has recommended that students learn to value mathematics and become confident in their ability to make sense of new problem situations and trust their mathematical thinking (NCTM, as cited in McLeod, 1992). Likewise, the National Research Council (1989) has emphasized the need to change societal beliefs and attitudes about mathematics (McLeod & Ortega 1993; NRC, as cited in McLeod, 1992) and has conducted numerous studies on the future of mathematics education, which found that the United States public tends to believe that achievement in mathematics is due to an individual's innate ability rather than one's effort (McLeod & Ortega 1993; NRC, 1989; NRC, as cited in McLeod, 1992).

Through the years the boundaries between the affective and cognitive domains have become increasingly blurred (Schoenfeld, 1992) and research on teaching problem solving has recommended the need for more studies on affect (Silver, 1985). While affect began to emerge as a significant theme in teaching problem solving, researchers attempted to incorporate affective factors into cognitive theories (McLeod, 1992; McLeod & Ortega, 1993). In particular, Mandler (1984, 1989) described how affective factors could be applied to a cognitive theory of problem solving (McLeod, 1992; McLeod & Ortega, 1993) and theorized that most affective factors emerge as a result of emotional responses that occur when an individual's plans are interrupted (McLeod, 1992; McLeod & Ortega 1993).

There are three aspects of the affective domain that individuals experience through their study of mathematics (McLeod, 1992; McLeod & Ortega, 1993). First, individuals possess a set of beliefs about mathematics and themselves, which in turn contribute to their affective responses to mathematics (McLeod, 1992; McLeod &

Ortega, 1993). Second, individuals will experience positive and negative emotions while learning mathematics, as a result of the interruptions and blockages that naturally occur (McLeod, 1992; McLeod & Ortega, 1993). Third, individuals will develop positive or negative attitudes toward mathematics based on their repeated experiences with similar types of mathematics problems or parts of the mathematics curriculum (McLeod, 1992; McLeod & Ortega, 1993). Accordingly, these three characteristics of the affective domain correspond to the three subdomains of affect: beliefs, attitudes, and emotions (Hart, 1989; Lester, Garofalo, & Kroll 1989; McLeod, 1992; McLeod, as cited in Feldman, 2003; McLeod & Ortega, 1993).

McLeod (1989) described the affective domain as "a wide range of feelings and moods that are generally regarded as something different from pure cognition. Beliefs, attitudes, and emotions are terms that express the range of affect involved in mathematical problem solving" (pp. 245-246). According to the literature these subdomains vary in their level of stability, with beliefs and attitudes being generally stable, while emotions can change quickly (McLeod, 1989, 1992; McLeod & Ortega, 1993). Conversely, beliefs, attitudes, and emotions differ in their level of intensity, with beliefs and attitudes being generally mild, while emotions are more intense (McLeod, 1989, 1992; McLeod & Ortega, 1993). Furthermore, these affective subdomains also vary in the time it takes for them to develop and in the extent to which cognition is involved in the affective response (McLeod, 1989, 1992; McLeod & Ortega, 1993). Beliefs develop over an extended period of time and are mainly cognitive in nature, whereas emotions can appear and disappear suddenly and

possess a much stronger affective component (McLeod, 1989, 1992; McLeod & Ortega, 1993).

As noted earlier by virtue of the Putnam Fellows' success on the Putnam Competition, individuals who have been named Putnam Fellows multiple times would be categorized as "expert" problem solvers. What are the characteristics of "expert" problem solvers? To answer that question, the mathematics research community has examined the differences between the problem-solving behavior of expert and novice problem solvers for over four decades. Through his research studies, Schoenfeld (as cited in DeFranco, 1996) revised the definition of expert as it pertains to the problem-solving domain and identified the attributes that constitute problem-solving expertise. The characteristics of expert problem solvers include: domain knowledge (the knowledge base a problem solver brings to a problem), problem-solving strategies (heuristic techniques), metacognitive skills (the ability to comprehend, control, and monitor one's own thought processes, the ability to allocate one's resources, etc.), and a set of beliefs (affect and cognitive belief systems) about mathematics and problem solving (DeFranco, 1996; Schoenfeld, 1992; Schoenfeld, as cited in DeFranco, 1996).

Over the past few decades problem solving has occupied a substantial part of the K-12 mathematics curriculum (Cai & Lester, 2010; MAA, 1983; NCTM, 1980a, 1980b, 1989, 1991, 2000; NGAC & CCSSO, 2010; NRC, 1989, 2001). Therefore, it would be beneficial to the fields of mathematics and mathematics education to examine the factors that have contributed to the Putnam Fellows' abilities as

"expert" problem solvers, and in turn use these findings to improve the problem-solving performance of K-12 students.

Since Putnam Fellows are expert problem solvers and go on to have extraordinary careers in mathematics or mathematics-related fields, it is important to understand the characteristics that led these individuals to becoming Putnam Fellows. In order to do so, the Walberg Educational Productivity Model was used as a framework to study the Putnam Fellows in this study.

The Walberg Educational Productivity Model

Campbell and Wu (1996) have conducted a number of research studies that examined the Mathematics Olympiad programs in the United States as well as in other countries. Through a series of studies, Campbell (1996a) designed a questionnaire using other instruments such as: the Johns Hopkins University Study of Mathematically Precocious Youth (Follow-up Questionnaires); the Westinghouse Talent Search winners (After High School Follow-up Questionnaire); the Longitudinal Study of American Youth; and Campbell's international instruments (Campbell, 1996a; Campbell and Connolly, 1987; Campbell and Connolly, as cited in Campbell, 1996a). As part of his investigation of mathematics achievement in pre-collegiate students, Campbell and Wu (1996) adapted the Walberg Educational Productivity Model as the theoretical framework for their Mathematics Olympiad studies (Walberg, 1984a, 1984b, 1986; Walberg, as cited in Campbell & Wu, 1996).

Walberg has synthesized thousands of empirical studies in the construction of his nine-factor educational productivity model (see Figure 1) (Campbell & Wu, 1996). The Walberg Educational Productivity Model depicts aptitude, instruction, and the environment as the major causal influences to learning (Campbell & Wu, 1996; Iverson & Walberg, 1982; Walberg, 1984a, 1984b, 1986; Walberg, as cited in Campbell & Wu, 1996; Walberg & Marjoribanks, 1976). In their research, Campbell and Wu (1996) adapted the Walberg Educational Productivity Model (see Figure 2), by subsuming five of the global Walberg factors and expanded the number of variables within the home factor to include: family processes, socio-economic status, and the number of parents living at home. Additionally, Campbell and Wu (1996) expanded the motivation factor to include: mathematics and science self-concepts, general self-concept, and two attribution factors (ability and effort).

As part of a research study of the American Mathematics Olympians, Campbell (1996a, 1996b) examined the factors that contribute to or impede the development of the Olympians' talent in mathematics, and investigated the contributions Olympians make to the fields of mathematics and science. The Mathematics Olympiad study revealed that the four most important factors that contribute to the Olympians' mathematical talent include: the home, the school, the Olympiad Program, and mentoring (Campbell, 1996b).

Figure 1

Walberg Educational Productivity Model Depicting the Causal Influences on Student Learning (Campbell & Wu, 1996; Walberg, 1984b)

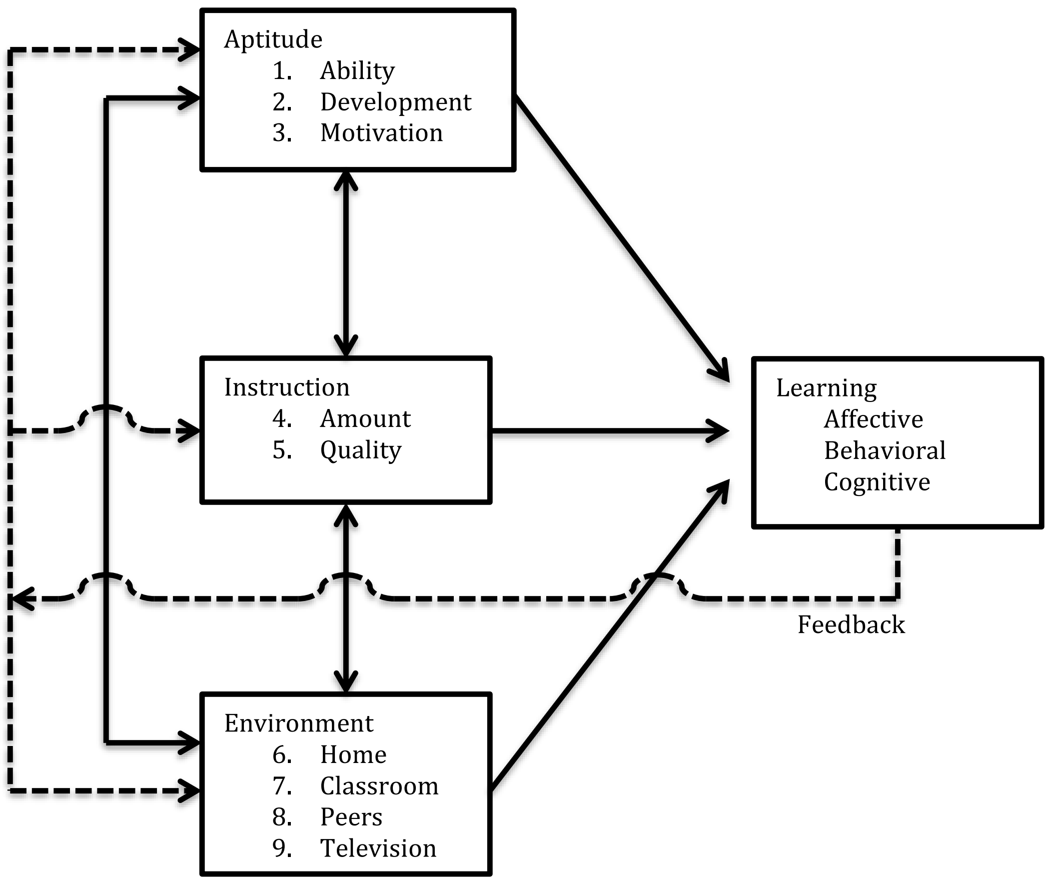
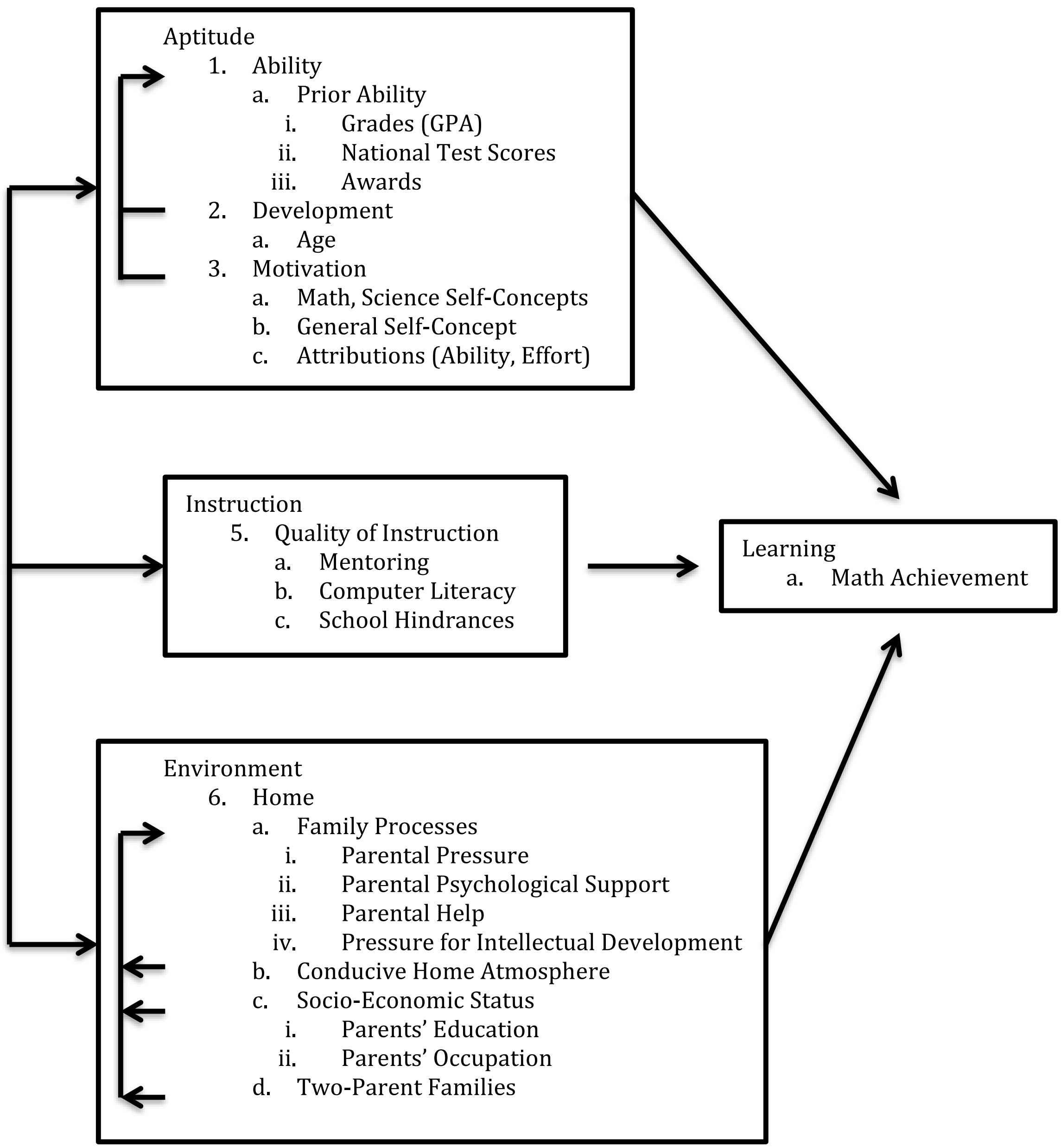


Figure 2

Interconnections Analyzed Within Campbell and Wu's (1996) Adaptation of the Walberg Educational Productivity Model



The Olympiad study showed that most Olympians grew up in households with supportive and resourceful parents, where a stimulating learning environment existed and learning was highly valued (Campbell, 1996b). In school, Olympians experienced high levels of academic achievement, had confidence in their mathematical abilities, and believed effort was more important to their success than ability (Campbell, 1996b). Most Olympians reported that the Olympiad Program positively changed their attitudes toward mathematics and furthered the development of their mathematical talent, which led to their admittance to selective colleges and universities where their mathematical talent could be better developed (Campbell, 1996b). Lastly, the Olympians who received mentoring as undergraduate and graduate students achieved an exceptionally high level of academic productivity, as demonstrated by their numerous publications and patents (Campbell, 1996b).

The Mathematics Olympiad study also revealed that Olympians attend the most prestigious colleges and universities in the United States, and go on to earn advanced degrees, including the doctorate or law degrees (Campbell, 1996b; Campbell & Walberg, 2010). After completing their formal education many Olympians remain in academics where they teach at the college or university level, conduct research, publish scholarly articles and books, and present research papers (Campbell, 1996b; Campbell & Walberg, 2010). Still other Olympians enter the workforce as computer programmers, computer systems analysts, engineers, lawyers, and scientists (Campbell, 1996b; Campbell & Walberg, 2010).

Therefore, this study guided by the Walberg Educational Productivity Model investigated: 1) the personal experiences Putnam Fellows identify as influential in their success on the Putnam Examination, 2) the formal educational experiences Putnam Fellows identify as influential in their success on the Putnam Examination, 3) the role the affective domain plays in the development of a Putnam Fellow, and 4) the role the cognitive domain plays in the development of a Putnam Fellow.

Chapter III

Methods and Procedures

The exploratory nature of this research required that information gathered during the study be analyzed qualitatively. Presented in this section is a description of the: (a) selection of participants for the study, (b) materials used in the collection of data, (c) procedures for data collection, and (d) procedures for data analysis.

Participants

The study took place during the spring, summer, and fall months of 2014. In order to participate in this study each subject must have been named a Putnam Fellow during two or more Putnam Competitions.

A list of 74 individuals, who have been named a Putnam Fellow four, three, or two times throughout the 74 Putnam Competitions held between 1938 and 2013 inclusive, was generated from the Mathematical Association of America's results published annually in *The American Mathematical Monthly*. The Putnam Fellows on the list were contacted beginning with the four-time competition winners, followed by the three-time and two-time winners, through e-mail, mail, and/or telephone invitations requesting their participation in this study. A total of 57 Putnam Fellows were contacted of which 25 individuals expressed a willingness to participate in the study; five people declined to participate; and 27 individuals did not respond. In

regards to the remaining 17 Putnam Fellows, five people could not be located and 12 individuals are deceased. The 25 participants in the study were all males and attended eight different colleges and universities in the United States and Canada at the time they were named Putnam Fellows.

Demographic information on the participants (24 subjects responded) was collected from a questionnaire (see Appendix H), which included questions on: number of parents living at home; birth order and age span between siblings; parents' level of education; and parents' occupation. Analysis of the demographic questions found that 22 participants reported growing up in a household with two parents living at home (see Table 1), and 19 indicated that they were the first-born children in small families (2.25 children) where the average age span between siblings was 4.12 years (see Table 2). In regards to parents' level of education, the subjects indicated that 22 fathers and 21 mothers had earned college degrees (see Table 3). Lastly, 22 participants reported their parents' occupation (see Table 4).

Table 1

Number of Parents Living at Home

Number of Parents Living at Home	**Two**	**One**
Number of Putnam Fellows	22	2

Table 2

Putnam Fellows' Birth Order

Birth Order	**1st of 1**	**1st of 2**	**1st of 3**	**1st of 4**	**2nd of 2**	**2nd of 3**	**3rd of 3**	**5th of 5**
Number of Subjects	5	10	2	2	2	1	1	1

Table 3

Parents' Level of Education

Level of Education	**Post Doc**	**M.D.**	**Ph.D.**	**Master's**	**Bachelor's**	**High School**
Father	1	2	8	4	7	2
Mother	0	1	4	5	11	3

Table 4

Parents' Occupation

Occupation	**Father**
Chemist	1
Computer Programmer	1
Engineer	4
Mathematician	1
Military	2
Office Manager	1
Physicist	1
Professor	5
Research Scientist	1
Software Engineer	1
Surgeon/Medical Doctor	2
Technician	1
Writer/Editor	1

Occupation	**Mother**
Accountant	1
Computer Programmer	1
Consultant	1
Engineer	2
Homemaker	6
Librarian	1
Mathematician	1
Professor	2
Psychologist	1
Legal Secretary	1
Surgeon/Medical Doctor	1
Teacher	4

Materials

In this study, data came from the interview questions (see Appendix H), which are described in this section. The purpose of the questions posed by the researcher was to gather information from the participants about the factors and characteristics, (i.e., personal experiences, formal educational experiences, and the role of the affective and cognitive domains), which have contributed to their success as Putnam Fellows. Sources for the questions were adapted from Campbell's (1996a, 1996b) study of the American Mathematics Olympians and DeFranco's (1996) study of the mathematical problem-solving expertise of male Ph.D. mathematicians. Follow-up questions were interjected as needed to pursue or clarify a participant's response.

Procedures for Data Collection

The data in this study was collected using oral and written interviews that were conducted during the spring, summer, and fall months of 2014. In this study, each participant was sent a copy of the 18-item questionnaire (see Appendix H), through e-mail or the mail, which allowed each Putnam Fellow the opportunity to review and give thought to the questions prior to the interview. Because of the subjects' physical locations, work schedules, personal and professional commitments, and summer travel plans, data was collected through interviews

conducted over the telephone, on the computer using Skype, and/or through e-mail written responses.

The purpose of the interview was to collect data regarding the factors and characteristics that contributed to the success of the participants as Putnam Fellows. The Interview Questionnaire contained 18 questions that focused on the personal and formal educational experiences that the subjects identified as influential in their success on the Putnam Examination (e.g., the role their parents played, the role of an influential event or individual, the role of an influential teacher, participation in enrichment classes or summer programs in mathematics, participation in mathematics contests and competitions, and preparation for the Putnam Examination) as well as the role the affective and cognitive domains (e.g., confidence, feelings experienced when solving a Putnam problem, motivation, Pólya-type heuristic techniques, the use of analogous cases, and the role of intuition) played in the participants' development as Putnam Fellows.

In collecting the data, eight subjects consented to participate in audio-recorded interviews, whereas 17 subjects elected to provide their responses in writing via e-mail. Each of the eight Putnam Fellows who were audio recorded participated in an in-depth, semi-structured interview that took the form of a dialogue or conversation with a purpose (Erlandson, Harris, Skipper, & Allen, 1993). The eight participants who were interviewed over the telephone or on the computer through Skype, were audio-recorded using Piezo software and the interviews ranged in length from 45 to 90 minutes each. The 17 participants, who elected to answer the Interview Questionnaire in writing, returned their written responses

through e-mail. The written responses were formatted into Microsoft Word documents that ranged in length from two to four pages each. To clarify or elaborate on participants' responses, two follow-up telephone calls and six follow-up e-mail communications were made to eight Putnam Fellows.

Procedures for Data Analysis

The audio compact discs from the eight subjects who participated in the telephone and Skype interviews were transcribed and this information along with the written responses from the 17 subjects who responded to the 18-item questionnaire in writing were used to answer the research questions. The data was analyzed qualitatively due to the exploratory and descriptive nature of this study. Data analysis "is the process of bringing order, structure, and meaning to the mass of collected data… Qualitative data analysis is a search for general statements about relationships among categories of data; it builds grounded theory" (Marshall & Rossman, 1995, p. 111). Qualitative analysis requires that the data collected be organized and condensed into manageable elements that can be interpreted to allow categories, themes and patterns to emerge and hypotheses to be identified and tested against the data (Marshall & Rossman, 1995; Rossman & Rallis, 2003). In this study, the process of data analysis occurred concurrently with the process of data collection, which allowed the researcher to regulate data collection strategies and test out emerging ideas against the new data that was collected (Marshall & Rossman, 1995).

In order to answer the research questions, the eight audio-recorded interviews were transcribed (a sample interview transcription can be found in Appendix I) in preparation for the coding of this data. The eight transcribed interviews, together with the 17 questionnaires that were completed in writing, were then formatted into text matrices. Every significant statement or complete thought within the transcripts was color-coded according to the coding category it represented (see Figure 3). During the peer debriefing meetings, the peer debriefer examined samples of the text to check the inter-rater reliability of the coding (Thomas, 2006). A within-case analysis was employed for each participant to identify themes and patterns with respect to the factors and characteristics that have contributed to the success of the Putnam Fellows. In order to accomplish this, the rows within the coded matrix for each individual participant were sorted by the categories and subcategories of personal experiences, formal educational experiences, and learning domains. Building upon each within-case analysis and using the data entered into the text matrices, a cross-case analysis was conducted to identify similarities and differences across participants with respect to the categories and subcategories of personal experiences, formal educational experiences, and learning domains. The purpose of cross-case analysis is to enhance generalizability and deepen understanding and explanation of the phenomenon being studied (Miles & Huberman, 1994). Finally, themes and patterns that emerged across cases were organized into summary theme tables, one for each subcategory of personal experiences, formal educational experiences, and learning domains. This thematic analysis of the data allowed the researcher to identify the

major recurrent patterns within the data across all cases of personal experiences, formal educational experiences, and learning domains.

Figure 3

A Sample of the Line-by-Line Coding of the Interview Data

Participant	**Question 4: Can you describe a teacher or teachers who have influenced you to help become a Putnam Fellow?**	**Code**
17	I had many teachers and coaches who helped me over the years; none specifically helped with the Putnam exam. I should single out [redacted], the head coach of the IMO (International Mathematical Olympiad) team for the [redacted] years in which I participated. He had been working with me on writing up cleaner solutions; at the [redacted] IMO, he read over one solution of mine, sighed, and remarked that this was what people's papers looked like right before they went to start winning Putnams for [redacted]. His remark gave me a lot of confidence.	FE1 FE2 AD1

Coding Legend

Red: Personal Experiences
Blue: Formal Educational Experiences
Green: The Affective Domain
Purple: The Cognitive Domain

FE1: The participant had helpful teachers and coaches.
FE2: The subject participated in mathematical competitions.
AD1: The participant had a lot of confidence.

The procedures for data analysis for each of the research questions are presented next.

Research Question 1

> What personal experiences do Putnam Fellows identify as influential in their success on the Putnam Examination?

In order to answer research question 1, the transcriptions (i.e., the eight transcribed interviews, together with the 17 questionnaires that were completed in writing), were formatted into a text matrix (see Appendix J). Throughout the coding process, the researcher developed inductive codes as the data was examined to identify meaningful phrases and sentences. Every significant statement or complete thought within the transcripts was color-coded in red according to the category or subcategory of personal experiences (i.e., households conducive to learning; influential family members and family friends; an interest in and a talent for mathematics at a young age; and access to educational resources) it represented.

Research Question 2

> What formal educational experiences do Putnam Fellows identify as influential in their success on the Putnam Examination?

In order to answer research question 2, the transcriptions were formatted into a text matrix (see Appendix K). Throughout the coding process, the researcher developed inductive codes as the data was examined to identify meaningful phrases

and sentences. Every significant statement or complete thought within the transcripts was color-coded in blue according to the category or subcategory of formal educational experiences (i.e., influential teachers and individuals in academics; participation in mathematical contests and competitions; access to released mathematical contest and competition problems; and participation in extracurricular mathematics training) it represented (see Figure 3).

Research Question 3

> What role does the affective domain play in the development of a Putnam Fellow?

In order to answer research question 3, the transcriptions were formatted into a text matrix (see Appendix L). Throughout the coding process, the researcher developed a priori codes before the data was examined to identify meaningful phrases and sentences. Every significant statement or complete thought within the transcripts was color-coded in green according to the category or subcategory of the affective domain (i.e., beliefs about confidence; beliefs about natural ability, aptitude, and talent in mathematics; beliefs about having an interest in mathematics and liking mathematics; beliefs about the role of intuition; beliefs about talent; feelings experienced when solving Putnam problems; and motivation) it represented (see Figure 3).

Research Question 4

What role does the cognitive domain play in the development of a Putnam Fellow?

In order to answer research question 4, the transcriptions were formatted into a text matrix (see Appendix M). Throughout the coding process, the researcher developed a priori codes before the data was examined to identify meaningful phrases and sentences. Every significant statement or complete thought within the transcripts was color-coded in purple according to the category or subcategory of problem-solving strategies (Pólya-like heuristics: alternative methods to solve mathematics problems; similarities between Putnam problems and other mathematics problems; and analogous cases as a strategy to get a feel for Putnam problems) it represented (Olkin & Schoenfeld, 1994; Pólya, 1988).

Chapter IV

Results

Introduction

The purpose of this study was to investigate the characteristics that contributed to the success of Putnam Fellows and to see if these Putnam Fellows share some of the same characteristics of "expert" problem solvers as defined in the literature (DeFranco, 1996; Schoenfeld, 1992; Schoenfeld, as cited in DeFranco, 1996). Qualitative methods were employed to answer the general questions, what personal experiences and formal educational experiences do Putnam Fellows identify as influential in their success on the Putnam Examination and what role do the affective and cognitive domains play in the development of a Putnam Fellow?

In order to answer these questions, 25 male Putnam Fellows who attended eight different colleges and universities in the United States and Canada at the time they were named Putnam Fellows were selected to participate in this study. Data was collected over a period of seven months and included oral and written interviews. Eight subjects consented to participate in audio-recorded interviews, whereas 17 subjects elected to provide their responses in writing using e-mail. The transcriptions of the eight audio-recorded interviews and the 17 written responses to the questionnaire served as the data, which was analyzed using qualitative techniques in order to answer the research questions. The number in the parentheses, following each participant's response, corresponds to the item number

on the questionnaire (see Appendix H). Results of the data analysis are reported next.

Research Question 1

> What personal experiences do Putnam Fellows identify as influential in their success on the Putnam Examination?

In order to answer research question 1, all of the interview protocols were reviewed and coded. The coded data was then sorted to identify salient themes and patterns within each category for each participant. Emerging themes were organized into a partially-ordered meta matrix and a cross-case analysis of the data was employed. From the matrix, themes and patterns were identified across cases. All data supported by eight or more participants was reported. Results of the thematic analysis are reported next.

Personal Experiences

In this study, participants were asked to reflect on their personal experiences beginning with their earliest childhood memories, including stories retold by family members and family friends, and continuing up until the time they first participated in the Putnam Competition.

In addressing themes within the personal experiences of the participants in this study, four subcategories of experiences emerged as themes and were examined. These included: (a) being raised in households that were conducive to

learning, (b) having influential family members and family friends, (c) showing an interest in and a talent for mathematics at a young age, and (d) having access to educational resources.

<u>Households Conducive to Learning</u>

Twenty-two of the participants in this study reported being raised by parents who valued academic achievement and who provided encouragement and support in learning mathematics. For example, Participant 1 stated, "Both parents encouraged me strongly in my academic success and interest in science and math" (Q.2). Whereas Participant 2 responded, "They [parents] were imbued with the Jewish tradition, which places the highest value on study and knowledge. In matters of education, my parents were psychologically very supportive" (Q.2). Similarly, Participant 7 indicated, "They [parents] encouraged me implicitly to go into a career in the sciences" (Q.2).

Likewise, Participant 15 expressed,

> I guess one important one is not pushing me in any direction, and allowing me to pursue what I found interesting, and then supporting me in that. Driving me around places if need be or buy books for me if need be (Q.2).

Whereas Participant 17 responded,

> My parents did not directly teach me math beyond elementary school, but they created an environment where I was able to flourish. They encouraged my interest in math from an early age. They made clear that my primary job was to do well in school (Q.2).

Similarly, Participant 18 indicated,

> My parents were the first to notice my mathematical talent and to encourage me to develop it. At first, by teaching me arithmetic and basic algebra well before the normal age for learning it, later by encouraging me to read and do mathematics on my own and to study with other teachers. As for the home atmosphere, it's hard to say that this was particularly directed toward making me a Putnam Fellow as opposed to promoting education in general and in mathematics in particular (Q.2).

Influential Family Members and Family Friends

Twelve participants in this study reported that family members and family friends were influential in helping them learn mathematics beginning at a young age. For example, Participant 12 stated, "My brothers were the people who most influenced me regarding math competitions. Both were older and had participated in competitions before me. One brother in particular frequently encouraged me to learn advanced math and solve problems" (Q.3). Whereas Participant 19 responded, "They [parents] recognized my love and talent for mathematics at a very young age (Kindergarten) and always encouraged me. My mom taught me fractions and algebra while I was in elementary school" (Q.2). Similarly, Participant 25 indicated,

“Dad spent most hours working and lent almost exclusively financial support. ‘I'm just the cook,’ he likes to say. The focus and meticulosity it takes to endure the contest was developed over many years under Mom's example and guidance” (Q.2).

Likewise, Participant 5 expressed,

> An important early mentor was a family friend who introduced me to Pascal's Triangle and Newton's Method for square and cube roots, and similar goodies. My grandmother also taught me the basics of arithmetic when I was about four. I was visiting grandma for maybe a month or two, don't recall exactly. My fourth birthday was during the visit. I believe she was a/the librarian at nearby University of [redacted] in [redacted]. I think she had been a teacher, but was now semi-retired. She wasn't going to work while I was visiting. I assume she enjoyed teaching me. The material was basic reading and arithmetic. I started out reading a few pages per day of the primer, and eventually finished the last third or so in one day. There were flash cards with words. I don't recall numeric flash cards. I think I learned addition, subtraction and some of multiplication and division in this first visit, but my memory is hazy. For a long period, a year or two, I could only do single-digit times multi-digit multiplication, and ‘short division’, one-digit into multi-digits. There were several visits, for varying lengths of time. Sometimes my sister would come; she was 1.5 years younger. We played Monopoly with the neighbor kids, and I read through the long rulebook (there was a ~40 page pamphlet) and understood the majority of it, but I was probably five or six by then. Most of us could do the basic money handling. I doubt we did decimals, or figured ‘mortgage’ interest. I recall learning two-digit by two-digit multiplication from a baby-sitting teenager, sometime before age seven. (I'm basing the date on where I lived at the time.) Sometime later, I'd worked out generalizing short division to several-digit divisors, and was later surprised to see formal long division amounted to the same scheme, but writing down the numbers I'd been working with mentally. My mother was also a teacher of grade-school kids, but I don't recall any explicit teaching from her except a failed attempt at French. She must have helped with my vocabulary though, and the rules for possessives and tenses. I recall asking her once ‘what's a hundred times a hundred times

> a hundred?' and she replied promptly 'a million'. I don't recall learning any other math from her (Q.4).

Whereas Participant 11 responded,

> I got interested in math very young. When I was a small child, I don't know how old, my grandmother gave me, I don't remember this, but I'm told my grandmother gave me an audio recording of a song, which was just the multiplication tables. So I memorized the multiplication tables when I was small, so I could do arithmetic for as long as I can remember. And I guess which is unusual, obviously to be able to do arithmetic long before Kindergarten, because I started early. So I always knew I was years and years ahead of my classmates. And then because I was years ahead, instead of learning in a normal way, my father taught me things that I needed to know to learn more math (Q.3).

Similarly, Participant 20 indicated,

> My parents helped and contributed in a number of ways. One thing I should say, not specifically in regards to this question, but you ask on a number of questions, what contributed to your being a Putnam Fellow? I did a lot of math contest stuff in high school and college, and so there was not one particular thing. I don't think of taking the Putnam is itself a discrete event. I think that's one part of a general career interest in math problem solving that I developed for a long time. So what role did my parents play? I think they did quite a lot, both of my parents. They already had a technical background, so they were interested in getting me – they were naturally equipped to get me to learn about math and science early on. Starting as far back as I can remember, age two, three, they were teaching me how to add and multiply, and by the time I was six they were giving me books of logic puzzles to learn from. And since I was interested, they continued to get me books to learn math from at home. And so, that's sort of the obvious thing. They continued to do that more or less throughout my childhood, although you know, as I got older I became more self-sufficient. By the time I was in high school, I had either other people supplying resources for me or I was finding stuff on my

> own. My parents were certainly encouraging all the time, and they took an interest in what I was learning as well (Q.2).

Likewise, Participant 21 expressed,

> The fact that my dad was a professional mathematician had a significant impact on my early mathematical experience. In addition to providing me with the opportunity to discuss more advanced topics with him outside of school, I was essentially homeschooled in mathematics after about second grade. I was not homeschooled, but I was effectively homeschooled in mathematics. I mean, essentially the setup was that during the time when everybody else in my class were doing math with the teacher, my parents had assigned me readings from various high school textbooks. I would go work on those, and work on problems associated to those, on my own. So effectively, most of the instruction was coming from books and effectively all the directions were coming from my parents. So this wasn't done for any other subject (Q.2).

An Interest in and Talent for Mathematics at a Young Age

Fifteen participants in this study reported that they became aware of their interest in and talent for mathematics at a young age. For example, Participant 2 stated, "My interest in mathematics developed spontaneously, starting to emerge around the age of ten" (Q.3).

Whereas Participant 3 responded,

> Here is one case of a very early influence. It was in third grade I first became aware of my interest in and talent for mathematics. The next year, while in fourth grade, I was part of a 'play' being put on by the school, involving

> classes at all levels. While waiting for rehearsal one day, a ninth-grade girl somehow became aware of my interests and taught me the simplest case of the binomial theorem – the formula for (x + y)^2 (Q.3).

Similarly, Participant 5 indicated,

> I was always competitive and good at math. I went directly from wanting to be a fireman at age four – five, to knowing that I'd be a mathematician. I'm actually a blend of mathematician and algorithm developer, but computers were just coming in as I was growing up (Q.3).

Likewise, Participant 10 expressed,

> My father told me that when I was little I loved counting green stamps, which were given out as a bonus when you bought gas. He would always ask how many we got. One time he was startled at how quickly I counted them. He asked how I did it so fast. Apparently I described a method I used to count them quickly, which was a sort of primitive method of multiplication I had come up with (Q.3).

Whereas Participant 14 responded,

> My father encouraged me to try the Putnam unofficially while I was in high school (and taking college mathematics classes, so I knew most of the material covered by the exam). I scored zero as a freshman, because I didn't know much about the competition and didn't know how to write up the two problems I had solved, but as a sophomore, I solved six problems and my score was an unofficial 19th place, and I did even better as a junior and senior. But I first realized I was good at mathematics competitions when I watched the competition organized by [redacted] College (where my father was a professor) for high school students when I was in grades four through seven, and realized that I could have competed with the students even then (Q.3).

Similarly, Participant 20 indicated,

> I think I remember the first time proving something that I would call a theorem was when I was ten years old and was thinking about for no particular reason that I can remember, I was thinking about perpendicular bisectors as sides of triangles. And I thought, 'Gee, if I draw the perpendicular bisectors of the three sides, they all seem to meet in the point.' And I realized why that is. And I said, 'Well okay, so if the perpendicular bisector of AB and the bisector of AC meet somewhere, then that point must be the same distance from all three of the corners, so it is also on the bisector of BC.' And so then after that I was like, 'Hey, I can prove things.' And at some point, I don't remember whether it was then or some time later my dad would have a habit of asking me when I came home from school, he would say, 'Did you prove any theorems today?' You know I think it was something I did as a hobby for a long time. The first time that it really struck me that I was doing more of this than a large majority of people my age was when I starting actually competing in contests, this would have been in the eighth grade, I took the American, the exam that is now called the AMC 12, at the time it was called the AHSME, American High School Mathematics Examination, just because the local high school where I was taking geometry at that time was administering it. This was just an activity that the math club did and it turned out it was the first of several rounds (Q.4).

<u>Access to Educational Resources</u>

Eleven participants in this study reported having access to educational, printed materials (i.e., mathematics textbooks, puzzle books, and encyclopedias) at home during their childhood. For example, Participant 5 stated, "The main support was valuing academic achievement, and connecting me with a couple of math folks. She [mother] helped me get my own copy of Hardy and Wright's *An Introduction to the Theory of Numbers* when I was about 12" (Q.2).

Whereas Participant 10 responded,

> My father had some advanced textbooks on mathematics that he allowed me to look at around high school. However, he claimed that I mostly acquired my mathematical ability on my own. My parents did encourage me to attend math contests once I became interested in them. They also always had encyclopedias available to me, at least from around the time of middle school on (Q.2).

Similarly, Participant 14 indicated,

> I know that I've always been well ahead of the mathematics that was being taught in schools, partly because of what I had been able to do on my own, reading books. I'd always enjoyed mathematical puzzle books, things like Martin Gardner's columns from *Scientific American* and book collections of them. I own most of those books (Q.3).

Likewise, Participant 16 expressed,

> I think they [parents] certainly helped me gain an advanced education in math when I was living at home. I mean when I was off in college, I was on my own, but they certainly had a lot of influence in getting me access to mathematical resources when I was in high school (Q.2).

Research Question 2

> What formal educational experiences do Putnam Fellows identify as influential in their success on the Putnam Examination?

In order to answer research question 2, all of the interview protocols were reviewed and coded. The coded data was then sorted to identify salient themes and

patterns within each category for each participant. Emerging themes were organized into a partially-ordered meta matrix and a cross-case analysis of the data was employed. From the matrix, themes and patterns were identified across cases. All data supported by eight or more participants was reported. Results of the thematic analysis are reported next.

Formal Educational Experiences

In this study, participants were asked to reflect on their formal educational experiences beginning with their earliest memories of schooling, including stories retold by family members and family friends, and continuing through their participation in the Putnam Competition.

In addressing themes within the formal educational experiences of the participants in this study, five subcategories of experiences emerged as themes and were examined. These included: (a) having influential teachers and other individuals in academics and other formal educational settings, (b) participating in mathematical contests and competitions prior to the Putnam Competition, (c) having access to released mathematical contest and competition problems and solutions, (d) participating in extracurricular mathematics training, and (e) not receiving coaching or preparatory classes through their college or university.

Influential Teachers and Individuals in Academics

Nineteen participants in this study reported having strong K -12 mathematics teachers, coaches, and mentors. For example, Participant 3 stated, "In grades eight and nine I had a fabulous math teacher, Miss [redacted], who was incredibly effective and inspiring for all ability levels. She gave me a wonderful foundation in algebra and geometry" (Q.4). Whereas Participant 10 responded, "I had a combination of teachers and professors who were interested in nurturing my talent, which I think helped a lot" (Q.18). Similarly, Participant 13 indicated, "The best teachers were those who pointed me to reading material beyond the traditional classes" (Q.4). Likewise, Participant 17 expressed, "I had many teachers and coaches who helped me over the years" (Q.4).

Whereas Participant 9 responded,

> Well, the main one would be in high school, the leader of our mathematics, team in the local competitions, and his name was [redacted]. And yeah, he certainly was very helpful in my continued progress through the mathematics competitions. Although of course, this was before the Putnam part of it (Q.4).

Similarly, Participant 15 indicated,

> I had a teacher in grade seven who taught math and English. And he was an incredible teacher and he really communicated the love of everything, of all subjects, and then of course the two he taught. And he actually did show me

more than a few things. And he really was incredibly encouraging. I think that made a huge difference (Q.4).

Likewise, Participant 19 expressed,

> [redacted] taught mathematics at [redacted] and got me involved in all the contests. [redacted], [redacted] – two other teachers at [redacted] who encouraged me with problem solving. [redacted] (University of [redacted]) – taught an advanced polynomials correspondence course for high school students and was otherwise very active in running advanced math programs for high school students. [redacted] – ran a problem-solving correspondence course designed to prepare students for the IMO (International Mathematical Olympiad). [redacted] was four years ahead of me, very much a mathematical role model, and very active in problem-solving training and the IMO. The most important point is that there wasn't one specific teacher, but rather a collection of teachers over a long period of time (Q.4).

Whereas Participant 20 responded,

> I had a number of mentors during my childhood who encouraged me to pursue interests in mathematics, especially in competitive problem solving and who recommended sources of material to practice on, things to learn, and so forth. So probably, not probably, the most influential person I would say was [redacted], currently at [redacted] College, also visiting professor at [redacted], who while I was in high school, was assistant professor of mathematics at [redacted]. She started there I think after my freshman year of high school, which was when I had just qualified for the U.S. team to the International Math Olympiad for the first time. And so I had been involved in a number of other contests and I was on the local team for the American Regents Math League [ARML], which is a team contest that you've probably heard about from other people you've talked to. So at some point I got a very long and detailed e-mail from a professor who I had not met yet, actually congratulating me on what I was doing, and expressing lots of enthusiasm for the [redacted] Math Circle, which was a program that she was

> then just starting and encouraging me to participate. And so that was one of the biggest things that she has done in the last few years is started this Math Circle program all over the U.S. She comes from [redacted] where this is a long-standing tradition to have these extra after-school or weekend math programs to go to. And so she and several others in the [redacted] started importing this tradition to the U.S. and to the [redacted] Circle, which meets on weekends, or it did at that time. And I think it still does. It is much larger now than it was then. So they were just getting started at that time. And so she asked me to take part, take sort of an organizational role. So my job initially was as the coordinator for the monthly contests. And they would have lectures on one or another sort of math topics that you wouldn't ordinarily see in the classroom every week. And they also had these contests, which were run on a monthly basis. Problems were distributed and you would go home and write solutions to them and then submit them. And they said, so the way that she put it was like, 'We can't let you compete in this contest, so why don't we have you organize it instead?' So I was in charge of putting together official solutions and administering grading. And then in subsequent years I gave some lectures at the Math Circle as well. And for me that was a great way to meet other people of my age who were interested in mathematics, so I made a number of friends that way. And of course it was also a good way to learn new stuff. And so she was the most influential person I would pick out. There were a number of others like [redacted], at the University of [redacted], who also was involved in organizing the Math Circle, as well as several other extracurricular math events for secondary students in the [redacted]. He I think was approaching our ARML team and he gave me a number of, a long list of reading suggestions to work on problem solving. So those are the two main people who come to mind. Yeah, [redacted] was the first one and [redacted] is the second (Q.3).

<u>Participation in Mathematical Contests and Competitions</u>

Nineteen participants in this study reported taking part in mathematical contests and competitions (e.g., the Annual Mathematics Competition, the United States of America Mathematical Olympiad, and the International Mathematical

Olympiad), prior to the Putnam Competition. For example, Participant 4 stated, "The McClatchy News Services, which owned *The Sacramento Bee*, *Fresno Bee* and other newspapers sponsored a math contest for high school students. I did well but don't recall any details now" (Q.6). Whereas Participant 8 responded, "I participated in the USA Mathematical Olympiad one year. I don't recall any others" (Q.6). Similarly, Participant 13 indicated, "Yes, starting in seventh grade I participated in local math competitions in the Greater [redacted] area, and later I participated in the USA Mathematical Olympiad and the International Mathematical Olympiad" (Q.6). Likewise, Participant 22 expressed, "I didn't do MATHCOUNTS, but I did all other major math competitions in high school, including the [redacted] Math Olympiad, the USA Math Olympiad, and the International Math Olympiad" (Q.6).

Whereas Participant 5 responded,

> There was something called the Future Engineers, organized as a Sputnik response, and they organized math contests for a few years around 19[redacted]. It was mostly high school students, but I participated as a sixth grader. [redacted] Junior College had a citywide competition that I entered in high school, and the Math Association also ran its own national math contest. This seemed to have vanished by the 19[redacted]s, when I looked for it for my daughter. I never heard of the Math Olympiads in a serious way before college, although I must have seen them mentioned occasionally on TV news programs (Q.6).

Similarly, Participant 11 indicated,

> The other person was a math teacher at a neighboring high school. He's actually quite an important figure in the world of math competitions, his name is [redacted], and I think he's still alive, there's a *Wikipedia* article about him. And I was in a math league competition, his team was very strong, my team was very strong, and he found out that I was a member of the high school math team when I was probably in eighth grade. He noticed that I was doing well and he recommended me, as I understand, he never told me this, but later I found this out, for the Math Olympiad program, for the USA Mathematical Olympiad. And you want to thank God for that program. I was enormously, better prepared than anybody who hadn't been to the program. That program was excellent (Q.3).

Likewise, Participant 12 expressed,

> I was on math teams every year from seventh to 12th grade. I took the AHSME [American High School Mathematics Examination] those same years. I took the AIME [American Invitational Mathematics Examination] in 11th and 12th grades, and the USAMO [United States of America Mathematical Olympiad] in 10th, 11th, and 12th grades. I participated in the International Mathematical Olympiad after 11th and 12th grades (Q.6).

Whereas Participant 20 responded,

> Well I wouldn't say throughout my formal education. I didn't do MATHCOUNTS. I had a bad experience with a locally organized math contest when I was in fifth grade and so I didn't participate in competitions for a little while after that. But then starting in eighth grade is when I joined the local high school math club, which participated in a number of other, sorts of informal regional contests, as well as the AHSME [American High School Mathematics Examination] sequence. And then, I guess starting then, once I realized that I was enjoying this, and I was doing well, I sort of started participating in every contest that I could find. So there is the AMC [Annual

> Mathematics Competition] contest, and the IMO [International Mathematical Olympiad], there was ARML [American Regions Mathematics League], there was something called American High School or USA Mathematical Talent Search that was run out of, I can't even remember who was organizing it or whether it still exists. At some point, I'm sure there are others that I'm forgetting about. You know that was basically my career when I was in high school. I heard, 'Who does math problems for fun?' There were enough strands and outlets of them, enough agencies organizing these things that I participated in, and there is no way I am going to successfully remember all of them. So that was not the Westinghouse, but I did also participate in that, so that was my senior year. So I think there were both, there were two research-oriented science contests. There had formally been a Westinghouse one. Now there was one run by Intel and another one run by Siemens at the time that I was a high school senior. I competed in the Intel one. I used the topology paper I had written at [redacted] ([redacted]) for that (Q.6).

<u>Access to Released Mathematical Contest and Competition Problems</u>

Thirteen participants in this study reported working through released examinations as a way to prepare for the Putnam Competition. For example, Participant 3 stated, "Basically by studying old Putnam exams, i.e., attempting to solve the former problems. My college ([redacted]) did not at the time offer any prep classes or coaching for the competition" (Q.7). Whereas Participant 11 responded, "By far the biggest thing was that I went through all the Putnam problems and tried solving them all. I didn't succeed in solving all of them, but I solved most of them. And that was the main thing" (Q.7). Similarly, Participant 13 indicated, "I went through all the Putnam problems from previous years" (Q.7). Likewise, Participant 16 expressed, "I don't believe we had Putnam preparation classes. I think I did work

some previous exams. I think I had a book of the previous ones. And I worked through some of the old exams" (Q.7).

Whereas Participant 20 responded,

> So there wasn't a whole lot of preparation that I did. I mean organized preparation. I prepared for it [the Putnam] the same way that I prepared for other contests, which was basically working on, working through problems from past years. One thing that makes it hard to answer the questions about the Putnam contest is by the time I was taking the Putnam it was not a big deal from my point of view. You know like there are a lot more of these high school contests than there are college-level contests. And the Putnam is basically the only one of its kind in the U.S. or at least it was at the time that I was doing it. I don't know if that's still the case since I don't really live as much in this world now. And it was also, I had done the IMO [International Mathematical Olympiad] and IMO problems are much harder than Putnam problems, so by the time that I was taking the Putnam I was sort of like, 'Okay, you know I've developed these skills beyond the point that it is necessary here, so I can just do a little bit of extra practice', which I did each year that I took the Putnam, but it was not something that I needed to invest a whole lot of time in it (Q.7).

<u>Participation in Extracurricular Mathematics Training</u>

Twenty-three participants in this study reported taking part in extracurricular mathematics training (e.g., after school, on weekends, and during the summer months) in addition to their normal high school program of studies. For example, Participant 4 stated, "The summer after 11th grade I went to JESSI [Junior Engineers and Scientists Summer Institute] in [redacted] and the summer after 12th grade I went to an NSF [National Science Foundation] sponsored program at [redacted]

[redacted]" (Q.5). Whereas Participant 6 responded, "I learned to write proofs taking a college course in number theory during the summer when I was in high school" (Q.4). Similarly, Participant 12 indicated, "After 10th, 11th, and 12th grades, I participated in the training sessions for the International Math Olympiad" (Q.5).

Likewise, Participant 5 expressed,

> My high school offered a 'fifth year of math' during the summer between my junior and senior years. Topics were analytic geometry, limits, and other pre-calculus. Maybe some simple calculus, I don't recall. My high school signed me up for a Saturday morning math program at a local college, which I went to for two years. It was for high school students, and had snippets of college level math – extension fields and similar (Q.5).

Whereas Participant 11 responded,

> Well those Math Olympiad programs [Mathematical Olympiad Summer Program]. So the summer after my eighth grade year all the way through high school. And so of course, I had an enormous advantage over anybody who didn't have that background, when it came to the Putnam. Enormous, it's completely unfair. I mean, if you hadn't been trained with that material you really don't have any chance with someone who has that background (Q.5).

Similarly, Participant 15 indicated,

> The University of [redacted], with the people that did all the competitions, had a one-week summer program that I went to every year. And once I was on the Olympiad team, we would have a week of Olympiad training and there would be an extra week of work as well. Although I think with that one by

> the end, it was less that I could get something out of it that someone told me, and more just being among peers and working together was very productive (Q.5).

Likewise, Participant 22 expressed,

> I attended numerous Olympiad training camps back in high school – they were probably the most helpful to my participation in math competitions. Yeah, so for high school in the summertime I go to the summer camps for training in the Olympiads. And that was very helpful, so there were peers of similar level and there were also coaches who were specialized for math competitions, so that was very helpful. And during the school year when I'm at school, mostly you know reading books on my own and reading articles I follow from the Internet (Q.5).

<u>College or University Did Not Offer Preparatory Classes</u>

Ten participants in this study reported that their college or university did not offer any coaching or practice sessions as preparation for the Putnam Competition. For example, Participant 6 stated, "I made no special preparation. There were then no classes for this at [redacted]" (Q.7). Whereas Participant 12 responded, "I practiced solving problems from previous Putnams on my own. My school did not offer preparation classes at that time" (Q.7). Similarly, Participant 18 indicated, "There was very little formal preparation for the Putnam Competition at my undergraduate institution, [redacted]" (Q.7).

Likewise, Participant 2 expressed,

> And there was no preparation offered at the University of [redacted] for Putnam exams. I was asked to try the exam, presumably because I had done well in first-year classes. As a freshman (19[redacted] – 19[redacted]), I began to work on the problems in the *American Mathematical Monthly*. My calculus teacher occasionally added a Putnam-level problem (optional) to our homework. Being able to solve some of these 'extracurricular' problems, and discussing them with a few like-minded fellow students, reinforced my confidence and appetite for taking up such challenges.
>
> A few months before the 19[redacted] competition, the famous Paul Erdős gave two months of lectures at [redacted] on number theory. One of the problems he talked about showed up on the competition. That helped! (Q.7).

Whereas Participant 4 responded,

> When I was at [redacted] (19[redacted] – 19[redacted]) attitude toward the Putnam was laid back. Math professors described what it was and you could sign up to take it or not. There was no preparation for the exam. Maybe that has changed (Q.7).

Similarly, Participant 7 indicated,

> No preparation. At University of [redacted], there were no classes to prepare people for the exam. I took it only because the professor in one of my courses, [redacted] who happened to be involved in administering the exams, announced that we could register for it (Q.7).

Research Question 3

What role does the affective domain play in the development of a Putnam Fellow?

In order to answer research question 3, all of the interview protocols were reviewed and coded. The coded data was then sorted to identify salient themes and patterns within each category for each participant. Emerging themes were organized into a partially-ordered meta matrix and a cross-case analysis of the data was employed. From the matrix, themes and patterns were identified across cases. All data supported by eight or more participants was reported. Results of the thematic analysis are reported next.

The Affective Domain

The affective domain has been described using a number of constructs, which include beliefs, attitudes, and emotions (Hart, 1989; McLeod, 1989, 1992; McLeod, as cited in Feldman, 2003). In this study, participants were asked to reflect on the role that affect plays in solving Putnam problems (i.e., confidence in their ability to solve Putnam problems, feelings they experienced when they solve Putnam problems, motivation for their success on the Putnam examination, a description of the qualities, characteristics, or factors that contribute to an individual becoming a Putnam Fellow, the role intuition plays in solving Putnam problems, and whether a Putnam Fellow's extraordinary talent is innate or can be taught).

In addressing themes within the role the affective domain plays in the development of the participants in this study, seven subcategories of affect emerged as themes and were examined. These included: (a) beliefs about confidence, (b) beliefs about natural ability, aptitude, or talent in mathematics, (c) beliefs about having an interest in mathematics and liking mathematics, (d) beliefs about the role that intuition plays in solving Putnam problems, (e) beliefs about talent being innate or taught, (f) feelings experienced when solving Putnam problems, and (g) motivation for success on the Putnam Examination.

Beliefs About Confidence

Sixteen participants in this study reported being confident in their ability to solve Putnam problems. For example, Participant 5 stated, "Yes. The majority of the problems involve 'spotting the trick', and I'm fairly good at that. Not perfect though" (Q.8). Whereas Participant 9 responded, "Well, yes, quite so. I have looked at them over the years and I'm usually able to solve the majority of them still" (Q.8). Similarly, Participant 17 indicated, "Yes. I'm not as 'in shape' as I was, but at the time I was confident that I could get at least five of the six questions in a session; now I'd say at least four" (Q.8). Likewise, Participant 19 expressed, "Yes, and this is important. Being confident helps you focus on approaches to the problems that are more likely to lead to solutions" (Q.8). Whereas Participant 21 responded, "Yes. Generally speaking when I have looked at recent exams I have been able to figure out how to solve the vast majority of the problems relatively easily" (Q.8). Similarly,

Participant 22 indicated, "Yes, especially when in college when I was actively preparing for the competition" (Q.8).

Likewise, Participant 11 expressed,

> Yeah, I can certainly solve most Putnam problems now, sure. I wouldn't guarantee that I could solve every problem on a given exam, but most, certainly the great majority of Putnam problems I can solve. If I were to take the exam today, I believe that I would probably be a Putnam Fellow. I'm not certain about that, but I would certainly stand a great chance in doing so (Q.8).

Although it does not rise to the level of a theme, six participants in this study reported having no confidence in their ability to solve Putnam problems. For example, Participant 1 stated, "No – each one was its own challenge. Solving half of the problems was a big accomplishment" (Q.8). Whereas Participant 7 responded, "This was more than [redacted] years ago. I've not looked at Putnam problems since then" (Q.8). Similarly, Participant 8 indicated, "No, because I haven't done any for over [redacted] years" (Q.8). Likewise, Participant 24 expressed, "No. In such kinds of competitions, nobody can be sure how hard a problem can be and how much time it takes to solve it" (Q.8).

Whereas Participant 18 responded,

> I had, and have, no particular confidence in my ability to solve every Putnam problem, but only to make reasonable attempts at many types of such

> problems. Any confidence that you can solve all of the Putnam problems is rapidly upset by reality, so it only makes sense to try to cultivate the attitude that you will do everything you are capable of and hope for the best (Q.8).

Beliefs About Natural Ability, Aptitude, or Talent in Mathematics

Fifteen participants in this study reported having a natural ability, aptitude, or talent in mathematics as factors that contribute to an individual becoming a Putnam Fellow. For example, Participant 12 stated, "There are three things: a natural problem-solving ability, adequate knowledge of math, and practice solving problems" (Q.11).

Whereas Participant 10 responded,

> I don't feel I can speak for others. It helped a lot that I wasn't very social and enjoyed math so much, so I spent a lot of time alone working on math problems. There is some sort of natural aptitude, which I certainly had, but also a willingness to devote yourself to working on problems is important as well (Q.11).

Similarly, Participant 11 indicated,

> Speed is certainly one of them. If you're not a person who thinks quickly, you're never going to do very well. Memory certainly plays a role. And of course you have to have a taste and maybe a flair for mathematics. One thing, which I have to say, I think that like any competitive activity, to do well at a high level in math competitions you probably need both some natural talent and some degree of practice. And my feeling about the Putnam exam, at least it was when I was a student and it may be more competitive now, but

my feeling about it is that to do well you need to have a very unusual degree of natural ability, but not an extraordinary one. I can't put a number on it, one in a thousand maybe less, there may be more people than one in a thousand who could be trained to win the Putnam exam, it's certainly not a one in a million kind of talent that you need by any means, and it might be one in a hundred, it's just I don't know. I had a very unusually good exposure. My father was a [redacted]; a [redacted] is basically a [redacted]. I got involved in math competitions. At a very early age I was given very high-level training. I had access to books. I really had as good a preparation as you could have, short of the kind of thing you would get in maybe in Russia, where they have special schools for people who are drilled in this kind of thing. But, I think that there are many people who could have been Putnam winners if they had the kind of exposure to it that I did. Not everybody I mean, but many, many people (Q.11).

Likewise, Participant 15 expressed,

Certainly there is some natural talent coming into it. Many people are great mathematicians without being good in that particular way. And many people are good in that particular way, without having the talents that are good for being on the Putnam, which are not the same as the talents for other things, although there is a relation. So certainly some natural talent, but also the good fortune of seeing things that you remember, not because you memorize things or learn things, but because you get to develop a way of thinking, that's a huge advantage. And probably persistence, a willingness not to, a fearlessness to both tackle hard problems and not afraid of being wrong, and not being afraid of failure. The median on the Putnam is, in fact it is often zero, means that people who take it, all the thousands of people who take it, and it's impressive, and if someone gets a point, they're doing really well against, I mean they're self-selected, many of the best people in North America are taking it. And so getting a single point is a big deal. So I think fearlessness in both tackling a problem and, also in the kind of person who would do things that isn't a safe bet, those are good characteristics I think (Q.11).

Beliefs About Having an Interest in Mathematics and Liking Mathematics

Nine participants in this study reported having an interest in mathematics and liking mathematics as factors that contribute to an individual becoming a Putnam Fellow. For example, Participant 14 stated, "An interest in studying mathematics for its own beauty, not as a computational tool, and the ability to look at a problem in many different ways" (Q.11).

Whereas Participant 4 responded,

> I guess liking and being good at math and also liking a challenge in math are important. I know that I like to find the simple way to do things and that may help since, when I was still looking at Putnam problems, they frequently became simple if you looked at them the right way (Q.11).

Similarly, Participant 5 indicated,

> Wide reading of puzzle books, Martin Gardner columns, and some serious study. I think the main thing is enough interest to think about problems all the time, make up your own, and a few supportive adults. Having sufficient interest to stick with math is some kind of obsession, bordering on pathological. It makes you something of an oddball among your friends. One thing to mention – my high school was large, 5,500 kids (four years). They had an honors program for the brightest, which was the first time I encountered a quorum of bright kids for longer than an afternoon. This might have helped at a psychological level (Q.11).

Likewise, Participant 20 expressed,

> Obviously an interest in mathematics is important. One has to care about the subject enough to invest the time to develop one's skills. There are also lots of ways of being interested in mathematics or for that matter being good at mathematics. And so the Putnam is a particular kind of skills involved in mastering a number of quick problem-solving techniques. And so, how would I describe the quality that's involved in that? I guess it involves an academic interest as well as a certain form of speed or agility. I know people who are better at mathematics than I ever was, but would never be able to solve a contest problem in an hour, just because the way that they think about things is by sort of understanding them deeply and getting to see a big picture over a very long period of time. That's the only thing that stands out in regards to the Putnam particularly and that I think is maybe one reason that I typically have done well is attention to detail. So I hear that, although I haven't been in the room to confirm this firsthand, I hear the Putnam is graded very harshly and that basically you fall on a scale of zero to 10 right, but if you have a solution the big graders recognize immediately as being a correct and complete solution you get tens. And if there's a small hole then you get two or three, or if it's a correct solution but it's sloppily written and they can't immediately tell if it's a correct solution you get two points or zero points. And I think that my score each year basically corresponded to the number of problems that I tried to solve when I walked out of the test room, so I get the sense that I tend to be more careful about writing complete proofs and about writing them in a clear and organized way than most people who sort of reason intellectually at about the same level that I do. I mean it's a useful kind of skill to have as a mathematician, although it can also be hazardous (Q.11).

Although it does not rise to the level of a theme, three participants in this study also reported luck as a factor that contributes to an individual becoming a Putnam Fellow. For example, Participant 24 stated, "First of all, you need to devote a lot of time on problem solving since your childhood. That means trading off a lot

of fun (sports, social, etc.). Then, you need some talent. Finally, you need to be lucky" (Q.11).

Whereas Participant 1 responded,

> Frankly, my working hypothesis is that success in math (and perhaps most fields) is one-third lucky genetics, born with good genes; one-third hard work, 10,000 hours is a figure often quoted, determination to succeed; and one-third plain luck (Q.11).

Similarly, Participant 17 indicated,

> One thing I haven't mentioned is luck. Suppose that a bunch of people can solve, on average, eight or nine out of 12 Putnam problems. The day of the contest, some of them will solve 10; a few may solve more. They'll be the Putnam Fellows. A useful analogy would be success at the Olympics; it is based on a combination of athleticism, technique, and practice, but we all know stories where luck played a role (Q.18).

<u>Beliefs About the Role of Intuition</u>

Nineteen participants in this study reported that intuition plays an important role in solving Putnam problems. For example, Participant 6 stated, "A lot. Since there is limited time, it [intuition] helps to guess which approach is likely to be more fruitful" (Q.16). Whereas Participant 7 responded, "A big role, I'm sure" (Q.16). Similarly, Participant 8 indicated, "I would say it [intuition] plays a large role" (Q.16). Likewise, Participant 16 expressed, "I think it [intuition] plays a very big

role, because it’s what determines what you try, right? It’s what determines what path you decide to set off on” (Q.16).

Whereas Participant 2 responded,

> ‘Examine analogous cases’ is too mechanical a description of what is mainly a subconscious process. ‘Intuition’ is a better word. Once they have grabbed you, mathematical phenomena, and problems in particular, take on a life of their own. You think about them more-or-less involuntarily, trying to understand them from different angles and with different approaches. So the first successful insights are often not the last (Q.14).

Similarly, Participant 17 indicated,

> Intuition is very important. You have to be able to guess which approaches are likely to work in the time allotted. But I would include in this both the hard-to-quantify innate intuition about math and the more concrete experience one gets through practice (Q.16).

Likewise, Participant 18 expressed,

> Intuition, to me, means the ability to go in the right direction from the start, rather than to waste time doing the wrong thing. On a timed contest this is vital, because there is never enough time to try everything that could possibly work. In research, on the other hand, an afternoon spent trying a possible approach to a problem and concluding that it doesn’t help is perfectly acceptable, and indeed hard to avoid, though still the best mathematicians probably do this less often than the rest of us (Q.16).

Whereas Participant 21 responded,

> Good intuition for the problems is probably the most important. To do well you really want to be able to solve most of the easy problems almost instantaneously (Q.11). I have developed really good intuition and usually after reading the problem two or three techniques come to mind and very often one of them works (Q.13). Intuition plays a tremendous role. To do really well on the Putnam, you need to solve problems quickly. In order to do that you need to have intuition that will get you to a solution without a lot of trial and error (Q.16).

Similarly, Participant 22 indicated,

> It plays a big role. Having done a lot of practice problems, I have a sense of what kinds of techniques will work on each type of problem, so that many dead end approaches can be eliminated quickly early on (Q.16).

Likewise, Participant 25 expressed,

> What is intuition? Insofar as it denotes the kind of non-rigorous 'hunches' used to supplement mathematical reasoning, it plays a role everywhere: in reading the problem, selecting an approach, finding ways to translate vague ideas into math, deciding which steps are worth writing and which are too obvious, and even scouring the final proof for the scent of logic gone awry (Q.16).

<u>Beliefs About Talent</u>

Sixteen participants in this study reported that their extraordinary talent as a Putnam Fellow is partly innate and partly due to effective teaching. For example,

Participant 10 stated, "Like most things, it is a combination. It takes innate talent, but also a lot of work and preparation" (Q.17). Whereas Participant 19 responded, "Both. You need natural talent to begin with, but specific problem-solving skills can and must be learned with a lot of training and a lot of problems" (Q.17). Similarly, Participant 22 indicated, "Some innate talents, but rigorous training is necessary" (Q.17). Likewise, Participant 24 expressed, "[Talent is] 25% innate, 25% can be taught, and 50% giving up other opportunities (socialization, dating, or a chance to win a competition in physics)" (Q.17).

Whereas Participant 3 responded,

> It [talent] is certainly partly innate, but given this it can certainly be 'cultivated'. By the latter term I suppose I mean it can be taught, but really I mean that it can be greatly improved with practice – and with learning and studying more mathematics. One often gets lots of extra 'practice' helping friends with their homework (Q.17).

Similarly, Participant 9 indicated,

> Well, I guess there are two parts of that. One is just innate ability, which I'm not sure I can describe, and then the other part is experience, just coming. Doing one of these competitions for the first time you're not going to do nearly as well as if you've looked at problems like this many times before, either in practice sessions or by taking the exams before, so it definitely needs both of those. I really don't think I can describe what I meant by innate ability, but I think it's necessary, as well as the experience (Q.11).

Likewise, Participant 11 expressed,

> No, I mean so again, let me repeat what I've said before. I think that there's some degree of natural ability, which would be needed before you could hope to train a Putnam Fellow. But I don't think it's an extraordinary degree of natural ability. I think it's unusual, but not extraordinary. And then of course I had excellent training. There may be some people who are so talented that they don't need that training, but I was not one of them. Well, it's not so much harder to be a multiple winner as it is, I mean, again, I was ready to take the Putnam exam from my freshman year. So I had four real chances to score in the top five, right? So, some people they don't know enough math maybe, to do it until they get to their senior year, then of course that would be a different story. But a lot of the people it seems to me, that I knew who were interested in this, had as good a chance of winning in their freshman year as in their senior year. So I really don't think it's extraordinary. I have to say, I've given it some thought over the years, just watching what happens to different students in different situations and I think that mathematical ability is much more teachable than is generally thought. Not to say that everybody can become research mathematicians, obviously I don't think that's true. But much more can than I think is generally appreciated. And I think a lot of people who are badly taught when they were young, later on think of themselves as not having talent. And I think that's kind of the mistake of what's going on, I think. And that's my feeling (Q.17).

Whereas Participant 14 responded,

> Much of the mathematics behind the Putnam can be taught, as can many of the techniques, and I have taught them. However, the ability to specifically do Putnam problems, which requires identifying the right technique quickly, is not easy to teach (Q.17).

Similarly, Participant 15 indicated,

> I think it's a combination of two things, of both. I mean a random person will have talents of some sort and they may not be these talents, so absolutely something innate must be there, but that's absolutely not enough. I guess taught is less the word I would use, than talents can be developed. And I mean it's not surprising that people who do well tend to have been fortunate, in terms of growing up educationally advantaged, so it's not surprising that people who have access to the right sort of nurturing can have advantages in that way. But I think you need something unusual to be a Putnam Fellow, because you have to be a little bit at the tail end of the curve, but I think a very large number of people can and often do develop their problem-solving skills in a serious way, in mathematical or otherwise (Q.17).

Although it does not rise to the level of a theme, four participants in this study reported that their extraordinary talent as a Putnam Fellow is purely innate. For example, Participant 25 stated, "I was adding at two and reading a calculus book at ten. If that isn't innate talent I don't know what is" (Q.17).

Whereas Participant 2 responded,

> How do you explain the few NBA stars that emerge from the thousands of devoted college basketball players? The world-class musicians who emerge from the tens of millions who work hard on mastering an instrument? The top scientists whose achievements dwarf even those of other active researchers at high-level universities? I have coached Putnam students over the years, and can attest that outstanding performance cannot be taught.
> I believe that exceptional talent in any field is innate, but needs to be developed in a social and cultural context (Q.17).

Feelings Experienced When Solving Putnam Problems

Eighteen participants in this study reported experiencing positive feelings when they solve Putnam problems. For example, Participant 1 stated, “It’s fun to succeed” (Q.9). Whereas Participant 2 responded, “I became intrigued – if not obsessed – by problems, and the pleasure (not fear!) of conquering them” (Q.3). Similarly, Participant 3 indicated, “I think ‘aesthetic joy’ describes it best. Also a certain sense of ‘power’ ” (Q.9). Likewise, Participant 6 expressed, “Excitement” (Q.9). Whereas Participant 17 responded, “I'd say the main feeling is pride at coming up with the solution, and the aesthetic pleasure at seeing how it fits together” (Q.9).

Similarly, Participant 10 indicated,

> I feel a bit of exhilaration when tackling a difficult math problem, especially when I suddenly realize how the pieces fall into place. I don’t remember how Putnam problems were particularly different, except they usually involved great cleverness and they always had a clean solution if you did them right (Q.9).

Likewise, Participant 15 expressed,

> That’s the reason I’m a mathematician, because I get to do this and work on problems and get that feeling of reward when it works out. And I know that with Putnam problems, when I do things at [redacted] for students, I try to encourage lots of people to take part, because the goal is not just to win, but those people who go and tackle these problems, they’ll learn to be fearless

> and not be bothered by the fact that they are not going to get every problem. I feel like that carries over into life in a big way and that the people who are happy to push themselves and try to deal with problems they've never seen before, even if there's no guarantee of success and no one tells them how to do it, then those are people who I think are going to be really impressive in the long run in whatever they do. And so I absolutely have the same feeling. And of course once you try these hard things and you think you can't do them and then when you get one and it's beautiful, that really just lights the fire (Q.9).

Whereas Participant 16 responded,

> There's a certain kind of very deep satisfaction that can be hard to find in other realms of life. A sense of when one has gotten it all the way down to the bottom, which is of course very pleasing (Q.9).

Similarly, Participant 22 indicated,

> I feel excitement and joy. And if the solution is nice, I feel a sense of aesthetic beauty. The smell of excitement, a sense of accomplishment you know when I solve a difficult problem. I think back when I was in high school, or in college doing the Putnam, you know solving these problems, doing well on these contests were definitely the high joys of my life at that point. It was a very exhilarating experience to do well on these competitions (Q.9).

Although it does not rise to the level of a theme, five participants in this study reported that what they felt when they solved a Putnam problem depended on the problem. For example, Participant 25 stated, "I hope not to feel anything. Part of the joy of maturity is being able to plow through problem sets without the attendant excitements and fears disrupting my concentration" (Q.9).

Whereas Participant 9 responded,

> The best word for it would be satisfaction, in that 'Hey, now I understand how that works.' Yeah, and the other feelings probably depend on how difficult the problem was, if it's something I see how to do immediately then I probably don't feel much about it, but if it's something that I've been working on for a while, and then I see the key idea, and then, 'Okay, oh now it all works out, that's exactly the way it works', then yeah, there's definitely more satisfaction at that point (Q.9).

Similarly, Participant 19 indicated,

> Always happy. A more specific answer very much depends on the problem. Sometimes not much else for easy problems, sometimes a tremendous feeling of accomplishment for difficult problems that were hard to solve, sometimes surprise or amazement for problems with counter-intuitive answers, sometimes the excitement of discovery for problems hiding some amazing little mathematical fact that I had never seen before, sometimes awe when a particularly beautiful solution emerges, and sometimes relief for a problem whose answer ends up being mostly grunge work (Q.9).

Likewise, Participant 20 expressed,

> It really depends on the problem. So I think for problems that I enjoy working on, it's very similar to what you described. There is the process of getting an idea and then exploring it and trying to build on it, and seeing whether it can be developed into a solution. There's a feeling of anticipation of building excitement that sort of supplies the motivation to continue working. And then when I solve a problem there can be a feeling of admiration, maybe partly narcissistically for myself, but mostly for the beautiful or surprising piece of mathematics that I have just gotten to see and understand. This is something that I experience if the problem or the line of reasoning is something new to me. So for the easier problems, you know for

> some of them, I just look at the problem and I say, 'Oh, I know how to do this. I just need to calculate *xyz*', and then there's not much feeling to it. But if I can look at a problem and know what I need to do then I'm just like, 'Okay, I've got a job to do. I'll do it.' Sometimes there can even be something sort of ugly or tedious in the calculation and then there's even a feeling of displeasure. And then it's not maybe so much satisfaction or aesthetic joy or admiration when the problem is solved, so much as just you know relief that the job is over (Q.9).

<u>Motivation</u>

Eight participants in this study reported extrinsic motivation as the reason for their success on the Putnam Examination. For example, Participant 17 stated, "I am a competitive person – I wanted to win. I also knew that my performance on the Putnam would help me in my subsequent career" (Q.10).

Whereas Participant 11 responded,

> When you are actually taking the exam, of course, it's the thrill of the chase you know. It's more to it than that. I knew most of the people who, it's a small world at least it was a small world for people who are kind of interested in such things. And so you're competing with your friends and rivals and of course you want to do well (Q.10).

Similarly, Participant 12 indicated,

> I had previously done well on the USAMO [United States of America Mathematical Olympiad] a couple of times, and I didn't want people to think I was getting dumber. So a better question is what motivated me to do well on the USAMO. I suppose I probably wanted to impress people (Q.10).

Likewise, Participant 16 expressed,

> I think like a lot of college kids I was fairly competitive. I liked to win. You know, college kids often are thinking a lot about their grades, thinking a lot about how they compare with other students, both academically and in other ways. And I think the Putnam certainly feeds that, certainly draws on that (Q.10).

Whereas Participant 20 responded,

> What motivated me? So to be honest, I guess by the time I was taking the Putnam it was pretty mercenary. Like somebody's going to give me a couple thousand dollars for solving problems on a Saturday and doing a good job of it, 'Sure, I'll take the money.' I mean there was also I guess a sense of identity, you know, like I had sort of built a sense of myself as somebody who solves math problems for a living and so this was sort of one of the natural things to do (Q.10).

Although it does not rise to the level of a theme, five participants in this study reported intrinsic motivation as the reason for their success on the Putnam Examination. For example, Participant 7 stated, "I liked solving problems; I guess that was the motivation" (Q.10). Whereas Participant 15 responded, "I like doing math and it was fun. That's probably the fast, the simplest reason" (Q.10). Similarly, Participant 19 indicated, "Just a love of problem solving. Again, the question is mis-phrased: the motivation to be successful was spread out over the years leading up to the Putnam Examinations, not during the examinations themselves" (Q.10).

Likewise, Participant 9 expressed,

> Well, I don't think I needed external motivation; it was just mainly interest in the problems themselves. I know they awarded cash prizes, but that had no influence at all on my desire to solve the problems, so it was really just interest in the math itself (Q.10).

Again, although it does not constitute a theme, five additional participants in this study reported both intrinsic and extrinsic motivation as the reasons for their success on the Putnam Examination. For example, Participant 3 stated, "The aesthetic joy of solving the problems played a large role, but also I think I am naturally competitive in certain directions" (Q.10). Whereas Participant 13 responded, "Mainly I enjoyed solving the problems. Receiving recognition was a bonus" (Q.10).

Similarly, Participant 21 indicated,

> I think it was mostly that I enjoyed the problems. Though trying to get my name in the awards was also a motivating feature. Sure, I enjoy it. I mean it's a puzzle. You can try, I mean it's sort of a thing, which is not obvious what to do, but you can feel around and find new and interesting ideas, which sort of give you tools that you can use to try and work at this thing. And then eventually you have to take these ideas and assemble them in their right way and suddenly you can get things to work out. I mean it involves being clever, it involves thinking about interesting ideas, and yeah, it feels great when you've actually solved something (Q.10).

Likewise, Participant 25 expressed,

> When I first took the Putnam, I could give the 'Mt. Everest answer': 'Because it's there.' There was an intense joy in unlocking the puzzles put forth by other mathematical minds. As I approached my [redacted] and final Putnam, I was beginning to get weary of the problems and their irrelevance to research mathematics. Accordingly, I went in with the altruistic goal of bringing more glory to [redacted] (Q.10).

Research Question 4

> What role does the cognitive domain play in the development of a Putnam Fellow?

In order to answer research question 4, all of the interview protocols were reviewed and coded. The coded data was then sorted to identify salient themes and patterns within each category for each participant. Emerging themes were organized into a partially-ordered meta matrix and a cross-case analysis of the data was employed. From the matrix, themes and patterns were identified across cases. All data supported by eight or more participants was reported. Results of the thematic analysis are reported next.

The Cognitive Domain

The characteristics of expert problem solvers include: domain knowledge (the knowledge base a problem solver brings to a problem), problem-solving strategies (heuristic techniques), metacognitive skills (the ability to comprehend,

control, and monitor one's own thought processes, the ability to allocate one's resources, etc.), and a set of beliefs (one's worldview) about mathematics as a domain (DeFranco, 1996; Schoenfeld, 1992; Schoenfeld, as cited in DeFranco, 1996). In this study, participants were asked to reflect on the role that cognitive factors play in solving Putnam problems (i.e., reworking and using alternative methods to solve mathematics problems, identifying general strategies or techniques used to solve Putnam problems, and examining analogous cases and other methods to get a feel for Putnam problems).

In addressing themes within the role the cognitive domain plays in the development of the participants in this study, three subcategories of cognition emerged as themes and were examined. These included: (a) using alternative methods to rework previously solved mathematics problems, (b) recognizing Putnam problems as being similar to previously solved mathematics problems, and (c) examining analogous cases as a strategy to get a feel for Putnam problems.

Alternative Methods to Solve Mathematics Problems

Eighteen participants in this study reported using alternative methods to rework previously solved mathematics problems in their everyday work, but not during the Putnam Competition because of the time constraint.

For example, Participant 7 stated,

> I guess this refers to math problems in general, not just Putnam problems. I look for alternative solutions when I feel that the first solution is longer than should be necessary, or doesn't sufficiently explain why the result is true. Sometimes, it's just by accident that I notice an alternative solution (Q.12).

Whereas Participant 10 responded,

> When competing on a math contest, you use the first solution you get because there is rarely time to go back and clean it up. In real life, it is almost always preferable to think about clearer ways to present an argument, preferably one with as few steps as possible. My published work (in [redacted], not math, but closely related) puts a strong emphasis on clear, simple presentation, preferably with as complete an argument as possible (Q.12).

Similarly, Participant 13 indicated,

> If the solution is difficult to explain, or seems overly complicated, that is often a sign that a simpler solution exists. During the Putnam Competition itself, there is usually not much time for searching for alternative solutions, but even so, a little advance planning on how to write up a solution can save time, and can lead to a simpler solution that may be more convincing and less prone to errors (Q.12).

Likewise, Participant 14 expressed,

> I don't do this on the Putnam because of the time limit, but I do this regularly in research, in order to get a better understanding of the problem and develop related work. One of the papers in my Ph.D. thesis was based on a determinant identity, which was easy to prove by linear algebra. Since both determinants counted random walks, I expected that there should be a

> combinatorial correspondence between the walks, and eventually wrote up a proof. Another example was a combinatorics lecture at a conference I attended; at the break an hour later, I was one of four people who came up to the speaker with four different ways of looking at the problem leading to four different generalizations (Q.12).

Whereas Participant 15 responded,

> I guess I do it right away. Sometimes it will happen right away depending on the time or often it will happen later when I'm not working directly, I'm just musing about it or I'm walking down the street and I'm thinking about it and I realize another idea might work or sometimes it will happen when I'm working on another problem and suddenly I realize this idea or I'm thinking about something else and I also may realize that something else is related and that's when I'll have the alternate solution, not quite intentionally going after alternative solutions all the time, but sometimes it just comes after the fact (Q.12).

Similarly, Participant 17 indicated,

> On a timed contest, I move on. In real life, I keep trying to find a clean solution. One reason is the aesthetics of a beautiful proof. Another is that I feel joy and satisfaction whenever I solve a math problem two different ways and get the same answer (Q.12).

Likewise, Participant 18 expressed,

> In a contest there is almost never time for this. However, in research it is quite an important activity. The first proof that one comes up with for almost any statement is almost never a particularly good one, and further thinking usually simplifies and shortens the proof while often strengthening the statement as well (Q.12).

Although it does not constitute a theme, four additional participants in this study reported never using alternative methods to rework previously solved mathematics problems. For example, Participant 24 stated, “No. After solving a problem, I feel like it's gone, and I don't want to think about it any more” (Q.12).

Whereas Participant 20 responded,

> So I think my general habit would not be to solve a problem more than once. I think usually when I was working on solving contest problems, systematically my sense of each problem is you’ve got a problem and you solve it and then it’s done. It’s like somebody gives you a cookie and you eat it and now the cookie is not there anymore, you’re not going to eat it again. I mean, maybe one reason for that is sort of, imagine that a lot of problems there are sort of a key idea that you have to get in order to solve them, and once you have them you know there might be other solutions, but they would likely involve variations on the same idea and so like going back and looking for a different way to solve a problem maybe, typically wouldn’t reveal all that much, wouldn’t reveal new knowledge and there wouldn’t be the same sort of instant gratification, the sort of same sense of excitement that you get from solving a problem because now it’s not new anymore and so the shine has sort of worn off. Sometimes people would give me a problem and specifically say, ‘There’s a couple of different ways of solving the problem’, or even like the [redacted] guy I mentioned earlier, gave me a particular problem, he said, ‘There’s lots of ways of doing this. There is a graph theory way. There is a complex integration way’ and those clues, and I would start thinking about, ‘Okay, what’s the graph theory way of doing this? What’s the complex integration way of doing this?’ (Q.12).

<u>Similarities Between Putnam Problems and Other Mathematics Problems</u>

Twelve participants in this study reported recognizing similarities between Putnam problems and other mathematics problems they have previously

encountered. For example, Participant 6 stated, "Thinking of similar problems, trying special cases" (Q.13). Whereas Participant 24 responded, "I would try to remember a similar problem I have seen before, and attempt the same method to solve it" (Q.13).

Similarly, Participant 9 indicated,

> Okay, the first thing to do is ask, does it look someway familiar, is it like something I've seen before? If so, then that is a strong hand as to what direction to go. And after that it's very important to know that I understand the question and in particular, probably write down some special cases and see whether they work, and how they work, and quite often once you go through two or three cases, you see a general pattern, which leads you to the proof of the full statement, so that's I guess what I would say. It's as close as I could look to a strategy. In general, anything more than that, it would depend on the particular type of problem (Q.13).

Likewise, Participant 11 expressed,

> Well, first of all I try to recognize that it is something that I've seen before, which as I've said I can very often do. Sometimes it takes a little bit of work before you see how it relates to something you've seen before, but usually, or maybe for half the Putnam problems, I look at it and I'm pretty sure that it's like a particular thing that I've seen before. After that, if you've read this book *How to Solve It* then you know what the basic things are. You try to specialize it; you try to generalize it; you try to think of an analogous problem; you can write hypotheses. You know the drill, so I do all those things (Q.13).

Whereas Participant 19 responded,

> The first strategy is to essentially pattern-match the problem to previous problems to see if there is a specific approach that is likely to work. The second strategy is to strip away the specific wording of the problem to get at the underlying mathematical concepts, which makes it easier to (a) focus on what actually needs to be proved, (b) find alternate formulations of the problem that may be more amenable to a proof. The third strategy is to look for creative new approaches. So, it's kind of like approaching the problem as a doctor, as a mechanic, or as an artist (Q.13).

Analogous Cases as a Strategy to Get a Feel for Putnam Problems

Fifteen participants in this study reported examining analogous cases as a strategy to get a feel for a Putnam problem. For example, Participant 3 stated, “I certainly do try to consider analogous cases to get a feeling for a problem” (Q.14). Whereas Participant 5 responded, “Similar strategy. Working de novo is a bad strategy for a competition; it sucks up too much time” (Q.14). Similarly, Participant 6 indicated, “Analogous cases, yes” (Q.14). Likewise, Participant 21 expressed, “Sometimes this technique is useful on the harder problems” (Q.14).

Whereas Participant 10 responded,

> Yes, if a problem is similar to one I’ve seen before, this is the first thing I do. When that doesn’t work, the nature of the problem often suggests a strategy. In some cases, when you have no idea how to prove something in general, you start testing it to see why it might be true (Q.14).

Similarly, Participant 20 indicated,

> Yeah, that's I guess one that I'm sure how to answer. You know, I think anything that you said that starts, 'Typically mathematicians solve problems this way', would probably apply to me also. But it depends on what the problem is, and sort of how, whether it invokes some particular tool in my toolbox, where I can just say, 'I know exactly what to do, I'll solve it this way' or whether, if I don't know what to do then I probably will try to find things that I have seen before or that I know how to reason about, that look similar to that problem (Q.14).

Although it does not constitute a theme, five additional participants in this study reported using simpler cases as a strategy to get a feel for a Putnam problem. For example, Participant 14 stated, "I tend not to look at analogous cases, but at simple cases; if a problem asks to prove a result for sequences of length 2014, I try sequences of length six or eight" (Q.14).

Whereas Participant 19 responded,

> I wouldn't say 'analogous'; I would say 'simpler'. A specific and effective version of the 'mechanic' approach above [Q.13] is to look at specific simple cases, try to prove those, and then see if the solution generalizes. A canonical example is if a problem asks you to prove something for all positive integers N, then a good start is to prove it for $N = 1, 2, 3...$ and see if a pattern emerges (Q.14).

Similarly, Participant 24 indicated,

> I would say that solving a problem, as a mathematician in research, is very different from solving a problem as a contestant. As a mathematician, you want to try some simple special cases first to see if the problem statement makes sense and if it is worth your time to study it. As a contestant, you simply trust the exam committee (Q.14).

Summary

Results of the thematic analysis of this study are presented in the summary theme tables that follow.

Table 5

Summary Theme Table for the Personal Experiences Putnam Fellows Identify as Influential in Their Success on the Putnam Examination

Themes	**Personal Experiences**
Households Conducive to Learning	Eighty-eight percent (*n* = 22) of the participants reported being raised by parents who valued academic achievement and who provided encouragement and support in learning mathematics.
Influential Family Members and Family Friends	Forty-eight percent (*n* = 12) of the participants reported that family members and family friends were influential in helping them learn mathematics beginning at a young age.
An Interest in and Talent for Mathematics at a Young Age	Sixty percent (*n* = 15) of the participants reported that they became aware of their interest in and talent for mathematics at a young age.
Access to Educational Resources	Forty-four percent (*n* = 11) of the participants reported having access to educational, printed materials (i.e., mathematics textbooks, puzzle books, and encyclopedias) at home during their childhood.

Table 6

Summary Theme Table for the Formal Educational Experiences Putnam Fellows

Identify as Influential in Their Success on the Putnam Examination

Themes	Formal Educational Experiences
Influential Teachers and Individuals in Academics	Seventy-six percent (*n* = 19) of the participants reported having strong K - 12 mathematics teachers, coaches, and mentors.
Participation in Mathematical Contests and Competitions	Seventy-six percent (*n* = 19) of the participants reported taking part in mathematical contests and competitions (e.g., the Annual Mathematics Competition, the United States of America Mathematical Olympiad, and the International Mathematical Olympiad), prior to the Putnam Competition.
Access to Released Mathematical Contest and Competition Problems	Fifty-two percent (*n* = 13) of the participants reported working through released examinations as a way to prepare for the Putnam Competition.
Participation in Extracurricular Mathematics Training	Ninety-two percent (*n* = 23) of the participants reported taking part in extracurricular mathematics training (e.g., after school, on weekends, and during the summer months) in addition to their normal high school program of studies.
College or University Did Not Offer Preparatory Classes	Forty percent (*n* = 10) of the participants reported that their college or university did not offer any coaching or practice sessions as preparation for the Putnam Competition.

Table 7

Summary Theme Table for the Role the Affective Domain Plays in the Development of a Putnam Fellow

Themes	Affective Domain
Beliefs About Confidence	Sixty-four percent (n = 16) of the participants reported being confident in their ability to solve Putnam problems.
Beliefs About Natural Ability, Aptitude, or Talent in Mathematics	Sixty percent (n = 15) of the participants reported having a natural ability, aptitude, or talent in mathematics as factors that contribute to the development of a Putnam Fellow.
Beliefs About Having an Interest in Mathematics and Liking Mathematics	Thirty-six percent (n = 9) of the participants reported having an interest in mathematics and liking mathematics as factors that contribute to the development of a Putnam Fellow.
Beliefs About the Role of Intuition	Seventy-six percent (n = 19) of the participants reported that intuition plays an important role in solving Putnam problems.
Beliefs About Talent	Sixty-four percent (n = 16) of the participants reported that their extraordinary talent as a Putnam Fellow is partly innate and partly due to effective teaching.
Feelings Experienced When Solving Putnam Problems	Seventy-two percent (n = 18) of the participants reported experiencing positive feelings when they solve Putnam problems.
Motivation	Thirty-two percent (n = 8) of the participants reported extrinsic motivation as the reason for their success on the Putnam Examination.

Table 8

Summary Theme Table for the Role the Cognitive Domain Plays in the Development of a Putnam Fellow

Themes	**Cognitive Domain**
Alternative Methods to Solve Mathematics Problems	Seventy-two percent (*n* = 18) of the participants reported using alternative methods to rework previously solved mathematics problems in their everyday work, but not during the Putnam Competition because of the time constraint.
Similarities Between Putnam Problems and Other Mathematics Problems	Forty-eight percent (*n* = 12) of the participants reported recognizing similarities between Putnam problems and other mathematics problems they have previously encountered.
Analogous Cases as a Strategy to Get a Feel for Putnam Problems	Sixty percent (*n* = 15) of the participants reported examining analogous cases as a strategy to get a feel for a Putnam problem.

Chapter V

Discussion

This chapter will present: (a) an overview of the study, (b) a discussion of the results in light of current research, (c) implications of the study, and (d) recommendations for future research.

Overview of the Study

The William Lowell Putnam Mathematical Competition is an intercollegiate mathematics competition, administered by the Mathematical Association of America (MAA), for undergraduate college and university students in the United States and Canada and is regarded as the most prestigious and challenging mathematics competition in North America (Alexanderson, 2004; Grossman, 2002; Reznick, 1994; Schoenfeld, 1985).

Students are eligible to compete in the Putnam Competition a maximum of four times and throughout the history of the 75 competitions students have participated in the Putnam a total of 140,314 times (see Appendix A). The students who earn the five highest scores on the examination, whether competing as part of a team of three people or as individuals, are named Putnam Fellows, and these contestants are not ranked by their scores, but are named alphabetically.

Over the course of the 75 Putnam Competitions, there have been 280 Putnam Fellows, and counting repeated winners, these individuals have received this award a total of 393 times (see Appendix B). Among the 280 Putnam Fellows, eight students have earned this distinction four times; 21 individuals have won this award three times; 47 students have received this title two times; and 204 individuals have earned this accolade one time (see Appendix C). Therefore, given the number of students who compete in the Putnam Competition each year, to be named a Putnam Fellow is a remarkable accomplishment while being named a Putnam Fellow multiple times is an extraordinary achievement.

After finishing their undergraduate studies, most Putnam Fellows further their education by completing graduate school, and then go on to make significant contributions to academia and industry. For example, many Putnam Fellows have become college and university professors of mathematics, physics, and computer science, while others have worked in industry as mathematicians, physicists, chemists, and engineers, as well as for the United States federal government in the Department of Energy, the Department of Transportation, and the Census Bureau. In addition, Putnam Fellows have also received some of the most prestigious awards in their respective fields including: the Nobel Prize in Physics, the National Medal of Science, the International Medal for Outstanding Discoveries in Mathematics (the Fields Medal), the Abel Prize, and the Albert Einstein Award in theoretical physics (Alexanderson, 2004; Gallian, 2004, 2014; Grossman, 2002; MAA, 2008). Furthermore, some Putnam Fellows have served as presidents of the American Mathematical Society or the Mathematical Association of America (Gallian, 2004,

2014), as well as members of the National Academy of Sciences, the American Academy of Arts and Sciences, and the National Academy of Engineering (Alexanderson, 2004; Gallian, 2004, 2014).

As noted above, the Putnam Competition has played a central role among mathematical competitions in the United States and Canada, as well as having an impact in the field of mathematics over the past several decades. Moreover, becoming a Putnam Fellow requires strong problem-solving skills and the ability to quickly analyze and understand the structural relationships in the problems. Consequently, because the Putnam Fellows earn the highest scores on the examination and the Putnam Competition is regarded as the most prestigious and challenging mathematics competition in North America (Alexanderson, 2004; Grossman, 2002; Reznick, 1994; Schoenfeld, 1985), these individuals can be classified as exceptional problem solvers. Therefore, the purpose of this research was twofold. First, because Putnam Fellows are expert problem solvers and go on to have extraordinary careers in mathematics or mathematics-related fields, it is important to understand the characteristics that led these individuals to becoming Putnam Fellows. Second, to see if these Putnam Fellows share some of the same characteristics of "expert" problem solvers as defined in the literature (DeFranco, 1996; Schoenfeld, 1992; Schoenfeld, as cited in DeFranco, 1996).

The study took place during the spring, summer, and fall months of 2014. The subjects selected for this study (n = 25) were Putnam Fellows who had won the Putnam Competition four, three, or two times. The 25 participants were all males and attended eight different colleges and universities in the United States and

Canada at the time they were named Putnam Fellows. Of the 25 individuals who participated in the study, five have been named Putnam Fellows four times; seven have earned this distinction three times, and 13 have won this award two times. The Putnam Fellows were contacted beginning with the four-time competition winners, followed by the three-time and two-time winners, through e-mail, mail, and/or telephone invitations requesting their participation in this study. Each subject was also sent a questionnaire (see Appendix H), via e-mail or through the mail, which allowed each participant the opportunity to review and reflect on the questions prior to the interview.

Information gathered during this study came from audio-recorded interviews conducted over the telephone or through Skype and through written e-mail responses. The purpose of the interviews was to collect data from the participants about the factors and characteristics, (i.e., personal experiences, formal educational experiences, and the role of the affective and cognitive domains), which have contributed to their success as Putnam Fellows. The sources for the questions included Campbell's (1996a, 1996b) study of the American Mathematics Olympians and DeFranco's (1996) study of the mathematical problem-solving expertise of male Ph.D. mathematicians. Follow-up questions were interjected as needed to pursue or clarify a participant's response. In collecting the data, eight subjects consented to participate in audio-recorded interviews, whereas 17 subjects elected to provide their responses in writing using e-mail. To clarify or elaborate on participants' responses, two follow-up telephone calls and six follow-up e-mail communications

were made to eight Putnam Fellows. The audio-recorded interviews were transcribed and analyzed using qualitative methods of analysis.

To analyze the interview data, a line-by-line coding of the data was employed and every significant statement or complete thought within the transcripts was color-coded according to the coding category it represented (see Figure 3). A within-case analysis was employed for each participant to identify themes and patterns with respect to the factors and characteristics that have contributed to the success of the Putnam Fellows. In order to accomplish this, the rows within the coded matrix for each individual participant were sorted by the categories and subcategories of personal experiences, formal educational experiences, and learning domains. Building upon each within-case analysis and using the data entered into the text matrices, a cross-case analysis was conducted to identify similarities and differences across participants with respect to the categories and subcategories of personal experiences, formal educational experiences, and learning domains. Finally, themes and patterns that emerged across cases were organized into summary theme tables, one for each subcategory of personal experiences, formal educational experiences, and learning domains. This thematic analysis of the data allowed the researcher to identify the major recurrent patterns within the data across all cases of personal experiences, formal educational experiences, and learning domains. This information was used to answer research questions 1, 2, 3, and 4.

Discussion of the Results

The purpose of this study was to collect information about the factors and characteristics that contribute to the success of Putnam Fellows. In order to identify the categories of personal experiences, formal educational experiences, the affective domain, and the cognitive domain that the Putnam Fellows attributed as being influential in their success on the Putnam Examination, each participant was asked a series of 18 semi-structured questions (see Appendix H). Transcribed interview data was coded and then sorted to extract themes and patterns across cases. A discussion of the findings for each research question follows next.

Research Question 1

What personal experiences do Putnam Fellows identify as influential in their success on the Putnam Examination?

Four themes of personal experiences emerged as a result of this study. First, subjects indicated growing up in households that were conducive to learning and being raised by parents who valued academic achievement and who provided encouragement and support in learning mathematics was a key component to their success. As noted by Participant 17,

> My parents did not directly teach me math beyond elementary school, but they created an environment where I was able to flourish. They encouraged my interest in math from an early age. They made clear that my primary job was to do well in school (Q.2).

Second, participants expressed having influential family members and family friends who helped them learn mathematics throughout their childhood. For example, Participant 21 stated,

> The fact that my dad was a professional mathematician had a significant impact on my early mathematical experience. In addition to providing me with the opportunity to discuss more advanced topics with him outside of school, I was essentially homeschooled in mathematics after about second grade (Q.2).

Third, participants believed that having an interest in and talent for mathematics at a young age was influential in their success on the Putnam Examination. For example, Participant 3 noted,

> It was in third grade I first became aware of my interest in and talent for mathematics. The next year, while in fourth grade, I was part of a 'play' being put on by the school, involving classes at all levels. While waiting for rehearsal one day, a ninth-grade girl somehow became aware of my interests and taught me the simplest case of the binomial theorem – the formula for (x + y)^2 (Q.3).

For many of the Putnam Fellows their interest in mathematics and their ability to successfully do mathematics continued on throughout their academic careers.

Finally, Putnam Fellows reported having access to educational resources such as mathematics textbooks, puzzle books, and encyclopedias at home during their childhood. For example, Participant 5 said, "The main support was valuing academic achievement, and connecting me with a couple of math folks. She

[mother] helped me get my own copy of Hardy and Wright's *An Introduction to the Theory of Numbers* when I was about 12" (Q.2). Many participants believed this provided a rich environment for learning mathematics.

The importance of the family and the home environment are factors that played an important role in the development of the Putnam Fellows' mathematical talent, which subsequently contributed to their success on the Putnam Competition. These findings are consistent with Campbell's (1996b) American Mathematical Olympiad research studies. According to Campbell (1996b), the participants in his study attributed the home atmosphere as being critical to the development of their mathematical talent. Campbell's (1996b) research found that most Olympians grew up in households with supportive and resourceful parents, where a stimulating learning environment existed and learning was highly valued. Similarly, the participants in this study reported growing up in households surrounded by family members and family friends who valued academics and provided encouragement and support in helping them learn mathematics throughout their childhood.

Research Question 2

What formal educational experiences do Putnam Fellows identify as influential in their success on the Putnam Examination?

Subjects reported formal educational experiences as a second characteristic that played a significant role in their success on the Putnam Examination. Four themes of formal educational experiences emerged as a result of this study. First, subjects indicated having influential teachers, coaches, and mentors in academics

during their formative K-12 years. For example, Participant 3 stated, "In grades eight and nine I had a fabulous math teacher, Miss [redacted], who was incredibly effective and inspiring for all ability levels. She gave me a wonderful foundation in algebra and geometry" (Q.4).

Next, participants expressed the importance of taking part in mathematical contests and competitions (e.g., the Annual Mathematics Competition, the United States of America Mathematical Olympiad, and the International Mathematical Olympiad), throughout the childhood years leading up to their participation in the Putnam Competition. As noted by participant 12,

> I was on math teams every year from seventh to 12th grade. I took the AHSME [American High School Mathematics Examination] those same years. I took the AIME [American Invitational Mathematics Examination] in 11th and 12th grades, and the USAMO [United States of America Mathematical Olympiad] in 10th, 11th, and 12th grades. I participated in the International Mathematical Olympiad after 11th and 12th grades (Q.6).

Third, participants responded that solving Putnam problems from previous years' examinations was a valuable way to prepare for the Putnam Competition.

Finally, Putnam Fellows reported taking part in extracurricular mathematics training (e.g., after school, on weekends, and during the summer months) in addition to their normal high school program of studies. On this topic Participant 11 indicated,

> Well those Math Olympiad programs [Mathematical Olympiad Summer Program]. So the summer after my eighth grade year all the way through

> high school. And so of course, I had an enormous advantage over anybody who didn't have that background, when it came to the Putnam (Q.5).

The importance of having strong K-12 mathematics teachers and university mathematics professors, participating in mathematical contests and competitions, having access to and practicing previous Putnam Examination problems, and receiving extracurricular mathematics training are all factors that played an essential role in the development of the Putnam Fellows' mathematical talent, which subsequently contributed to their success on the Putnam Competition. These findings are consistent with Campbell's (1996b) American Mathematical Olympiad research studies, which found that the Olympiad training program was an important stimulant to the development of the Olympians' success in mathematics. Further, most Olympians in the study believed that the Olympiad Program fostered the development of their mathematical talent. As noted above, this is very similar to the Putnam Fellows' beliefs in this study.

Research Question 3

What role does the affective domain play in the development of a Putnam Fellow?

Participants identified affect as a third factor that played a role in their development as a Putnam Fellow. Seven themes of the affective domain emerged as a result of this study. First, subjects indicated being confident in their ability to solve Putnam problems. For example, Participant 9 stated, "Well, yes, quite so. I have looked at them over the years and I'm usually able to solve the majority of

them still" (Q.8). Similarly, Participant 19 responded, "Yes, and this is important. Being confident helps you focus on approaches to the problems that are more likely to lead to solutions" (Q.8). Second, participants expressed that possessing a natural ability, aptitude, or talent in mathematics as factors that contribute to winning the Putnam Competition. As noted by Participant 12, "There are three things: a natural problem-solving ability, adequate knowledge of math, and practice solving problems" (Q.11). Third, subjects believed that having a strong interest in mathematics and enjoying doing mathematics are characteristics they thought contributed to their success. Fourth, Putnam Fellows believed that intuition plays an important role in solving Putnam problems. For example, Participant 25 stated,

> What is intuition? Insofar as it denotes the kind of non-rigorous 'hunches' used to supplement mathematical reasoning, it plays a role everywhere: in reading the problem, selecting an approach, finding ways to translate vague ideas into math, deciding which steps are worth writing and which are too obvious, and even scouring the final proof for the scent of logic gone awry (Q.16).

Fifth, subjects thought that their extraordinary talent as a Putnam Fellow is partly innate and partly due to effective teaching. On this subject Participant 3 stated,

> It [talent] is certainly partly innate, but given this it can certainly be 'cultivated'. By the latter term I suppose I mean it can be taught, but really I mean that it can be greatly improved with practice – and with learning and studying more mathematics. One often gets lots of extra 'practice' helping friends with their homework (Q.17).

Sixth, participants indicated experiencing certain aesthetic or positive feelings after solving a Putnam problem. For example, according to Participant 22,

> I feel excitement and joy. And if the solution is nice, I feel a sense of aesthetic beauty. The smell of excitement, a sense of accomplishment you know when I solve a difficult problem. I think back when I was in high school, or in college doing the Putnam, you know solving these problems, doing well on these contests were definitely the high joys of my life at that point. It was a very exhilarating experience to do well on these competitions (Q.9).

Finally, individuals in the study reported extrinsic motivation as an incentive for their success on the Putnam Examination.

Over the years, research (DeFranco, 1987, 1996; Schoenfeld, 1985) on mathematical problem-solving expertise found that experts possess a wide-range of attributes that include: 1) strong domain knowledge, 2) effective problem-solving strategies (e.g., Pólya-like heuristics), 3) metacognitive skills (i.e., selecting strategies and solution paths to explore or abandon, the allocation of one's resources on a problem, etc.), and 4) a set of beliefs or world view of mathematics that impacts one's behavior on a problem. Further, Schoenfeld (1985) realized that mathematical behavior on a problem, which appears to be solely cognitive in nature, may in fact be influenced by affective components. As noted by DeFranco (1996), "beliefs regarding problem solving (e.g., perseverance, confidence, motivation, interest, etc.) contribute significantly to an individual's performance on a problem" (p. 205).

A research study (DeFranco, 1996) involving two groups of male Ph.D. mathematicians (group A-8 who have achieved national or international recognition

within the mathematics community and group B-8 who have not achieved such recognition) found that: 1) group A mathematicians outperformed the group B mathematicians on a set of four mathematics problems even though both groups had sufficient knowledge to solve the problems, 2) the problem-solving skills or strategies acquired by group A and group B mathematicians can best be described as productive and minimal, respectively, 3) similarly, the metacognitive skills acquired by group A and group B mathematicians can best be described as productive and minimal, respectively, and 4) the belief systems acquired by group A and group B mathematicians can best be described as productive and counterproductive, respectively. For example, with respect to the difference in belief systems of the two groups, subjects in group A were confident and self-assured in their ability to solve problems (seven out of eight believed they were expert problem solvers) while subjects in group B were not confident in their ability to solve problems (e.g., only one out of the eight subjects believed he was an expert problem solver). According to DeFranco (1996), confidence and motivation were some of the characteristics mentioned as qualities of expert problem solvers and in a similar way, the Putnam Fellows in this study clearly felt confident in their ability to solve the problems on the Putnam Examination and motivated to do well on the problems.

Research Question 4

What role does the cognitive domain play in the development of a Putnam Fellow?

Subjects in this study reported cognition as a fourth characteristic that played a role in their development as a Putnam Fellow. Three themes of the cognitive domain emerged as a result of this study. First, subjects indicated using alternative methods to rework previously solved mathematics problems in their everyday work, but not during the Putnam Competition because of the time constraint. As noted by Participant 15,

> I guess I do it right away. Sometimes it will happen right away depending on the time or often it will happen later when I'm not working directly, I'm just musing about it or I'm walking down the street and I'm thinking about it and I realize another idea might work or sometimes it will happen when I'm working on another problem and suddenly I realize this idea or I'm thinking about something else and I also may realize that something else is related and that's when I'll have the alternate solution, not quite intentionally going after alternative solutions all the time, but sometimes it just comes after the fact (Q.12).

Second, participants expressed an ability to recognize similarities between Putnam problems and other types of mathematics problems they have previously solved.

Finally, individuals in the study reported examining analogous cases as a strategy to get a feel for a Putnam problem. For example, Participant 9 stated,

> Okay, the first thing to do is ask, does it look someway familiar, is it like something I've seen before? If so, then that is a strong hand as to what direction to go. And after that it's very important to know that I understand the question and in particular, probably write down some special cases and see whether they work, and how they work, and quite often once you go through two or three cases, you see a general pattern, which leads you to the proof of the full statement, so that's I guess what I would say. It's as close as I could look to a strategy. In general, anything more than that, it would depend on the particular type of problem (Q.13).

Whereas Participant 11 responded,

> Well, first of all I try to recognize that it is something that I've seen before, which as I've said I can very often do. Sometimes it takes a little bit of work before you see how it relates to something you've seen before, but usually, or maybe for half the Putnam problems, I look at it and I'm pretty sure that it's like a particular thing that I've seen before. After that, if you've read this book *How to Solve It* then you know what the basic things are. You try to specialize it; you try to generalize it; you try to think of an analogous problem; you can write hypotheses. You know the drill, so I do all those things (Q.13).

The importance of using alternative methods to rework previously solved mathematics problems, recognizing similarities between Putnam problems and other mathematics problems, and examining individual cases of a problem or recalling analogous cases as a strategy to get a feel for a Putnam problem are factors that played an essential role in the development of the Putnam Fellows' mathematical talent. These findings are consistent with the characteristics of "expert" problem solvers as defined in the literature (DeFranco, 1996; Schoenfeld, 1992; Schoenfeld, as cited in DeFranco, 1996). For example, as noted above, in a research study on mathematical problem-solving expertise DeFranco (1996) found

that “the most important characteristics or qualities of an expert problem solver include: experience, knowledge of mathematics, the use of analogies, confidence, perseverance and motivation” (p. 205). Further, when asked to describe the general strategies that would be most useful in solving a problem the subjects cited recalling and using analogous problems and examining individual or special cases of a problem (DeFranco, 1996). Also, on the issue of using alternative methods to solve a problem subjects indicated they routinely rework problems (DeFranco, 1996). Clearly, the strategies used to solve problems by the Putnam Fellows are consistent with research literature in the way experts solve problems.

Implications of the Study

The purpose of this study was to try to understand the factors and characteristics that contribute to an individual becoming a Putnam Fellow, and in turn use the findings to help K-12 teachers foster a climate that helps children become better problem solvers. To help students achieve this goal the findings of this study indicate that: 1) students need a rich home environment, which supports and encourages children to begin learning mathematics at a young age, 2) K-12 mathematics teachers need to identify mathematically talented students and take an active role in nurturing their talent, 3) K-12 mathematics teachers should facilitate the access and participation of more students in mathematics contests and competitions, and 4) educators also need to encourage students to pursue

extracurricular opportunities in mathematics and recommend or provide them with additional sources of material for practice.

For example, Participant 10 stated,

> In my public high school (a good public high school in [redacted], [redacted]), Mr. [redacted] regularly took students to compete in local math competitions, as well as encouraging them to participate in national math contests. A friend of mine recommended my name to him. When I won the first contest I went to, he did everything he could to encourage me to do more, including giving me many old exams from a variety of sources. I went through all of them and practiced lots of problems (Q.4).

Next, the participants in this study believed that hard work and time spent solving problems played a critical role in their success in mathematics competitions, which subsequently led them to win the Putnam Competition and be named Putnam Fellows. The participants also had parents and teachers who provided them with encouragement and support. For this reason, parents and teachers should encourage and support children with their study of mathematics. Moreover, teachers should emphasize the importance of problem solving and encourage students to believe in their abilities, which in turn will help students gain more confidence and achieve greater success.

Finally, problem solving has been a core component of reform initiatives in the United States mathematics curriculum for the last 50 years. At the center of the reform initiatives in mathematics are the use of Pólya-type heuristic techniques, which are important to the development of students' problem-solving abilities. For

this reason, teachers at the K-16 level need to provide students with opportunities to learn problem-solving strategies by modeling heuristic techniques. With the prominence of science, technology, engineering, and mathematics (STEM) initiatives in education, as well as future employment opportunities in STEM occupations, learning to solve problems is critically important to the future of our children and the future of our country.

Recommendations for Future Research

This study was exploratory and much work needs to be done to fully understand the factors and characteristics that contribute to the success of Putnam Fellows. One question raised involves the small number of women who participate in the Putnam Competition. Throughout the history of the 75 competitions, the preponderance of winners have been males, and although there have been some female Putnam Fellows, a comprehensive examination of why so few female students participate in the Putnam Competition is recommended. A similar study that investigates the characteristics of the female Putnam Fellows might shed some light on any obstacles, which prevent a greater number of women from participating in the Putnam Competition. Moreover, a study that investigates the characteristics that make female Putnam Fellows successful and compares these attributes to male Putnam Fellows is recommended.

A second question raised involves the problem-solving strategies exhibited by Putnam Fellows during complex problem solving. Therefore, to gain a more

complete understanding of a Putnam Fellow's conceptual and procedural knowledge of a particular problem and to better understand the problem-solving strategies they use, a study involving Putman Fellows using think-aloud protocols as they solve Putnam problems is recommended.

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Appendix A

Number of Students Participating in the Putnam Competition by Year

Year	Number
1938	163
1939	200
1940	208
1941	146
1942	114
1946	67
1947	145
1948	120
1949	155
1950	223
1951	209
1952	295
1953	256
1954	231
1955	256
1956	291
1957	377
1958 Spring	430
1958 Fall	506

Year	Number
1959	633
1960	867
1961	1,094
1962	1,187
1963	1,260
1964	1,439
1965	1,596
1966	1,526
1967	1,592
1968	1,398
1969	1,501
1970	1,445
1971	1,569
1972	1,681
1973	2,053
1974	2,159
1975	2,203
1976	2,131
1977	2,138

Year	Number
1978	2,019
1979	2,141
1980	2,043
1981	2,043
1982	2,024
1983	2,055
1984	2,149
1985	2,079
1986	2,094
1987	2,170
1988	2,096
1989	2,392
1990	2,347
1991	2,325
1992	2,421
1993	2,356
1994	2,314
1995	2,468
1996	2,407

Year	Number
1997	2,510
1998	2,581
1999	2,900
2000	2,818
2001	2,954
2002	3,349
2003	3,615
2004	3,733
2005	3,545
2006	3,640
2007	3,753
2008	3,627
2009	4,036
2010	4,296
2011	4,440
2012	4,277
2013	4,113
2014	4,320
Total	140,314

Data compiled from the results of the William Lowell Putnam Mathematical Competition, published annually in *The American Mathematical Monthly* by the Mathematical Association of America.

Appendix B

Number of Putnam Fellows by Year

Year	Number
1938	5
1939	5
1940	5
1941	5
1942	5
1946	5
1947	5
1948	6
1949	5
1950	5
1951	5
1952	5
1953	5
1954	5
1955	5
1956	5
1957	5
1958 Spring	5
1958 Fall	5

Year	Number
1959	8
1960	5
1961	5
1962	5
1963	5
1964	5
1965	5
1966	5
1967	5
1968	5
1969	5
1970	6
1971	6
1972	6
1973	5
1974	5
1975	5
1976	6
1977	5

Year	Number
1978	5
1979	5
1980	5
1981	5
1982	5
1983	5
1984	5
1985	5
1986	6
1987	6
1988	5
1989	6
1990	5
1991	5
1992	5
1993	6
1994	5
1995	5
1996	6

Year	Number
1997	6
1998	5
1999	6
2000	5
2001	5
2002	5
2003	5
2004	5
2005	6
2006	5
2007	6
2008	5
2009	5
2010	5
2011	5
2012	5
2013	5
2014	6
Total	393

Data compiled from the results of the William Lowell Putnam Mathematical Competition, published annually in *The American Mathematical Monthly* by the Mathematical Association of America.

Appendix C

Putnam Fellows' Winning Year(s)

Fellow	Year(s)	Fellow	Year(s)
1	1938	25	1946, 1947
2	1938	26	1947
3	1938	27	1947, 1949
4	1938	28	1947
5	1938, 1939	29	1947, 1948
6	1939	30	1948
7	1939	31	1948
8	1939, 1940, 1941	32	1948
9	1939	33	1948
10	1940	34	1948, 1949, 1950
11	1940, 1941, 1942	35	1949, 1950
12	1940	36	1949
13	1940	37	1949
14	1941	38	1950
15	1941	39	1950
16	1941	40	1950
17	1942	41	1951
18	1942	42	1951, 1952, 1953
19	1942	43	1951
20	1942	44	1951
21	1946	45	1951
22	1946	46	1952
23	1946	47	1952
24	1946	48	1952

Fellow	Year(s)
49	1952
50	1953
51	1953
52	1953, 1959, 1960
53	1953
54	1954
55	1954
56	1954
57	1954
58	1954, 1956
59	1955, 1956
60	1955, 1957
61	1955, 1956
62	1955
63	1955
64	1956, 1957
65	1956
66	1957
67	1957
68	1957
69	1958 (Spring)
70	1958 (Spring)
71	1958 (Spring)
72	1958 (Spring), 1958 (Fall)
73	1958 (Spring)
74	1958 (Fall), 1959
75	1958 (Fall)
76	1958 (Fall)

Fellow	Year(s)
77	1958 (Fall)
78	1959
79	1959
80	1959
81	1959
82	1959
83	1959
84	1960
85	1960
86	1960
87	1960
88	1961, 1962
89	1961
90	1961, 1962
91	1961, 1962
92	1961
93	1962
94	1962
95	1963
96	1963
97	1963
98	1963
99	1963
100	1964, 1965
101	1964
102	1964
103	1964
104	1964

Fellow	Year(s)
105	1965
106	1965
107	1965
108	1965
109	1966
110	1966
111	1966
112	1966, 1967
113	1966
114	1967
115	1967
116	1967
117	1967
118	1968, 1969, 1970, 1971
119	1968, 1969
120	1968
121	1968
122	1968
123	1969
124	1969, 1970
125	1969, 1970
126	1970
127	1970
128	1970, 1971, 1972, 1973
129	1971
130	1971
131	1971
132	1971, 1972

Fellow	Year(s)
133	1972
134	1972
135	1972
136	1972
137	1973, 1975
138	1973
139	1973
140	1973
141	1974, 1975
142	1974
143	1974
144	1974
145	1974
146	1975
147	1975
148	1975, 1976
149	1976
150	1976
151	1976
152	1976, 1978
153	1976
154	1977, 1978
155	1977
156	1977
157	1977, 1981
158	1977
159	1978, 1979, 1980
160	1978

Fellow	Year(s)
161	1978
162	1979
163	1979
164	1979
165	1979
166	1980, 1982, 1983
167	1980
168	1980
169	1980
170	1981, 1982, 1983
171	1981
172	1981, 1983
173	1981
174	1982, 1983, 1984
175	1982
176	1982
177	1983
178	1984
179	1984
180	1984, 1987
181	1984
182	1985
183	1985
184	1985, 1986
185	1985, 1986, 1987, 1988
186	1985
187	1986, 1987, 1988
188	1986

Fellow	Year(s)
189	1986, 1987, 1988
190	1986
191	1987
192	1987
193	1988
194	1988, 1989, 1990, 1991
195	1989
196	1989
197	1989
198	1989
199	1989
200	1990, 1992
201	1990
202	1990
203	1990, 1991
204	1991
205	1991
206	1991, 1992
207	1992, 1993
208	1992
209	1992
210	1993
211	1993, 1994, 1995
212	1993
213	1993, 1994, 1995
214	1993, 1994, 1995
215	1994, 1996
216	1994

Fellow	Year(s)
217	1995
218	1995
219	1996
220	1996
221	1996
222	1996, 1997
223	1996
224	1997
225	1997, 1998
226	1997
227	1997, 1998, 2000
228	1997
229	1998
230	1998, 2001
231	1998
232	1999
233	1999
234	1999, 2000
235	1999
236	1999
237	1999
238	2000, 2001, 2002, 2003
239	2000
240	2000, 2002
241	2001
242	2001
243	2001, 2002, 2003, 2004
244	2002

Fellow	Year(s)
245	2002
246	2003
247	2003, 2004
248	2003, 2004, 2005, 2006
249	2004
250	2004, 2005, 2007
251	2005
252	2005
253	2005
254	2005, 2006
255	2006
256	2006
257	2006, 2008, 2009
258	2007
259	2007, 2008, 2010, 2011
260	2007, 2009
261	2007
262	2007, 2008, 2009
263	2008, 2010, 2011
264	2008
265	2009
266	2009, 2011
267	2010
268	2010
269	2010
270	2011
271	2011, 2012, 2013
272	2012

Fellow	Year(s)
273	2012
274	2012, 2013
275	2012, 2013, 2014
276	2013, 2014

Fellow	Year(s)
277	2013, 2014
278	2014
279	2014
280	2014

Data compiled from the results of the William Lowell Putnam Mathematical Competition, published annually in *The American Mathematical Monthly* by the Mathematical Association of America.

Appendix D

Winning Teams by College or University

F = Fall; S = Spring; T = Tie Score

College or University (Total Number of Top Five Winning Teams)	First Place	Second Place	Third Place	Fourth Place	Fifth Place
Brooklyn College (6)	1939 1941 1948	1952 1963	1946		
California Institute of Technology (33)	1950 1962 1964 1971 1972 1973 1975 1976 1983 2010	1959 1967 1979	1958 F 1961 1974 1977 1982 2009 2011	1957 1963 1998 2000 2003	1965 1970 1978 1988 1996 2004 2008 2013
Carnegie Institute of Technology Carnegie Mellon University (7)		2011 2013	1949 1987	1946	2012 2014
Case Institute of Technology Case Western Reserve University (4)	1978			1964 1976 T	1959
City College of New York (5)		1953 T		1942 1948 T 1949 1951	
Columbia University (5)		1956 1957	1938 1940 T 1947		
Cooper Union Institute of Technology (2)			1940 T 1951		

College or University (Total Number of Top Five Winning Teams)	First Place	Second Place	Third Place	Fourth Place	Fifth Place
Cornell University (9)	1951 1954	1953 T 1994 1995	1957	1958 F	1960 1992
Dartmouth College (2)		1962			1961
Duke University (12)	1993 1996 2000	1990 1997	1999 2001 2002 2003 2004 2005		2007
Harvard University (60)	1947 1949 1953 1955 1956 1957 1958 F 1965 1966 1982 1985 1986 1987 1988 1989 1990 1991 1992 1994 1995 1997 1998 2001	1950 1951 1954 1958 S 1960 1980 1993 1999 2003 2006 2009 2014	1948 1952 1962 1964 1967 1971 1972 1981 1984 1996 2000 2010	1959 1961 1969 1973 1978 2013	1975

College or University (Total Number of Top Five Winning Teams)	First Place	Second Place	Third Place	Fourth Place	Fifth Place
	2002 2005 2007 2008 2011 2012				
Harvey Mudd College (2)			1991		2003
Illinois Institute of Technology (1)				1970	
Kenyon College (1)				1955	
Massachusetts Institute of Technology (45)	1968 1969 1979 2003 2004 2009 2013 2014	1939 1946 1961 1964 1965 1966 1970 1998 2000 2001 2010 2012	1941 1942 1954 1960 1975 1994 1995 2006 2007 2008	1952 1956 1967 1974 1976 T 1993 1997 2005	1963 1971 1972 1977 1986 1987 2011
McGill University (1)				1948 T	
Miami University (1)			1993		
Michigan State University (5)	1961 1963 1967			1960 1968	
Mississippi Women's College (1)			1939		

College or University (Total Number of Top Five Winning Teams)	First Place	Second Place	Third Place	Fourth Place	Fifth Place
New York University (1)			1950		
Oberlin College (1)		1972			
Polytechnic Institute of Brooklyn (3)	1958 S 1959				1958 F
Princeton University (28)	2006	1981 1985 1987 1988 1989 1996 2002 2004 2005 2007 2008	1976 1979 1997 1998	1965 1975 1977 1983 1984 1992 1994	1966 1973 1982 1995 2009
Queen's University (3)	1952		1956	1962	
Rensselaer Polytechnic Institute (1)			2014		
Rice University (4)		1969	1988	1985	1989
Stanford University (10)			2013	1979 1981 1991 2007 2008 2009 2011	2001 2002
Stony Brook University (1)				2012	
Swarthmore College (1)				1972	

College or University (Total Number of Top Five Winning Teams)	First Place	Second Place	Third Place	Fourth Place	Fifth Place
University of British Columbia (2)		1973			1974
University of California, Berkeley (11)	1960	1938	1985 1986	1953 1987 2001 2002 2010	1964 1980
University of California, Davis (3)	1984 T	1977		1971	
University of California, Los Angeles (3)			1968 2012		1962
University of Chicago (11)	1970	1971 1974 1975	1966 1969 1973	1980	1983 1999 2006
University of Kansas (1)					1968
University of Manitoba (1)				1958 S	
University of Maryland (2)			1980		1981
University of Michigan (4)				1966 1999	1967 1993
University of Pennsylvania (3)		1941	1963	1947	
University of Toronto (18)	1938 1940 1942 1946	1948 1949 1955 1958 F 1992	1958 S 1959 1965 1970	1950 1954 1995 2006	2000

College or University (Total Number of Top Five Winning Teams)	First Place	Second Place	Third Place	Fourth Place	Fifth Place
University of Waterloo (19)	1974 1999	1968 1982 1991	1978 1983 1989 1990 1992	1988 2004 2014	1979 1985 1994 1998 2005 2010
Washington University (11)	1977 1980 1981 1984 T	1976 1978 1983 1986		1996	1990 1997
Yale University (11)		1940 1942 1947	1955	1982 1986 1989 1990	1969 1984 1991

43 Colleges and Universities Total Number of Top Five Wins	First Place	Second Place	Third Place	Fourth Place	Fifth Place
355	76	75	75	73	56

Data compiled from the results of the William Lowell Putnam Mathematical Competition, published annually in *The American Mathematical Monthly* by the Mathematical Association of America.

Appendix E

Number of Putnam Fellows by College or University

College or University	Number of Putnam Fellows
Armstrong State College	1
Brooklyn College	5
California Institute of Technology	26
Carnegie Institute of Technology	3
Case Western Reserve University	4
City College of New York	10
College of Saint Thomas	1
Columbia University	8
Cooper Union	1
Cornell University	5
Dartmouth College	2
Duke University	6
Fort Hays Kansas State College	1
George Washington University	1
Harvard University	104
Kenyon College	2
Massachusetts Institute of Technology	66
McGill University	1
Michigan State University	5
New York University	3
Polytechnic Institute of Brooklyn	3
Princeton University	21
Purdue University	2
Queen’s University	1
Reed College	1

College or University	Number of Putnam Fellows
Rice University	3
Rose-Hulman Institute of Technology	1
San Diego State College	1
Simon Fraser University	1
Stanford University	3
Swarthmore College	1
Union College	1
University of Alberta	2
University of British Columbia	1
University of California, Berkeley	16
University of California, Davis	2
University of California, Los Angeles	2
University of California, Santa Barbara	2
University of Chicago	10
University of Detroit	1
University of Manitoba	1
University of Maryland, College Park	1
University of Minnesota, Minneapolis	3
University of Missouri, Rolla	1
University of North Carolina	1
University of Pennsylvania	3
University of Pittsburgh	1
University of Santa Clara	1
University of Toronto	23
University of Virginia	1
University of Washington, Seattle	1
University of Waterloo	8
Washington University, St. Louis	6

College or University	Number of Putnam Fellows
Wesleyan University	1
Williams College	1
Yale University	10

Total Number of Colleges and Universities	Total Number of Putnam Fellows
56	393

Data compiled from the results of the William Lowell Putnam Mathematical Competition, published annually in *The American Mathematical Monthly* by the Mathematical Association of America.

Appendix F

Women Awarded the Elizabeth Lowell Putnam Prize by Year(s)

PF = Putnam Fellow

Woman	Year(s)
1	1992
2	1994
3	1995, 1996 PF, 1997
4	1999
5	2001, 2002 PF
6	2003 PF, 2004 PF
7	2005, 2006, 2007
8	2008
9	2010
10	2011
11	2013

Data compiled from the results of the William Lowell Putnam Mathematical Competition, published annually in *The American Mathematical Monthly* by the Mathematical Association of America.

Appendix G

Putnam Fellows' Awards, Honors, and Professional Appointments

Awards and Honors	Professional Appointments
• Alfred P. Sloan Research Fellow – Alfred P. Sloan Foundation • American Academy of Arts and Sciences Fellow • American Association for the Advancement of Science Fellow • American Mathematical Society Fellow • American Philosophical Society (APS), Philadelphia, Pennsylvania • André-Aisenstadt Prize – Centre de Recherches Mathématiques • Banco Bilbao Vizcaya Argentaria (BBVA) Foundation Frontiers of Knowledge Award in the Basic Sciences • Ben Fusaro Award – Society for Industrial and Applied Mathematics (SIAM) • Canadian National Scrabble Champion • Claude E. Shannon Professor • David & Lucile Packard Foundation Fellow • Davidson Institute for Talent Development • Fields Medal – International Congress of Mathematicians, International Mathematical Union • Frank and Brennie Morgan Prize for Outstanding Research in Mathematics by an Undergraduate Student – American Mathematical Society, Mathematical Association of America, and Society for Industrial and Applied Mathematics (SIAM) • Gabriella and Paul Rosenbaum Foundation Fellow	• Chair of the Board of Trustees of the Institute for Pure and Applied Mathematics at [redacted] • Chair of the Mathematics Department at [redacted] • Cryptographer • Cryptologist • Freelance Writer • Mathematician • National Security Agency • Postdoctoral Researcher • Professor of Computer Science and Engineering • Professor of Economics • Professor of Mathematics • Professor of Physics • Quantitative Analyst

Awards and Honors	Professional Appointments
• Gates Cambridge Scholarship – Bill and Melinda Gates Foundation • George Pólya Prize – Society for Industrial and Applied Mathematics (SIAM) • Gilbert de Beauregard Robinson Prize – Canadian Mathematical Society • Guggenheim Fellow – John Simon Guggenheim Memorial Foundation • [redacted] University Junior Fellow • Intel Science Talent Search • International Mathematical Olympiad – Gold, Silver, and Bronze Medalists • Jeffrey – Williams Prize – Canadian Mathematical Society • Leroy P. Steele Prize – American Mathematical Society • Longuet – Higgins Prize – The Technical Committee on Pattern Analysis and Machine Intelligence of the Institute of Electrical and Electronics Engineers (IEEE) • MacArthur Fellow – MacArthur Foundation • Machtey Award – Institute of Electrical and Electronics Engineers (IEEE) • Mathematical Sciences Research Institute • Miller Professor • MoMath Masters Tournament – National Museum of Mathematics • National Academy of Sciences • National Defense Science and Engineering Graduate Fellow – American Society for Engineering Education • National Medal of Science – National Science Foundation (NSF)	

Awards and Honors	Professional Appointments
• National Science Foundation Graduate Research Fellow • National Scrabble Champion • Norwegian Academy of Science and Letters, Oslo, Norway • Pat Goldberg Memorial Best Paper Award – [redacted] Research • Presidential Early Career Award for Scientists and Engineers – National Science Foundation • Research Science Institute Scholar • Royal Society, London, United Kingdom • Scripps National Spelling Bee • Shaw Prize – The Shaw Prize Foundation, Hong Kong • Simons Fellow – Simons Foundation • Symposium on Principles of Database Systems (PODS) Best Paper Award • United States of America Mathematical Talent Search • Westinghouse Science Talent Search • William Chauvenet Prize – Mathematical Association of America • Wolf Prize – The Wolf Foundation, Israel • World Scrabble Champion	

Data compiled from University of Minnesota Duluth, Putnam Fellows' Career Paths (Gallian, 2015)

Retrieved from: http://www.d.umn.edu/~jgallian/putnamfel/PF.html

Appendix H

Interview Questionnaire

1. Demographics
   a. While you were growing up, how many parents were at home?
   b. What is your birth order (e.g., "n^{th} out of N") and what is the age span between you and your siblings?
   c. What is/was your father's level of education and occupation?
   d. What is/was your mother's level of education and occupation?

2. What role (e.g., financial, parental influence, psychological support, help with schoolwork, access to educational resources, progress monitoring and time management, conducive home atmosphere, etc.) did your parents play in your success as a Putnam Fellow?

3. Can you tell me a story about an event or an individual who influenced you to become a Putnam Fellow?

4. Can you describe a teacher or teachers who have influenced you to help become a Putnam Fellow?

5. Beyond traditional mathematics classes, did you participate in any enrichment classes or summer programs in mathematics?

6. Did you participate in mathematics competitions (e.g., MATHCOUNTS, the United States of America or Canadian Mathematical Olympiad, etc.) throughout your formal education?

7. How did you prepare to take the Putnam Examination? Did you participate in practice sessions or receive coaching as preparation for the Putnam Competition? Did your college or university offer preparatory classes for the Putnam Competition? Please explain.

8. Are you confident in your ability to solve Putnam problems? Please explain.

9. What do you feel (e.g., excitement, aesthetic joy, fear, etc.) when you solve a Putnam problem?

10. When you took the Putnam Examinations, what motivated you to be successful? Please explain.

11. Please describe the qualities, characteristics, or factors that you think contribute to an individual becoming a Putnam Fellow.

12. After solving a mathematics problem, when do you rework and use or not use alternative methods to solve the problem? Why?

13. When you first read a Putnam problem, what general strategies or techniques do you think you would use to help you toward the solution of the problem?

14. Typically when mathematicians begin to solve problems they examine analogous cases to get a feel for the problem. When you solve a Putnam problem, do you use a similar strategy or are there other methods you employ?

15. Do you still do Putnam problems? Why? Why not?

16. What role does intuition play in solving Putnam problems?

17. Do you believe your extraordinary talent as a Putnam Fellow is innate or can it be taught?

18. To be a multiple Putnam Competition winner is an extraordinary achievement. Is there anything you would like to share that I did not ask that might shed light on your success as a Putnam Fellow?

Appendix I

Sample Interview Transcription

Subject: Participant 20

Skype Interview: July 16, 2014

1. Demographics

    a. While you were growing up, how many parents were at home?

    Two parents.

    b. What is your birth order (e.g., "n^{th} out of N")?

    I'm the [redacted] of [redacted].

    c. And what is the age span between you and your [redacted]?

    There are [redacted] years between my [redacted] and me.

    d. Does your [redacted] possess mathematical talent like yourself?

    [redacted] doesn't undertake it competitively or didn't undertake it competitively like I did. And [redacted] interest is not as developed as mine, but [redacted] learns math for fun, so I can't say [redacted] not interested.

    e. What is/was your father's level of education and occupation?

    Yeah, he's a [redacted]. He has a [redacted], a [redacted] actually. He's a, I guess he calls himself a [redacted]. He does [redacted] in the [redacted].

    f. What is/was your mother's level of education and occupation?

    My mother is a [redacted] in [redacted] and she is a [redacted].

2. What role (e.g., financial, parental influence, psychological support, help with schoolwork, access to educational resources, progress monitoring and time management, conducive home atmosphere, etc.) did your parents play in your success as a Putnam Fellow?

My parents helped and contributed in a number of ways. One thing I should say, not specifically in regards to this question, but you ask on a number of questions, what contributed to your being a Putnam Fellow? I did a lot of math contest stuff in high school and college, and so there was not one particular thing. I don't think of taking the Putnam is itself a discrete event. I think that's one part of a general career interest in math problem solving that I developed for a long time. So what role did my parents play? I think they did quite a lot, both of my parents. They already had a technical background, so they were interested in getting me – they were naturally equipped to get me to learn about math and science early on. Starting as far back as I can remember, age two, three, they were teaching me how to add and multiply, and by the time I was six they were giving me books of logic puzzles to learn from. And since I was interested, they continued to get me books to learn math from at home. And so, that's sort of the obvious thing. They continued to do that more or less throughout my childhood, although you know, as I got older I became more self-sufficient. By the time I was in high school, I had either other people supplying resources for me or I was finding stuff on my own. My parents were certainly encouraging all the time, and they took an interest in what I was learning as well. I am trying to think what else I can say. Like you mentioned financial support. Obviously my parents gave me a home to live in, food to eat and stuff, because that's what parents do, but I don't think that learning math is a particularly expensive activity. We didn't have individual tutoring. I guess there are some privately run, math-oriented summer camps, although I didn't participate in any of those. I went to a private school from age five until ten. And so that was from the pre-kindergarten year until fourth grade, and then after that I went to public school.

3. Can you tell me a story about an event or an individual who influenced you to become a Putnam Fellow?

Well I can certainly think of a few individuals who were influential. Please wait while I think about how to organize this. Yeah, so an event is hard to pin down, but certainly I can tell you about an individual. I had a number of mentors during my childhood who encouraged me to pursue interests in mathematics, especially in competitive problem solving and who recommended sources of material to practice on, things to learn, and so forth. So probably, not probably, the most influential person I would say was [redacted], currently at [redacted] College, also visiting professor at [redacted], who while I was in high school, was assistant professor of mathematics at [redacted]. She started there I think after my freshman year of high school, which was when I had just qualified for the U.S. team to the International Math Olympiad for the first time. And so I had been involved in a number of other contests and I was on the local team for the American Regents Math League [ARML], which is a team contest that you've probably heard about from other people you've talked to. So at some point I got a very long and detailed e-mail from a professor who I had not met yet, actually congratulating me on what I was doing, and expressing lots of enthusiasm for the [redacted] Math Circle, which was a program that she was then just starting and encouraging me to participate. And so that was one of the biggest things that she has done in the last few years is started this Math Circle program all over the U.S. She comes from [redacted] where this is a long-standing tradition to have these extra after-school or weekend math programs to go to. And so she and several others in the [redacted] started importing this tradition to the U.S. and to the [redacted] Circle, which meets on weekends, or it did at that time. And I think it still does. It is much larger now than it was then. So they were just getting started at that time. And so she asked me to take part, take sort of an organizational role. So my job initially was as the coordinator for the monthly contests. And they would have lectures on one or another sort of math topics that you wouldn't ordinarily see in the classroom every week. And they also had these contests, which were run on a monthly basis. Problems were distributed and you would go home and write solutions to them and then submit them. And they said, so the way that she put it was like, 'We can't let you compete in this contest, so why don't we have you organize it instead?' So I was in charge of putting together official solutions and administering grading. And then in subsequent years I gave some lectures at the Math Circle as well. And for me

that was a great way to meet other people of my age who were interested in mathematics, so I made a number of friends that way. And of course it was also a good way to learn new stuff. And so she was the most influential person I would pick out. There were a number of others like [redacted], at the University of [redacted], who also was involved in organizing the Math Circle, as well as several other extracurricular math events for secondary students in the [redacted]. He I think was approaching our ARML team and he gave me a number of, a long list of reading suggestions to work on problem solving. So those are the two main people who come to mind. Yeah, [redacted] was the first one and [redacted] is the second. I can e-mail those to you, which is probably easier than spelling them out over the phone.

4. Can you describe a teacher or teachers who have influenced you to help become a Putnam Fellow?

I am being silent while I think about this. As far back as I can remember I sort of knew that I was doing stuff that my classmates hadn't learned. You know like when I was in second and third grade, we had this middle school teacher. I was in this integrated elementary and middle school, so we would have this middle school teacher who would come like once a week and give me some extra math sessions, because what we were doing in class wasn't stimulating enough. I can sort of remember one fragment or another. I think I remember the first time proving something that I would call a theorem was when I was ten years old and was thinking about for no particular reason that I can remember, I was thinking about perpendicular bisectors as sides of triangles. And I thought, 'Gee, if I draw the perpendicular bisectors of the three sides, they all seem to meet in the point.' And I realized why that is. And I said, 'Well okay, so if the perpendicular bisector of AB and the bisector of AC meet somewhere, then that point must be the same distance from all three of the corners, so it is also on the bisector of BC.' And so then after that I was like, 'Hey, I can prove things.' And at some point, I don't remember whether it was then or some time later my dad would have a habit of asking me when I came home from school, he would say, 'Did you prove any theorems today?' You know I think it was something I did as a hobby for a long time. The first time that it really struck me that I was doing more of this than a large majority of people my age was when I starting actually competing in contests, this would have been in the eighth grade, I took the

American, the exam that is now called the AMC 12, at the time it was called the AHSME, American High School Mathematics Examination, just because the local high school where I was taking geometry at that time was administering it. This was just an activity that the math club did and it turned out it was the first of several rounds. So there was that and then there was the American Invitational Math exam, and then there was the USA Math Olympiad. And I got back my results from the USA and I made it through the first two rounds without a whole lot of fuss. And then I took the USAMO, which was a pretty hard test. And I got back my results and it was like, 'Well you solved two out of six problems, only 30 people in the country have done this well.' So I thought, 'Gee, you know, maybe I'm doing better at this than I thought. Maybe these are skills that I should try to actively develop rather than just doing it purely for fun.' So I don't know whether that exactly answers your question, but those are sort of a few episodes that stand out in my memory.

5. Beyond traditional mathematics classes, did you participate in any enrichment classes or summer programs in mathematics?

Yeah, so there was the [redacted] Math Circle, which I mentioned, which started up in my tenth grade year. And I was pretty consistently involved in that in tenth, eleventh, and twelfth grade. And as far as summer programs go, there was the Math Olympiad Program, since I was on the IMO team, so I went to a training program for that, and that would have been somewhere after ninth grade, tenth grade, and twelfth grade. In eleventh grade, I skipped out on that to do a different summer program called [redacted]. It's a program that takes high school students interested in an area of science and gives sort of a six-week mini-research experience at [redacted]. And so I was partnered with an [redacted] grad student working in algebraic topology, who sort of gave me some stuff that he was interested in to think about, and I wrote a paper that came out of that.

6. Did you participate in mathematics competitions (e.g., MATHCOUNTS, the United States of America or Canadian Mathematical Olympiad, etc.) throughout your formal education?

Well I wouldn't say throughout my formal education. I didn't do MATHCOUNTS. I had a bad experience with a locally organized math contest when I was in fifth grade and so I didn't participate in competitions for a little while after that. But then starting in eighth grade is when I joined the local high school math club, which participated in a number of other sorts of informal regional contests, as well as the AHSME [American High School Mathematics Examination] sequence. And then, I guess starting then, once I realized that I was enjoying this, and I was doing well, I sort of started participating in every contest that I could find. So there is the AMC [Annual Mathematics Competition] contest and the IMO [International Mathematical Olympiad], there was ARML [American Regions Mathematics League], there was something called American High School or USA Mathematical Talent Search that was run out of, I can't even remember who was organizing it or whether it still exists. At some point, I'm sure there are others that I'm forgetting about. You know that was basically my career when I was in high school. I heard, 'Who does math problems for fun?' There were enough strands and outlets of them, enough agencies organizing these things that I participated in, and there is no way I am going to successfully remember all of them. So that was not the Westinghouse, but I did also participate in that, so that was my senior year. So I think there were both, there were two research-oriented science contests. There had formally been a Westinghouse one. Now there was one run by Intel and another one run by Siemens at the time that I was a high school senior. I competed in the Intel one. I used the topology paper I had written at [redacted] ([redacted]) for that. Yes it's called American Regents Math League. I'm just looking at my browser right now to see if I can remember where the website is, so that you can find more. Well if I can quickly give you a pointer that will save you an hour of searching, then I might as well do it. But ARML is the name and it should be pretty easy to find.

7. How did you prepare to take the Putnam examination? Did you participate in practice sessions or receive coaching as preparation for the Putnam Competition? Did your college or university offer preparatory classes for the Putnam Competition? Please explain.

So there wasn't a whole lot of preparation that I did. I mean organized preparation. I prepared for it [the Putnam] the same way that I prepared for other contests, which was basically working on, working through problems from past years. One thing that makes it hard to answer the questions about the Putnam contest is by the time I was taking the Putnam it was not a big deal from my point of view. You know like there are a lot more of these high school contests than there are college-level contests. And the Putnam is basically the only one of its kind in the U.S. or at least it was at the time that I was doing it. I don't know if that's still the case since I don't really live as much in this world now. And it was also, I had done the IMO [International Mathematical Olympiad] and IMO problems are much harder than Putnam problems, so by the time that I was taking the Putnam I was sort of like, 'Okay, you know I've developed these skills beyond the point that it is necessary here, so I can just do a little bit of extra practice', which I did each year that I took the Putnam, but it was not something that I needed to invest a whole lot of time in it. Let's see, so my taking of the Putnam was a little bit unusual in that I took it for the first time when I was in [redacted]. So I grew up in [redacted], [redacted]. I took the Putnam at [redacted] when I was a [redacted]. I did this because I knew that the rules are such that you're only allowed to take the Putnam four times. And let's see how do I put these – say these thoughts in logical order? So I took the Putnam when I was in [redacted] because I was like, 'Well I know enough math so that I can do this and there are prizes, so why not?' Most [redacted], I mean it is rare for people to take the Putnam [redacted] because you're only allowed to take it four times, so people usually wait until they're [redacted]. My reasoning at the time was probably by the time I'm in my [redacted] I'll have some other things that I feel like doing that day and that will conflict with the Putnam. So chances are if I take the exam [redacted], I'm not really losing one of my [redacted]. And I also sort of reasoned at the time, now I've been doing these kinds of problems, sort of day in and day out for several years probably. By the time I'm [redacted], they're aren't as many contests going on, I'll be doing other things with my time, and maybe my problem solving skills will have already reached their high point and will be starting to fall, which I think also turned out to be an

accurate prediction in the sense that I was certainly not putting in as much time probably, not as much creativity into problem solving after starting college. So I mentioned that just because you asked what sort of systematic training there was. And so I answered sort of like two different questions, two different answers that I can give, because I took the Putnam [redacted]. So while in [redacted], I took it at [redacted]. And I'm trying to remember what I did in terms of preparation, because they do have a Putnam preparation class. I don't think that I was enrolled in any formal capacity. I think I might have gone to a few sessions sporadically. And then as an [redacted] they did not have a Putnam preparation class. The department's official position was, 'We don't need to train our students because they're good enough already.' So the first year or two I remember having a few sporadic sessions together with Putnam teammates where we would pick out specific previous years of Putnam and go through them together, but I don't think that I did anything really organized beyond that.

8. Are you confident in your ability to solve Putnam problems? Please explain.

   If the answer is yes or no, then I would say yes. I am probably less confident now, than I would have been 10 years ago.

9. What do you feel (e.g., excitement, aesthetic joy, fear, etc.) when you solve a Putnam problem?

   It really depends on the problem. So I think for problems that I enjoy working on, it's very similar to what you described. There is the process of getting an idea and then exploring it and trying to build on it, and seeing whether it can be developed into a solution. There's a feeling of anticipation of building excitement that sort of supplies the motivation to continue working. And then when I solve a problem there can be a feeling of admiration, maybe partly narcissistically for myself, but mostly for the beautiful or surprising piece of mathematics that I have just gotten to see and understand. This is something that I experience if the problem or the line of reasoning is something new to me. So for the easier problems, you know for some of them, I just look at the problem and I say, 'Oh, I know how to do this. I just need to calculate *xyz*', and then there's not much feeling to it. But if I

can look at a problem and know what I need to do then I'm just like, 'Okay, I've got a job to do. I'll do it.' Sometimes there can even be something sort of ugly or tedious in the calculation and then there's even a feeling of displeasure. And then it's not maybe so much satisfaction or aesthetic joy or admiration when the problem is solved, so much as just you know relief that the job is over.

10. When you took the Putnam examinations, what motivated you to be successful? Please explain.

What motivated me? So to be honest, I guess by the time I was taking the Putnam it was pretty mercenary. Like somebody's going to give me a couple thousand dollars for solving problems on a Saturday and doing a good job of it, 'Sure, I'll take the money.' I mean there was also I guess a sense of identity, you know, like I had sort of built a sense of myself as somebody who solves math problems for a living and so this was sort of one of the natural things to do.

11. Please describe the qualities, characteristics, or factors that you think contribute to an individual becoming a Putnam Fellow.

You're suddenly getting much more abstract here. I want to say, what do I want to say? Obviously an interest in mathematics is important. One has to care about the subject enough to invest the time to develop one's skills. There are also lots of ways of being interested in mathematics or for that matter being good at mathematics. And so the Putnam is a particular kind of skills involved in mastering a number of quick problem-solving techniques. And so, how would I describe the quality that's involved in that? I guess it involves an academic interest as well as a certain form of speed or agility. I know people who are better at mathematics than I ever was, but would never be able to solve a contest problem in an hour, just because the way that they think about things is by sort of understanding them deeply and getting to see a big picture over a very long period of time. That's the only thing that stands out in regards to the Putnam particularly and that I think is maybe one reason that I typically have done well is attention to detail. So I hear that, although I haven't been in the room to confirm this firsthand, I hear the Putnam is graded very harshly and that basically you fall on a scale of zero to

10 right, but if you have a solution the big graders recognize immediately as being a correct and complete solution you get tens. And if there's a small hole then you get two or three, or if it's a correct solution but it's sloppily written and they can't immediately tell if it's a correct solution you get two points or zero points. And I think that my score each year basically corresponded to the number of problems that I tried to solve when I walked out of the test room, so I get the sense that I tend to be more careful about writing complete proofs and about writing them in a clear and organized way than most people who sort of reason intellectually at about the same level that I do. I mean it's a useful kind of skill to have as a mathematician, although it can also be hazardous.

12. After solving a mathematics problem, when do you rework and use or not use alternative methods to solve the problem? Why?

So I think my general habit would not be to solve a problem more than once. I think usually when I was working on solving contest problems, systematically my sense of each problem is you've got a problem and you solve it and then it's done. It's like somebody gives you a cookie and you eat it and now the cookie is not there anymore, you're not going to eat it again. I mean, maybe one reason for that is sort of, imagine that a lot of problems there are sort of a key idea that you have to get in order to solve them, and once you have them you know there might be other solutions, but they would likely involve variations on the same idea and so like going back and looking for a different way to solve a problem maybe, typically wouldn't reveal all that much, wouldn't reveal new knowledge and there wouldn't be the same sort of instant gratification, the sort of same sense of excitement that you get from solving a problem because now it's not new anymore and so the shine has sort of worn off. Sometimes people would give me a problem and specifically say, 'There's a couple of different ways of solving the problem', or even like the [redacted] guy I mentioned earlier, gave me a particular problem, he said, 'There's lots of ways of doing this. There is a graph theory way. There is a complex integration way' and those clues, and I would start thinking about, 'Okay, what's the graph theory way of doing this? What's the complex integration way of doing this?'

13. When you first read a Putnam problem, what general strategies or techniques do you think you would use to help you toward the solution of the problem?

    I probably am not going to have different answers from many other people unless, I mean it really depends a lot on what the problem is, so sometimes there are problems where I can see 'aha' there is a particular tool that I'm going to need, if not, then try to either prove special cases, you know, like I don't know, right? If there is a problem that says, 'Prove this statement for all positive integers', then the sort of obvious thing to do is try to prove it for $n = 1$, and then $n = 2$, and $n = 3$, and see if you find the pattern. Sometimes when there's no easy such place to start, then maybe I would just sort of start by writing down all of the observations that I could make about the object and the problem. And sometimes some strategy is, well I can go look at a problem and say, 'Okay, this looks like something that I want to prove by contradiction.' You know, it's hard to say things of much more detail than those kinds of, sort of scenarios, kind of all answers, unless I have a particular problem in front of me.

14. Typically when mathematicians begin to solve problems they examine analogous cases to get a feel for the problem. When you solve a Putnam problem, do you use a similar strategy or are there other methods you employ?

    Yeah, that's I guess one that I'm sure how to answer. You know, I think anything that you said that starts, 'Typically mathematicians solve problems this way', would probably apply to me also. But it depends on what the problem is, and sort of how, whether it invokes some particular tool in my toolbox, where I can just say, 'I know exactly what to do, I'll solve it this way' or whether, if I don't know what to do then I probably will try to find things that I have seen before or that I know how to reason about, that look similar to that problem.

15. Do you still do Putnam problems? Why? Why not?

    So if the question is, 'Do I look at a Putnam exam and solve the problems on it?' I have not done that for several years. And I have not done this with other contests, so I think I also haven't looked at like the IMO for the last

couple of years, but for pretty much every year until maybe two years ago, I was doing that. And I still write problems for contests, so I send a few problems to the USAMO or the IMO each year. A couple of my problems have been used on those exams as well. Let's see, I mean the other thing that I could say is, as an [redacted] I do things sort of on a day-to-day basis, they're not problems that I would put on the exam because they generally involve too much notation to state conveniently, but they sometimes have the feel of contest problems. You know, here's some cute mathematical statement that I want to prove, and the proof might involve doing something a little bit surprising and the techniques are relatively all memory.

16. What role does intuition play in solving Putnam problems?

So I'm not sure how to interpret this unless it's a dichotomy or unless I'm asked to make a comparison between intuition and something else. You know I think problem solving is a lot about intuition, it's like any other creative field that you know, as you learn more and more you acquire the ability to look at something and say, 'Gee, I think this will work.' You can look at a problem and say, 'Gee, I think that I'll be able to prove this statement by induction or no, I don't think that I'll be able to prove this by induction.' And you just sort of have that feeling and it's hard to articulate in full detail exactly why you get the sense from, based on comparisons between this and other things in your memory that sort of invoke or you know you can look at a problem and say, 'This is likely to be a hard problem or likely to be an easy problem.' I think there's quite a lot of intuition involved. It's probably, so I'm not sure that it's true for the Putnam specifically, but certainly for other competitions. Like for the IMO say, I would say that intuition probably plays a smaller role relative to accumulated knowledge than it would have say 20 or 30 years ago, just because the amount of accumulated knowledge, and theorems, and vocabulary that is competing in these things, mastery is much, much higher than it used to be, just because once they're gotten, once there got to be a lot of these contests, as well as things like the Internet where you could access a lot of, a large body of knowledge cheaply and easily then it got to be a lot of stuff that sort of let people know. And writers of these problems always want to write problems that are dependent on intuition or creativity and not dependent on pre-existing knowledge to whatever extent they're able, so that it gets harder and harder as the sort of fruit gets plucked away. And that's the sort of widely known tools become expensive enough that

they're not that many more problems that require new insight to solve or that require only insight.

17. Do you believe your extraordinary talent as a Putnam Fellow is innate or can it be taught?

    You know there's a couple of ways of interpreting that question. So one way is, you know, if you're a high school student trying to get to be good at problem solving, can you do it? And I would say, 'Yes, certainly.' I'm, probably not the right, the best person to ask for an opinion about this sort of thing, you know you want to talk to people who have done systematic educational studies or perhaps you know more about this than I do. Do I speak from my own personal experience rather than you know from doing any controlled experiments or large-scale field studies? My sense is, you know, yes you can certainly learn to be good at problem solving to an extent, but probably very early influences make a difference. I think that yes, the fact that I was learning arithmetic when I was three years old and thinking about logic puzzles when I was six, was undoubtedly important for me. If I had not learned any math until I was in middle school and then tried to become an IMO team member, it would have been a lot harder. So then that's sort of one interpretation, maybe it's not exactly what you had in mind. Another question, another way I could interpret your question is simply, you know, do I think that problem-solving ability is at least partially genetically predetermined? And that, I don't see any fundamental principles of the universe that would say that it can't be, but it also seems very hard to prove one way or the other just because any test that you would do for that is likely to mix in so many other cultural factors. And also might not be all that useful of a question to ask in the first place.

18. To be a multiple Putnam Competition winner is an extraordinary achievement. Is there anything you would like to share that I did not ask that might shed light on your success as a Putnam Fellow?

    I'm thinking about this. No, I don't think that I have anything earth shaking to add that hasn't already come up in this conversation, sort of like as a high school and college student I was kind of the guy who solved math puzzles for a living. I didn't do a whole lot else. I guess I did do a little bit else, but not at

that level. I worked on contest problems everyday, at some point accumulated the 10,000 hours or whatever. And at some point it was just sort of like, you know, 'This is my job' not exactly a nine to five job, but it was sort of like, 'This is what I know how to do and this is what I do.' And I stress I sort of pretty quickly got sick of celebrating it, sort of like, you know, 'Let me do my thing.' You know, I feel like that I've already said, you know, I think there's a whole lot of different ways that one can be successful in mathematics never mind, you know, talking about lots of ways that one can be successful in life. And you know, there's contest problem solving with sort of one particular idiosynchronatic kind of skill and I happen to be one person who developed that skill. You know, it's not even necessarily a particularly strong predictor of research success in mathematics, you know, I guess you probably know that better than I do if you have been looking at the subsequent careers of Putnam Fellows.

Appendix J

Text Matrix for Research Question 1

What personal experiences do Putnam Fellows identify as influential in their success on the Putnam Examination?

Participant	(PE1) Households Conducive to Learning	(PE2) Influential Family Members and Family Friends	(PE3) An Interest in and a Talent for Mathematics at a Young Age	(PE4) Access to Educational Resources
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Appendix K

Text Matrix for Research Question 2

What formal educational experiences do Putnam Fellows identify as influential in their success on the Putnam Examination?

Participant	**(FE1) Influential Teachers and Individuals in Academics**	**(FE2) Participation in Mathematical Contests and Competitions**	**(FE3) Access to Mathematical Contest and Competition Problems**	**(FE4) Participation in Extracurricular Mathematics Training**
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Appendix L

Text Matrix for Research Question 3

What role does the affective domain play in the development of a Putnam Fellow?

Participant	(AD1) Beliefs About Confidence	(AD2) Beliefs About Natural Ability, Aptitude, and Talent in Mathematics	(AD3) Beliefs About Having an Interest in Mathematics and Liking Mathematics	(AD4) Beliefs About the Role of Intuition
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What role does the affective domain play in the development of a Putnam Fellow?

Participant	(AD5) Beliefs About Talent	(AD6) Feelings Experienced When Solving Putnam Problems	(AD7) Motivation
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Appendix M

Text Matrix for Research Question 4

What role does the cognitive domain play in the development of a Putnam Fellow?

Participant	(CD1) Alternative Methods to Solve Mathematics Problems	(CD2) Similarities Between Putnam Problems and Other Mathematics Problems	(CD3) Analogous Cases as a Strategy to Get a Feel for Putnam Problems
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