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Substantive Theories of Epistemic Justification: An Exploration of Formal Coherence Requirements

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Michael Hughes, PhD

University of Connecticut, 2015

ABSTRACT. Are there formal coherence constraints governing categorical belief? If so, what are they? Those who answer the first question affirmatively typically hold that categorical belief is governed by logical consistency and closure principles. However, such principles are difficult to maintain in the face of the epistemic inconsistency paradoxes. The debate on this issue usually revolves around the question of whether deductive logic can be afforded a significant enough role in guiding rational inquiry. We shall take up these questions from a different angle. Various substantive theories of justified belief have been thought to carry commitments to logical consistency and closure principles (e.g., coherence theories of epistemic justification, permissibility theories of justification, etc...). On the one hand, such commitments about the nature of justified belief might explain why we should be committed to consistency and closure principles, or they might be taken as a *reductio* of the theories in question. Our primary aim will be to determine what, if any, formal coherence requirements can be derived from plausible substantive commitments regarding the nature of justified beliefs.

Substantive Theories of Epistemic Justification:
An Exploration of Formal Coherence Requirements

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APPROVAL PAGE

Doctor of Philosophy Dissertation

Substantive Theories of Epistemic Justification: An Exploration of Formal Coherence
Requirements

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CHAPTER 1

Formal Coherence Norms and Categorical Belief - The Basics

The primary concern of this dissertation will be to determine whether there are reasons for holding that there are logical consistency and logical closure requirements that follow from substantive analyses of epistemic justification. The starting point for our inquiry is two notions of belief and corresponding notions of justified belief. One notion is about an attitude toward propositions that comes in degrees, the other is an attitude that is binary or categorical. We shall refer to these as *degrees of belief* and *categorical belief*, respectively. Some of the central questions in contemporary mainstream epistemology pertain to the relationship between these two notions of belief, and whether there are formal coherence requirements governing them. In answer to these questions, there is a certain naïve view about the formal coherence norms to which categorical belief is subject, and the relationship between categorical belief and degrees of belief that initially seems intuitive. While the naïve view is, as the name suggests, highly problematic, it is nevertheless a good place to start when trying to understand what if any formal coherence requirements might govern categorical belief (the question that will occupy the center of this dissertation), as it brings into focus some of the theoretical challenges that one faces when trying to answer this question.

1.0.1. A Naïve View. The naïve view I have in mind is comprised of three core theses.¹ The first thesis is that an agent's degrees of belief are subject to the laws of probability. Let us call this *Probabilism*. The idea behind Probabilism is that an agent's degrees of belief in a proposition correspond to how probable an agent thinks it is that a proposition is true. One's degree of belief is rational insofar as it is proportional to the strength of the evidence in support of the proposition that is available to the agent.² And, according to the thesis, when one's beliefs are proportioned

¹The idea that the paradoxes we focus on problematize the view I have in mind certainly doesn't originate with me. Kyburg (1961) sets out the paradox to show that principles at least very close to those I describe below are incompatible. For a slightly different way of framing the principles that the lottery paradox is intended to undermine, see Wheeler (2007).

²For arguments in favor of probabilism, see Ramsey (1928), Hájek (2008), Joyce (1998, 2009), Jeffrey (1990, 2004). And for an extended discussion of the relationship between probabilism and deductive cogency constraints on categorical belief, see Christensen (2004).

to the available evidence, one's degrees of belief would be representable by a function that satisfies the classical probability axioms (throughout we shall assume that we are dealing with classical probability functions satisfying Kolmogorov's axioms).³

The second core thesis of the naïve view is that categorical beliefs are subject to certain deductive cogency constraints (I am here following Christensen's (2004) terminology). More precisely, the thesis is that there is a global consistency requirement on epistemically justified beliefs, and that justified beliefs are closed under recognized logical entailment. Of course, we could spend a great deal of time worrying about how exactly to formulate the target closure principle. One might hold that justification is closed under recognized logical entailment, or add additional restrictions to the antecedent of the closure principle. We will sidestep many of these worries, by primarily focusing on a deductive consistency requirement. It is the weaker and more plausible of the two requirements (We shall assume throughout that the logic is classical). The primary motivation for a logical consistency requirement is that it helps to explain the normative role of classical logic in guiding us in our belief forming practices, and helping us to see when a rational change in view is in order.

The third thesis of the naïve view pertains to the relationship between justified categorical belief and rational degrees of belief. A thesis held by many epistemologists is that justified categorical belief and an agent's rational degrees of belief are related by a threshold principle:

Threshold Principle: An agent, S , is justified in (categorically) believing a proposition, p , when S is rational to have a degree of belief in p above a threshold, t .⁴

Such a principle entails that justified categorical belief is just a special case of having a rational degree of belief, and thus explains in a straightforward way how the norms governing the two notions are connected. The underlying picture is that we aim to proportion of degrees of belief in p to the evidential or epistemic probability of p , given the epistemic position we occupy with respect

³See Section 1.4 for a formalization of the Kolmogorov (1956) axioms.

⁴This is sometimes referred to as 'The Lockean Thesis.' See Foley (1992, 2009), James Hawthorne (2009), and Christensen (2004) for thorough discussions of this thesis.

to p .⁵ Since we aim to believe only propositions that are true, we believe those propositions that are highly probable, given one's total evidence.

Now, the three theses aren't naïve because each principle is counter-intuitive when considered individually, but rather because however much *prima facie* intuitive appeal each thesis may enjoy on its own, the combination of these three principles stand in a deep tension with a fallibilist theory of epistemic justification.⁶ The main way to see why a fallibilist will have difficulty adopting the naïve view is made clear by considering the epistemic inconsistency paradoxes.

1.0.2. Epistemic Inconsistency Paradoxes. There are two familiar epistemic inconsistency paradoxes that make plain the problems with the naïve view. The first paradox was discovered by Henry Kyburg (1961), and is generally known as the lottery paradox. The second paradox we shall consider is D.C. Makinson's (1965) preface paradox. There are versions of both paradoxes that show that a fallibilist threshold principle entails that it is possible for each member of an inconsistent set of propositions to enjoy an epistemic probability that exceeds that threshold. But, then the agent will be justified in believing an inconsistent set of claims, and so the deductive cogency thesis is subject to counter-example. We consider many different variations of the paradoxes in chapters to come.

⁵Throughout I will talk of one's epistemic position as a way to try to stay neutral between various theories of evidence, justification, etc... that might impact exactly what factors are relevant for determining the degree of belief that one should assign to a proposition. We shall refer to the probability one should assign, given the relevant facts, the epistemic probability for an agent. When the agent is proportioning her degree of belief to the evidence, her degree of belief and her epistemic probabilities will align. When we speak of an agent's *rational degree of belief*, we will mean degrees of belief that align with the agent's epistemic probabilities. This is to allow that one might be more confidence in the truth of some proposition than one's evidence warrants.

⁶Leitgeb (2014) has recently proposed a radical way to avoid the tension, which we shall set to one side for the present discussion. In a nutshell, Leitgeb's (2014) proposal is to allow for the threshold to be determined by features of the probability function itself, features that will always ensure that an agent's beliefs satisfy closure and consistency principles. Leitgeb observes that for any probability function Pr , there exists a range of values such that as long as the threshold is above those values, closure and consistency principles will be satisfied. If we hold that the only acceptable thresholds fall within that range, then the three theses of the naïve are jointly satisfiable. This rescues the naïve view from having any external contradictions, but the solution comes at some significant costs, one of which Leitgeb helpfully summarizes: 'The range of permissible choices of threshold in the Lockean thesis codepends on the agent's degree of belief function P .' (2014, p. 149) This sort of codependence strikes me as highly implausible, and it seems especially implausible to me that ordinary agents consider anything like whether the threshold in question will yield closure and consistency principles before determining how high their degree of belief must be before they count as having a categorical belief. This I find to be the most significant problem that Leitgeb raises for the view, and it leads me to be suspicious of the view. At the same time, I am not ready to formulate anything approaching a clear objection based on this codependence, and thus think the best I can do is leave this particular proposal as something to consider in future research.

1.0.3. Kyburg's Lottery Paradox. Kyburg's paradox is just this. Suppose there is a lottery with n tickets, and that the tickets will be drawn at random, yielding exactly one winner. We suppose that we are absolutely certain about these parameters. Now, we can define the problematic set of lottery propositions as follows:

$$l_i = i^{th} \text{ ticket in the lottery is a loser.}$$

The claim that the lottery has a winner is materially equivalent to: $\neg(l_1 \wedge \dots \wedge l_n)$, which is just the claim that it is not the case that all of the tickets will lose. Now, by hypothesis: $Pr(\neg(l_1 \wedge \dots \wedge l_n)) = 1$. And, we have it that

$$Pr(l_i) = \frac{n-1}{n}.$$

For any threshold t where $t < 1$, there is an n s.t. $t < \frac{n-1}{n}$. Thus, by letting n be large enough, each member of the following set has a probability exceeding t :

$$\text{Lottery Set} = \{l_1, \dots, l_n, \neg(l_1 \wedge \dots \wedge l_n)\}.$$

Now, this set of claims is logically inconsistent and transparently so, i.e., any agent with minimal deductive reasoning skills will recognize said inconsistency. And yet, according to the threshold principle, an agent whose credence function aligns with the probabilities (both epistemic and presumably objective probabilities) of each proposition, an agent will believe each member of the Lottery Set and be rational in doing so.

1.0.4. Mackinson's Preface Paradox. D.C. Mackinson's Preface Paradox provides a similar lesson, but in some ways the initial problem is very different. For starters, Mackinson's paradox is not initially stated in precise formal terms the way that the lottery paradox is. Instead, Mackinson simply asks us to consider a historian who has written a lengthy text. In the text, she makes a large number of assertions, each of which she has carefully investigated and for which she has as good of evidence as one can reasonably expected to have for any thing she asserts. Of course, most of the assertions, like most propositions about the empirical world beyond one's immediate perceptual state, are about matters that one cannot be absolutely, positively certain. The author, recognizing her fallibility on the matters she is discussing, confesses to her readers that she believes that at least some errors, i.e., false assertions, remain to be discovered in the body of the text. The

author says this on the basis of a historical track record of other historians failing to reach a level of perfection, and the recognition that she is no more likely to have obtained perfection as any of those historians who have preceded her. Her claim in the preface seems eminently reasonable and fairminded, and yet they too pose a problem. The historian believes each of the claims she has made in her book, but she also believes her preface. And, these claims are logically inconsistent with one another.

1.0.4.1. *Various Versions the Preface Paradox.* There are a variety of ways of revising the preface example to make the challenge it poses to the deductive cogency thesis even more robust. Which version will be relevant for our purposes may depend on the particular theories of justification that we wish to consider. When presented as a counter-example to certain formal principles connecting categorical belief and degrees of belief, the paradox is often presented in terms of a large set of probabilistic independent propositions with a low joint probability.⁷

But the paradox can also be set up as a skeptical challenge to all of one's beliefs by being formulated in terms of agent's total belief system. That is to say, instead of considering an author's claims in the body of some book, we can consider the large body of claims that an individual accepts. It is clear that for large systems of belief, there is an extremely high chance that it will contain at least a few false propositions.⁸ Hence, most ordinary people should be in position to reflect on their own fallibility, and the large number of claims they accept, and recognize that their almost certainly a few false beliefs among them. But then if they come to accept this claim, they will arrive at an inconsistent set of judgments. Hence, the preface paradox can be set up to suggest that virtually all agents, not just historians who write lengthy books, are rational to hold at least some inconsistent beliefs.

1.1. Taxonomy of Responses to the Inconsistency Paradoxes

Now, all epistemologists can agree that the inconsistency paradoxes demonstrate that there is something fundamentally wrong with the naïve view. There is not agreement, however, on what exactly is wrong with the naïve view, what lessons should be drawn from the inconsistency paradoxes, nor

⁷This is the version discussed by Olsson (1998), Lehrer (1990), which we shall consider in detail in Chapter 2.

⁸This version will be our primary focus in Chapter 3 when we assess permissibility solutions to the inconsistency paradoxes.

what those paradoxes tell us about the formal coherence norms (if there are any) governing categorical belief. While there is a vast number of ways of dividing up the reactions philosophers have to the inconsistency paradoxes, the most general, and for our purposes important, division is between philosophers who defend deductive cogency principles in the face of the paradoxes, and those who are willing to countenance justified inconsistent beliefs in at least some cases – they need not accept that one can be justified in believing the propositions for both paradoxes.⁹ We shall refer to the former views as *deductive cogency solutions*, and the latter *inconsistency solutions*.

There is a third position that falls somewhere in between the standard reactions that should not be overlooked. A variety of epistemologists have recently defended, so called, *permissibility solutions* to the lottery paradox, a view according to which one can have permission to believe each member of inconsistent set of claims at a time, but one can never be justified in believing all members of that set at one time.¹⁰ According to proponents of *permissibility solutions* to the inconsistency paradoxes, the permissions to believe inconsistent claims is similar to permissions we might have to perform actions that are acceptable when done individually, but not collectively. It is clear, for instance, that certain legal permissions, like the permission to drink and the permission to drive do not agglomerate into a legal permission to drive drunk. Proponents of permissibility solutions hold that the high probability of lottery propositions provide us with permission to believe each of them, but these permissions do not agglomerate, so we are not permitted to believe them all at once. Permissibility solutions thus agree with defenders of deductive cogency constraints that there is a rational obligation to avoid inconsistent belief, while siding with proponents of an inconsistency response that one can have justification for each member of an inconsistent set of claims. Before setting out the aims of the dissertation, it will be helpful if we briefly review the central theoretical challenges to each kind of response to the epistemic inconsistency paradoxes.

1.2. Deductive Cogency Solutions - The Skeptical Challenge

The main challenge for those who wish to defend a deductive cogency solution is to provide an explanation for why one cannot be justified in accepting each member of an inconsistent set of claims,

⁹I should also note that there are, of course, views that do not fall neatly onto either side of this division. Some epistemologists like Jeffrey (1990, 2004) are eliminativists about categorical belief, and thus reject the presupposition that there exists categorical beliefs that might be subject to certain formal coherence requirements.

¹⁰The view has since been argued for by Ross (2003), Douven (2008), Kroedel (2012, 2013a, 2013b).

and to do so in a way that avoids collapsing one's theory of justification into infallibilism/skepticism, i.e., entailing that all but epistemically certain claims are justified. It should already be clear how deductive cogency responses to the preface paradox threaten to collapse one's view into skepticism. The grounds for denying that one cannot be justified in believing the preface proposition, the claim that one has at least some false beliefs, must not apply equally well to the propositions in the body of the book. But the threat of skepticism looms just as large in the case of the lottery paradox, and it will be informative to see how one of the most common approaches to the lottery paradox winds up collapsing into skepticism for the same reason.

There are a family of responses I have in mind, which come very close to the naïve view we considered above. They simply hold that the threshold principle was mistaken, and that a nearby principle provides the actual bridge between rational degrees of belief and justified belief. These proposals have been explained by Douven (2012) as follows: '[some epistemologists] have thus sought to formulate variants of [the threshold principle] that let high probability still defeasibly warrant acceptance, where the defeater is supposed to apply as selectively as possible to lottery propositions' (2012, p.55).¹¹ Douven & Williamson (2006) suggest that such solutions can be represented schematically:

NJ-Schema: p is rationally acceptable if $Pr(p) > t$, unless defeater D holds of p .

The goal of such solutions is for D to only apply to a very limited set of propositions, and, in particular, those propositions that would otherwise provide a counter-example to deductive cogency constraints on epistemically justified beliefs. The challenge, of course, is to provide a precise specification of D that delivers a logical consistency requirement without applying to the majority of propositions about which we are justified in believing, but not rational to be epistemically certain about. And there are, of course, a variety of kinds of properties that one might rely upon to define D . Douven and Williamson (2006) note that many have attempted to define a defeater condition in terms of probabilistic and logical relations between propositions, and Douven and Williamson show that all such proposals collapse into infallibilism and thus confront the skeptical problems known to be associated with infallibilism.

¹¹See Douven & Williamson (2006, 758) for similar remarks.

Douven's (2002) solution is representative of the sort of approaches that belong to this family, and considering it will help us see ways in which the lottery might be generalized. Douven and Williamson (2006, p. 759) explain Douven's (2002) proposed defeater condition as follows:

being a member of a probabilistically self-undermining set, where a set of propositions Φ with cardinality $|\Phi|$ is defined to be *probabilistically self-undermining* iff for all $\varphi \in \Phi$: $Pr(\varphi) > t$ and $Pr(\varphi|\Phi - \varphi) \leq t$ (where $\Phi - \varphi$ is the conjunction of all members of Φ except φ).¹²

The virtue of this proposal is that it provides a general explanation for why we cannot be justified in believing an inconsistent set of claims. Every inconsistent set will either have members whose probabilities fall short of the threshold, or else it contains some subset that is probabilistically self-undermining in the sense defined above. The problem with this condition, and the others like it, is that the lottery paradox can be easily generalized so that this condition applies to any arbitrarily proposition that is less than epistemically certain. For discussion in later chapters, it will be useful to consider a generalized version of the lottery paradox that demonstrates why the above proposal is unacceptable.

1.2.1. Generalizing the Lottery Paradox - An Example. Here is how Douven and Williamson (2006, p. 760) generalize the lottery argument for Douven's proposal. In order to generalize the lottery for Douven's (2002) proposal, let p be any arbitrarily chosen proposition that enjoys epistemic credentials that make it a good candidate for being epistemically justified, but that falls short of being epistemically certain (that is, $Pr(p) < 1$). Now, let there be a standard lottery composed of lottery propositions, $l_1 - l_n$, defined as above where $Pr(l_i) = n^{-1}/n > t$ for each $1 \leq i \leq n$. Next, we form generalized lottery propositions as follows:

$$G_i = l_i \vee \neg p.$$

And, now note that the set: $\{G_1, G_2, \dots, G_n\} \cup \{p\}$ is a probabilistically self-undermining set in Douven's (2002) sense. Consequently, if being a member of such a set is sufficient to prevent a proposition p from being epistemically justified, then it follows that p must not be justified, despite

¹²Douven and Williamson (2006, p. 759) also consider a number of attempts to define a defeater condition very close to this one. These proposals include Pollock (1995), and Ryan (1996). Lehrer's (2000) solution is similar, but isn't purely probabilistic.

its solid epistemic credentials.¹³ And, of course, there was absolutely nothing special about p , other than that it wasn't epistemically certain, and so we arrive at the result that only epistemically certain propositions are not subject to a generalized lottery argument on Douven's (2002) proposal.

1.2.2. Probabilistic Defeaters and Skepticism. Now, Douven and Williamson (2006) and Smith (2010) establish that any defeater condition defined exclusively in terms of probabilistic and logical relations between propositions that aimed to rescue consistency and closure principles will succumb to some version of a generalized lottery argument.¹⁴ That is, a purely probabilistic condition will either allow for some cases of justified inconsistent belief, or else rule out the possibility of justified belief in propositions that are less than epistemically certain. This means (i) we can rule out all attempts to argue for deductive cogency principles in terms of purely probabilistic epistemic conditions on belief that are fallibilistic, and (ii) that we should be on the look out for ways that attempts to define defeater conditions might succumb to similar generalizations of the lottery paradox.

It is worth noting briefly what these results will mean for the views we shall consider in later chapters. Both Lehrer (2000) and Fantl and McGrath's (2002, 2009) views of epistemic justification can be looked at as providing defeater conditions to be plugged into Douven and Williamson's NJ-Schema. On both views, one can accept that there is a high probability requirement, but also hold that additional conditions besides having a high probability need to be met in order for a proposition to be justified. The key thing is that the defeater conditions employed in Lehrer's (2000) theory of personal justification, and that the pragmatic conditions on justified belief employed in Fantl and McGrath's (2002, 2009) pragmatic encroachment theory both rely on decision-theoretic resources that go well beyond the probabilistic relations considered by Douven and Williamson (2006) and Smith (2010). Consequently, the generalized lottery arguments do not immediately apply to either of these proposals, and so the question of whether Lehrer or Fantl and McGrath's proposals rule out justified inconsistent belief and/or are subject to a generalized lottery argument is something that needs to be investigated.

¹³This same example will apply to the simplified version of Pollock's (1995) and Ryan's (1996) proposals that Douven and Williamson (2006) entertain.

¹⁴Douven and Williamson's (2006) results are limited to probability spaces that are uniform and finite. Smith (2010) extends the result to infinite and atomless probability spaces. See Chandler (2010) for a philosophical discussion of the significance of the limits of Douven and Williamson's (2006) results.

Another thing to note is that Douven and Williamson's (2006) results restrict the plausible application of probabilistic measures of coherence (Something that we shall consider in Chapter 4). One cannot employ probabilistic measures of coherence to define a defeater condition a la Douven and Williamson's (2006) schema without either failing to rule out justified inconsistent belief, or else entailing infallibilism. Nevertheless, probabilistic coherence measures may be employed in a principle underwriting a permissibility solution, i.e., a principle that holds that one should only believe propositions that are globally coherent. If the probabilistic measure of coherence entailed a logical consistency constraint, then such a principle may perhaps be employed by coherentists to explain the non-agglomeration of epistemic permissions. And, it is for this reason that we might consider whether probabilistic measures of coherence entail that consistency is required for coherence.

1.2.3. Permissibility Solutions - The Explanatory Challenge. Permissibility solutions are importantly different from deductive cogency responses in that they do not hold that inconsistent sets of claims must contain at least some members that lack justification. In fact, one of the core assumptions that all permissibility solutions to the lottery paradox share in common is the assumption that each lottery proposition for a large enough lottery is justified, i.e., one has permission to believe each lottery proposition. It is just that one doesn't have permission to believe all of them at any one time. Permissibility solutions thus do not seek to identify some defeater condition that fits Douven and Williamson's schema, and the permissibility theorist thus doesn't owe us any explanation as to why inconsistent sets of claims must contain at least some members that lack justification. The explanation the permissibility theorist does owe us is why inconsistency is something that epistemic rationality require us to avoid. While the permissibility solution may avail herself of principles that appeal exclusively to the logical and probabilistic relations between propositions without automatically succumbing to problems posed by a generalized lottery, the permissibility solution still must avoid principles that have implausible skeptical implications. But we shall wait to consider the permissibility theorist's explanations and potential skeptical problems until we turn our full attention to permissibility solutions in Chapter 3.

1.3. Inconsistency Solutions - The Role of Logic Challenge

In order to set the stage for our later exploration, we will need to spend some time getting clear on what has often been presented as the central challenge to inconsistency solutions to the lottery

and preface paradoxes: that is the challenge of accounting for the role of logic in rational inquiry. Some epistemologists have thought that if the principles of epistemic rationality tolerate some inconsistency (i.e., one can accept an inconsistent set of claims without being irrational), then we cannot assign logic any important role to play in constraining rational belief or in guiding rational inquiry. And, yet, the practice of presenting deductive arguments seems to be an important means of rationally persuading both ourselves and others to change our beliefs. But how can an argument, like a *reductio ad absurdum*, provide one with grounds to undergo a rational change in view if one can be justified in accepting an inconsistent set of claims? Christensen, I think sums up the essential worry well while describing the views of Pollock (1986, 1995) and Kaplan (1996, p. 97), two staunch defenders of deductive cogency principles:

Thus, for both writers, the challenge of accounting for the rational force of arguments should be understood as the challenge of accounting for the way in which rational belief seems to be conditioned synchronically by deductive logic. (Christensen, 2004 p. 80)

It is this challenge that most commonly leads philosophers to persist in holding that we need some account of belief or epistemic justification that delivers deductive cogency requirements.

However, proponents of inconsistency solutions to the lottery or the preface have explanations to offer for the normative force of arguments. The explanation they can give depends in part on how far their theories of justified belief diverge from the naïve view we considered at the very outset. Some epistemologists think that the only thing wrong with the naïve view was the assumption that categorical belief is subject to deductive cogency constraints. They thus hold that justified categorical belief is connected to rational degrees of belief by a threshold principle, and that degrees of belief are subject to probabilistic coherence constraints.¹⁵ Such epistemologists can follow Christensen (2004) in accounting for the normativity of logic by appealing to the ways that logic constrains rational degrees of belief. Christensen denies that there are any global coherence requirements on categorical belief because he favors eliminativism for categorical belief. But, some epistemologists hold that categorical belief is related to degrees of belief via a threshold principle

¹⁵In effect, such a position would be to accept the view presented by Christensen (2004), but to resist Christensen's insistence that categorical belief has no significant theoretical role to play in our theory or rational belief and rational agency. This seems to be the position of Weintraub (2001), Fantl and McGrath (2009) (See Ch. 5 for an extended discussion).

and yet insist that categorical beliefs nevertheless shouldn't be eliminated from our theory of rational agency. Weintraub presents a compelling case for why 'categorical belief should survive the Bayesian revolution,'¹⁶ even if one accepts a threshold account of belief. She also presents provides a clear presentation of why *reductio* arguments can be rationally persuasive that fit well with the view we shall consider below. And, Fantl and McGrath (2009) put forward a their theory that can be understood as non-eliminativist threshold view (something we shall consider in great detail in Chapter 5).

But proponents of inconsistency solutions aren't wed to a threshold principle, or probabilism about degrees of belief. Easwaran and Fitelson (in press) and Easwaran (Manuscript) show how one can derive formal coherence constraints on categorical belief from some highly plausible evidentialist principles on epistemic justification. They thus show that holding that categorical belief is a *sui generis* kind, i.e., that categorical belief is not merely some special kind of degrees of belief, is compatible with holding that there are formal constraints on belief in the sense that whether an agent's beliefs satisfy these constraints depends solely on the logical relations between the contents of the agent's beliefs. Thus, in either case, the inconsistency solution can be combined with plausible explanations for how logic impinges on epistemic rationality, why logical consistency is relevant to being rational, and for how logical arguments can be thought to synchronically constrain justified belief. We shall take the different sorts of explanations inconsistency proponents can give for explaining the role of logic in turn, starting with the explanation available to those who accept a threshold principle.

1.4. Probabilist Accounts of the Role of Logic

We start by considering the explanation for the normative force of arguments that the following two principles can deliver.

(Threshold Principle): binary belief or the norms of binary belief supervene on degrees of belief or the norms governing rational degrees of belief in the sense that to be rational to believe p one must be rational to assign p a probability above a certain threshold.

¹⁶Weintraub (2001, p. 440)

(Probabilism about Degrees of Belief): The laws of probability provide basic synchronic formal coherence constraints on degrees of belief.

Throughout this dissertation, we shall appeal to *the laws of probability, probability functions*, etc... . We here set out the Kolmogorov axioms that we shall use to interpret probability functions. We assume that a probability function is any function, Pr , that is defined over a Boolean algebra of propositions that satisfies the following three axioms Kolmogorov (1956).

Axiom 1: For all propositions p , $Pr(p) \geq 0$.

Axiom 2: $Pr(\top) = 1$ (all tautologies are maximally probable).

Axiom 3: For all p_1, p_2 , if $p_1 \wedge p_2 \vdash \perp$, then $Pr(p_1 \cup p_2) = Pr(p_1) + Pr(p_2)$.

We could strengthen the third axioms to hold that probability functions are countably additive, i.e., that it holds over all sets of mutually exclusive propositions that are countable. Such a strengthening is consistent with all the discussion to come, but unnecessary. Thus, we shall stick to the simpler assumption.

Now, we might ask: What explanation does the probabilist have for our tendency to think that inconsistent beliefs are some how flawed, and that an agent with inconsistent beliefs is under rational pressure to undergo some kind of change of view? Assuming the threshold for rational belief is below 1, probabilism does not entail a general logical consistency requirement. But, as observed by Kyburg (1970), it does entail that accepting a small set of inconsistent claims is rationally unacceptable. In particular, for any threshold t , the probabilist must accept an n -wise logical consistency requirement on rational belief:

For all n such that $\frac{n-1}{n} < t$, S is not rational to believe all the members of any set B where $|B| \leq n$ and B is logically inconsistent.

Such a principle will entail that small sets of inconsistent beliefs are rationally unacceptable for the simple reason that one's degree of confidence in all of the members of the set cannot meet the threshold t . So, for most of everyday reasoning where we are only entertaining relatively small sets

of propositions, inconsistency amongst the sets will entail that an agent cannot rationally believe all of them.¹⁷

Now, it is doubtful that there is one threshold t that holds in absolutely all epistemic contexts. What factors play a role in determining the threshold is certainly up for debate. In Chapter 5, we shall consider one proposal for answering the threshold problem that strikes me as highly plausible (Fantl and McGrath's (2009) pragmatic encroachment theory).

1.4.1. Arguments as Guides to Belief. As was reflected in Christensen's explanation of Kaplan and Pollock's view above, a common thought is that arguments somehow play a guiding role in what we should believe. Of course, limited consistency requirements show how logic can play a role in guiding us away from certain patterns of belief. But many have thought that deductive arguments can also serve to guide us toward certain beliefs. Paradigmatically, when trying to persuade ourselves or some interlocutor to add a new proposition, q , to one's stock of beliefs, we regularly try to identify premises p_1, \dots, p_n that one already accepts and that deductively entails q . Is there any principle that would explain how logic can serve as such a guide in this manner? The most obvious principle, and commonly defended in answer, is a multi-premise epistemic closure principle, i.e., a principle that says that justified belief is closed under known logical entailment.¹⁸ Obviously, such a principle is difficult to reconcile with a threshold principle.¹⁹ To see how the probabilist can explain the manner in which logic can serve as a guide to belief, it will be informative to consider why one who accepts a multi-premise epistemic closure principle cannot accept that one can be justified in holding an inconsistent set of claims.

A proponent of multi-premise epistemic closure will argue that if one can be justified in accepting an inconsistent set of claims, then one is justified in believing any claim that logically follows from that inconsistent set. But, since inconsistent sets entail any arbitrary proposition whatsoever,

¹⁷See Christensen (2004, p. 26) for further discussion of n -wise consistency requirements and their intuitiveness and theoretical usefulness.

¹⁸See Hawthorne (2004, 2005) for a discussion of the merits of closure principles. See Harman (1986) for arguments against the view that one should expand one's beliefs when one discovers that one's beliefs have a certain logical entailment.

¹⁹Difficult but not conceptually impossible. Leitgeb (2014) has demonstrated that one can combine a threshold principle with a closure principle by making the threshold for a particular probability function at least partially dependent on whether the threshold will yield a set of categorical beliefs that satisfy deductive cogency principles. This dependence seems counter-intuitive to me, but I have nothing further to say on the matter. Dialectically, I suppose that if one is inclined to follow Leitgeb, then whatever motivations for a consistency requirement that one might find from substantive analyses of epistemic justification could be taken reason to prefer Leitgeb's solution to the inconsistency paradoxes. Alas, I don't see that there is much more to say about his proposal.

it follows that someone with inconsistent beliefs is justified in believing any arbitrary conclusion whatsoever! Now, this is clearly unacceptable. Dialectically, such an argument isn't going to be rationally persuasive to the proponent of an inconsistency solution precisely because she wouldn't find closure plausible. But for our current purposes, it is more important to note a lesson Mylan Engel says can be drawn from this sort of defense of a logical consistency requirement. He says,

Such an objection [to the possibility of justified inconsistent belief] is, however, fundamentally misguided, for while it is true that everything follows *validly* from an inconsistent set of premises, nothing follows *soundly* from such a set of premises, and only apparently sound arguments should be our guide. (1991, p. 127)²⁰

His diagnosis of the argument for a consistency requirement carries over to any epistemic closure principle of the form:

Validity Closure Principle: VCP: If S justifiably believes all $p \in X$ and recognizes that the argument from X to q is valid, then S is justified in adding q to her stock of beliefs (or else subtract from her beliefs in X).²¹

But his observation also suggests a principle in the neighborhood that can be adopted, even by agents who sometimes rationally accept a logically inconsistent set of claims. We just replace the notion of validity with soundness to get:

Soundness Closure Principle: SCP: If S justifiably believes all $p \in X$ and is justified in thinking the argument from X to q to be sound, then S should be justified in adding q to her stock of beliefs (or else subtract from her beliefs in X).²²

²⁰If one were a paraconsistent logician one might try to diagnose the problem with the argument by rejecting the validity of the particular argument form in question, namely, explosion. It may well be that certain instances of explosion are invalid in certain contexts, for instance, in the case of the semantic paradoxes. Ultimately, I think Steinberger (forthcoming) has made a compelling case that if there are motivations for denying explosion, they cannot be traced to the inconsistency paradoxes that we will be focusing on. Consequently, I am not sympathetic to an adoption of paraconsistent logic in the context of the epistemic inconsistency paradoxes. We shall thus set such a solution to the side throughout.

²¹Ultimately, this means that we shall be rejecting any of the strong bridge principles considered by Macfarlane (2004). That one believes a set of premises and sees that they entail some conclusion in no way warrants, obliges, or gives them a good reason to accept the entailed claim in question.

²²Again, this is not to disagree with Steinberger (forthcoming): Such a principle would be far too weak to serve the purpose of the paraconsistent logician who wants to motivate a rejection of classical logic via the epistemic inconsistency paradoxes.

The key idea here is that deductive arguments from premises that one accepts provide reasons to accept the conclusion only if the argument appears to be a sound argument. This principle is much weaker than the sort of closure principle needed to motivate a deductive consistency requirement. And, just as importantly, it is roughly the form of a principle that the probabilist can straightforwardly accept and explain.

1.4.2. Unpacking SCP. Before we consider how a probabilist can explain SCP, we should first note the way in which SCP is weaker than VCP, and alternative revisions to VCP that one might impose. We have already seen one sort of case where SCP is weaker than VCP, namely, in the case of inconsistent beliefs. Take any arbitrary claim, p , VCP entails that one who justifiably believes an inconsistent set of claims is justified in accepting p (assuming she recognizes the validity of ex falso). SCP offers no such route to justifiably believing p , and is more intuitive on that score.

Now, do the differences between the two principles arise only in cases where agent's accept inconsistent sets of claims? The answer is clearly no, and the way to see the difference is to consider a situation where an agent accepts a large set of beliefs, like in the preface case, but where she remains agnostic about the conjunction of the set of claims. That is to say, her beliefs are the set:

$$\{p_1, \dots, p_n\} \text{ where } Pr(p_1 \wedge \dots \wedge p_n) < t \text{ where } t \text{ is the threshold for rational acceptance.}$$

In a standard Preface case, we suppose that the small possibilities of error amongst the p_1, \dots, p_n aggregate to the point where she accepts a Preface proposition, that is to say, the proposition that the set contains at least some false claims. But for illustrations sake, let us suppose that the possibilities of error merely aggregate to the point where S is rational to suspend judgment about $p_1 \wedge \dots \wedge p_n$. That is to say, she doesn't, as in the Preface case, believe the conjunction to be false, but her confidence is shaken to the point where she isn't willing to commit to the conjunction either. For simplicities sake, let us suppose that the conjunction is no more and no less likely to be true than its negation, i.e., $Pr(p_1 \wedge \dots \wedge p_n) = .5$. If we are allowing that it can be rational for her to reject the conjunction when the set almost certainly contains errors, (i.e., accept the Preface proposition), then it should be equally plausible that for less risky sets she might merely suspend judgment about the conjunction.²³

²³The doxastic state we are ascribing to the agent is plausible insofar as we are not assuming a closure principle of the form VCP.

Now, in such a case, we have a situation where the agent's beliefs are not inconsistent. Nevertheless, SCP and VCP's treatments of the case come apart in dramatic ways. To see this, note that our agent might easily come to recognize the validity of the argument from $\{p_1, \dots, p_n\}$ to q , and our two principles will have very different implications for what the agent should come to believe. According to VCP, the agent will thereby place herself in position to justifiably believe q , despite the fact that her evidence makes the conjunction as likely to be false as it is to be true. VCP thus entails that in such cases the agent should come to believe the conjunction and q .

What follows from SCP in such a case? It all depends on whether we think the agent should recognize the argument as sound. She recognizes the validity of the argument, and for simplicities sake, we can suppose that she is utterly certain about the arguments logical validity. The only thing then that could stand in the way of her judging the argument to be sound is for her to fail to judge that all of the premises of the argument are true (that is, she must judge the premise set free of error). How confident should she be that the argument is sound? Given our description of her doxastic state, her confidence in the soundness of the argument should track her confidence in the truth of the conjunction of the premises. By our initial description of the case, her confidence in the conjunction should be no higher than her confidence in the falsity of the conjunction. Thus, her attitude toward the claim that the argument is sound should match her attitude toward the claim that the conjunction is true. And, as we noted at the outset, her attitude towards the conjunction is one of suspension of judgment. Thus, SCP does not entail that the agent is in position to justifiably believe q .

The main difference between this case and the Preface Paradox (where she is considering an *ex falso* argument to an arbitrary conclusion) is that, in this more recent case, our agent does not definitively judge the argument to be unsound, but rather merely fails to judge the argument sound. For the purposes for evaluating SCP, this makes absolutely no difference, as it allows in both cases for the agent to lack justification for the logical consequence of an argument that she recognizes to be valid, and that proceeds from premises that she believes to be true.

In general, the lesson from the foregoing is that an agent is rationally obliged to accept the logical consequences of any set of claims that she accepts if she (a) recognizes the argument to be valid, and (b) judges that the premise set is all true (in the sense that she judges the premise set to be free of errors, and so their conjunction to be true).

1.4.3. Probabilist's Account of SCP. How does a probabilist explain SCP? The first thing to note is that there is an analogous probabilistic principle constraining an agent's degrees of belief. This is seen by first noting important logical constraint on probability functions:

Probabilistic single premise Closure: If p logically entails q , then $Pr(q) \geq Pr(p)$.²⁴

Now, given that in classical logic any finite set X entails q if and only if $\bigwedge X$ entails q , it follows that

If X logically entails q and X is finite, then $Pr(\bigwedge X) \leq Pr(q)$.

From these two observations, it is clear that the probabilist can endorse a probabilistic formal coherence constraint on degrees of belief that is stronger than SCP. In effect, it says that an agent's degree of belief in the truth of q must be as high as one's degree of belief in the conjunction of a set of claims that entail q . If the probabilist endorses a threshold principle relating full belief to degrees of belief, we get that:

If X logically entails q , then S is rational in fully believing $\bigwedge X$ only if she is rational in fully believing q (where by 'rational' we here simply mean that her degrees of belief are probabilistically coherent and respect the threshold principle).

This principle is stronger than SCP in the sense that it places no requirement on the agent recognizing the soundness nor validity of the argument from $\bigwedge X$ to q . In effect, it entails that an agent is required to accept the logical consequences of a set if she accepts the conjunction of the claims in the set, and is subject to this rational obligation even if she fails to recognize the entailment.

One might want to weaken the antecedent by adding a recognition of the entailment clause, especially for logically complex propositions that might be out of cognitive reach for the agent. Alternatively, one might simply observe that the requirement is supposed to hold for an ideal rational agent. At any rate, we can bracket the concern of unrecognized entailments by noting that in cases where an agent does recognize that there is a valid argument from X to q , then, in the probabilist's view, the only thing that could stop an agent from being obliged to accept the logical consequences of some set of proposition X is if she fails to assign the conjunction some probability

²⁴Christensen (2005, pp. 81-88) points out a variety of ways that such a principle does explanatory work in terms of explaining how arguments can impact an agent's degrees of beliefs, and thus one's categorical beliefs.

above the threshold for rational acceptance. This is just to fail to fully believe the claim that the premises of the argument are likely to be jointly true, and so fail to judge that the argument is sound. Thus, the probabilist can hold that SCP is a true principle.

The probabilist thus has an explanation for how we are guided by deductive arguments. I agree with Christensen that it is highly unlikely that one will be able to put forward an example that shows that the probabilist cannot account for the normative role of logic. As he puts it,

For any such example will have to be one in which we think that it is rational for someone to believe the conclusion of an argument based on the argument's premises, where all the premises are necessary to reach the conclusion, and yet where we also think that it's not rational for her to be confident that the premises are all true! (2004, p. 95)

It is very difficult to imagine what such a case would look like. And, thus, if one accepts probabilism and a threshold principle, then I am inclined to think that one can explain why arguments appear to be rationally persuasive.

1.5. Easwaran and Fitelson's Accuracy Coherence Constraints

One may, of course, be inclined to reject probabilism for a wide variety of reasons. Some obvious reasons include the fact that one might find the claim that degrees of belief should satisfy probabilistic coherence constraints to be lacking in philosophical motivation.²⁵ Or one might think the assumption that we have such finely grained propositional attitudes to be overly idealized, and thus implausible as providing formal coherence constraints that govern the beliefs or degrees of belief of actual people.²⁶ Whatever the reasons are that one might reject probabilism about degrees of belief, the question we turn to next is how the proponent of an inconsistency solution to the lottery paradox could account for the appearance that there are formal coherence constraints on categorical belief. In particular, we shall now review Easwaran and Fitelson's (in press) accuracy-coherence constraints on categorical belief.

²⁵See Easwaran and Fitelson (2012) for discussion of issues relating to Joyce's (1998) epistemic motivations for probabilism. See Hájek (2008) for a discussion of problems with the standard arguments given in favor of probabilism.

²⁶See Weintraub (2001) for a good discussion of problems regarding over idealization of degrees of belief.

1.5.1. Global Versus Local Norms. The starting point for understanding the basis for Easwaran and Fitelson's formal requirements begins with an observation about different strategies one might take in trying to derive formal coherence constraints on binary belief. To begin with, we need to note there are two different kinds of norms that might apply to belief (or degrees of belief) *global* as opposed to *local* norms. They explain that they intend to defend a coherence requirement that is a global norm on sets of beliefs:

Coherence is a global property of a judgment set in the sense that it depends on properties of the entire set [of judgements] in a way that is not (in general) reducible to properties of individual members of the set. (In Press, p. 2)

Generalizing, a *global norm* on a doxastic state is a norm that depends on the relations between the members of the set of the relevant kind of doxastic state, whereas a norm is *local* when the satisfaction of the norm depends on the properties of an individual instance of the relevant kind of doxastic state. So, for example, a norm that says that one's degree of belief in p should be proportioned to the evidence, qualifies as a local norm on one's degree of belief. A norm of the form:

(TN) One should believe a proposition p if and only if p is true (or likely to be true).

counts as a local norm on binary belief. Global norms, on the other hand, entail that certain patterns of binary belief or degrees of belief are rationally (un)acceptable (rationally speaking). For example, both a logical consistency requirement on sets of beliefs and probabilistic coherence requirements on degrees of belief are global in the intended sense.

With this distinction between local and global in hand, we can note one of the key aspects of the probabilist's derivation of SCP and other global formal requirements on binary belief. The probabilist strategy, I considered above, was to derive global formal requirements on binary belief from global formal requirements on an agent's degree of belief. The potential problem with such a view is that there are a variety of reasons one might be skeptical about accepting the global coherence norms on degrees of belief on which the derivation crucially depended. So, now, the obvious question to ask is this, if we aren't going to be able to derive formal coherence norms from

probabilistic constraints on degrees of belief, then what strategy is available. There seem to be two options available:

- (S1) Identify distinct global coherence requirements on degrees of belief, and derive global coherence requirements for binary belief from those.
- (S2) Derive global coherence requirements from local requirements on binary belief.

While both options will need to be taken seriously, the strategy taken by Easwaran and Fitelson is (S2). The next step to seeing how they derive formal coherence constraints on binary belief is by considering the sorts of local norms on binary belief that seem relevant to the derivation of formal coherence constraints on binary belief.

1.5.2. Local Norms on Binary Belief.

Alethic Norm and Logical Consistency. The easiest way to appreciate how one might derive a global coherence constraint on binary belief from a local norm on binary belief is to consider a well-worn strategy embraced by many proponents of deductive cogency requirements. To that end, Easwaran and Fitelson note that some might endorse the following local alethic norm on binary belief:

- (TB) All agents S should (at any given time t) have full beliefs that are true. (in press, p. 4)

There are a variety of ways that one might interpret (TB), but we needn't worry too much about how it is to be understood. Easwaran and Fitelson note that it seems to clearly describe the accuracy-norm for binary belief. That is, an agent should believe p only if p is true on pain of having inaccurate beliefs. And as Easwaran and Fitelson observe,²⁷ from (TB), one can derive a global consistency requirement on belief.²⁸ In particular, if one holds that an agent who has any false beliefs violates (TB), then any agent who has inconsistent beliefs will thereby have some false beliefs, and thus violate (TB). Thus, (TB) entails a consistency requirement.

²⁷Easwaran and Fitelson suggest that it doesn't matter how this principle is interpreted. I think it is clear that if one endorses it as expressing a normative ideal, then it doesn't provide the basis for the derivation of a global consistency requirement on rational belief.

²⁸In Chapter 3, we shall consider a variety of arguments for a consistency requirement from principles in the neighborhood of (TB).

As Easwaran and Fitelson point out, and will become especially clear in Chapter 3, (TB) isn't remotely plausible as an epistemic condition on rational belief. That is to say, just because an agent has some belief p that is false and thereby violates (TB), it hardly follows that the agent has done something irrational. For agents like ourselves who must rely on fallible evidence and who must engage in risky belief forming practices (if we are to have beliefs about contingent empirical matters at all), preface considerations establish that it is all but certain that even the most circumspect amongst us will inevitably be in violation of (TB). For these sorts of reasons, Easwaran and Fitelson, thus, conclude that while (TB) might serve as a kind of correctness norm on belief, i.e., someone violating (TB) does so on pain of being incorrect about some things, satisfaction of (TB) does not express a requirement on rational belief.

(Begin Parenthetical Remark) There may well be factive norms that are relevant for assessing an agent's beliefs and assertive practices.²⁹ And, it may well be that there are external conditions on knowledge that go beyond the evidential conditions relevant to the assessment of an agent's rationality. But I agree with Easwaran and Fitelson that there is a clear sense in which it would be inappropriate to use factive norms as assessments of an agent's rationality. Any two people with exactly the same information available to them, whose evidence makes a proposition, p , equally likely to be true, and whose beliefs enjoy whatever other epistemic credentials are relevant to assessing the rationality of belief will be equally (ir)rational if they both judge p to be true. Assuming that agents can be rational in believing propositions on the basis of fallible evidence (evidence that fails to logically entail the truth of the target belief), one agent might be correct, while the other is not, despite the fact that their beliefs are equally rational. We shall assume that the sort of rationality in question is an internalist notion, and this is an assumption shared by all of the substantive theories of justified belief that we shall consider in later chapters. Hence, principles like (TB) seem highly inappropriate for assessing the kind of rationality with which we are concerned. (End Parenthetical Remark)

While deriving a consistency requirement for justified belief from (TB) is implausible, it should be clear how we might go about deriving a global norm on belief from a local one. If violating the global norm for a set of beliefs guarantees that one violates the local norm for some individual belief, then the latter entails the former. While (TB) is highly implausible as a condition on rational

²⁹See Littlejohn (2010, 2012) for arguments in favor of the factivity of the sort of justification required for knowledge.

belief, Easwaran and Fitelson note that there are distinct evidential norms that are highly plausible conditions on rational belief.

Evidential Norm on Rational Binary Belief. Fitelson and Easwaran note that when assessing the rationality of an agent's beliefs, evidential principles seem far more plausible. In fact, they note that the following principle seems to express a constraint on rational belief that many epistemologists can get behind, even if they reject probabilism:³⁰

Evidential Norm for Full Belief (EB): All agents S should (at any given time t)
have full beliefs that are supported by the total evidence. (In Press, p. 4)

Easwaran and Fitelson's strategy for deriving formal synchronic coherence constraints on justified belief are analogous to the strategy for deriving cogency requirements from (TB). In effect, what they show is that violation of certain formal global requirements on binary belief entail the violation of (EB). We shall now consider a number of global requirements on binary belief that can be derived from (EB). In order to do so, we shall have to review a variety of formal definitions that are needed to state Easwaran and Fitelson's accuracy-coherence norms.

1.5.3. Agenda Sets - Opinionated Sets of doxastic States. To start out with, the first thing to note is that their accuracy-coherence norms are defined on opinionated sets of beliefs, represented as \mathbf{B} , where \mathbf{B} is a set of judgments on a finite set of propositions (propositions are represented as classical, possible worlds), \mathcal{A} , which Easwaran and Fitelson refer to as *Agendas* (sets of propositions on which an agent has a belief agenda). Letting $B(p)$ stand for an agent S believes p , and $D(p)$ stand for S disbelieves p , we can say that any set of opinionated judgements, $\{B(p_1), \dots, B(p_n), D(q_1), \dots, D(q_m)\}$ constitutes an a belief set on an agenda set, $\mathcal{A} = \{p_1, \dots, p_n, q_1, \dots, q_m\}$, where certain conditions are satisfied. In their *opinionated* framework, they impose the following three constraints on beliefs and/or denials on an agenda set:³¹

Accuracy Condition. $B(p)$ [$D(p)$] is accurate if and only if p is true [false].

Incompatibility. $B(p) \Rightarrow \neg D(p)$.

Opinionation. $B(p) \vee D(p)$ (for all judgments in an agenda \mathcal{A}).

³⁰Most epistemologists will accept (EB) as a necessary condition on rational or justified belief. Where there will be disagreement is whether or not there are additional norms on belief, norms that might rule out justified inconsistent beliefs.

³¹See Easwaran and Fitelson (in press, pp. 11-12) for further explanation.

The accuracy and incompatibility conditions are self-explanatory. The opinionation condition requires comment. To be clear, what opinionation is meant to capture is that there are only two sorts of doxastic states that might belong to agenda state, belief and disbelief (it isn't to insist that one's theory of the world is prime). One might want to hold that other doxastic states like suspension of judgment could also be part of an agenda set.³² This is important since one might think that suspending judgment about certain propositions, while believing others, might in a sense be incoherent, in a way that guarantees that one has violated a local evidential norm for some proposition. To keep things simple, we shall consider Easwaran and Fitelson's framework on the assumption that an agent's belief set is opinionated. The primary novel application of their formal coherence conditions that shall be developed in Chapter 4 will work best under this assumption.

With the basic elements of an agenda set in hand, we are now in position to define a variety of different sorts of synchronic formal coherence norms on full belief that can be derived from local and very basic evidentialist norms on belief. The key thing to note is that all of the norms we shall define are *formal* in the sense that satisfaction of the norm supervenes on the logical relations between the contents of the judgments contained in an agenda set. Now, there are a variety of different tactics we can pursue to introduce the various formal coherence norms that Easwaran and Fitelson derive from (EB). I think it best to start with the most general and fundamental formal norm that can be derived from (EB).

1.5.4. Possible Probablification Norm. How much do we need to unpack the evidentialist condition on full belief in order to be able to derive any sort of formal norms for full belief. According Easwaran and Fitelson, little has to be assumed about the nature of evidence, or the sorts of probabilistic relations in play for evaluating how well an agent's total evidence supports a proposition for us to be able to identify some fairly basic formal coherence norms that hold for full binary belief. On this subject, they say the following:

While there is considerable disagreement about the precise content of the Evidential Norm for full belief (EB), there is widespread agreement (at least, among evidentialists) that the following is a necessary condition for satisfying (EB).

³²Easwaran (Manuscript) pursues a generalization of this framework in a way that relaxes the assumption of opinionation. Once that assumption is relaxed one can give exactly the same arguments for why arguments seem rationally persuasive as are given by probabilists.

Necessary Condition for Satisfying (EB). **B** satisfies (EB), i.e., all judgments in **B** are supported by the total evidence, only if : (\mathcal{R}) There exists some probability function that probabilifies (i.e., assigns probability greater than $1/2$ to) each belief in **B** and dis-probabilifies (i.e., assigns probability less than $1/2$ to) each disbelief in **B**. (In Press, p. 15)

The basic idea then is that if it is not possible for each of one's (dis)beliefs to be (dis-)probabilified by a single probability function, then it can't possibly be the case that each of one's judgments is supported by one's total evidence. The underlying thought seems to be that one's evidence supports a proposition when it makes it likely to be true, i.e., highly probable.³³ So, if the logical relations that hold between a set of propositions one believes are such that no probability function could possibly assign each proposition a high probability (where being less than $1/2$ is an absolutely uncontroversial threshold for failing to be highly probable), then it can't be the case that one's total evidence supports each of one's full beliefs in the sense of making them highly likely to be true (and likewise for one's disbeliefs and making them likely to be false). One whose beliefs violate \mathcal{R} are thereby guaranteed to contain at least some judgments that are not supported by one's evidence, and thus violate the local norm (EB).

\mathcal{R} is less restrictive than a logical consistency requirement. Agents whose beliefs sets are constituted by the full belief in all lottery propositions, can satisfy \mathcal{R} , and the same goes for an agent who accepts all of the propositions she has written in her book, and the preface proposition, which states that the body of the book contains at least some error.³⁴ Intuitively, each member of these sets enjoy a great deal of evidential support, as they are highly likely to be true on one's total evidence.³⁵ Thus, belief in lottery and preface propositions, as described, satisfy (EB). But, clearly, not all sets of inconsistent beliefs will be able to satisfy \mathcal{R} or (EB). For instance, assuming we are

³³Easwaran and Fitelson (in press, footnote 40) note that there is considerable disagreement over the nature of the probability function appropriate for evaluating which claims are supported by one's total evidence. There are various internalist and externalist options for the probability function that one might think relevant. They note that we can put worries about the exact nature aside because the formal constraint merely holds that whatever kind of probability function is relevant, the possibility claim should hold once it is plugged in to (EB).

³⁴Easwaran and Fitelson (in press, footnote 38) do not endorse that one can be justified in believing lottery propositions. They just make the above observation that beliefs in lottery propositions do not violate \mathcal{R} .

³⁵Note that it doesn't follow that one is justified in believing all members of an inconsistent set like the lottery from the foregoing observations. On Lehrer's theory, for instance, \mathcal{R} constitutes a constraint on sets of justified beliefs. It is just that he thinks there are additional constraints that rule out justified inconsistent belief.

limiting our focus to standard probability functions on propositions (that are classical, possible worlds), no probability function can probabilify p and $\neg p$ at the same time. Hence, an agent S whose judgment set is constituted by $\{B(p), B(\neg p)\}$ will necessarily violate \mathcal{R} . And, the intuitive underlying thought is that insofar as one's total evidence supports p by making p likely to be true and thus accurate, one's total evidence must also undermine one's belief in $\neg p$ by making the content likely to be false. Thus, if one's belief that p satisfies (EB), it follows that one's belief that $\neg p$ violates (EB), and vice versa.

The example is also useful for indicating how logical relations between members of the agenda set are relevant to the satisfaction of \mathcal{R} and thus (EB). The relationship between (EB) and \mathcal{R} should be clear enough. Let us now turn to a variety of other formal coherence norms that Easwaran and Fitelson demonstrate to be entailed by \mathcal{R} .

1.5.5. Accuracy-Dominance Avoidance Principles. There are a variety of accuracy-dominance avoidance norms that can be shown to be strictly weaker than \mathcal{R} , which will be useful both in terms of having the full theoretical space before us. And, it will be especially important for many of the later applications we will explore in Chapter 4 where we will investigate the relationship between probabilistic measures of coherence and these various formal coherence norms.

Easwaran and Fitelson's idea is that just as we can define decision-theoretic (weak) and (strong) dominance avoidance principles that say that one ought not perform an action if it is dominated by an alternative action in terms of utility, so too one might define dominance avoidance principles that say that one ought not hold a set of beliefs if there is an alternative set that dominates it in terms of accuracy.³⁶ They explain that the recipe for defining accuracy-dominance avoidance principles contains three basic ingredients. They explain their three key steps as follows:

Step 1. Say what it means for a set \mathbf{J} of type \mathfrak{J} to be perfectly accurate (at a possible world w). We use the term "vindicated" to describe the perfectly accurate set of judgments of type of type \mathfrak{J} , at w , and we use \mathbf{J}_w to denote the vindicated set.

³⁶The more direct analogy is that there weak dominance principles are for full belief what De finitte's dominance avoidance principles are for degrees of belief.

Step 2. Define a measure of distance between judgment sets, $d(\mathbf{J}, \mathbf{J}')$. We use d to gauge a set \mathbf{J} 's distance from vindication at w [viz., $d(\mathbf{J}, \mathbf{J}_w)$].

Step 3. Adopt a fundamental epistemic principle, which uses $d(\mathbf{J}, \mathbf{J}_w)$ to ground a (synchronic, epistemic) coherence requirement for judgment sets \mathbf{J} of type \mathfrak{J} . (In Press, p. 11)

Following through on the steps is relatively straightforward. The least controversial step, step one, is to define the vindicated set, \mathbf{B}_w , as follows:

$$\mathbf{B}_w = \{O(p) \mid O(p) = B(p) \text{ and } p \text{ is true, or } O(p) = D(p) \text{ and } p \text{ is false.}\}$$

This just says that the vindicated set contains all beliefs that have true contents and all disbeliefs that have false contents. The choice of the distance function between judgment sets is trickier in that there are wide variety of choices one might make. To begin with and to keep things simple, Easwaran and Fitelson propose the simple (though possibly naïve) distance measure that just counts the number of judgments on which the two sets disagree. They explain in a more precise fashion:

“In particular, if you want to know how far your judgment set \mathbf{B} is from vindication at w [i.e., if you want to know the value of $d(\mathbf{B}, \mathbf{B}_w)$] just count the number of mistakes you have made at w .” (In Press, p. 12)

To put it formally, distance from perfect accuracy at a world, w , is given as follows:

$$d(\mathbf{B}, \mathbf{B}_w) = |\mathbf{B}| - |\mathbf{B} \cap \mathbf{B}_w|.$$

With these two elements in hand, Easwaran and Fitelson are able to defend weak and strict accuracy dominance avoidance principles. We start with their definition of what it is for a set of judgments to be weakly accuracy dominated:

Weak Accuracy Dominance Avoidance (*WADA*): A set of beliefs, \mathbf{B} , *avoids weak accuracy dominance* if and only if there does not exist an alternative belief set \mathbf{B}' such that:

- (i) $(\forall w)[d(\mathbf{B}', \mathbf{B}_w) \leq d(\mathbf{B}, \mathbf{B}_w)]$, and
- (ii) $(\exists w)[d(\mathbf{B}', \mathbf{B}_w) < d(\mathbf{B}, \mathbf{B}_w)]$. (In Press, p. 13)

Put informally, a set of beliefs is weakly dominated just in case there is another belief set that (i) is at least as close to the vindication set at all possible worlds, and (ii) is closer to the vindication set at some possible worlds. Easwaran and Fitelson note that we can use the following definition to express a condition equivalent to weak accuracy dominance avoidance as follows:

DEFINITION 1.1. S is a *witnessing set* if (a) at every world w , at least half of the judgments in S are inaccurate; and, (b) at some world more than half of the judgments in S are inaccurate. (In Press, p. 14)

They prove that a set \mathbf{B} is weakly dominated if and only if it contains a witnessing set (p. 14). Thus, we can also understand a weak dominance avoidance principle as requiring that one avoid belief sets that contain witnessing sets.

And there is related notion of strict, as opposed to weak, accuracy-dominance that they explain as follows:

Strict Accuracy-Dominance Avoidance (SADA). \mathbf{B} is not strictly dominated in distance from vindication. Or, to put this more formally (in terms of d), there does not exist an alternative belief \mathbf{B}' set such that: $(\forall w)[d(\mathbf{B}', \mathbf{B}_w) < d(\mathbf{B}, \mathbf{B}_w)]$. (In Press, p. 17)

So, they say a set is strictly accuracy-dominated when there is another set that is closer to the vindication set at all worlds. Based on the choice of the distance measure employed in the definitions, it is helpful to keep in mind that a belief set is weakly accuracy-dominated when there is a belief set that contains fewer inaccurate beliefs at some worlds, and contains less inaccurate beliefs at all worlds. A belief set is strictly accuracy-dominated just in case there is another set that contain fewer inaccurate judgments at all worlds. Again, there is a corresponding notion of a witnessing set for strict dominance, namely:

DEFINITION 1.2. S is a *witnessing set*₁ if (a) at every world w , more than half of the judgments in S are inaccurate. (p. 21)

Again, they prove that a belief set is strictly dominated just in case it contains a *witnessing set*₁ (p. 22). The proof of the claim that dominance and containing witnessing sets of the respect kinds

is a matter of noting that if one reverses the judgments of a witnessing set, and keeps the rest the same, one arrives at a dominating set, and if B dominates B' , then one can take the set over which B and B' disagree and that must form a witnessing set of the relevant kind.

Now, before going on to consider the third step in the process of defining a formal coherence norm on full belief, we should pause to note that there is an obvious third option for defining a witnessing set, namely:

DEFINITION 1.3. S is a *witnessing set*₂ if (a) at every world w , at least half of the judgments in S are inaccurate.

We can then define a third kind of witnessing set avoidance principle that is stronger than WADA and SADA. Easwaran and Fitelson prove that satisfying (\mathcal{R}) requires that one avoiding belief sets that contain any of these three sorts of witnessing sets.

For the moment, let us pause to note the relationship between WADA and SADA. Any belief set that is strictly dominated is also weakly dominated, but not vice versa. Thus, the third step is to adopt a formal coherence norm that requires that agent's belief sets ought not be dominated in either sense. Easwaran and Fitelson define such a constraint as follows:

(NDB) All (opinionated) agents S should (at any given time t) have sets of full beliefs (and disbeliefs) that are non-dominated. (p. 14)

Now, Easwaran and Fitelson demonstrate that (NBD) can be derived directly from (\mathcal{R}) . In fact, Easwaran and Fitelson show that avoidance of a *witnessing set*₂ is necessary in order to avoid violating (\mathcal{R}) . Of course, we have it that avoidance of containing a *witnessing set*₂ requires avoidance of containing *witnessing set*, which in turn requires avoidance of a *witnessing set*₁, and we know the latter two are equivalent to (WADA) and (SADA) respectively. Thus, Easwaran and Fitelson prove that if a belief set is (at least) weakly dominated, then it violates (\mathcal{R}) . Thus, any belief set that is dominated is such that at least some of the beliefs in the set must lack support from one's total body of evidence. This, it seems to me, is strong enough reason to think that (NBD) states an uncontroversial global formal coherence requirement for full belief, and all of the formal coherence norms that are entailed by (NBD).

1.5.6. Generalizing (\mathcal{R}) . Easwaran and Fitelson note that (\mathcal{R}) is easily generalized, as it is an instance of the following parametric family of coherence requirements that one might impose on full beliefs. They explain the family as follows:

Parametric Family of Probabilistic Requirements Between (\mathcal{R}) and **(CB)**

(\mathcal{R}_r) There exists a probability function Pr such that, for every $p \in A$:

(i) **B** contains $B(p)$ iff $Pr(p) > r$, and

(ii) **B** contains $D(p)$ iff $Pr(p) < 1 - r$,

where $r \in [1/2, 1)$. (p. 17)

Now, (\mathcal{R}) is the norm one get when $r = 1/2$, and Easwaran and Fitelson observe that (\mathcal{R}) is probably the strongest norm that is entirely uncontroversial for (probabilistic) evidentialists. But it is quite plausible that in almost epistemic contexts, the amount of evidential support needed to suffice for one to be rationally in accepting a proposition is well above $1/2$. That is to say, in almost all contexts, we think an agent requires evidence that makes a proposition highly likely to be true, not just more likely to be true than its negation, in order for an agent to be rational in fully believing a proposition. But, supposing that some instance of (\mathcal{R}_r) is operative in all contexts or perhaps holds for some agent (one who accepts pragmatic encroachment might hold that the evidential threshold depends on the practical interests of the subject of a justified belief ascription, as opposed to the ascriber's context), and allowing that r may vary from contexts to contexts or practical situation to practical situation, we can arrive at potentially stronger consistency requirements that are operative in a context. In fact, (\mathcal{R}_r) provides perfectly analogous justification for restricted n -wise consistency requirements that we earlier derived for degree of belief probabilists. That is to say, we have it that:

For all n such that $\frac{n-1}{n} < r$, if S believes all the members of any set B where $|B| \leq n$ and B is logically inconsistent, then S 's beliefs fail to satisfy (\mathcal{R}_r) .

Now, this holds generally. Whether one's beliefs being inconsistent make one violate some norm of rationality operative in her context depends on whether (\mathcal{R}_r) is a norm of rational belief in that context. In general, in contexts where a high degree of evidential support seems necessary for an agent's beliefs to be justified, inconsistency amongst a small set of the agent's beliefs will be rationally unacceptable.

1.5.7. Brief Summation. Easwaran and Fitelson’s framework thus provides an alternative, but equally plausible explanation for the *prima facie* plausibility of a logical consistency requirement as the explanation provided by probabilism, and shows that we can identify certain formal coherence constraints on belief that are more permissive than a logical consistency requirement. If one wants to insist that arguments must offer a guide to belief, then analogous explanations to those we offered for probabilist’s can be provided by the framework developed by Easwaran (Manuscript) in which the assumption of opinionation is dropped. Easwaran’s framework offers analogous single premise closure principles that would provide exactly the same sorts of explanation for why an agent would be rationally required to close her beliefs under arguments she recognized to be valid and have premises that are jointly true. I see no reason to think that there is any role that logic must play that cannot be explained either via probabilism, or by Easwaran and Fitelson’s formal coherence constraints.

Nevertheless, many epistemologists will remain unconvinced. The one kind of challenge to inconsistency solutions to the epistemic inconsistency paradoxes that I don’t think have met fully satisfactory answers are arguments based on substantive analyses of epistemic justification. That is to say, even if proponents of inconsistency solutions can account for the role of logic in rational inquiry in some manner along the lines we have considered above, some epistemologists will think that there are substantive properties of epistemic justification that entail a logical consistency requirement.

1.6. Aim of Dissertation

The aim of this dissertation is to address some of the most prominent arguments for consistency requirements that are based on substantive theoretical commitments about the nature of epistemic justification. Our primary concern will be to try to determine whether there are good theoretical motivations for thinking that we need stronger logical consistency requirements on rational belief than those available to probabilists and Easwaran and Fitelson’s accuracy dominance avoidance principles. We shall focus most of our attention on whether there exist theoretical motivations for accepting a global consistency requirement that might arise from particular substantive theories of justified belief (e.g. we will consider whether coherentists have a special reason to accept a global

consistency requirement). The reason for this focus is twofold. First, I want to consider theoretical reasons one might have for adopting a consistency requirement that haven't been thoroughly explored. Much of the literature on the inconsistency paradoxes revolves around the question of the normativity of logic, and opponents of deductive cogency solutions have provided compelling replies to arguments for deductive cogency requirements based on the assumption that deductive cogency principles are needed to explain the role of logic in rational inquiry. As we have seen, proponents of inconsistency solutions have compelling replies. Second, certain theories have not only been said to carry with them commitments to particular solutions to the lottery and preface paradoxes, but have also been objected to on the basis of what they imply with regard to the acceptability or unacceptability of inconsistent claims (e.g., coherence theories of epistemic justification). Given that we cannot do an exhaustive survey of all theories of epistemic justification, we shall narrow our focus to theories of justification where there is some controversy as to whether or not the theory entails a consistency requirement, and where our understanding of the theory will be improved by investigating what, if any, formal coherence requirements on justified belief follow from the substantive commitments of the theory. We shall finish our introduction by laying out the primary substantive theories to be considered with a brief explanation of why they have been thought to motivate a logical consistency requirement on justified belief.

1.6.1. Overview of Substantive Theories to be covered.

Chapters 2: Lehrer's Coherence Theory of Justification. So, what theses about the substantive nature of epistemic justification and/or the nature of categorical belief might require one to reject an inconsistency solution to the lottery and preface paradoxes? The paradigmatic example is supposed to be coherentist epistemology. It has been claimed by various epistemologists that coherentists have a special commitment to a global consistency requirement on justified belief. Jonathan Kvanvig (2012) provides a clear expression of this view:

...there is one version of the problem of the relationship between justification and truth that is, to my mind, the most pressing difficulty coherentism faces: it is the problem of justified inconsistent beliefs. In a nutshell, there are cases in which our beliefs appear to be both fully rational and justified, and yet the contents of the

beliefs are inconsistent, often knowingly so. This fact contradicts the seemingly obvious idea that a minimal requirement for coherence is logical consistency.

Kvanvig is certainly not alone in thinking this. This mantra that logical consistency is a minimal requirement for coherence has been repeated time and again by epistemologists considering the nature of the coherence relation (Foley (1979), Bonjour (1985), Olsson (1998), Hansson (2006)), Lehrer (2000, 2003), though there are, of course, dissenters (Lycan 1996, 2012). And, this assumption that coherence requires consistency has motivated key elements of some attempts to develop a precise and systematic account of the coherence relation, as in Lehrer's (1979, 2000,) coherence theory of personal justification and knowledge.

Despite these repeated claims, it is still a wide open question whether coherentists really have substantive theoretical motivations for holding that consistency is a necessary condition of coherence, and thus accepting a global consistency requirement on rational belief. Consequently, this is one of the motivations for rejecting an inconsistency solution to the lottery and preface paradox that we shall investigate. We shall pursue this question by considering whether the prominent analyses of the coherence relation actually entail that coherence requires consistency, as it is often assumed. Our investigation on this matter will be broken up into two parts. In Chapter 2, we shall examine Lehrer's coherence theory of personal justification, and the explanation his theory is supposed to offer for why one is not justified in accepting inconsistent claims in cases like the preface and the lottery.

Chapter 3: Permissibility Theories. While many have thought that coherentism provides motivation for adopting a deductive cogency solution to the lottery and preface, Glenn Ross (2002, 2013) and Igor Douven (2008, 2012) have suggested that coherentist epistemologists like Lehrer (2000) and Harman (1986), respectively, have set out substantive reasons for why we should accept some version of the permissibility solution. And, Kroedel (2012, 2013) has suggested that, in general, a view of epistemic justification as having a rational permission to believe is naturally combined with holding that there is a general obligation to avoid inconsistent beliefs. Much of the debate over the plausibility of permissibility solutions to the inconsistency paradoxes turns on whether there is an explanation as to why permissions fail to agglomerate in cases like the lottery or the preface (Littlejohn 2012, 2013), and why it is, exactly, that one should avoid inconsistent justified belief. In Chapter 3, we shall review the basics of the permissibility solution and then survey the various

theoretical motivations that are supposed to provide us with reasons to think that inconsistency is something that we have an obligation to avoid.

Chapter 4: Inconsistent Belief and Probabilistic Coherentism. In this chapter, we turn to more recent attempts to develop alternative formal characterizations of the coherence relation in terms of probabilistic measures of mutual agreement and/or mutual support. Only very recently have formal epistemologists investigated whether coherence requires consistency on the proposed ways of formally characterizing the coherence relation. To my knowledge, the first and only survey of this topic was taken up in William Roche’s (2013) “Coherence and probability: A probabilistic account of coherence,” and his investigation is limited to the study of one particular kind of case. Thus, there is much to be learned about the kinds of inconsistency that may or may not be coherent on various probabilistic characterizations of the coherence relation. In Chapter 4, we shall take up a careful study of coherence measures and try to isolate the factors that determine whether consistency and other less restrictive formal coherence constraints are necessary for coherence on probabilistic measures of the coherence relation.

Chapter 5: Pragmatic Encroachment Theory. There is one final theory of epistemic justification that we shall consider, one that has been attacked on the grounds that it leads to excessive tolerance of inconsistent belief, and that is the pragmatic encroachment theory defended by Fantl and McGrath (2002, 2009, 2010, 2012a, 2012b). In a nutshell, Fantl and McGrath’s (2009, 2002) pragmatic encroachment theory holds that one is justified in believing a proposition, p , in a decision-context only if she is rational to act and prefer as if p is true in that decision-context. Ross and Schroeder (2012) object to Fantl and McGrath’s proposal on the grounds that it allows small sets of inconsistent claims, just two propositions in some cases, to all be justified. This worry is reinforced by Stanley and Hawthorne’s (2008) objection that Fantl and McGrath’s pragmatic conditions lead to epistemic closure failure *because* these pragmatic conditions fail to be closed under conjunction introduction. These objections are difficult to square with Fantl and McGrath’s (2002, Appendix) proof that their pragmatic conditions are closed under modus ponens, which, in conjunction with other closure principles that can be proven in analogous fashion, would seem to indicate that their pragmatic theory of epistemic justification actually entails a consistency requirement. Thus, at present, I think it is an open question as to what sorts of inconsistent beliefs can be tolerated on Fantl and McGrath’s proposal. Since Fantl and McGrath have formalized their pragmatic

conditions on justified belief, it is possible for us to undertake a careful study of their pragmatic conditions to determine when we can be justified in accepting inconsistent claims on their account.

Ironically, once we have a charitable interpretation of Fantl and McGrath's proposal before us, we shall see that it does rule out a wide variety of cases of inconsistent belief, and can buttress the explanations already available to inconsistency theorists as to why inconsistent belief is to be avoided in most cases. In a nutshell, it follows on their view (once charitably reconstrued) that we should not accept an inconsistent set of claims when the truth or falsity of any maximally consistent subset of the claims is of practical relevance to us, i.e., when the actions that would be best for us depend on the truth or falsity of any such conjunction. Their view of epistemic justification thus implies that inconsistency is only to be tolerated when the claims are not pragmatically relevant in a certain sense. Together with the explanations for the normativity of logic that follow from some fairly plausible evidentialist conditions discussed by Easwaran and Fitelson (in press), I think we can arrive at a compelling explanation for how consistency is relevant to epistemic justification.

The Main Theses to be Defended. Undoubtedly, some philosophers have very strong views regarding the tenability of deductive cogency principles. And, some philosophers will regard a sound argument that a theory requires (or forbids) the tolerance of some inconsistent belief to be a reductio of that theory.³⁷ My own view is that some logical inconsistency ought to be tolerated, and think that Christensen (2004), Weintraub (2001), Foley (1979) among many others have provided compelling arguments to that effect. But my intention here is not to try to establish that deductive cogency principles must be rejected on all theories of epistemic justification. I remain open to the possibility that certain theoretical commitments could provide one with compelling reasons to disagree on this matter, and recognize that there are certain substantive theories of epistemic justification that may lead one to reasonably judge that I am mistaken.

What I will argue is that each of the theories of justification that we shall consider can be combined with an inconsistency solution to the lottery and preface paradoxes, and that none of the theories in question provide anything approaching a compelling reason to adopt a consistency requirement. If my argument is successful, then we will have strengthened the case for an inconsistency solution considerably, since the theories we shall consider are some of the best contenders for providing a substantive basis for accepting a global consistency requirement on belief. In the process, I think

³⁷Stanley and Hawthorne (2008) reject Fantl and McGrath's (2002) view, at least partially, for this reason.

we shall show that coherentist and pragmatic theories of epistemic justification face no serious objections from the inconsistency paradoxes. Permissibility theories of epistemic justification, on the other hand, enjoy no special theoretical advantage over rival theories that have been claimed on their behalf.

CHAPTER 2

Lehrer's Coherence Theory and Inconsistency

2.1. Introduction:

Keith Lehrer has been one of the most prominent defenders of a coherence theory of epistemic justification and knowledge. One of the basic assumptions motivating many key elements of Lehrer's theory is the thought that coherentists ought to reject the possibility of justified inconsistent acceptances in cases like the lottery and preface paradox.¹ Ultimately, Lehrer holds that there are doxastic states he calls 'acceptances' whose functional role is to aim exclusively at truth and the avoidance of falsehood. According to Lehrer, the manner in which acceptances play their role provides a special motivation for accepting a consistency requirement on rational acceptance. Lehrer's account of why one ought not hold inconsistent acceptances in cases like the lottery and the preface is similar to the probabilistic approaches we considered in Chapter 1. Lehrer holds that a high subjective probability is one factor that helps to determine whether an agent is justified in accepting a proposition, but epistemic justification also depends on the absence of certain kinds of defeat relations. The way Lehrer conceives of it, to be justified in accepting p , one must be in position to answer all objections that a skeptic might put forward as reason not to accept p . For Lehrer, this is just what it takes for a proposition to cohere with one's background system of acceptances.

¹The *acceptance* and *belief* distinction is not insignificant for Lehrer, and is a bit idiosyncratic. For Lehrer (1990, 2000), acceptances are meta-doxastic states, functional states with distinctively epistemic goals. Pascal Engel (2012) provides a concise explanation of Lehrer's distinction, which we shall take as authoritative:

Acceptance, like belief, is a functional state, but a second-order one: it has a belief-content as its object, but it is not a belief. It is a meta-level state, involving conscious reflection and reasoning. It is defined in terms of some aim or purpose of a subject, namely the purpose is attaining truth and avoiding error, through reasoning and evaluation of one's own justifications. Belief, in contrast may be defined in terms of a purpose (it "aims" at truth, as one says) but not necessarily (many beliefs are not aimed at truth). Belief involves mostly the processing of information at the first-order level that is not necessarily conscious or available directly to reasoning or inference. (p. 18)

Given that Lehrer holds that it is acceptances rather than beliefs that are central to epistemological investigation and the evaluation of an agent's epistemic rationality, we shall focus on this notion throughout this chapter. The question we shall thus focus on is whether one can be epistemically rational in having inconsistent acceptances on Lehrer's view. See Lehrer (2000B) and Engel (2012) for more discussion and clarifications of Lehrer's account of this distinction.

According to Lehrer, the lottery and preface paradoxes are not cases where an agent is justified in accepting an inconsistent set of propositions because there are objections to lottery and preface propositions that a skeptic might put forward, which cannot be answered. In this chapter, we shall scrutinize Lehrer's claim that his coherence theory provides an explanation as to why one would not be rational to accept inconsistent claims in these cases.

Ultimately, I shall argue that Lehrer's explanations for why we should avoid inconsistency falls short. It falls short because Lehrer's account of personal justification either tolerates inconsistent acceptances in nearby alternatives to the standard lottery and preface paradoxes, or else rules out justified inconsistent acceptances in a way that dissolves into skepticism. The reason Lehrer's view, as officially formulated, does not rule out inconsistent acceptances is somewhat surprising. There are two basic facts that have gone unnoticed by all of Lehrer's defenders and critics. The first is that a set of propositions can be inconsistent, and yet when considered individually, each member of that set can constitute evidence for, rather than, in Lehrer's sense, an objection to each of the other members of that set. The second observation overlooked by Lehrer's critics is that Lehrer's coherence condition on justified belief is not closed under logical entailment, not even single premise logical entailment. This leads to the surprising result that Lehrer's view invalidates single premise closure of justification. Whether single premise closure failure is a virtue or vice for a theory is open to debate, but single premise closure is actually essential to the sort of solution that Lehrer's account is supposed to provide to the inconsistency paradoxes. I shall further show that any attempt to recover single premise closure while retaining the central elements of Lehrer's theory of acceptances will yield radically skeptical results.

2.2. Lehrer's Epistemology

In this section, we shall consider, in broad outline, Lehrer's theory of personal justification. To fully understand Lehrer's theory of personal justification, we shall need to consider the precise details of Lehrer's account of what it takes for a claim to cohere with one's system of acceptances. In a nutshell, Lehrer's view is that a proposition is justified just in case it coheres with one's acceptance system. He makes this claim formally precise, but before we get to the technical definitions that Lehrer relies on, it will be helpful to consider the intuitive gloss Lehrer provides. Lehrer explains

that his analysis of personal justification can be intuitively understood in terms of the following heuristic:

The justification game is played in the following way. The claimant presents something she accepts as true. The critic may then raise any objection to what the claimant presents. If what the claimant accepts is something that is more reasonable for her to accept than the critical objection, that is, if the objection cited by the critic is answered, then the claimant wins the round. If all objections raised by the critic are answered, then the claimant wins the game. If she wins the game, she is personally justified in accepting what she presented; if not, she is not personally justified. (2000, p. 132)

Lehrer goes on to explain that being justified isn't a matter of having actually played the game, but rather the game serves as a kind of heuristic we can use to determine whether or not an agent is justified in believing a proposition. The idea being that if an agent is in position to defend a claim against objections (by her own lights, not necessarily by the lights of her critics), then the claim coheres with her acceptance system and she is personally justified in accepting the proposition.

How does this view of justification constitute a kind of coherentism? The game can be thought of as a test of the internal coherence of the claimant accepting a claim as true. To see this, consider what it means for S to lose at the justification game when trying to defend accepting p ? It means that, by S 's own lights, there is some objection that cannot be answered, i.e., that there is some challenge to the rational acceptability of p that cannot be adequately responded to. In other words, losing the game entails that accepting p conflicts with one's inclination to accept another claim that conflicts with p . Thus, intuitively, a belief is supposed to cohere with one's background system of acceptances just in case accepting a proposition stands in no such conflict. Why is this notion of personal justification internalist in the sense that justification is determined by relations internal to an agent's cognitive system? The key thing to note is that when comparing the reasonableness of a claim p and an objection o that a critic might offer as reason not to believe p , we are not considering how reasonable the critic (who may be of a more skeptical mind) views p and o , but rather how reasonable p and o are by the subject S 's lights.²

²This is how the game avoids radical skepticism. The claimant need not be in position to put forward a defense that would be persuasive to the radical skeptic. Rather, she just needs to be in position to persuade herself that the

Informally, Lehrer's account of personal justification can be summed up as follows. An agent S is justified in believing p just in case p coheres with S 's acceptance system. An acceptance of p coheres with S 's acceptance system just in case S has an adequate response to any objections that might be put forward against her acceptance of p . Of course, whether it really follows that on Lehrer's theory, one cannot be justified in believing an inconsistent set of claims turns on the details of how we understand the key technical notions employed in Lehrer's analysis of justification. In particular, we need to consider three things: (a) what it takes for one claim to be more reasonable than another, (b) what it takes for one claim to constitute an objection to another, and (c) we also need to get clear on the kinds of responses a claimant is permitted to make when a critic puts forward an objection. In the next section, I will provide a detailed overview of each of the technical notions that Lehrer employs, so that we can then evaluate his claim that his view rules out justified inconsistent belief (at least in cases like the lottery and the preface).

2.2.1. Technical Overview of Lehrer's Theory of Personal Justification. Since all of the technical concepts used to explicate Lehrer's theory of justification are defined in terms of how reasonable it is for an agent to accept a proposition, the place to start in giving a technical definition of personal justification is the notion of reasonableness that Lehrer relies on in his theory. The target notion of reasonableness for Lehrer is the degree to which it is reasonable for an agent S to accept a proposition, p , for the sake of accepting what is true and avoiding what is false.³ According to Lehrer, this epistemic notion of reasonableness cannot be reduced to a question of probability as it is influenced by a variety of factors (prominent among which is the degree of informativeness of a proposition). In general, Lehrer holds that reasonableness can be understood as the expected epistemic utility an agent has for choosing to accept a proposition. Lehrer explains how this idea can be formally represented:

radical skeptic's objections are less reasonable to believe than her common sense judgments about the existence of an external world, etc... .

³That is to say, we are only considering the epistemic value of an acceptance, and not the potential practical benefits that may come from accepting or believing a proposition. We shall consider pragmatic theories of epistemic justification in Chapter 5 where practical factors may play a role in determining how much evidence is required for justified belief. There we shall be looking at theories that employ decision-theoretic tools that resemble those that Lehrer employs. The key difference is that Lehrer is only concerned with trying to achieve one's epistemic goals, and the epistemic utility that is gained or lost by choosing to accept or reject a proposition. The pragmatic theories considered in Chapter 5 also allow for the amount of practical harms that might be derived from holding false beliefs to play some role in determining the evidential standards posed on belief.

Suppose that we could specify what value or, more technically, the positive utility we assign to accepting some specific hypothesis, h , if h is true and represent that by ' $Ut(h)$ ' and, similarly, the negative utility we assign to accepting h , if h is false, and represent that by ' $Uf(h)$.' If we ask ourselves how reasonable it is to accept h in the interests of accepting what is true and avoiding accepting what is false, we must take into account the values of both $Ut(h)$ and $Uf(h)$. There are just two outcomes of accepting h , that h is true and that h is false, and we must take into account what value we attach to each of these outcomes. There is, however, another factor to consider, namely, how probable each of these outcomes is. So, if we let ' $r(h)$ ' represent the degree of reasonableness of accepting h , and ' $Pr(h)$ ' represent the probability of h being true and ' $Pr(\sim h)$ ' as the probability of h being false, we obtain the following formula for computing the reasonableness of h :

$$r(h) = Pr(h)Ut(h) + Pr(\sim h)Uf(h). \text{ (2000, p. 145-146)}^4$$

Thus, using standard decision-theoretic tools, Lehrer maintains that we can formally represent an agent's comparative judgments of how reasonable it would be to accept competing claims.⁵ With

⁴I have cleaned up Lehrer's formalization of a probability function. He represents a probability function as ' $p(h)$,' which I have changed to ' $Pr(h)$ ' to fit with more common contemporary notation, and to avoid potential ambiguity in the formalism.

⁵It is worth noting that not everyone thinks that this provides a plausible way to formulate Lehrer's theory of personal justification. Wolfgang Spohn (2002) rejects this approach in favor of ranking theory, which he develops with the aim of understanding comparative judgments of reasonableness. He doesn't provide much explanation for why he rejects Lehrer's account other than that he doesn't find epistemic decision-theory particularly plausible, and that probabilistic approaches don't do well in the face of the lottery paradox. Of course, here we want to know why coherentists would be committed to a theory of justification that rules out logical inconsistency, and this is something Lehrer claims to supply. Spohn doesn't say why Lehrer's argument that his account rules out justified inconsistent beliefs in cases like the lottery fails, and so even he might take the discussion here as helpful for seeing why we should accept his claim that Lehrer's account doesn't avoid the lottery paradox.

Ultimately, Spohn opts for a formulation of Lehrer's account in terms of a ranking theory that assumes a logical closure and consistency requirement on belief. This means his account doesn't provide an explanation for why coherentists must adopt a consistency requirement, as it is something that is just assumed in the framework for evaluating comparative reasonableness. Since we are primarily interested in what might explain a coherentists commitment to a consistency requirement on justified belief, I am thus inclined to stick to Lehrer's original formulation of his theory, however troubled epistemic decision-theory might be. We will thus evaluate Lehrer's claim that it provides the means to explain why one is not justified in believing logically inconsistent statements in cases like the lottery paradox. Similarly, others have objected to this sort of formalization of Lehrer's account and some of the formal properties that Lehrer's theory possesses, which we shall presume throughout. In particular, Bender and Davis (1989) have argued that Lehrer's analysis of reasonableness is incoherent. They may well be right, but if they are, then I think it becomes even less plausible that anything like the argument Lehrer presents as a solution to the lottery and preface paradoxes will hold up to careful scrutiny. Ultimately, I will follow Olsson's (1998) interpretation of Lehrer's view because it provides Lehrer's account with the very best chance of delivering a valid argument from the substantive analysis of the coherence relation to a consistency requirement on acceptances. Dialectically, the point of this is to show that even on Olsson's most charitable interpretation, Lehrer's argument for a consistency requirement fails.

Lehrer's analysis of reasonableness in hand, we can now try to unpack Lehrer's technical definition of each of the key concepts employed in the justification game.

Now, according to the justification game, an agent must be in position to respond to any objections that a critic might put forward against the claim one is inclined to accept. In order to understand what this condition implies, we need to know what makes one claim an objection to another? Lehrer offers the following technical definition:

Lehrer's Definition of Objection: '*o* is an objection to *p* for *S* on the basis of the evaluation system of *S* at *t* if and only if it is less reasonable for *S* to accept that *p* on the assumption that *o* is true than on the assumption that *o* is false on the basis of the system *X* at *t*.' (2000, p. 131)

Given Lehrer's analysis of reasonableness, Lehrer's analysis of an objection can be simplified greatly. In fact, as Olsson (1998) points out, leaving reference to a subject, time and evaluation system implicit, Lehrer's analysis of an objection is reducible to the following in the context of the lottery and preface paradoxes:

A proposition *o* is an objection to *p* if and only if $Pr(p|o) < Pr(p)$.

In other words, *o* is an objection to *p* just in case *p* is negatively dependent on *o*. It is easy to verify that this simplification preserves the objection-relation in the contexts we are considering (i.e., the two accounts of Lehrerian objections are extensionally equivalent).⁶ When *p* is negatively dependent on *o*, taking *o* as true lowers the likelihood of the positive outcome $Ut(p)$. Taking *o* as false increases the likelihood of a positive outcome. Hence, *p* will be more reasonable to believe when assuming *o* is false than when assuming *o* is true in such a case.⁷ The rest of the cases, that

⁶It is put forward in Olsson (1998) as the obvious interpretation of Lehrer, but Olsson doesn't there explain why this is a natural simplification of Lehrer's proposal. Ultimately, Olsson argues for a revision of Lehrer's account of justification that appears to also solve the standard lottery in exactly the same manner as Lehrer's solution. As a consequence, my revised version of the lottery will undermine Olsson's (1998) solution for exactly the same reasons. For alternative interpretations and problems, see Spohn (2002) and Bender and Davis (1989).

⁷One assumption that I am making here is that the values of $Ut(p)$ and $Uf(p)$ remain stable on the assumption that *o* is true (or false). Technically, Lehrer is free to insist that the positive value we assign to *S* accepting *p* when *o* is true may increase or decrease depending on the content of *o*. In the context of developing a theory of inductive inference, Lehrer (1990, pp. 312-316) considers a theory of epistemic utility that at least implicitly suggests a way in which we might interpret the utility of accepting *p* when *p* is true on the assumption that *o* is true, as distinct from the utility of accepting *p* when *p* is true on the assumption that *o* is false. In that framework, we might hold that *p*'s informative content changes depending on the assumptions we are making. Adopting this proposal would open up the possibility that our simplified analysis of a Lehrerian-objection is not equivalent to Lehrer's intended objection relation. However, Lehrer himself seems to be assuming the simplification I am accepting in cases like the lottery and

is, the cases where p is either positively dependent on o , or probabilistically independent of o , go exactly as you would expect. In these cases, on the assumption that o is true, the chance of a positive outcome goes up or remains the same, and the chance of a negative outcome goes down or remains the same, while the reverse holds when one assumes o is false. Thus, we can understand Lehrerian objections in terms of claims standing in negative dependence relations.⁸

With a grasp on Lehrer's analysis of the objection relation, we come to Lehrer's account of what it takes to be able to adequately respond to an objection. In the explication of the justification game, Lehrer explains one way an objection can be answered, namely, an objection is answered when the target claim is more reasonable to accept than the objection. To illustrate, consider a simple case like:

Claimant: I have hands.

Critic: You do not have hands.

In this sort of case, one might answer the critic by noting that the first claim is far more probable and reasonable to accept than the critic's claim. Lehrer explains this formally as follows:

Lehrer's Definition of an Objection being Answered: 'An objection o to p is answered for S on X at t if and only if o is an objection to p for S at t and it is more reasonable for S to accept p than to accept that o on X at t .' (2000, p. 131)

In other words, objections are answerable when one's belief is more reasonable than the objection put forth to one's belief relative to one's acceptance system. But Lehrer goes on to explain that, on his view, there is another way to successfully respond to a critic's objection. One can put forward a

the preface, since it is the negative dependence of the propositions in question that leads Lehrer to conclude that the propositions in question are objections to each other. If the simplified analysis of objections doesn't hold generally, we have the fallback position that Lehrer seems to assume it in cases like the lottery and the preface paradoxes. And, so, like Olsson (1998) we shall assume it for those contexts. And if we cannot safely assume this analysis is correct in these contexts, then this would call into question Lehrer's explanation for why the various propositions Lehrer thinks are objections to the preface and lottery propositions are, in fact, objections.

Moreover, it would seem to me a *reductio* against analysis of an objection if it entailed that a proposition o constitutes an objection to p despite the fact that o makes p more probable. If Lehrer were to insist that my simplification was wrong because it fails to be coextensive, then it would seem that there could be cases where o is an objection to p despite being positively related to p . Thus, I take it that if the simplification fails, we have good reason to be suspicious of Lehrer's analysis. Thus, we shall assume the simplification going forward.

⁸In earlier work, Lehrer refers to objections as 'competitors,' which reflects the symmetry of the 'is an objection to' relation. If q is an objection to p , then p is an objection to q . They, thus, both threaten the acceptability of the other, and, hence, *compete* for acceptance.

claim that reconciles the tension between an objection and the target claim. According to Lehrer, objections can also be neutralized, which he explains as follows:

Lehrer's Definition of an Objection being neutralized: '*n* neutralizes *o* as an objection of *p* for *S* on *X* at *t* if and only if *o* is an objection to *p* for *S* on *X* at *t*, but the conjunction of *o* and *n* is not an objection to *p* for *S* on *X* at *t*, and it is as reasonable for *S* to accept the conjunction of *o* and *n* as to accept *o* alone on *X* at *t*.' (2000, p. 136)

The intuition behind neutralizing an objection is perhaps best illuminated by an example Lehrer provides. Suppose a claimant and critic put forward the following claims:

Claimant: I see a zebra.

Critic: People sometimes dream they see zebras. (Lehrer 2000, p. 135)

In such a case, the critic's claim represents a way in which the claimant might be in error in thinking she is seeing a zebra, and thus the target claim is negatively dependent on the critic's objection (that one sees a zebra is more likely to be true on the assumption that no one ever merely dreams they see zebras than it is on the assumption that people sometimes do). The critic's claim thus constitutes an objection in Lehrer's sense. And, it is quite natural to think that the critic's objection is highly reasonable to accept, perhaps just as reasonable as the Claimant's initial assertion (I hedge because I find it difficult to arrive at clear intuitions when comparing the reasonableness of competing propositions). After all, the critic's claim is almost certainly true. In this case, it might seem improper for the claimant to respond to the critic by insisting that her belief that she sees a zebra is more reasonable than the critic's claim that people sometimes dream they see zebras. In this case, the critic's objection needs to be addressed in a different manner. So, rather than meeting the critic's objection head on by denying that it is as reasonable to accept as the target claim, one might try to disarm the reason the critic provides for doubting the target claim by putting forward additional claims that screen-off the negative dependence between the target claim and the critic's objection. In this particular case, Lehrer suggests that the claimant might go on to offer the following rejoinder as a means to neutralize the objection.

Claimant: I am not dreaming.

This claim reconciles the tension between the critic's objection and the target claim in the sense that the conjunction:

Zebra Conjunction: People sometimes dream they see zebras, but I am not dreaming

does not make it any less likely that one sees a zebra (the reasonableness of the initial claim when probabilities of the initial claim are conditioned on the conjunction are as high as they are before conditioning on the conjunction). So, while the first conjunct (the critic's objection), when considered alone, expresses a way that the claimant might be in error in her belief that she sees a zebra, the conjunction of the critic's objection and the claimant's rejoinder do not. If the critic is not dreaming, then it is not possible that she is dreaming that she sees a zebra. Hence, the critic's statement is not an objection on the assumption that the claimant is not dreaming. Thus, one who is reasonable in accepting that she is not dreaming but people sometimes dream they see zebras has a straightforward response to the critic in this case.

To be able to neutralize an objection in this manner requires that the conjunction be as reasonable to accept as the objection itself. How could this be? Questions of comparative reasonableness does not always reduce to questions of subjective or epistemic probability. A proposition p might be more reasonable than a proposition q , even if p is less probable than q , in cases where $Ut(p) > Ut(q)$ or $Uf(p) > Uf(q)$. Since reasonableness does not reduce to subjective probability, it is possible, on Lehrer's view, that a conjunction can be as reasonable as its conjuncts, so long as the decrease in probability is compensated for by an increase in epistemic utility. And so the claimant's rejoinder may neutralize the objection in this case, just as long as it is as reasonable for the claimant to accept the conjunction of the critic's objection and her rejoinder as it is reasonable for her to accept the critic's objection.

We now have covered all of the essential technical definitions employed in Lehrer's formulation of his theory of justification. His technical definition follows from the justification game and the technical definitions we considered above.

Lehrer's Definition of Justification: S is justified in accepting that p at t if and only if everything that is an objection to p for S on the basis of the acceptance system of S at

t is answered or neutralized on the basis of the acceptance system of S at t . (Lehrer 2000, p. 137).

Again, the general idea is that an acceptance is justified for an agent when the acceptance coheres with the rest of one's acceptance system, and cohering with one's acceptance system is to be understood in terms of there being no un-neutralizable objections to one's belief that are more reasonable relative to one's acceptance system than the belief in question. Or put slightly less technically, p coheres with one's acceptance system when there aren't any objections to p that are just as reasonable to believe as p itself, except, perhaps, those objections that can be reconciled with the acceptance of p .⁹ With the account of justification in hand, we can consider Lehrer's solution to the lottery.

2.2.2. Reasonableness as Probability. Before considering Lehrer's solution to the epistemic inconsistency paradoxes, we consider at least one more key feature of the account. Lehrer notes that "...if the utility of accepting two claims are the same, then comparative reasonableness of accepting one in comparison to the other will be determined solely by the probabilities" (2000, p. 146). So, comparative reasonableness of two propositions reduces to a question of which proposition has greater probability in cases where the two propositions enjoy the same epistemic utilities. That is to say, if q and p are such that $Ut(p) = Ut(q)$ and $Uf(p) = Uf(q)$, then $r(p) \geq r(q)$ if and only if $Pr(p) \geq Pr(q)$. This means that in cases where it is clear that the propositions' epistemic utilities ought to be the same for an agent, then we only need to attend to the probabilities of the propositions in question. Lehrer holds that these conditions are satisfied in the case of lottery propositions, and seems to assume it for the preface as well. Let us now turn to Lehrer's solution to the lottery.

2.3. Lehrer's Argument that His View Avoids the Inconsistency Paradoxes

2.3.1. The Lottery Paradox.

⁹It should also be noted that the proposal ought not to be confused with formal information theory. Davis and Bender (1989) provide a convincing argument that Lehrer's account conflicts directly with standard accounts of *information* and the use of probability in those accounts.

Lehrer's argument that lottery propositions aren't justified: According to Lehrer, his theory provides an answer to the question: why are we not justified in believing lottery propositions? Lehrer explains by having us consider the justification game when played by a claimant aiming to defend a lottery proposition. The claimant puts forward her claim:

Claimant: The number one ticket has not won.

The critic then puts forward her objection:

Critic: The number two ticket has not won. (2000, p. 147)

He goes on to say:

The critic has produced an objection to my claim because, by definition, o is an objection to p just in case it is more reasonable to accept that p on the assumption that o is false than on the assumption that o is true. (2000, p. 147)

In a standard lottery case, $Pr(l_1|l_2) < Pr(l_1)$, and so each lottery proposition being true makes each other lottery proposition slightly less likely to be true.¹⁰ And, as Lehrer notes, in the standard lottery, on the assumption that l_2 is false, i.e., that the second ticket is the winning ticket, l_1 is guaranteed to be true. Thus, l_1 is negatively dependent on l_2 in just the way that our simplified analysis of the objection relation requires. He goes on to explain why the claimant cannot respond to the critic in this case:

the utilities of accepting the two claims, mine and the critic's, are obviously the same, and therefore the comparative reasonableness of the two claims is the same. Consequently, the critic's claim is not answered—it is as reasonable as mine—and it cannot be neutralized either. (2000, p. 147)

In short, the argument is that lottery propositions are objections to each other, and are such that they cannot be answered or neutralized. Lehrer concludes that they are not justified.

Glenn Ross (2003, 2012) has challenged the last step in the solution by questioning why lottery propositions aren't their own self-neutralizers. That is to say, he asks why it isn't the case that

¹⁰The negative dependence of l_1 on l_2 seems to be the only feature that provide reason to think the former is less reasonable to accept on the assumption of the latter.

the conjunction of lottery propositions are more reasonable to accept than their conjuncts? If $r(l_1 \wedge l_2) \geq r(l_2)$, then lottery propositions could serve as their own self-neutralizers. Lehrer (2003) responds that he has a fallback position that his view can account for differing intuitions in cases like the lottery, and that whether one thinks lottery propositions are justifiable may vary with one's judgments of reasonableness (which for Lehrer will depend on the epistemic utilities one assigns to the conjunction of lottery propositions).¹¹

As will become clear below, the case against Lehrer's solution can be strengthened significantly beyond the question put forward by Ross. Ultimately, even if we grant Lehrer's analysis of coherence and the objection relation, the fallback position is unavailable because Lehrer's theory of epistemic justification fails to explain why we cannot rationally have inconsistent acceptances in variants on the standard lottery paradox. The fact is that his strategy for answering the lottery paradox turns on inessential features of the way the lottery is standardly presented: we assume a probability function where lottery propositions are negatively dependent on each other. If lottery propositions are not negatively dependent on each other, then they do not count as objections to each other, and so can't stand in each other's way of being justified. Below we shall turn to consider such problems.

2.3.2. Avoiding the Generalized Version of the Lottery Paradox. It would be natural at this point to be suspicious of Lehrer's solution on the grounds that we have already seen that purely probabilistic solutions succumb to a generalization of the lottery paradox. That is to say, purely probabilistic defeater solutions either tolerate some inconsistency, or else entail that one can only be justified in believing propositions that are absolutely certain. Since Lehrer's solution to the lottery seems to reduce to considerations of the probabilistic relations between lottery propositions, it is reasonable to think that his view suffers from the same problem. Why then doesn't it immediately follow that Lehrer's view succumbs to the generalized version of the lottery and entails that only propositions with probability 1 are justified? The short answer is that while the comparison of reasonableness of lottery propositions reduces to considerations of probabilistic relations, it doesn't follow that considerations of the reasonableness of all propositions reduces to probabilistic relations.

¹¹As far as I can see, Ross has offered no reason to think Lehrer's fallback position is undermined by the arguments in Ross (2003, and 2012), but rather has simply put forward an alternative characterization of reasonableness to motivate a competing solution to the lottery.

And, thus, Lehrer's defeater condition is not definable in terms of purely probabilistic features of propositions.

It will be useful to quickly review how the generalized lottery paradox argument would be advanced against Lehrer's position and what kinds of replies are available on Lehrer's theory of justification. We can make Douven and Williamson's generalization of the lottery concrete by starting with some ordinary proposition that appears to be justified:

Computer: I am using a computer as I write this.

Next, consider a lottery where lottery propositions are more probable than *Computer*. Let l_i be our standard lottery proposition: the i^{th} ticket is a losing ticket. Then, following Douven and Williamson (2006), we can form generalized lottery propositions as follows:

$$g_i = l_i \vee \neg \textit{Computer}.$$

Now, we get the generalized lottery set: $G = \{g_1, \dots, g_n, \textit{Computer}\}$. This is clearly an inconsistent set of claims, and so if Lehrer's view is supposed to rule out justified inconsistent beliefs, it must rule out some of these propositions. So, why can't an agent be justified in believing each member of G ? Lehrer can make exactly the same response to the g_i 's as he does to the l_i 's of a standard lottery. The g_i 's are objections to each other with the same epistemic utility, and so their comparable reasonableness reduces to their comparative probabilities. Since these are the same, one cannot be justified in believing the g_i 's.

But what about *Computer*? Now, it would be a real problem if the same argument could be made to show that *Computer* is not justified. This would show that Lehrer's view solves the lottery paradox by collapsing into infallibilism in just the same way that the probabilistic proposals we considered in the last chapter did, i.e., by ruling out justified belief in all ordinary propositions like *Computer*. However, such a simple refutation of Lehrer's solution to the inconsistency paradoxes is not possible. The reason is that while the comparative reasonableness of the g_i 's reduces to a comparison of their probabilities, it is not fair to assume this about *Computer*. *Computer*, after all, contains very different informational content from the g_i 's, something that makes an important contribution to $Ut(\textit{Computer})$ and $Uf(\textit{Computer})$. Comparing the reasonableness of accepting *Computer* to the reasonableness of accepting generalized lottery propositions does not reduce to

their probabilities on Lehrer's account. And so, despite the fact that the lottery propositions are more probable, it is possible to hold that *Computer* may be more reasonable to accept.

It thus seems as though Lehrer's solution can avoid such a straightforward and simple refutation from a generalized version of the lottery paradox. It remains to be seen whether Lehrer's solution is successful for all versions of the lottery, and for all cases of inconsistent beliefs. Before we consider how to generalize Lehrer's solution, we turn to his response to the preface paradox, and a few formal problems that Lehrer's solution faces that have been pointed out by Olsson (1998).

2.3.3. The Preface. The preface paradox can be formulated in a variety of ways.¹² For now, we shall focus on the version of the paradox that Lehrer responds to in order to see how that version of the paradox is supposedly avoided by Lehrer's view (It shall become clear that it doesn't really matter what version of the paradox we focus on as Lehrer's solution will succumb to the same general problems). The simplest way to formulate the paradox is to start with a very large set of probabilistically independent atomic propositions $\{p_1, \dots, p_n\}$. Suppose some agent S accepts each member of this set. (and each is very likely to be true relative to S 's acceptance system). Now, consider the preface proposition:

$$\text{IMP: } (\neg p_1 \vee \dots \vee \neg p_n)$$

For an agent with a very large number of acceptances the probability of IMP could be extremely high (it will be equal to $1 - [Pr(p_1) \cdot Pr(p_2) \cdot \dots \cdot Pr(p_n)]$, since we are assuming independence). And so if a high likelihood of truth were sufficient for justification, then one could be justified in holding all of one's atomic acceptances, and yet also be justified in accepting IMP. But, of course, on Lehrer's view, a high probability is insufficient for one to be justified in accepting a preface proposition. In addition to being very likely to be true, it is also necessary that IMP cohere with S 's acceptance system, i.e., S must be in position to win the justification game when defending IMP against potential critics and their objections to IMP. Lehrer's solution to the preface paradox is to insist that one cannot be in position to win the justification game for IMP.

Before considering Lehrer's ultimate defense of the claim that his coherence theory does not entail that one can be justified in believing a proposition like IMP, it will be useful to briefly review Mylan

¹²See Makinson (1965) for the original presentation. See Easwaran and Fitelson (in press) for an alternative, and perhaps more convincing, formulation of the paradox.

Engel's (1991) argument that IMP is justified on Lehrer's account. Engel (1991) makes two basic observations. The first is that when considering the expected epistemic utility of believing IMP, accepting IMP has positive expected epistemic utility for reasonable assignments of the epistemic utility of accurately accepting IMP (this is something Lehrer seems happy to concede).¹³ That is to say, when we plug IMP into Lehrer's formula for calculating reasonableness

$$r(IMP) = Pr(IMP) \cdot Ut(IMP) + Pr(\neg IMP) \cdot Uf(IMP) > 0$$

for a large enough preface set even if the negative epistemic utility obtained from believing IMP in situations where IMP is false greatly outweighs the positive epistemic utility obtained from believing IMP in situations where IMP is true. This is because for any values for $Uf(IMP)$ and $Ut(IMP)$ we might choose, we can allow the preface set to be large enough so that $Pr(IMP)Ut(IMP) > Pr(\neg IMP)Uf(IMP)$. The upshot of this is supposed to be that if one is choosing between three doxastic options: accepting IMP, remaining agnostic about IMP, and accepting $\neg IMP$, the choice with the highest epistemic utility is believing IMP (This assumes that remaining agnostic yields neither positive nor negative epistemic utility). Thus, in terms of choices aimed at maximizing expected epistemic utility believing IMP is supposed to be well motivated by Lehrer's analysis of reasonableness.

The other important claim that Engel defends is that even though each of the p_i in S 's preface set constitute objections to IMP, Engel insists that for a large enough set of contingent propositions, we can be assured that IMP is more probable than any of its objections (1991, p. 122).¹⁴ Engel concludes from this that since IMP has a higher epistemic probability, it is at least as reasonable as the objections in the preface set. Engel thus concludes that one could defend IMP against a critic's objections and win the justification game, thereby establishing that one is justified in believing IMP on Lehrer's view.

Lehrer offers the following response to Engel (1991) and the preface paradox:

¹³See Engel (1991, p. 122-124) for his defense of this claim.

¹⁴A terminological note: Lehrer's coherence theory, in most of its detail, has remained the same, but his terminology has not. For most of his career, Lehrer talked not of claims being objections to each other, but rather claims being competitors. That is to say, when claims stand in negative evidential relations where one claim is less reasonable to believe conditional on the assumption that the other is true, Lehrer held that those claims competed for acceptance. Thus, when Olsson (1998) talks of competitors he means claims that are objections to one another.

That statement IMP competes with each p_j , and, as Engel says, may be more probable than each p_j . But, and this is what Engel failed to notice, IMP also competes with the disjunction of 9,999 of p_j s. IMP competes with the disjunction, D9,999, which says either p_1 , or p_2 , or ... or $p_{9,999}$, that is, the disjunction of all but one of the 10,000 independent statements. Why? Because it is less reasonable to accept IMP if I assume that D9,999 is true than if I assume that it is not. D9,999 is a tough competitor, and IMP must beat this competitor as well as any other disjunction of 9,999 of the original statements, and some of these are going to be more probable than IMP. So, contrary to Engel, IMP does not beat all these competitors, and we are not justified in terms of coherence with an acceptance system in accepting IMP. It does not beat all of its competitors. I agree that for some purposes it may be reasonable to accept an inconsistent set of propositions. But the set of things that one is justified in accepting in the quest for truth and the attainment of knowledge must be consistent. (Lehrer 1991, pp. 132-133)

A terminological clarification is needed. In earlier presentation of his theory, Lehrer calls claims that a critic puts forward in the justification game 'competitors', and holds that a proposition is justified just in case all competitors offered by a critic can either be answered or neutralized. 'Competitor' and 'objection' can thus be used interchangeably.

Now, we can strengthen Lehrer's point by noting that the most probable competitor of IMP is the disjunction of all members of the preface set, namely:

Postscript: $p_1 \vee \dots \vee p_n$.¹⁵

The key observation is that Postscript and IMP are competitors, or objections to one another, and Lehrer's claim is that Postscript is more reasonable than IMP and that it cannot be neutralized. Lehrer concludes that IMP is thus unjustified and his theory avoids countenancing inconsistent beliefs in the preface as well as the Lottery.¹⁶

¹⁵'Postscript' is a good name for this proposition in that if IMP is supposed to be a proposition corresponding to the fact that S has made some inaccurate judgment, then the Postscript says he made at least one correct judgment.

¹⁶Lehrer initially offers a couple of different arguments against accepting IMP. See Engel (1991) and Lehrer (1991) for discussion of earlier versions of the arguments wherein Lehrer's two earlier responses to the preface paradox are laid out and seemingly refuted. Also see Olsson (1998) for a continued discussion of Lehrer's work on the preface paradox and a purported proof that Lehrer's theory rules out JIB in cases like the preface.

2.4. Olsson's Observations

Erik Olsson (1998) suggests a straightforward way to understand Lehrer's view in contexts where questions of comparable reasonableness can be reduced to comparisons of propositions subjective probabilities (an assumption Lehrer seems to make when comparing lottery and preface propositions). According to Olsson, Lehrer's account of justification can be defined as follows for such contexts:

Definition 1: The sentence p is Lehrer-acceptable relative to the probability assignment P if and only if (i) $0 < P(p) \leq 1$ and (ii) $P(p) > P(c)$ for all sentences c such that $P(p|c) < P(p)$. (1998, p. 35)

In effect, this simplifies Lehrer-acceptability to some degree, and thereby helps to bring into focus a few formal problems with Lehrer's solution to the inconsistency paradoxes.

First, Olsson makes an observation that problematizes Lehrer's solution in a very fundamental way:

Observation 1: If (i) $P(p) \leq P(q) < 1$, and (ii) $P(p \& q) = P(p)P(q)$, then p is not Lehrer-acceptable relative to P . (1998, p. 40)¹⁷

Olsson's argument for this proceeds by noting that if we let r be the proposition $(p \wedge q) \vee \neg p$, then r is an objection to p , i.e., $P(p|r) < P(p)$ and $P(r) > P(p)$.¹⁸ This observation carries with it radically skeptical implications. What it means is that one cannot be justified in believing any two independent propositions, p and q , if the epistemic utilities of believing p and believing q are the same, since it will follow that comparing their degrees of reasonableness will automatically reduce to comparing their probabilities.

Olsson goes on to note that things are actually much worse for Lehrer than this first observation suggests. What it implies is that in contexts where only relative probabilities are necessary to evaluate comparative reasonableness of propositions then we get the following result:

¹⁷I will from here on use *Lehrer-acceptable* as a technical notion referring to Lehrer's general notion of acceptability but only in contexts where comparisons of reasonableness reduce to comparison's of subjective probabilities. This is in keeping with Olsson's usage.

¹⁸The technical proof is actually quite straightforward, so I will leave it as an exercise for the reader.

If two proposition p and q , each with a probability less than 1, are both acceptable relative to P then $P(p|q) > P(p)$. (1998, p. 41)

The argument for this is that if they are independent, then we get this result from observation 1. If they are negatively relevant to each other, then at least one is no more probable than the other and so that proposition is unacceptable. Thus, the coherence constraint implied by Lehrer's proposal is extremely strong in such contexts. It is not just that acceptable propositions must not be in tension with one another, but that they must be pairwise mutually supportive of one another.

Olsson notes that this then has highly skeptical implications for Lehrer's account of acceptability in cases like the preface. Ultimately, Lehrer offers no response to the fact that IMP is an objection to, or competes with, each p_i in the preface set, and thereby threatens the acceptability of the p_i 's. In fact, both Lehrer and Engel maintain that IMP is more probable than the p_i 's. If comparing the reasonableness of the p_i 's to IMP turns on their subjective probabilities, then a highly skeptical conclusion follows: none of the propositions in the preface set are Lehrerian-acceptable. Thus, Lehrer's solution to the preface does seem to collapse into skepticism. Ultimately, Olsson concludes that Lehrer's solution cannot stand as it has been formulated.

2.4.1. Olsson's Solution. Rather than take his observations to thoroughly refute Lehrer's proposal, Olsson maintains that the solution is to revise Lehrer's analysis of acceptability. The basic lesson Olsson takes away from his observations is that Lehrer's account of coherence ought not require that one be in position to answer all objections, since such a requirement implies skepticism. He notes that the problem in the preface case is that IMP is the basis for one not being able to justifiably believe any of the p_i 's, and yet itself unacceptable in virtue of succumbing to an objection. Olsson asks, "...If IMP is not itself acceptable why should it have any influence over the acceptability of the p_i 's?" (p. 43). He goes on to explain his reply:

IMP presents indeed an objection to the p_i 's but, being itself unacceptable, an objection which is overruled.

Lehrer's acceptability definition states that all competitors must be beaten by a given sentence for that sentence to be acceptable. According to the previous line of reasoning, that definition should be modified to say that a sentence in order to be acceptable must beat, not all competitors, but the competitors that

are themselves acceptable. Now, clearly we cannot define acceptability in terms of acceptability. Instead we must find a way to avoid circularity. How this can be done using an inductive definition is the subject of the next section. (1998, p. 43)

Olsson's solution is to set out a recursive definition of acceptability that is meant to capture the idea that a proposition is acceptable just in case one is in position to answer all objections to that proposition that are not overruled.

Olsson's proposal requires a few technical definitions. First, we let pCq stand for p competes with q (note that this relation is symmetric, so we could also read this as p and q are competitors). Here is Olsson's essential definitions:

Let M and N be sets of sentences. We define:

- (i) $\text{Max}(M) = \{p \in M \mid P(p) \geq P(q) \text{ for all } q \in M\}$
- (ii) $\text{NoComp}(M) = \{p \in M \mid \text{there is no } q \in M \text{ such that } qCp\}$
- (iii) $\text{Comp}(N, M) = \{p \in N \mid pCq \text{ for some } q \in M\}$ (1998, pp. 43-4)

Olsson goes on to explain that

$\text{Max}(M)$ is the set of maximally probable sentences in M . $\text{NoComp}(M)$ is the set of sentences in M that lack competitors in that set. $\text{Comp}(N, M)$, finally, is the set of all sentences in N that compete with some sentence in M . (1998, p. 44)

And then he offers a refined notion of *acceptable* as follows:

P^* is the set of accepted sentences relative to P and a finite language L . U^* is the set of undecided sentences relative to P and L . P^* and U^* are defined inductively as follows:

$$P^0 = \emptyset$$

$$U^0 = \{L \setminus \{p \mid Pr(p) = 0\}\}$$

$$P^{n+1} = P^n \cup \text{NoComp}(\text{Max}(U^n))$$

$$U^{n+1} = U^n \setminus [\text{Max}(U^n) \cup \text{Comp}(U^n, \text{NoComp}(\text{Max}(U^n)))]$$

$$P^* = \bigcup \{P^n \mid n \in \mathbb{N}\}$$

$$U^* = \bigcap \{U^n \mid n \in \mathbb{N}\} \text{ (1998, p. 44)}$$

While this may look difficult to parse, the procedure for determining which sentences are P^* -acceptable is relatively straightforward. We begin with all sentences in the language being undecided save those that have probability zero. Then, at each stage, we throw those propositions into the acceptable set that are most probable or reasonable to accept that don't have any competitors that are just as reasonable. And we subtract from the set of undecided sentences two different groups of sentences. First, we subtract those sentences that are most reasonable that were remaining when we entered this stage. Some of these sentences will have been moved over into the pile of acceptable sentences, the rest in this group must compete with other members of this group and thus are all thrown out as sentences that have been beaten or overruled. The second group of sentences that are removed are any sentences that compete with those sentences that are the most probable or reasonable sentences that were remaining when we entered this stage. All such sentences have objections that haven't yet been overruled by this stage, and thus must be eliminated. This process continues until all sentences of the language have been decided.

This, in a nutshell, is Olsson's proposal, and he demonstrates that it stands at an advantage to Lehrer-acceptability in a variety of ways. For instance, Olsson considers a finite language where p and q are the only atomics and shows that his view delivers the desired verdicts in the preface and lottery cases for such a language, while avoiding the problem of independent propositions that he first observed (See Observation 1 above). Olsson's proof of this is simply to provide a particular probability function and go through the calculation of P^* in each case. For a language with exactly two atomic propositions, all of his results are easy to verify (Since the results aren't in dispute, I leave the interested reader to consult Olsson's proofs). In the process, Olsson establishes two further formal features of his account of acceptability worth reviewing.

Olsson proves that the following two observations hold:

Observation 2: All Lehrer-acceptable sentences are P^* -acceptable. (p. 48)

Olsson provides a rigorous formal proof, but the reason why the proof works can be stated informally. If p is Lehrer-acceptable, then $Pr(p) > Pr(c)$ for all c such that pCc . The only way $p \notin P^*$

is if at some stage of the process there is a c such that c hasn't been overruled, pCc holds, and $Pr(c) \geq Pr(p)$. By hypothesis, there can be no such c and thus $p \in P^*$ if p is Lehrer-acceptable.

The third observation Olsson makes will be key as we return our attention back to the question of whether a Lehrian account of acceptability rules out justified inconsistent beliefs. Olsson observes:

Observation 3: P^* -acceptable sentences do not compete. (1998, p. 51)

Again, it is fairly easy to explain why this must be true without reviewing Olsson's rigorous formal proof. The basic reason why is that if p and q competed and $P(p) \geq P(q)$, then either there is some stage where p is eliminated because there is a competitor c such that $P(c) > P(p)$, or else there is a stage where p is one of the most probable sentences that has been undecided. If the latter, then q is jettisoned into the group of unacceptable propositions, since all competitors to p would be so jettisoned at this stage.

The essential upshot then of both Lehrer and Olsson's theory of acceptance (at least in contexts where questions of reasonableness reduce to questions about subjective probabilities) is that one cannot be justified in accepting both p and an objection, o , to p . Why did we need to go through all of this? The answer is that it puts us in position to consider the most natural way to try to generalize Lehrer's solution to the lottery and the preface paradoxes.

2.4.2. Generalizing Lehrer's Solutions to the Lottery and the Preface? We have now seen how the Lehrer-Olsson account of acceptability is supposed to provide a resolution to the standard versions of the lottery and preface paradoxes, as well as side step generalized versions of the lottery paradox. While this goes some way to showing that Lehrer's view rules out justified inconsistent acceptances in a manner that doesn't collapse into skepticism, we are still a long way from establishing that justified inconsistent acceptances are not possible on Lehrer's view. Neither Lehrer nor Olsson offer anything like a generalized argument that for any inconsistent set of claims,

there will be some member of the set that cannot be justified.¹⁹ That is, we have no argument for thinking that the following hypothesis is true.

Hypothesis 1: *For any inconsistent set of propositions G where members of G are such that their comparable reasonableness reduces to a comparison of their subjective probabilities, it is necessarily the case that at least some members of G are Lehrer-unacceptable or P^* -unacceptable for S .*

But in order for this proposition to be true, the following proposition would also have to be true (it represents a necessary, though not sufficient condition for P^* -acceptability).

Hypothesis 2: *For any inconsistent set of claims, $A = \{a_1, \dots, a_n\}$ (where questions of reasonableness reduce to questions of subjective probability), there must be some $a_j \in A$ such that there exists some proposition c where $Pr(p|c) < Pr(p)$ and $Pr(c) \geq Pr(p)$.*

After considering Olsson's revision to Lehrer's account, I think we can see the most natural reason one might think an argument against JIB might be generalized from Olsson and Lehrer's solution to the lottery and the preface. As we saw, in both the standard lottery and preface sets, as well as in the generalized version of the lottery, inconsistent sets of claims were all such that at least some members of the inconsistent set were competitors with one another. Given that one cannot be justified in accepting claims and their competitors on Olsson's account (Observation 3) or Lehrer's in contexts where probabilistic relations determine reasonableness, one might think that we can establish Hypothesis 1, by showing that the following hypothesis is true:

Hypothesis 3: *For any inconsistent set of claims, $A = \{a_1, \dots, a_n\}$, there must be some $a_i, a_j \in A$ such that $P(a_i|a_j) < Pr(a_i)$.*

In other words, if every inconsistent set of propositions contained some propositions that are competitors, then both Lehrer and Olsson's accounts of acceptability would rule out one being justified in accepting inconsistent sets of claims.

¹⁹It must be emphasized that in trying to generalize from Lehrer's argument that his coherence theory rules out JIB in cases like the lottery and the preface, we must confront an essential feature of the examples as Lehrer construes them. The relevant propositions are all of the sort that allow for considerations of comparable reasonableness to reduce to comparison of those propositions' subjective probabilities. The trouble is that there is no guarantee that all inconsistent sets of propositions are going to be like this. So, in trying to generalize, we can at best see how the generalized argument is supposed to go for situations where the relevant propositions are such that we need only consider their probabilities when comparing the reasonableness of accepting those propositions. Nevertheless, it is worth pursuing the question whether Lehrer and Olsson's approaches necessarily rule out inconsistent beliefs in cases where the set of propositions' comparable reasonableness reduces to questions about relative subjective probabilities.

As far as I can see, this is the only obvious method available to try to generalize Olsson and Lehrer's solution. But is such a method viable? As we shall see, the answer is clearly no.

2.5. The Generous Lottery Paradox

Now we shall consider an alternative version of the lottery paradox that demonstrates that inconsistent sets do not always contain members that are objections to each other in Lehrer's sense. In other words, that lottery propositions constitute objections to one another in Lehrer's sense is an unnecessary artefact of the particular sort of lottery paradox Lehrer has chosen to focus on. All relevant features of the lottery paradox can be preserved while avoiding this fact. Let us call the following lottery a ***Generous Lottery***. The generous lottery is just like a standard lottery with one exception. First, a ticket is drawn to be the winning ticket. The authorities running the lottery announce that this ticket has been drawn and is guaranteed to be a winning ticket. But then, the authorities announce that they are feeling extra generous and want to allow for the possibility that all tickets will be declared winning tickets. Some randomizing mechanism is chosen to determine whether all tickets in the lottery will be declared winners. The randomizing mechanism is designed so that the probability that all tickets will be declared winners is $1 - r$. Thus, r is the probability that the only winning ticket is the one that was initially drawn.

Now, consider the probabilities of lottery propositions of such a lottery when there are n -many tickets. As constructed, this lottery is guaranteed to have at least one winner, so at least one lottery proposition is false (thus, $\sim (l_1 \wedge \dots \wedge l_n)$ is still known to be true). Now consider the lottery propositions for a generous lottery. First, let us define the proposition stating that the randomizing mechanism has not selected all tickets to be winning tickets:

NG: Not all tickets are declared winners.

The probability of NG is simply r . And now we let each of our lottery propositions be sentences claiming that some particular ticket doesn't win:

g_i : "The i^{th} ticket in the lottery loses."

What is the probability of g_i ? The calculation of g_i 's probability is straightforward. g_i is true if and only if NG is true and the i^{th} ticket was not initially chosen as a winner. And NG's being true and i^{th} ticket not being chosen are independent events. Thus, by multiplying their probabilities we get:

$$Pr(g_i) = r \cdot \frac{n-1}{n}.$$

The next thing we need to determine is whether each g_i counts as an objection to each g_j when $i \neq j$ in Lehrer's sense. To answer this question, we must first calculate the probability of g_i conditional on g_j . Now, if any lottery proposition is true, it follows that NG must be true. Thus, conditional on g_j , the probability of NG is 1. So, the probability of one lottery proposition conditional on another will be the chance that both were not chosen as winners out of all tickets minus the one we are conditionalizing on. In other words, the conditional probabilities reduce to the conditional probabilities of a standard lottery:

$$Pr(g_i|g_j) = \frac{n-2}{n-1}.$$

As we noted above, g_j is an objection to g_i in Lehrer's sense just in case $Pr(g_i) > Pr(g_i|g_j)$. But now at least if we choose the right r and n , then we can be sure that just the opposite is true, i.e., each lottery proposition constitutes evidence for each other lottery proposition.

THEOREM 2.1. *For any threshold $.5 < t < 1$, there exists r and n such that $Pr(g_i) > t$ and $Pr(g_i|g_j) > Pr(g_i)$.*

PROOF. Start by choosing n such that $\frac{n-2}{n-1} > t$. As explained above, $Pr(g_i) = r \cdot \frac{n-1}{n}$. By the density of the reals, there is some ε such that $t < t + \varepsilon < \frac{n-2}{n-1}$. We simply choose r such that $r \cdot (\frac{n-1}{n}) = t + \varepsilon$, i.e., let $r = \frac{t+\varepsilon}{(\frac{n-1}{n})}$, and we have it that $Pr(g_i) > t$ and $Pr(g_i|g_j) > Pr(g_i)$. \square

What the above theorem demonstrates is that there is a generous lottery such that none of the lottery propositions of a generous lottery are negatively dependent on each other, and thus none constitute objections to each other in Lehrer's sense. In such a case, a claimant in the justification game need not worry about the critic's objection that another generous lottery proposition is true because such claims are not objections to each other in Lehrer's sense (they actually provide reason

to increase one's confidence in the target claim). Thus, Lehrer's strategy for solving the standard lottery paradox does not apply to a generous lottery.

At this point, one might want to insist that Lehrer can respond that while lottery propositions need not be objections to each other, for any lottery proposition, g_1 , there is some conjunction of lottery propositions, $g_2 \wedge \dots \wedge g_k$, that will constitute an objection to g_1 in Lehrer's sense.²⁰ This is surely right, as we know that $Pr(g|g_2 \wedge \dots \wedge g_n) = 0$, but in order for such a conjunction to serve the critic's purpose of winning the game over the claimant, it will have to be the case that the conjunction, $g_2 \wedge \dots \wedge g_k$, is at least as reasonable as g_1 . Otherwise, on Lehrer's account, the objection can be answered, and thus fails to stand in the way of the claim being acceptable. But, as Ross (2003, 2012) has pointed out, if conjunctions of lottery propositions are as reasonable to accept as individual lottery propositions, then lottery propositions in the standard lottery are self-neutralizers. In which case, Lehrer's account of justification fails to explain why one cannot be justified in accepting lottery propositions in a standard lottery.

Unless some alternative principle can be invoked to explain why one is not justified in believing generous lottery propositions, Lehrer faces a dilemma. He cannot offer a uniform explanation for why we aren't justified in believing lottery propositions in the two cases. If his strategy applies in the one case, then it undermines the solution in the other. The same problem seems to arise for Olsson's updated proposal. After all, as Olsson shows, Lehrerian-acceptable propositions are P^* -acceptable. Since there seems to be no explanation to rule out accepting generous lottery propositions for Lehrer's account, the same issue holds for Olsson's as well.

2.5.1. Epistemic Closure to the Rescue?

There is one more reply that we need to consider. One might insist that in cases like this one, some kind of epistemic closure principle might provide Lehrer with grounds to deny that one can believe g_1 .²¹ In particular, one might insist on the following epistemic principle:

²⁰In other words, Lehrer's proposal could take a familiar form, namely that lottery propositions serve as collective defeaters for each other. A variety of proposals have been put forward that take this form, and Lehrer's has a distinct advantage of not succumbing to the generalizations of the lottery paradox put forward by Douven and Williams (2006) and Smith (2010).

²¹It was in conversation with Junyeol Kim that I came to recognize that such a reply is worth considering.

SPC: If S is not justified in accepting q and p entails q , then S is not justified in accepting p .²²

And then argue that this principle in conjunction with Lehrer's analysis of epistemic justification would rule out justified belief in generalized lottery propositions. How would such a principle help in this case? First, one might note that embedded in the generous lottery paradox are standard lottery propositions, namely, propositions of the form:

l_i : The i^{th} ticket was not chosen as the ticket that would-be the sole winner in the event that NG is true.

These propositions are standard lottery propositions, and thus stand in exactly the same negative dependence relation employed in Lehrer's argument that each l_i is not Lehrer-acceptable. Of course, if one's ticket was chosen as the would-be sole winner, then one's ticket cannot possibly be a loser (it is either a sole winner, or one of many). Thus, the g_i and l_i are related such that g_i is true only if l_i is true. And, hence, given that the l_i are not Lehrer-acceptable, it would follow from SPC and Lehrer's theory of epistemic justification that the g_i are not as well.

Now, the question is whether or not SPC holds for Lehrer's theory of epistemic justification, or if perhaps one might revise his analysis to incorporate this principle. It would be explanatorily convenient if an SPC principle of this sort followed directly from Lehrer's theory of acceptability. So, the next question we turn to is whether or not Lehrer can simply hold that a principle like SPC is true and then use it to argue that generous lottery propositions are unjustified.

2.6. single premise Closure Failure

Though he makes no note of this, it follows from several of Olsson's examples that P^* -acceptability is not closed under single premise entailment. In several cases, Olsson shows that propositions p

²²This, of course, is a naïve version of the principle that few philosophers would defend. Those who would argue for single premise closure would usually require a recognition condition on the entailment, and may instead frame the principle in terms of what an agent is in position to come to justifiably believe or accept by the performance of a competent deduction. No such qualifications are necessary for my main purpose here which is to show that the way in which Lehrer's view fails to validate single premise closure calls into question his solution to the preface paradox. However, I will say that it does seem counter-intuitive for Lehrer's view to entail that a conjunction can be acceptable because it coheres with one's background system of acceptances, but that each conjunct is not because they fail to cohere with one another.

and q are both P^* -acceptable, but that $p \vee \neg q$ and $\neg p \vee q$ are not. It is not just that we get failure of single premise closure in a few outlier cases. For Olsson's proposal, we have the following theorem.

THEOREM 2.2. *If propositions p and q are both P^* -acceptable and less than fully certain, then it follows that both $p \vee \neg q$ and $\neg p \vee q$ are not P^* -acceptable when $p \vee \neg q$ and $\neg p \vee q$ are less than fully certain.*

PROOF. This claim follows from the fact that p always competes with $\neg p \vee q$, q always competes with $\neg q \vee p$ when $p \vee \neg q$ and $\neg p \vee q$ are less than fully certain, and Olsson's observation 3, that P^* -acceptable claims don't compete. That p always competes with $\neg p \vee q$ follows from the fact that these claims cannot be jointly false. They are thus subcontraries, and as Meijs (2006, p. 239) points out, subcontraries are always negatively dependent on one another. The same considerations explain why q competes with $p \vee \neg q$. \square

Theorem 2.2 is in and of itself highly counter-intuitive and shows that there will be widespread failure of single premise closure over sets of propositions where questions of reasonableness reduce to questions of probability. And, the result holds for Lehrer's view in all cases where there isn't some neutralizer that can be put forward to neutralize the tension between the competing propositions. Hence, as officially formulated, neither view can appeal to a principle like SPC to avoid inconsistent justified belief in cases like the generous lottery paradox.²³ I am not going to argue that such a failure refutes Lehrer and Olsson's proposals. I do think it is noteworthy that their views do end up having to confront many of the same issues that views like Dretske's (2005) problem of abominable conjunctions (cases where an agent believes a conjunction, but not one of the conjuncts).²⁴ But, in the context of our present inquiry, the issue I want to focus on is what the failure of SPC means for Lehrer and Olsson's solutions to the inconsistency paradoxes.

At this point, it is worth noting that the failure of SPC doesn't just undermine Lehrer's proposed solution to the lottery paradoxes. There is an analogous problem for Lehrer's proposed solution to the preface paradox. To see how a problem might arise, first let us suppose that we have some argument that Lehrer and Olsson's views entail that one cannot be justified in accepting

²³It is open on Lehrer's proposal to appeal to neutralizing propositions, but then all of the real explanatory work would be done via neutralization. Since we don't get anything like a theory of neutralization, the explanation for why closure doesn't fail in all or most cases is left unaddressed by Lehrer's theory.

²⁴Also see Hawthorne (2005) for discussion of abominable conjunctions.

all members of some set $\{p_1, p_2, p_3\}$ because of certain negative dependence relations they stand in. Can we conclude from this that their views preclude one from being justified in accepting all members of $\{p_1 \wedge p_2, p_3\}$? If the former is an inconsistent set, we might hope to be able to infer this for the latter set, since it too will be inconsistent. But, given that SPC fails, the answer is clearly no. In fact, we can show that even if p_1 and p_2 stand in a tension with one another that makes it impossible to be justified in accepting both of them at the same time, there need be no tension between them that would rule out justifiably believing in their conjunction.

In order to show this, I will construct a counter-example for a very simple language with just two atomic propositions. So, let L_2 be a simple classical propositional language with standard syntax and semantics and standard Boolean connectives and atomic propositions p and q . Such a language will have exactly 16 distinct equivalence classes for sentences, one corresponding to each sentence in the follow set: $\{p \wedge \neg p, p \wedge q, \neg p \wedge q, p \wedge \neg q, \neg p \wedge \neg q, p, q, p \equiv q, p \equiv \neg q, \neg p, \neg q, p \vee q, p \vee \neg q, \neg p \vee q, \neg p \vee \neg q, p \vee \neg p\}$. In order to show that both notions of acceptability fail to be closed under single premise entailment, we will first show that the following theorem holds:

THEOREM 2.3. *If a probability function on L_2 is such that $P(p \wedge q) > 1/2$, then $p \wedge q$ is Lehrer-acceptable and P^* -acceptable.*

PROOF. We prove this by showing that there is no $r \in L_2$ such that $P(p \wedge q|r) < Pr(p \wedge q)$ and $Pr(r) > 1/2$, i.e., $p \wedge q$ has no competitors that are more probable than it. Let us first suppose $P(p \wedge q) > 1/2$. Now, for potential competitors in L_2 , we break up the sentences into two classes, those that are entailed by $p \wedge q$ and those that are not. It is a law of probability that if $p \wedge q$ entails r , then $P(p \wedge q|r) > Pr(p \wedge q)$. So, the only possible competitors are those sentences in L_2 not entailed by $p \wedge q$. Thus, the only potential competitors to $p \wedge q$ are the members of the following set: $Comp(p \wedge q) = \{\neg p \wedge q, p \wedge \neg q, \neg p \wedge \neg q, p \equiv \neg q, \neg p, \neg q, p \vee \neg q, \neg p \vee q, \neg p \vee \neg q\}$. These are all sentences that are potential competitors of $p \wedge q$. To complete the proof, we simply need to show that all members of $Comp(p \wedge q)$ have probability less than one half. Note that for all r , if $r \in Comp(p \wedge q)$ then r entails $\neg p \vee \neg q$, and thus $P(r) \leq P(\neg p \vee \neg q)$. Since $p \wedge q$ is logically incompatible with $\neg p \vee \neg q$, it follows that $P(p \wedge q) + P(\neg p \vee \neg q) \leq 1$. Thus, by the assumption that $P(p \wedge q) > 1/2$, it follows that $r \leq P(\neg p \vee \neg q) < 1/2$ for all $r \in Comp(p \wedge q)$ \square

Thus, another way to construct a counter-example to SPC for Olsson and Lehrer's accounts of rational acceptability is to choose any probability function P where $P(p) \cdot P(q) > P(p \wedge q)$ and $P(p \wedge q) > 1/2$. In other words, take any example where p and q are negatively dependent on each other relative to P , where the conflict between these two propositions cannot be neutralized. Then on both Lehrer and Olsson's views, at least one of the two propositions is unacceptable, since it has an objection that cannot be neutralized or answered. But, given Theorem 2.3 above, we know that their conjunction, $p \wedge q$, is both Lehrer-acceptable and P^* -acceptable. Such examples give us cases where $p \wedge q$ is rationally acceptable, but where one of the conjuncts is not.

Why does this pose a problem for the explanation for why IMP is not acceptable to belief? In a nutshell, given how likely it is that one has some true beliefs and some false beliefs, the conjunction of IMP and Postscript are very likely to be true. And while *IMP* might be a competitor to all other ordinary preface propositions, there is no reason to think that the conjunction of *IMP* and *Postscript* is a competitor to any other ordinary preface proposition. Hence, there is no reason to think that their conjunction is rationally unacceptable. In the next section, I construct an example illustrating this fact.

2.6.1. SPC and a Strengthened Preface Paradox. Recall that Lehrer's response to the preface paradox is to maintain that one cannot justifiably believe IMP because postscript constitutes an unanswerable and unneutralizable objection to IMP. Thus, we cannot believe all members of the following preface set: Original Preface Set = $\{p_1, \dots, p_n, \text{IMP}\}$. But again, we must ask, why can't we believe:

$$\text{Postscript\&IMP: } (p_1 \vee \dots \vee p_n) \wedge (\neg p_1 \vee \dots \vee \neg p_n).$$

For large enough n , $P(\text{Postscript\&IMP}) > k$ for whatever threshold k we might choose, and so Postscript&IMP is a prime candidate for SPC failure along the lines we discussed above. At the very least, it seems that we have no good reason to take SPC for granted nor to assume that Lehrer's view entails that one can't justifiably believe Postscript&IMP. And so, we need some argument for thinking we cannot believe all members of: Strengthened Preface Set = $\{p_1, \dots, p_n, \text{Postscript\&IMP}\}$.

So, let us consider a concrete case with four atomic propositions. That is, we let the Strengthened Preface Set be the set: $\{p, q, r, s, \text{Postscript}\}$. And then we set our probability function over the set be defined as follows:

<u>Sentence</u>	<u>Probabilities</u>
$p \wedge q \wedge r \wedge s$.23
$p \wedge q \wedge r \wedge \neg s$.134
$p \wedge q \wedge \neg r \wedge s$.134
$p \wedge \neg q \wedge r \wedge s$.134
$\neg p \wedge q \wedge r \wedge s$.134
$p \wedge q \wedge r \wedge s$.234
p	.632
q	.632
r	.632
s	.632
Postscript&IMP	.536

Now, the crucial thing is that every element of our Strengthened Preface Set provides support for every other element relative to this probability function. This can be seen from the fact that

$$Pr(p|q) = P(p|r) = \dots = P(r|s) = .788 > .632.$$

And, the fact that

$$P(p|\text{Postscript\&IMP}) = P(q|\text{Postscript\&IMP}) = P(r|\text{Postscript\&IMP}) = P(s|\text{Postscript\&IMP}) = .75$$

Finally, IMP and Postscript are both entailed by Postscript&IMP, so it follows that neither of them are objections to Postscript&IMP either. Thus, on such an example, none of the sentences in the set are competitors. The upshot is that there are versions of the preface paradox that problematize Lehrer's solution in much the same way that alternative versions of the lottery do. And, again, this

doesn't establish that one can have justified inconsistent beliefs on either view, but it does show that nothing they have said so far provides any reason to think one could not be so justified.

2.7. Revising Lehrer's Proposal

So far, we have seen that, as the theory is officially formulated, Lehrer's explanation for why lottery and preface propositions are not justified fails to generalize to all lottery and preface cases. And, we have seen that the problematic examples seem to turn on Lehrer's notion of personal justification failing to being closed under single premise entailment. But, we might ask, what if we revise the account of rational acceptability so as to ensure that rational acceptability is closed under single premise closure. For instance, one might revise Lehrer's account of rational acceptability as follows:

Rational Acceptability Principle (RAP): It is rational to accept p if p is Lehrer-acceptable, unless there is some proposition q entailed by p such that q is not Lehrer-acceptable (We could adopt an analogous revision to Olsson analysis of P^* -acceptable).

If (RAP) holds then we can explain the rational unacceptability of generous lottery propositions and the strengthened preface proposition by appealing to Lehrer's original explanation and then this principle. So, why not just adopt (RAP) and use it to generalize Lehrer's explanation for why inconsistent claims cannot all be rationally acceptable?

The main problem with this proposal is that the explanation that would be given for why one would not be justified in believing generous lottery propositions would seem to carry over for almost all propositions. The easiest way to see this is to consider one explanation for why either g_1 or g_2 is not rationally acceptable that can be given on Lehrer's account if we adopt (RAP). Suppose that x_1 entails a proposition y_1 and x_2 entails a proposition y_2 , such that:

- (i). y_1 and y_2 are subcontraries (i.e., if one of them is false the other must be true).
- (ii). Subcontraries constitute objections to each other in Lehrer's sense.²⁵
- (iii). $r(y_1) \geq r(y_2)$ or $r(y_2) \geq r(y_1)$. Thus, at least one of the two claims has a competitor that it is no more reasonable than.

²⁵Again, see Meijs (2006, p. 239)

(iv). There is no way to neutralize y_1 when y_1 is put forward as an objection to y_2 , nor is there a way to neutralize y_2 when it is put forward as an objection to y_1 .

Together conditions (i)-(iv) entail that either y_1 or y_2 isn't Lehrer-acceptable. If (RAP) holds, then it will then follow that at least x_1 or x_2 is not rationally acceptable. Let x_1 be g_1 , x_2 be g_2 , y_1 be l_1 , y_2 be l_2 and the above provides us with precisely the explanation we would expect for why we cannot be justified in accepting all generous lottery propositions. The only conditions that may need clarification are (i) and (iv). Thus, given this background knowledge, $l_1 \vee l_2$ are subcontraries. Explaining (i) is easy, in the case of the standard lottery paradox, the disjunction of any two lottery propositions is fully certain on the background knowledge that there will be exactly one winner. Explaining (iv) is not so easy, as we are not given a reason to think lottery propositions, as opposed to most other subcontraries, cannot be neutralized.

The problem with adopting (RAP) is that (i)-(iii) will hold for virtually all propositions in our language. Starting with p and q , we can choose any number of propositions. For instance, we could let y_1 be $p \vee \neg q$ and y_2 be $q \vee \neg p$. It follows that p entails y_1 , and q entails y_2 . These claims are subcontraries and are thus guaranteed to be negatively dependent, unless they are fully certain. So, conditions (i)-(iii) will hold in this case. But, even worse, we can even let x_1 and x_2 be the same proposition p , and then arrive at a countless number of subcontraries, y_1 and y_2 , that are entailed by p , thus arriving at a potential argument for thinking p is not rationally acceptable. In fact, for any proposition in the language r where neither $p \vee r$ and $p \vee \neg r$ are not fully certain, we can let y_1 be $p \vee r$ and y_2 be $p \vee \neg r$, and thus have identified propositions that are subcontraries and compete with each other according to Lehrer's proposal. As a consequence, steps (i)-(iii) of the explanation for why generous lottery propositions cannot all be rationally acceptable will carry over for almost all contingent propositions in an analogous manner. If we were to adopt a version of (RAP) for Olsson's proposal, on which appeals to neutralization have been eliminated in contexts where comparative reasonableness reduces to probabilistic relations, and thus condition (iv) is irrelevant, the view would collapse into radical skepticism for all such contexts.

Consequently, the rationality of almost all of our acceptances would be put in jeopardy by (RAP), since most propositions will entail a vast number of claims that stand in a conflict that needs to be neutralized. Perhaps, on the revised version of Lehrer's analysis, they could be neutralized, and

some theory can be developed that *explains* why the conflict between lottery propositions cannot be neutralized, while the conflict between most other subcontraries can be. But if we adopt (RAP), given that most claims that stand in the exact same tension as standard lottery propositions, we would need some principled reason for seeing why neutralization isn't possible in the case of lottery propositions. Without some principled reason for why lottery propositions are different from almost all other subcontraries, it doesn't look like the account would afford us with any explanation for why a coherentist should reject the possibility of justified inconsistent acceptances in the case of the lottery and the preface paradoxes without also rejecting the rationality of most ordinary acceptances.

2.7.1. Partial and Full Objections: Another Problem for Lehrer's Solution to the Lottery and Preface Paradoxes. Before concluding, we need to register one final problem with Lehrer's solution to the lottery and preface paradoxes. Up to this point, we have granted for the sake of argument, or at least we have left unchallenged, the plausability of Lehrer's analysis of the coherence relation. We have done so in order to show that Lehrer's account of coherence provides no substantive motivation for a consistency requirement. Even if we grant Lehrer's analysis of an objection and the nature of the coherence relation, there is still no argument from the substantive commitments of Lehrer's theory that will lead to the claim that one cannot be rational in accepting an inconsistent set of claims. But while granting Lehrer's analysis of the coherence relation was useful for dialectical purposes, we need to finish our discussion of his view by considering one of the glaring problems with his analysis of the coherence relation that is intimately related to his solution to the lottery and preface paradoxes. This will set up our focus on alternative analyses of the coherence relation to come in Chapter 4.

While in very general terms, I think the basic idea that coherence of one's beliefs might be understood as one's beliefs or acceptances being free from internal tension, there is a highly idiosyncratic aspect to Lehrer's analysis of the coherence relation to which we need to pay a bit of attention. The idiosyncrasy occurs in Lehrer's analysis of what it takes for one claim to constitute an objection to another. Now, in very general terms, Lehrer's analysis of this relation seems roughly correct. One claim constitutes an objection or competitor to another just in case the truth of each claim makes the other less likely to be true. And, in fact, Lehrer's account of objections expresses at least a rough equivalent of the formal account of epistemic defeaters defended in Mathew Kotzen's

(Manuscript) “A Formal Account of Defeaters”. As Kotzen notes, defeaters to some hypothesis H are the sorts of claims “that can lower our credence in H ” (p. 4). At a general level, a defeater can be analyzed as a claim that lowers the probability of one’s belief or acceptance, and this is coextensive with Lehrer’s formal analysis of the objection relation.

But, and this is where another problem for Lehrer’s solution to the inconsistency paradoxes arises, there is an important nuance that Kotzen emphasizes and that is central to the work on Bayesian confirmation measures (See Chapter 4 for a detailed discussion and references), which is not handled with much subtlety by Lehrer’s account of the coherence relation. While any claim d that lowers the probability of a claim p counts as a defeater for p on Kotzen’s analysis, Kotzen distinguishes between partial and full defeaters. Paraphrasing, a partial defeater, d , for a proposition, p , makes it less reasonable to believe or accept p , but a full defeater makes it unreasonable full stop. The question of how exactly we understand the relationship between partial and full defeaters is about as complicated as the question of how we should understand the relationship between partial and full belief. Many of the difficulties are brought out by Kotzen’s analysis. But what is clear is that Lehrer’s analysis of the objection or defeater relation is extremely coarse grained, and completely insensitive to the fact that disconfirmation or negative dependence is a relation that comes in degrees. Lehrer’s account of the coherence relation treats all objections as equals, regardless of how strong the degree of disconfirmation or negative dependence is between an objection and its target. No matter how weak the negative dependence between an objection and its target, the conflict between an objection and its target, on Lehrer’s view, gives *prima facie* reason to think one cannot believe both the objection and the target claim.

That this assumption is highly implausible has been brought out by some of the examples that we have already considered above. For instance, in our discussion of the failure of SPC for Lehrer’s theory, we saw that SPC fails in some instances precisely because a conjunction of two claims might be highly reasonable to accept despite the fact that there might be a very low degree of negative dependence between the two claims. In short, that two claims stand in an extremely weak negative dependence relation isn’t sufficient to make it unreasonable for someone to accept both claims. An obvious remedy to these problems would be to replace Lehrer’s analysis of objections in his definition of the coherence relation with an analyses that fits more closely with the idea that one

claim is a full, as opposed to a mere partial, defeater of another claim. Now, there are a variety of ways that we might go about defining the is-a-full-objection relation, perhaps the simplest being:

o is a full objection to p if $Pr(p|o) < Pr(p)$ and $Pr(p|o) < t$ where t is some threshold for rational acceptability.²⁶

This simple proposal seems rather plausible, and especially so if we let t be some low value like .5. The idea here being that one claim counts as an objection to believing another just in case p is more likely to be false than true conditional on the assumption that the objection is true. It is easy to verify that the proposal above would side step the problems with subcontraries that we observed above (all subcontraries are partial defeaters, but need not be full defeaters). I don't intend to argue for this analysis of full objections here, nor the account of the coherence relation that we would obtain by plugging this notion of full objection into Lehrer's account. But what must be noted is that as soon as we replace Lehrer's analysis of the objection-relation with a relation like that above, one that is sensitive to the fact that objections come in degrees, we lose any hope of arguing that lottery propositions must constitute full objections to one another. The same considerations apply to Lehrer's solution to the preface paradox. Thus, once we make this plausible revision, and acknowledge that objections come in different strengths, we need to seek out more sophisticated reasons for thinking that inconsistent sets of claims fail to cohere with one another.

It is for this reason that we shall take up a careful study of various proposed analyses of the coherence relation that are based on the fact that support and disconfirmation come in degrees, and thus that coherence between pairs of claims or sets of claims is also something that comes in degrees. What assumptions are needed to generate a consistency requirement on a coherence relation that is sensitive to the varying degrees of support and disconfirmation that propositions can stand in is something that we shall consider in great detail in chapter 4. I think it is safe to conclude that Lehrer's analysis of coherence does not provide a plausible way to argue for a consistency requirement on rational beliefs or rational acceptances.

²⁶This proposal is a formalization of an idea suggested by Kotzen (Manuscript, p. 4).

2.8. Conclusion

At this point, I think we can safely conclude that we have no explanation as to why a coherentist, committed to either version of Lehrer or Olsson's accounts of personal justification or rational acceptance should avoid justified inconsistent acceptances. The standard explanations fail on the official formulations of the theories, given that they provide no response to the generous lottery paradox, and strengthened preface paradox. And, given that virtually all claims entail a vast number of propositions that stand in the same tension as standard lottery propositions, i.e., entail a vast number of subcontraries, without some principled explanation for why lottery propositions are different from virtually all other subcontraries, we are left with no explanations for why using (RAP) to rule out justified inconsistent beliefs won't collapse the view into radical skepticism. I am thus inclined to conclude that Lehrer's theory furnishes us with no special motivation to avoid justified inconsistent beliefs, given the principles currently available to us. Perhaps the theory will some day be filled out in a way that would explain why a coherentist of this sort can adopt (RAP) and not be inclined toward radical skepticism. Until then, I am compelled to conclude that coherentists sympathetic to Lehrer's view should be sympathetic to the idea that inconsistent belief is rational in some cases.

I certainly don't think we can conclude at this point that coherentists don't have some special motivations to eschew inconsistency. In recent years, a number of epistemologists have suggested very different interpretations of the coherence relation, interpretations that are sensitive to the fact that mutual support and disconfirmation come in degrees. And, for all we have said, the possibility remains that there exist special motivations for proponents of these alternative coherence relations to endorse a consistency requirement. In Chapter 4, we shall turn to those accounts of the coherence relation to see whether or not they provide some special reason for coherentists to avoid inconsistency.

CHAPTER 3

Permissibility Solutions

In this chapter, we turn our attention to a distinctive strategy for trying to answer the epistemic inconsistency paradoxes, and to reconciling a high probability principle with a logical consistency requirement: the so-called permissibility solution to the inconsistency paradoxes. The basic idea behind the permissibility solution is to interpret an agent having epistemic justification for p in terms of an agent having permission to believe p .¹ According to some proponents of permissibility theories of epistemic justification, this provides us with a straightforward way to reconcile a high probability principle with a logical consistency requirement and epistemic closure principles. In this chapter, we shall consider permissibility accounts of epistemic justification that aim to avoid both horns of the dilemma posed by the inconsistency paradoxes, namely that we either have to give up on the idea that high probability is sufficient to confer epistemic justification or that we can sometimes be justified in holding an inconsistent set of beliefs.²

3.1. Permissibility Solutions Explained

Our first task is to understand how focusing on permissions to believe is supposed to afford us with the means to reconcile a high probability principles with a consistency requirement. The

¹The permissibility solution to the lottery paradox was first briefly outlined by Foley (1979, p. 251) and then more carefully stated by Harman (1986, p. 71). It is important to emphasize that both merely state the solution to be considered, neither endorse nor defend a permissibility solution. As far as I can tell, Glenn Ross (2003) was the first to take the idea seriously. Ross later reconsidered the solution in (2012). According to Ross, Lehrer's account of epistemic justification was compatible with the idea that an agent might be justified in accepting some, but not all lottery propositions. The view has since been argued for by Douven (2008) and Kroedel (2012, 2013a, 2013b). We should note that Douven (2012) presents one of the strongest challenges to certain versions of the permissibility solution, and so it is not clear whether we should understand the permissibility solution as a view Douven actually accepts. Kroedel (2012, 2013a, 2013b) is the one person whose defense of the permissibility solution hasn't wavered. Unsurprisingly, Kroedel's work will occupy the center of much of our attention throughout this chapter.

²While the version of the permissibility solution that Kroedel (2012, 2013a, 2013b) defends has us consider the possibility that epistemic justification is to be understood as a kind of permission, and while this is convenient for framing the discussion, one needn't accept the thesis that the only important normative notion when evaluating beliefs is whether one has a permission to believe. In fact, we could reframe the entire discussion exclusively in terms of whether high probability is sufficient to confer permission to believe, and whether one is obliged to avoid inconsistent beliefs. We shall stick closely to Kroedel's presentation for convenience sake only. Thus, we shall sidestep the difficult question of whether we can plausibly hold that epistemic justification really is best understood in terms of permissions, obligations or some other normative concept altogether.

first step to seeing how a permissibility approach is supposed to yield a reconciliation of a high probability principle and a consistency requirement is to make explicit how these principles are to be understood by a permissibility theorist. So, let us first consider the high probability permission principle. One idea that comes to mind is that a high probability requirement is to be understood as holding that if a claim is probable to a certain degree, then one is permitted to believe that claim. We put this in principle form as follows:

Probabilistic Permissibility Principle (PP) If $Pr(p) > t$ for S , it is rationally permissible for S to believe p .³

A consistency requirement is supposed to provide a limit on one's permissions. In particular:

Consistency Requirement on Permissions (CRP) It is not rationally permissible to believe a logically inconsistent set of propositions.⁴

In other words, a logical consistency requirement says that one is obliged to avoid inconsistent beliefs.

The next thing to note is that these principles are compatible with one another. Thomas Kroedel (2012, 2013a, 2013b) has pointed out that there is no conflict between (PP) and (CRP). (PP) entails that one is permitted to believe of each lottery ticket that it is a loser. Kroedel (2012, p. 58) observes that there is a narrow and wide scope interpretation of this claim.

Narrow Scope Permission (NSP): $Perm(Bel(l_1)) \& \dots \& Perm(Bel(l_n))$.

Wide Scope Permission (WSP): $Perm(Bel(l_1) \& \dots \& Bel(l_n))$.

According to the wide scope reading, one is permitted to believe all lottery tickets at the same time, whereas the narrow scope reading merely entails that one is permitted to believe each lottery proposition (though not necessarily at the same time).

³See Harman (1986, p. 71), Kroedel (2012, p. 59; 2013a, p. 106; 2013b, p. 452), Douven (2012), and Littlejohn (2012, p. 511; 2013, p. 233) for discussion of this principle.

⁴For defenses of such a principle, see Harman (1986, p. 71), Ross (2003, 2012), Kroedel (2012, 2013a, 2013b), and Douven (2008). For criticisms of such a principle, at least in the context of a permissibility theory of justification, see Littlejohn (2012, 2013).

Kroedel (2013a, p. 105) helpfully gives some intuitive examples to make it clear why the narrow scope reading need not entail the wide scope. Here is one such example he gives.⁵ Suppose you are at a party and granted permission to eat a slice of cake. Assuming none of the individual slices are claimed, you are permitted to eat the first slice, and you are permitted to eat the second slice, and so on. In other words, you have a narrow scope permission to have any slice of cake of your choosing. However, those permissions do not aggregate so that you are permitted to eat all of the cake. The narrow scope permissions to eat any slice of cake does not generate a wide scope permission to eat all of the cake.

The central idea behind the permissibility solution to the lottery is that we likewise cannot derive permission to believe all lottery propositions at the same time from the individual permissions we have to believe each of them. And, we can interpret a consistency requirement as holding that:

$$(\text{not-WSP}) \text{ Not-Perm}(Bel(l_1) \& \dots \& Bel(l_n)).$$

without thereby arriving at any conflict between our two basic principles. Since (PP) is most naturally read as entailing narrow scope permissions, the permissibility theorist thus maintains that she can do justice to both principles without any mutilation or theoretical sacrifice that all other approaches to the lottery require.

3.2. Overview of Objections to Permissibility Solutions

Now, the permissibility solution to the lottery paradox faces a variety of challenges. One very general challenge is trying to make sense of an epistemology of belief that makes permissions to believe its central focus. Huber (2014) presents the most detailed objection along these lines. What Huber points out is that an epistemology of belief that only has principles explaining the conditions under which permissions arise will be essentially vacuous. As Huber notes, an agent could qualify as ideally rational on such a theory even if her belief system corresponds to that of the radical skeptic, i.e., even if she has no beliefs at all (2014, p. 341). Without principles explicating doxastic obligations to go along with principles to compliment (PP) and (CRP) above, it is not clear how informative such a theory would be. In a nutshell then, a permissibility solution to the lottery

⁵The example is discussed in Kroedel (2012, p. 59) and Kroedel (2013a, p. 105), and considered by Littlejohn (2013, p. 510).

needs to be embedded in a more complex theory that explicates both doxastic obligations and permissions. Ultimately, I find the considerations Huber raises against permissibility solutions rather compelling, but don't think they are decisive. The permissibility theorist could reply that she has not committed herself to the claim that (PP) and (CRP) exhaustively characterize the principles governing permissions and obligations to believe. We would need to consider various combinations of principles that we might combine with the permissibility principles above to flush out a more complete theory.⁶ Such a study may well yield interesting and surprising results, but is unnecessary.

Ultimately, a general study of the sorts of principles about epistemic obligations that can be combined with (PP) and (CRP) above is unnecessary for one simple reason. Even if we grant that we can develop a theory of obligations and permissions that avoids the vacuity problems that Huber raises, we still won't find adequate motivation for accepting a consistency requirement on belief. We will, thus, grant for the sake of argument that we can make sense of epistemic justification as having a permission to believe in order to see whether granting this assumption really does afford us a solution to the inconsistency paradoxes. Our aim here is thus to evaluate whether, by treating justification as permission and noting that permissions do not agglomerate, the permissibility theorist really is in a better position to reconcile (PP) and (CRP). We shall thus primarily focus on challenges to the permissibility solution to the lottery paradox that can be made even after it has been granted that it is useful to focus on permissions to believe.

3.2.0.1. *Arbitrariness.* Now, there are several commonly made objections to the permissibility solution to the lottery paradox. The most common reasons given for rejecting a permissibility solution to the lottery paradox fall under the heading of objections from theoretical vices. That is

⁶There are some combinations that, at least on the face of it, seem like they might offer permissibility theorists some hope. For instance, it seems plausible to me that something like Dretske's (1971) conclusive reasons condition could be used to define a condition on belief that would engender obligations for an agent to believe in a way that would avoid Huber's concerns. In other words, the permissibility theorist might endorse a principle like the following:

Conclusive Reasons Principle: If S has a conclusive reason for believing p , then S is obliged to believe p .

Such a principle would not entail that an agent has an obligation to believe lottery propositions, and so this principle would not give rise to the vacuity problems that Huber charges. A permissibility theorist who accepts this principle would then note that, while we have an obligation to believe some propositions, those for which she has conclusive reasons, others, like lottery propositions are not obligatory, since one doesn't have conclusive reasons to believe them. Nevertheless, one might still insist they are permissible to believe, but not obligatory, and appeal to (PP) and (CRP) to explain why.

For discussion of Relevant alternatives principles that might also be used to define belief obligation principles along similar lines, see Stine (1976), Lewis (1996), DeRose (1996, 1999). See Hawthorne (2004) for an extended discussion of contextualist theories and their implications for the lottery paradox.

to say, many have rejected the permissibility solution to the lottery because it exhibits a theoretical property like *arbitrariness* or *ad hocness*. For instance, many epistemologists have objected to the fact that, on the permissibility solution, one can be rational in believing some lottery propositions while disbelieving or suspending judgment about others, despite the fact that all lottery propositions enjoy the same exact epistemic credentials. The permissibility solution allows arbitrary decisions about what to believe, and many epistemologists have outright rejected permissibility solutions as absurd on the grounds that such arbitrariness has no place in a theory of rational or justified belief.⁷

3.2.0.2. *Ad Hocness*. The other intimately related reason that some reject the permissibility solution to the lottery paradox is on the grounds that the proposal is ad hoc. Clayton Littlejohn (2012, 2013) has provided a precise version of the ad hocness objection to permissibility solutions. Littlejohn demands some explanation for why permissions do not agglomerate in lottery situations. Such an explanation is needed due to the fact that permissions only fail to agglomerate in certain cases. In many cases, permissions do agglomerate. For instance, if one of my neighbor grants me permission to borrow her lawn mower, and my other neighbor permits me to borrow her shovel, another her wheel barrow and so on, then in an ordinary circumstance, these permissions agglomerate. Were I to do a project around my place that required all three items, in normal circumstances, it seems clear that I can exercise all of these permissions at the same time. According to Littlejohn, the central challenge for the permissibility solution is to explain

...why this is a case in which you can't justifiably/permissibly take advantage of all the justifications/permissions you had before you started adding beliefs about lottery propositions to your belief set. (Littlejohn 2013, p. 233)

Littlejohn helpfully frames the problem as follows. There are two basic theses that the permissibility theorist is committed to:

(Start) Feel free to add at least one lottery belief to your belief set (i.e., you can justifiably believe a lottery ticket will lose).

⁷See Douven (2008, pp. 214-217) for further discussion of the history of such objections. Also, see Nelkin (2000, p. 377) and Foley (1979, p. 251) for examples of epistemologists who dismisses the permissibility solution for its arbitrariness. See Hawthorne (2004) for a discussion of parity reasoning, and for discussion of how arbitrariness tends to push us toward skepticism. Also, Smith (2014) for discussion of how arbitrariness infects our judgments about all uncertain propositions.

(Stop) Don't add all the lottery beliefs to your belief set (i.e., you can't justifiably believe each of the tickets will lose). (2013, p. 232)

I should add here that (Start) and (Stop) can be formally represented as follows:

(Start-Formalized) $Perm(Bel(l_1)) \& \dots \& Perm(Bel(l_n))$

(Stop-Formalized) $\text{not-}Perm(Bel(l_1) \& \dots \& Bel(l_n))$.⁸

The problem, says Littlejohn, is that there is no principle that would explain (Start) without undermining (Stop) (and vice versa). Littlejohn notes that the most natural way to explain (Start) is a principle like (PP) above. The version Littlejohn explicitly considers is the following:

(High-PJ) If the evidential probability of p is sufficiently high, you have justification to believe p . (2013, p. 233)

The trouble is this does not afford us with an explanation of (Stop). That is to say, it does not answer Littlejohn's main concern:

If [the evidential probability for each lottery proposition is] high enough for each of them, why can't you justifiably believe all of them? The rationale offered thus far supports (Start) but threatens to undercut (Stop). (2013, p. 233)

Without some explanation for (Stop), the permissibility theorist is not really in a position to defend a consistency solution to the lottery paradox; she is merely in position to endorse the compatibility of a consistency principle with her other commitments. Consequently, I think Littlejohn is right that the permissibility solution is only advantaged over rival approaches to the lottery paradox if there is some explanation for (Stop). These represent the core challenges to the permissibility solution.

⁸This formalization is meant to mirror Kroedel's (2013b, p. 453) formalization of his answer to the ad hoc objection.

3.2.0.3. *Answering the Objections.* While the arbitrariness of the permissibility solution may be somewhat counter-intuitive, that issue does not occupy the center of the controversy over the tenability of permissibility solutions. Douven (2008, pp. 214-218) provides several interesting, if not compelling, responses to the arbitrariness concern. First, he notes that the purported intuition that the tolerance of some arbitrariness in our theory of rational belief is unacceptable is not universally shared.⁹ Clearly, those who find the permissibility solution compelling do not find this intuition as moving as epistemologists who have dismissed such solutions on the basis of the intuition. Dialectically then, appeal to intuition alone will not provide the permissibility solution's proponents with reason to abandon their view. Second, he notes that, generally, when we think a theoretical vice like ad hocness or arbitrariness calls into doubt a theory, it is because we think that feature typically indicates that the theory in question is not likely to serve our theoretical goals. However, he argues that permissibility solutions are generally conducive to the achievement of our epistemic goals, and, thus, that this sort of concern does not apply in the case of the permissibility solution (How exactly it is supposed to serve our epistemic goal will be examined below). Third, he notes that it is almost universally accepted in discussions of different sorts of rationality, practical rationality for instance, that some arbitrariness is to be accepted. We think that sometimes it is rational for an agent to make an arbitrary choice about what to do when one has equally compelling options available and making some arbitrary choice means doing better at the attainment of our practical goals than making no decision at all. And, again, if a case can be made that making an arbitrary choice about what lottery propositions to believe means doing better with respect to our epistemic goals, then there may be a close analogy between Buridan's ass choosing between stacks of hay and an agent deciding to believe some but not all lottery propositions.¹⁰

⁹Ross (2003, 2012) and Kroedel (2012, 2013a, 2013b) seem to agree with Douven (2008). See Foley (1979, p. 251), and Nelkin (2000, p. 377) for examples of epistemologists who reject the permissibility solution due to its arbitrariness. Smith (2014) provides another reason not to be bothered by arbitrariness, namely that all beliefs are arbitrary in much the same way as lottery propositions.

¹⁰It is also worth noting that the arbitrariness worry doesn't just apply to lottery propositions. Smith (2014) argues that the sort of arbitrariness involved in believing of some lottery tickets that they are losers, while withholding belief about other lottery tickets, is common to belief in all propositions that have an evidential probability less than 1. Smith doesn't take this to show that such arbitrariness is acceptable, but his arguments demonstrate that dealing with such arbitrariness is actually a problem that all fallibilist epistemologists face. Ultimately, what he shows is that fallibilist epistemology requires us to discover some way of discriminating between the acceptability of propositions that doesn't depend on their evidential probabilities, or else we must learn to live with such arbitrariness. We can thus take the discussion that follows to constitute strategies that fallibilists might try to take in explaining why such arbitrariness can be acceptable in this more general debate.

Of course, Douven's responses to the arbitrariness objection are made more plausible if a case can be made that the agent who follows the permissibility solution does better than she would if she took an inconsistency or skeptical response to the lottery paradox, a case that Douven (2008) thinks he has successfully made.¹¹ This reflects one of the key ways that the explanatory worry and the arbitrariness worries are related. If, as Douven claims, one can show that the permissibility solution provides an optimal strategy for obtaining our epistemic goals, then this would seem to involve an explanation for why one ought not take an inconsistency approach to the lottery paradox, and thus answer why permissions do not agglomerate in cases like the lottery paradox.¹² Believing all lottery propositions does not serve our epistemic goals. In this way, the answer to the arbitrariness worry is intimately related to the ad hoc objection and the explanatory challenge posed by Littlejohn. If one can provide some explanation for why permissions don't agglomerate in cases like the lottery paradox, then one might explain why it is reasonable that one should make an arbitrary choice about what to believe in cases like the lottery.

Ultimately, if one can answer the explanatory challenge, then the answer to the objections to the permissibility solution might fall into place. We now consider what options are available for answering the explanatory challenge set out by Littlejohn.

3.2.0.4. *Taxonomy of Responses to The Explanatory Challenge.* As far as I can see, there are just a few basic strategies one might take to try to explain the non-agglomeration of epistemic permissions. One strategy that has been considered in a variety of places is to argue that epistemic

¹¹A similar defense of the permissibility solution is put forward in Ross (2003, 2012). Douven's (2012) presentation of the sequential lottery paradox presents a new set of concerns for those who would favor a permissibility solution of the lottery along the lines of that suggested by Harman (1986). So, it is unclear whether Douven still favors a permissibility solution. But what is clear is that none of the arguments in Douven (2012) call into question Douven's (2008) argument that there are certain advantages for the permissibility solution to the lottery. These supposed advantages thus deserve our careful consideration.

¹²One might wonder whether there is any analogy that can be made to the cake example that we considered earlier, or if the cases are radically different. I think the situation is very different in the original example. It seems like the nature of the permission is stipulated by the party host who grants one permission to eat exactly one slice of cake. In that case, there is no mystery as to why permissions do not agglomerate. However, given that the permission is not the result of some stipulation by one who can entitle one to a belief in a lottery proposition, the original analogy clearly doesn't hold.

However, I think it is clear that we can revise the cake example to make the analogy much closer. Consider the cake example again, but now suppose that we are evaluating whether or not, from a health-perspective, it would be permissible for one to eat a slice of cake. At least for some people, on some occasions, eating a single slice of cake is conducive to, or at least wouldn't interfere with, one's health goals. Then, it might be rationally permissible for one to eat a slice of cake. However, these permissions do not agglomerate because eating the entire cake would not be conducive to one's health goals. This seems closely analogous to a lottery situation if a case can be made that believing all lottery propositions is to the achievement of our epistemic goals as eating all the slices of cake is to our health goals. Whether such a case can be made we will need to consider below.

permissions do not agglomerate because accepting all or even most members of a lottery set is not conducive to the achievement of our epistemic goals.¹³ For taxonomic purposes, we shall call such strategies EGB explanations (for epistemic-goal-based explanations for non-agglomerations). There are different variants of such explanations, depending on the characterization of our epistemic goals. We shall consider a couple of concrete proposals in due course, but in the meantime, let me just note that the basic form such strategies take. We start with a characterization of some epistemic goal, and then from there argue that believing all (perhaps even most) lottery propositions will fail to be conducive to the attainment of that goal. Additionally, some argument must be given for the claim that believing some lottery propositions is more conducive to the achievement of one's epistemic goal. Together, these two arguments are supposed to provide complimentary motivations for accepting both (Start) and (Stop).

Another basic strategy for arguing against agglomeration is to hold that non-agglomeration of epistemic permissions follows from some more basic epistemic principles. The obvious sort of principle that one might appeal to is a substantive coherence principle. That is to say, if one is a coherentist, then there is some substantive notion of systematic or relational coherence that one thinks is relevant to the evaluation of the global rationality of a set of beliefs. As we discussed in Chapter 1, it is commonly assumed that the coherentist's substantive coherence relation is such that a set of propositions cohere in the substantive sense only if they form a consistent set. For this reason, it has been assumed that coherentists are committed to a non-agglomeration principle, and would explain non-agglomeration in terms of their substantive coherence condition on belief. As we have seen in the last chapter, we shouldn't take this assumption on faith. Lehrer's coherence theory, which was initially set out as a way to rule out justified inconsistent beliefs, employs an analysis of the coherence relation that is tolerant of some inconsistency. Consequently, whether a coherentist is really in position to defend a consistency requirement by way of the coherence relation is something that needs to be investigated in detail and depends on the exact nature of the coherence relation. Given the complexity of the various analyses of the coherence relation that have been proposed, we shall put off our discussion of substantive analyses of the coherence relation until Chapter 4, where we shall give that question our full attention. In Chapter 4, we shall survey the various attempts to explicate the substantive coherence relation in terms of various mutual support

¹³This is the strategy defended in Douven (2008), Ross (2003, 2012), and Kroedel (2013a, 2013b). Lehrer (1990, pp. 159-160) also provides an argument of this sort. See Engel (1991) for a reply.

and agreement relations in order to see which of the proposed analyses of the coherence relation really entails that inconsistent sets must be incoherent, and thus afford an immediate explanation for non-agglomeration.¹⁴

But, for now, we shall first consider attempts to explain non-agglomeration that avoid appeals to some substantive notion of coherence. After all, if more basic principles could be employed to explain non-agglomeration, then a consistency requirement need not be a special commitment of the coherentist. We shall here focus on explanations that appeal to basic evidential principles, closure principles and principles characterizing our epistemic goal. While it might seem convenient to organize the discussion around the types of principles that have been invoked to explain non-agglomeration, it will be more illuminating to organize the principles in terms of the problems they face. We shall begin with a simple principle that has been explicitly formulated by Littlejohn (2013), as it is both the simplest principle appealed to in this debate, and because it helps to illuminate one of the most general problems all attempts to explain non-agglomeration face.

3.3. Global Error Avoidance Principles

How might one try to explain the fact that one ought to avoid believing all lottery propositions, or any inconsistent set of propositions for that matter. As I have noted above, one common way to try to explain why one ought not to believe all lottery propositions is that to do so would mean failing to achieve one's epistemic goal. But how are we to characterize our epistemic goal? One somewhat neutral way to characterize the ideal we aim at is this:

Epistemic Goal (EG): We aim to believe a proposition if it is true, and avoid believing it if it is false.¹⁵

¹⁴It is always possible that a coherence relation could figure into an explanation with the aid of some overlooked auxiliary principles, even if coherence doesn't entail consistency. When we turn to the coherence based explanations for a consistency requirement in Chapter 4, we shall keep things simple by considering whether or not it is possible for inconsistent sets to be coherent.

¹⁵See Alston (2005, p. 32), Chrisman (2010), David (2001), Kvanvig (2005) and Lynch (2009) for precise characterizations of the idea that our epistemic goal is to believe what is true and avoid error, as well as discussion of some of the difficulties that come with treating truth as our epistemic goals.

Also see Lehrer (1990, pp. 159-160) and Engel (1991) for discussion of Lehrer's maxiverificity principle as a way of characterizing our epistemic goal, and the problems it faces. Lehrer argues for a consistency principle by arguing that our goal is to have an informative and error-free set of acceptances, and that having an inconsistent belief is incompatible with the achievement of this goal. Engel's (1991) response to Lehrer anticipates the problems that I raise for Kroedel's arguments in favor of a consistency requirement.

It is worth noting that many epistemologists would require that the things we believe be of practical or theoretical significance of some kind. This sentiment is expressed in many places, for examples, see Alston (2005, p. 32), Lehrer

That is to say, perfect attainment of the ideal would be believing every proposition that is true, while avoiding all propositions that are false.¹⁶ That this is our ideal, of course, doesn't mean that it is really our epistemic goal. Often the ideals we aim at might be hopelessly out of reach, so that we quite reasonably set the bar for our goals at some more attainable level. Anyhow, let us grant for the moment that this at least approximates our epistemic ideal, and factors into our epistemic goals. Then how would an explanation of non-agglomeration follow? Lehrer (1990, pp. 159-160) suggests that one might then want to avoid logically inconsistent sets of beliefs because having an inconsistent set of beliefs guarantees that one fails to obtain one's epistemic ideal.¹⁷ Let the principle be explicitly stated as:

Certain-Error Principle (CEP): If adding p to one's stock of beliefs would guarantee that one's belief set contained some error, then you should not believe p .

We can continue this line of thinking by observing that one needn't have logically inconsistent beliefs to be confident that one has failed to obtain one's epistemic ideal. An agent with a consistent set of beliefs might be almost certain to have at least some false beliefs in virtue of the fact that the risk of error aggregates over the propositions one believes. Thus, if inconsistent beliefs should be avoided in virtue of the fact that they guarantee failure to achieve one's epistemic ideal, then it is a small step to thinking that one ought to avoid belief sets that are nearly guaranteed to fall short of our epistemic ideal. This then pushes us in the direction of the principle that Littlejohn thinks the permissibility theorist is likely to appeal to in order to try to explain non-agglomeration:

(Risk-DJ) If the probability of acquiring an error-containing belief set would get too high by adding the belief that p to your belief set, you cannot justifiably believe p . (2013, p. 234)

(2000) and Harman (1986). All emphasize that little value should be assigned to believing trivial truths. Douven (2008, p. 207) rejects such a restriction. Ultimately, I think we can grant Douven his preferred interpretation of our epistemic goal, and refute his explanation for non-agglomeration.

¹⁶Of course, some epistemologists are not going to accept this characterization of the epistemic ideal, even if it is to be thought of only as an ideal. One might, for instance, not think that we aim to believe those propositions that are of little to no theoretical utility, either in epistemic terms or practical terms. Harman's (1986) observations about the limits to our cognitive capacities, for instance, might think that for agents like us, the ideal is to believe all of the *important* truths, and avoiding believe any falsehoods. Dialectically, it will be helpful to grant Douven the assumption that truth is our epistemic goal. Even if this is granted, we will still see that there is no plausible explanation for (Stop).

¹⁷See Engel (1991) for a reply. It is worth noting that Engel's reply to Lehrer anticipates the problems with the sorts of strategies we consider below that are taken up in Kroedel (2013b) and Littlejohn (2012, 2013).

Ultimately, if the permissibility theorist is to appeal to something like CEP, as opposed to (Risk-DJ), then she needs to be able to justify thinking that the difference between having almost no chance of attaining one's epistemic ideal, and no choice whatsoever is of large enough importance to motivate the rejection of (Risk-DJ) in favor of CEP. Given that the distance between a guarantee of error and near certainty of error is trivial for most people, this strikes me as a rather implausible position. Thus we shall proceed by considering how principles of this kind, focusing specifically on (Risk-DJ), would be employed in an explanation of the permissibility theorist's two central theses. So, let us turn to that now.

How does (Risk-DJ) explain the failure of permissions to agglomerate? Take any set where the set of propositions p_1, \dots, p_n is sufficiently risky so that the set is very likely to contain at least one false belief. Now, suppose that each individual claim has an evidential probability such that, by the lights of (High-PJ), one has a permission to believe each member of the set. One who accepts (Risk-DJ) might also accept (High-PJ), and thereby conclude that one has permission to believe each member of the set:

$$(\text{NSP}) \text{ Perm}(B(p_1)) \& \dots \& \text{Perm}(B(p_n)).$$

Next, imagine that an agent starts exercising her permissions and adding members of the set to her stock of beliefs. So, she adds p_1 , then p_2 , and so on. By hypothesis, at some point in this process, she reaches a stage where she believes $p_1 - p_m$, and adding p_{m+1} will make it so that her set of beliefs has a very high probability of error (i.e., a high probability of containing a false belief). At this point, (Risk-DJ) is supposed to entail that she must stop short of adding p_{m+1} to her stock of beliefs.¹⁸

3.4. Littlejohn's Objection to (Risk-DJ)

(Risk-DJ) would thus provide the basis for explaining why permissions do not agglomerate.¹⁹ The trouble, according to Littlejohn, is that (Risk-DJ) has highly implausible implications for cases like

¹⁸In response, Engel (1991) observes that this line of reasoning is not particularly plausible when applied to preface situations, given that agents with large sets of beliefs are already almost certain to have at least some false beliefs. Thus, there is a sense in which we are almost certainly to fall short of our epistemic ideal, anyhow, and so the foreclosure on our epistemic goal isn't a legitimate concern.

¹⁹A principle stating that we should avoid a doxastic state that is guaranteed to fall short of our epistemic ideal would provide a similar explanation for non-agglomeration. The key difference would be that they differ over the point at which one is in an epistemically unacceptable position. According to (Risk-DJ), the point at which one crosses over

the preface. For clarity's sake, we will need a substantial part of his argument before us. He objects as follows:

We can imagine epistemically conscientious students in our epistemology lectures who start to reflect about their own fallibility for the first time. They appreciate that there's an incredibly high probability that they have belief sets that contain errors. According to (High-PJ), they have sufficient justification to believe the following proposition:

(FB): There is at least one false belief in my present belief set.

We might imagine that prior to contemplating (FB), they had sufficient propositional justification for each of their beliefs. And we might imagine that each of their beliefs was justifiably held. (These are very good students!) Should they believe (FB)?

It seems obvious to me that they should. Indeed, it seems obvious to me that they know (FB). If they can justifiably believe (FB), they can justifiably take on a set of beliefs that obviously contains a falsehood. This is something they can easily work out for themselves. If they can justifiably believe (FB), we have a counterexample to (Risk-DJ). Without (Risk-DJ), I don't think we have any explanation as to why we can't justifiably make use of all the justifications we (allegedly) have to believe the lottery propositions. I take it that one lesson to take from the preface paradox is that the kind of evidential probability that (Risk-DJ) concerns has no bearing on whether the beliefs one holds one holds justifiably.²⁰ (2013, p. 234)

Now, there is some ambiguity in (Risk-DJ) and in Littlejohn's objection that makes it difficult to determine how exactly a preface case like that presented above constitutes a counter-example to (Risk-DJ). I do think Littlejohn's example exposes a decisive problem for (Risk-DJ), but not necessarily the one he explicitly states.

from doing something acceptable to unacceptable is when the conjunction of the contents is likely to be false. Max-
 iverification, on the other hand, only entails that one has crossed over from acceptable to unacceptable commitments
 when the conjunction of the contents is equivalent to a logical contradiction, or an impossible proposition.

²⁰I have added parenthesis to make reference to (Risk-DJ) and (FB) consistent.

In his argument against (Risk-DJ), Littlejohn implies that the counter-example to (Risk-DJ) turns on whether the students are permitted to add (FB) to their stock of beliefs. But it seems to me that whether the students add (FB) to their stock of beliefs is utterly immaterial to the question of whether they have violated (Risk-DJ). By hypothesis, Littlejohn has built into the scenario that the students have violated (Risk-DJ), prior to entering the classroom, and whether they consider their own fallibility or not. To see this, note that they are assumed to have a belief set that is highly likely to contain an error at the time they enter the classroom, otherwise the student's wouldn't be in position to know (FB). Thus, in the process of adding propositions to their stock of beliefs, prior to entering the classroom, they have already at some point added propositions that together are very likely to contain at least some errors. They have thus violated (Risk-DJ) at some point in the past. Thus, they have arrived at a set of beliefs that are impermissible (in the global sense that one isn't permitted to believe all of them at one time), prior to coming to the classroom and contemplating their fallibility.

The defender of (Risk-DJ) might then note that since one has already arrived at a stock of beliefs that almost certainly contains at least one error, adding (FB) to one's stock of beliefs doesn't really make much of a difference regarding how likely it is that one's belief set is in error. In fact, the only chance of error that including (FB) in one's stock of beliefs adds to one's overall chance of being in error will be whatever chance there was that one has a perfectly accurate belief set prior to adding (FB).²¹ Given that, by hypothesis, the students are in position to know this to be true, the chances of perfect accuracy must be slim to none. Consequently, the defender of (Risk-DJ) against Littlejohn's objection might point out that the antecedent clause of (Risk-DJ) isn't satisfied when one let's p be (FB). That is to say, she can deny that

The probability of acquiring an error-containing belief set would get too high by adding the belief that (FB) to your belief set.

She would deny that it *gets too high* by adding (FB) because it was too high in the first place. Adding (FB) does not *make* it too high, since it adds almost nothing to the chances of one's belief

²¹This strategy for defending (Risk-DJ) closely resembles Engel's (1991) reply to Lehrer's (1990) argument in favor of a consistency requirement. Engel argues that once one already has a belief set highly likely to contain errors, little is to be gained by suspending judgment about preface propositions like (FB).

set containing an error. Hence, even for the proponent of (Risk-DJ), there needn't be anything wrong with accepting (FB).

The defender of (Risk-DJ) might also point out that it is not uncommon for some general normative principle to fail to be applicable once it has been violated. Consider, for example, the following health recommendation about coffee consumption:

Coffee Consumption Principle (CCP): One should not consume massive amounts of coffee on a daily basis.

From a health standpoint, this principle seems to offer sound advice about what one should not do. Let us suppose for the sake of argument that it would be better if one avoided developing a caffeine addiction. However, now consider an avid coffee drinker who has consumed massive amounts of coffee on a regular basis for many years. At this point, it may well be the case that the caffeine addicts best available choice is to go on consuming coffee at their usual levels. Does this case provide a counterexample to the Coffee Consumption Principle? What it seems to show is that while this principle might be generally true, it is no longer true for a person once it has been violated. Perhaps something analogous can be said for (Risk-DJ).

That this is so might be easier to appreciate if we consider another closely analogous situation where an agent discovers that her belief set is inconsistent for the epistemologist who accepts a consistency requirement on rational belief. Suppose the students started reflecting on some set of complicated theories that they accept and discovered that those theories are logically inconsistent. Even according to the proponent of a consistency requirement, an agent who discovers the inconsistency of her belief set would be in good position to accept (FB) on that basis, even if (FB) is construed in a way to guarantee that one's belief system would be inconsistent. In so doing, the agent may have guaranteed that she has done something rationally impermissible, but the problematic belief in this case would not be (FB) itself. The proponent of the consistency requirement would be reasonable to hold that in accepting (FB), the agent would be a way of acknowledging that she has already done something rationally impermissible. Such an acceptance would, by the consistency proponent's lights, be the first step in acknowledging that she is under some pressure to undergo a rational change in view.

Thus, I don't think that a proponent of (Risk-DJ) need accept that one has done something impermissible by accepting (FB) after reflecting on the fact that her belief system is almost certainly in violation of (Risk-DJ). Rather, what she must insist is that if (FB) is something that the agent is in position to know, then this reflects the fact that, by the lights of (Risk-DJ), the agent's beliefs were globally impermissible (in the sense that the agent added propositions to her stock of beliefs that were deemed impermissible by (Risk-DJ) at the time they were added) before she entertained thoughts of her own fallibility. So, I don't think that the problem that Littlejohn's example shows is that a proponent of (Risk-DJ) cannot accommodate the intuition that the students are in position to rationally accept (FB). But I do think there is a decisive objection that can be brought to light by considering the example, and the objection is still in the spirit of Littlejohn's concern.

The real problem exposed by Littlejohn's preface example is that, according to (Risk-DJ), these diligent and thoughtful students have done something epistemically impermissible at the very outset of the class, just because they are inquisitive and have considered a wide variety of topics and come to hold a wide range of beliefs.²² Because they have rich belief systems that cover many important and complicated topics, (Risk-DJ) entails that these students have arrived at their beliefs in an epistemically impermissible way. Ultimately, the only way to avoid an epistemically impermissible set of beliefs, according to (Risk-DJ), is by being so conservative in forming beliefs that one believes only a small number of propositions that are nearly certain to be true. Few non-skeptical epistemologists think that the rational response to the preface paradox is to forego rich and complicated belief systems altogether. And, yet, this is exactly what (Risk-DJ) says. The principle is thus absurdly restrictive.

Ironically, there is a certain sense in which the principle is also too permissive to actually have any real teeth. Given the analogy that we have considered to the coffee example above, we have to wonder whether (Risk-DJ) is meant to apply to people who have already violated the principle. For instance, does it apply to ordinary people with risky and rich belief systems. If not, then the irony is that (Risk-DJ) in no way applies to the vast majority of ordinary agents, and thus provides no explanation for why we are obliged to avoid inconsistent beliefs in cases like the preface paradox. Thus, in the end, I think Littlejohn is right that (Risk-DJ) must be rejected.²³

²²This is a familiar objection made by Foley (1992), and discussed by Easwaran and Fitelson (in press, p. 5)

²³It is worth noting that (CEP) would not have quite these serious implications. But as I have noted, it is hard to see how one could motivate thinking that it is acceptable to be as near to certain that one has failed to achieve

3.5. Local Risk Avoidance Principle

Now, Kroedel has recently responded to Littlejohn's objection, claiming that the non-agglomeration of permission in the case of the lottery is derivable from a supposedly less controversial principle, namely:

(Low) If the probability that it is the case that p on my evidence is sufficiently low, then I'm not permitted to believe that p . (2013b, p. 453)

(Low) is a more plausible principle that seems to follow straightforwardly from our goal to avoid false beliefs. Where (Risk-DJ) is a global principle stating what combination of propositions can be accepted at a time, (Low) is a local principle stating conditions under which it is unacceptable to believe some individual proposition. In effect, what we saw is that a global principle is highly implausible precisely because the ideal of perfect accuracy is out of reach to agents like ourselves with rich and informative belief systems. But a principle like (Low) expresses a norm on belief that is highly plausible, and one that we would be wise to adopt.²⁴

Now, one can accept (Low) and yet also maintain that permissions agglomerate. That is to say, (Low) does not deliver the desired failure of permissions to agglomerate by itself. Kroedel's argument from (Low) to non-agglomeration assumes the following closure principle:

(P-Closure) If $Perm(Bel(p_1) \& Bel(p_2) \& \dots \& Bel(p_n))$, then $Perm(Bel(p_1 \& \dots \& p_n))$.²⁵

In other words, Kroedel assumes that if one is permitted to believe some set of propositions, then one is permitted to believe a proposition that is equivalent to the content of their conjunction. From these two principles, Kroedel suggests that we consider a set of propositions p , q where $Pr(p) > t$ and $Pr(q) > t$, but where $Pr(p \& q) < t$ where t is the threshold for permission to believe in both

the epistemic ideal is rationally permissible, but that negligible step to certain failure to meet the ideal is rationally impermissible.

²⁴It is worth noting that (Low) is not a principle that all epistemologists can accept. Anyone who accepts multi-premise epistemic closure and fallibilism is committed to the rejection of (Low) if she believes that one can be justified in believing a set of propositions that are likely to contain at least some falsehoods.

At the same time, (Low) is highly plausible, and should be thought of as a deontic analogue to Easwaran and Fitelson's (forthcoming) basic evidential principle EB that was introduced in Chapter 1. (Low) can thus be thought to motivate analogous accuracy-dominance avoidance principles for permissibility theorists.

²⁵This is a generalization of the closure principle Kroedel (2013b, 453) appeals to in his argument for non-agglomeration. Kroedel considers a case with just two propositions, but as Christensen (2005, p. 26) points out, restricted closure principles entail their generalizations through repeated applications of the restricted principle.

(High-PJ) and (Low). Assuming one accepts (Low) and (High-PJ), then we get from (High-PJ) that

$$(NS) \text{ } Perm(Bel(p)) \& Perm(Bel(q)).$$

It follows from (Low) that

$$(LB) \text{ not-}Perm(Bel(p \& q)).$$

And from LB and (P-Closure), we can thereby infer that

$$(Not-WS) \text{ not-}Perm(Bel(p) \& Bel(q))$$

And, thus, together (P-Closure) and (Low) entail that permissions do not agglomerate in this case or in cases like the lottery where the probability of the conjunction is far below the threshold for permission to believe.

Now, (P-Closure) and (Low) together entail that permissions do not agglomerate in lottery-like situations. However, the appeal to (P-Closure) and (Low) as a pair of propositions is no less problematic nor less controversial than appealing directly to (Risk-DJ) to explain the non-agglomeration of permissions in lottery-like situations. While Littlejohn has not responded to Kroedel's reply, it is fairly obvious how we should respond. Ultimately, the combination of (Low) and (P-Closure) face nearly the same problems posed by the preface for (Risk-DJ), and, in fact, entail some version of (Risk-DJ).

That (Low) and (P-Closure) have nearly the same counter-intuitive implications for agent's with large belief sets is implicit in Kroedel's proof that permissions do not agglomerate. If a set of propositions are highly likely to contain some falsehoods, i.e., $Pr(\neg(p_1 \& \dots \& p_n)) > t$ where t is the threshold for being likely to be true, then (P-Closure) and (Low) together entail that one has an obligation not to believe all members of the set. That is to say, (P-Closure) and (Low) together entail

$$\text{If } Pr(p_1 \& \dots \& p_n) < t, \text{ then not-}Perm(Bel(p_1) \& \dots \& Bel(p_n)).$$

The argument for this is exactly that provided by Kroedel for the claim that permissions do not agglomerate in lottery-like situations. From the fact that $Pr(p_1 \& \dots \& p_n) < t$ and (Low), it follows that $\text{not-Perm}(Bel(p_1 \& \dots \& p_n))$. And then by (P-Closure), we get that $\text{not-Perm}(Bel(p_1) \& \dots \& Bel(p_n))$. So, (P-Closure) and (Low) actually entail a stronger principle than (Risk-DJ), and have exactly the same counter-intuitive implications for the preface. (Low) and (P-Closure) together entail that all rational agents are obligated to keep their belief sets small enough so that they engender little or no risk of error. And as we have already noted, it seems wildly implausible to think that almost all ordinary people are under some epistemic obligation to purge their system of beliefs until they reach a point where their belief systems are likely to be free from error.

Consequently, we are faced with a clear choice. We must reject either (Low) or (P-Closure) (We might, of course, reject both). In either case, there is no sound argument to the conclusion that permissions don't agglomerate that has both of these principles as premises.

3.5.0.5. Accuracy-Optimization Explanation. From the proceeding reflections, the lesson to be drawn is that the explanation for non-agglomeration of permissions should not go by way of an epistemic principle that requires sacrificing our other epistemic goals (in the sense of prioritizing the avoidance of error to the extent that we are willing to embrace a near skepticism), or a principle that implies that a set of beliefs is globally rational only if it is highly probable that the set is perfectly accurate (formally, that the probability of the contents of the conjunction of the set is sufficiently close to 1). There is one last method for trying to explain non-agglomeration in terms of our epistemic goals that we shall need to consider. Douven (2008) defends a permissibility solution to the lottery paradox, and provides an argument for why accepting some, but not all, lottery propositions optimizes our attainment of our epistemic goals in cases like the lottery paradox.

Douven notes that there are generally two master ideals that have been claimed to characterize our two goals. We have already considered principles motivated by the first ideal, which Douven characterizes as follows:

(G2) We ought to aim 'at both believing only what is true and believing all that is true.'²⁶

²⁶Douven (2008, p. 206). Douven acknowledges that this view of our epistemic goal is set out in Lehrer (1974, p. 202), Rescher (1973, p. 21), Bonjour (1985, p. 8), and Foley (1992, p. 183; 1993, p. 19).

Since there is virtually no chance of perfectly realizing this goal, the best we can really do is to satisfy:

(G1) We ought to aim at '[amassing] a large body of beliefs with a favorable truth-falsity ratio'.²⁷

In other words, (G2) is at best a normative ideal, and one that we have no chance of perfectly satisfying. If we believe any propositions that are less than fully certain, there is some chance we will accept some false beliefs. If we believe only those propositions that are true, we will miss out on some true beliefs. Hence, Douven (2008, p. 207) points out that, from a practical standpoint, there is no difference between trying to satisfy (G2) and (G1). As he puts it, '...even if we try to accomplish (G2), we can do no better than to achieve (G1)' (2008, p. 207). At best, we can try to come close to (G2), and the way to do that is to aim to satisfy (G1) to the best of our abilities.

It is worth noting that (G1) is charitably read as a way to *roughly* approximate our epistemic goal or to track how close we are coming to the normative ideal, (G2). A more nuanced approach would be to acknowledge that some propositions are more theoretically important than others, and that accuracy of some beliefs may be of greater theoretical significance than others.²⁸ But, roughly speaking, when the propositions in question are on par with one another in terms of their theoretical importance, as it is reasonable to assume in cases like the lottery paradox, we can approximate how well an agent is doing with respect to the attainment of (G1) by considering (a) how large her belief set is, and (b) what the truth-to-falsity ratio of her belief set is over some set of propositions. In a case like the lottery, where most of the beliefs that we shall consider, in Lehrer's terms, have the same epistemic utility, this rough characterization of our epistemic goal seems adequate.²⁹ At least, let us grant for the time being that this is an adequate characterization for our epistemic goal in cases like the lottery.

According to Douven, an argument can be made that of the various doxastic states that one can take on in a lottery situation, the one that yields the most success with respect to our epistemic goal is when we believe most, but not all, lottery propositions. Non-agglomeration of justification

²⁷Douven (2008, p. 205). Douven acknowledges that this view is set out in Alston (1985, p. 59).

²⁸See footnote 16 above for discussion of references on this point.

²⁹For arguments sake, we can restrict out attention to a lottery case where no one lottery proposition is more theoretically significant than any other.

or permissions is thus to be explained as following from the fact that an agent who takes advantage of all of her epistemic permissions is going to be less epistemically successful (in the sense of (G1)) than she would be if she decided to take advantage of only some of her permissions. In the next section, we consider Douven's argument for a permissibility solution where permissions do not agglomerate.

3.6. Douven's Argument for the Truth Conductiveness of an Arbitrary Strategy

Douven's defense of the permissibility solution returns to Douven's (2008) observation (previously introduced in Chapter 1) that there are three different belief forming strategies that one might take in the case of the lottery paradox:

NJ-Solution: Withhold belief for all lottery propositions.

Inconsistency Solution: Believe all lottery propositions.

Permissibility Solution: Believe some, but not all lottery propositions.

These are the three basic strategies one can employ when faced with the lottery paradox, but unlike the others, the permissibility solution is itself open to variation. One might believe any number of proper subsets of lottery propositions. Douven picks a particular version of the permissibility strategy, namely where an agent believes of one lottery proposition that it is false, and the rest that they are true. Let us call this particular version of the permissibility solution the *arbitrary solution* (since one is arbitrarily selecting exactly one lottery proposition to disbelieve). Douven then argues that an agent who believes all but one lottery proposition (which she disbelieves) will do better with respect to the attainment of her epistemic goals than she would if she employed either of the other two strategies.

Even if Douven's claims about epistemic success are correct, this by itself does not yield an argument in favor of the Arbitrary Solution. In addition, Douven endorses the following principle:

(C^{cp}) If theories of justification J and J' are in all relevant respects alike except that J is more conducive to our epistemic goal than is J' , then J is to be preferred to J' . (2008, p. 210)

This principle doesn't assume that the only desiderata for evaluating a theory of justification is how epistemically successful an agent will be if she acts in accordance with theory J . The argument for the arbitrary solution is that the theory of justification J that permits an agent to arbitrarily choose which lottery propositions to believe/disbelieve is to be preferred if all other things are equal. This principle leaves open the possibility that other desiderata might outweigh the advantages of epistemic success. The question, of course, is whether it is truly the case that an agent who adopts the arbitrary strategy of forming beliefs will be more successful in the attainment of her epistemic goals. It is to this issue that we now turn.

3.6.1. Measuring Epistemic Success. According to Douven, the arbitrary solution possesses one significant advantage over its rivals: the belief set it generates has either a larger belief set, or else a much higher truth-to-falsity ratio than the belief sets generated by the alternative solutions. In effect, what Douven (2008) assumes is that if two belief sets have acceptably high rates of accuracy, other things being equal, then the larger of the two sets is more conducive to the achievement of our epistemic goals. The other assumption he makes is that if a set of beliefs is highly inaccurate (in the particular example he focuses on, there are as many false beliefs as true beliefs), then, other things being equal, there must be an alternative set of judgments that does better at the attainment of our epistemic goals. Using these two assumptions, Douven argues that the arbitrary solution optimizes our achievement of our epistemic goals.

Douven's argument for this thesis is as follows. First, let us start by considering NJ-solutions. NJ-solutions take a highly conservative, highly skeptical approach to lottery cases. All one is advised to believe, on the NJ-account, is that some ticket is a winning ticket and the rest are losers. The truth-to-falsity ratio of the belief set delivered by NJ-solutions is very high, but according to Douven, one thereby misses out on a large number of potential true beliefs that one might have if one was a bit bolder.

Douven points out that if one was willing to commit oneself to believing of at least some particular tickets that they are losing tickets, then one is likely to add many more true beliefs to her stock of beliefs. Of course, this strategy is likely to yield at least a few false beliefs. However, he notes that the one or two false beliefs one obtains by the arbitrary solution must be weighed against the many true beliefs that one also obtains. And, for large lotteries, generally we are talking a very

high ratio of true beliefs to false beliefs. Let the ticket be an n ticket lottery. Then if one guesses incorrectly, one will have arrived at $n-2$ true beliefs and 2 false beliefs (one will falsely believe the winning ticket is a loser and that that some losing ticket is a winner). For this reason, he goes on to explain:

More generally, pick any numbers a and b such that you would think $a : b$ is a favorable truth–falsity ratio (for example, you would find it acceptable if, on average, of every 1,000 beliefs you adopt one is false). Then there is a lottery such that, by believing of all but one of its tickets that they will lose and of the remaining one that it will win, you have with certainty added a collection of beliefs with a truth–falsity ratio greater than $a : b$ to your stock of beliefs. (2008, p. 211)

Douven concludes:

Thus, a lottery, provided it is large enough, appears to give us a good opportunity to take a non-negligible step toward our epistemic goal. Clearly, by prohibiting us to believe of even a single ticket of any given lottery that it is a loser, NJ-solutions do not allow us to benefit from such an opportunity. (2008, p. 211)

Douven does not conclude that this provides a reason, tout court, to prefer an arbitrary solution to the NJ-solution, but it does at least shift the burden to explain why the other factors outweigh the epistemic benefits derived from the arbitrary solution. Ultimately, this provides at least some *prima facie* reason for thinking that the permissibility theorist is right that we should be able to accept at least some lottery propositions. In other words, it provides some *prima facie* explanation for why we should be permitted to believe lottery propositions.

Of course, this does not yet provide any explanation for non-agglomeration. We have an explanation for why believing some lottery propositions might seem permissible, but not an explanation as to why we shouldn't utilize all of those permissions at a single time and believe all lottery propositions. Here is Douven's answer to the non-agglomeration problem:

So why not adopt all these beliefs? One answer is that if belief is closed under logical consequence, then by believing of each ticket that it will lose you will

automatically add all propositions – true and false ones – to your stock of beliefs (for believing of all the tickets that they will lose contradicts your belief that the lottery has a winner). The result is definitely not a body of beliefs with a favorable truth–falsity ratio. Of course the supposition that belief is closed under logical consequence is not a particularly realistic one. Still, it would seem that even given some weaker and more realistic closure principle – such as that belief is closed under recognized entailment – believing of all ten tickets that they will lose will result in a less than favorable truth–falsity ratio of your beliefs. (2008, p. 210)

Douven goes on to claim that the arbitrary solution has the advantage in terms of being conducive to our epistemic goal because it yields a higher truth-to-falsity ratio than the inconsistency solution. And, thus, one shouldn't believe all lottery propositions because doing so will mean doing worse at one's epistemic goal than one would if she stops short of believing all lottery propositions.

This, of course, would provide some support for non-agglomeration, in that holding an inconsistent set of beliefs would guarantee failure to optimize achievement of one's epistemic goal. Unfortunately, Douven does not provide any further explanation for why believing of each lottery proposition at a time will yield a less than favorable truth-to-falsity ratio, so this is something we need to now consider.

There are two steps toward evaluating Douven's claim. The first step is to restrict our attention to those propositions that are explicitly recommended by the two solutions (that is to say, when we don't extend our focus to the beliefs that are supposed to follow from some set of closure principles). First, we note that there are many propositions that both solutions recommend believing. For instance, both solutions recommend believing that some ticket will win, and any disjunction of distinct lottery propositions (at least one of any two of these propositions will be true, since at least one of the tickets will be a loser), and so on. Where the explicit recommendations of the two solutions differ is captured by the differences in what to believe about particular lottery propositions. The arbitrary solution explicitly recommends believing all members of the set:

$$\mathcal{B}_{Arb} = \{l'_1, l'_2, \dots, \neg l'_n, \}.$$

And, the inconsistency solution explicitly recommends believing all members of the set:

$$\mathcal{B}_{Inc} = \{l'_1, l'_2, \dots, l'_n\}.$$

Now, if we focus on just these propositions, it becomes clear that the inconsistency solution actually does better with respect to the achievement of (G1). This can be seen by observing a couple of facts. First, the fact of the matter is that unless $\neg l_n$ is true, the number of true propositions in \mathcal{B}_{Arb} is $n - 2$, whereas the number of true propositions in \mathcal{B}_{Inc} is guaranteed to be $n - 1$. Of course, there is only a $1/n$ chance that $\neg l_n$ is true, and so the expected number of true beliefs in \mathcal{B}_{Arb} is $(n + 1/n) - 2$, which, of course, is worse than the number of true beliefs in \mathcal{B}_{Inc} for $n \geq 3$. Since each set has the same number of propositions, the proponent of an inconsistency response to the lottery paradox can insist that her strategy is likely to have more true beliefs and a high truth-to-falsity ratio. Thus, it is reasonable to expect that, at least when we restrict our attention to just the lottery propositions themselves, one who adopts an inconsistency response will do better with respect to the attainment of one's epistemic goals, at least on the sort of naïve counting criteria Douven employs.

Thus, if there is any reason to think that the inconsistency solution delivers a doxastic state that does worse at the achievement of our epistemic goals than Douven's proposed arbitrary solution, it must be in virtue of the additional false beliefs that can be derived from some closure principle. Now, as Douven's notes, a naïve belief closure principle without any extra conditions on the belief set is highly implausible. I, for example, believe all of the axioms of Peano Arithmetic. I do not, however, believe all of the theorems that they logically entail – If only mathematical knowledge was so easy to come by.

So, Douven is right that a belief closure principle would need a recognition condition. This handles the problem of failing to know or believe the theorems of Peano Arithmetic, since the reason I lack many of the important arithmetic beliefs results from the fact that I fail to recognize that the relevant entailments. But is the principle really that plausible? Bear in mind that the invoked principle is not one holding that epistemic credentials for belief are transferred from premises to conclusions by valid arguments, but rather that, as a psychological matter, agents believe the

claims they recognize as entailments of their other beliefs.³⁰ The answer seems to be quite clearly no. Consider the sort of principle Douven's argument assumes:

Recognized Belief Closure Principle (RBCP) If $Bel(p)$ for all $p \in X$ and S recognizes that X entails q , then $Bel(q)$.

Such a principle seems highly implausible for a variety of reasons. The most obvious problem is captured by the very example at hand, namely that of agents with inconsistent beliefs.

Let us consider such an example. Suppose that I come to believe a logically inconsistent set of claims. For example, suppose I have come to believe two scientific theories, theories that are complicated enough so that the inconsistency between them escapes the notice of the scientific community for some time. It certainly doesn't follow that I thereby believe all propositions whatsoever. Likewise, I don't thereby come to accept patent absurdities like circles are square, or obvious falsehoods like the moon is made of cheese. One of the many problems with a naïve belief closure principle (a belief closure principle without a recognition condition) is that it has this as an implication. (RBCP) does not, but only so long as I don't recognize the inconsistency of my beliefs. But now suppose that I come to learn that the two theories are, in fact, logically inconsistent. Now, due to the complexity of the theories and my epistemic situation, I am not sure how to revise my beliefs. Each of the core tenets of each theory seem correct to me. We can imagine that, like in the case of the lottery, each of the core tenets is likely to be true given the total evidence available. And we can build into the scenario that I recognize this fact, so that my confidence in each claim remains high. It is just that I know that at least one of the claims of the theory is false, since the theory is inconsistent.³¹ Can I continue to hold my beliefs, despite their inconsistency, until I can figure out how to rationally change my beliefs? As a question of the psychological possibility, the answer seems to be plainly yes, but, surely, I don't thereby come to believe claims that are patently absurd, even though I

³⁰Other more normative epistemic closure principles in the neighborhood might not be subject to the criticisms I present, but they won't guarantee that an agent who adopts the inconsistency approach will have a belief system with an unacceptable truth-to-falsity ratio as Douven contends. After all, if the alternative principles are merely prescriptive, an agent can fail to do as she ought, and thus fail to close her beliefs under conjunction-introduction. And, in this case, given the arguments above, the agent who opts not to act in accordance with the norm would end up having more epistemic success than the agent who does. If Douven's argument from our epistemic goals can plausibly undermine our intuition that one ought to avoid arbitrary decisions in her beliefs, one might think a similar argument would be available for the normative epistemic closure principle.

³¹Of course, if I was a reasonable scientist, I probably would have held this belief prior to recognizing the inconsistency, given that most scientific theories haven't yet landed on a completely accurate description of the laws of nature.

recognize the logical validity of *ex falso quodlibet*. But, then, my beliefs are not always closed under recognized logical entailment, and (RBCP) is false.

Given that (RBCP) seems to be false, and a plausible counter-example is any situation where an agent persists in holding an inconsistent set of beliefs, there seems to be no reason to think that an agent who holds inconsistent beliefs about lottery propositions is going to accept all propositions. Without some belief closure principle, it is not clear why we should think that the agent will hold any beliefs beyond those specified explicitly in the formulation of the inconsistency solution. And as we saw, that belief set is more conducive to the achievement of our epistemic goals than Douven's arbitrary solution by the standards expressed in (G1).

The only possible argument that remains to be considered along these lines is that, perhaps, Douven was mistaken to formulate the closure principle in terms of a psychological notion like belief as opposed to some normative notion like epistemic justification. That is to say, perhaps, what Douven should insist is that there is a principle that holds that one is rationally obliged to close one's beliefs under recognized logical consequence. There are several problems with such a suggestion. The first is that we still are not going to arrive at a valid argument for the conclusion that an agent with inconsistent beliefs necessarily has a belief set that has an unacceptable truth-to-falsity ratio. As long as the principle is normative, the agent could simply go against her obligations and fail to form beliefs that her other beliefs oblige her to accept. As long as she doesn't believe these other propositions, Douven is wrong to assume the agent will end up with an unacceptable truth-to-falsity ratio for her belief set. The agent may very well be doing something rationally impermissible in holding an inconsistent set of beliefs, and these other principles may provide an explanation of non-agglomeration, but it won't be in virtue of the agent's doxastic state failing to be conducive to the achievement of our epistemic goals in the sense of (G1).³²

Second, appealing to a normative closure principle in the context of Douven's argument from one's epistemic goals is self-defeating. The argument assumes that a key criteria for evaluating belief forming strategies and epistemic principles is the degree to which they serve our epistemic goal of obtaining true beliefs and avoiding false beliefs. One of the commonly recognized problems with

³²I'm not suggesting that they do, in fact, provide such an explanation. It may well be the case that closure principles are required for certain functional or pragmatic roles of belief, and that such principles motivate a consistency requirement on belief. This is a possibility we shall take very seriously in Chapter 5. But the argument for consistency or closure would then be distinct from the arguments from our epistemic goals that are currently under consideration.

holding that justified belief is closed under logical consequence (perhaps with additional recognition conditions attached), and in particular over conjunction-introduction, is the fact that conjunctions are riskier and more likely to be false than their conjuncts.³³ Thus, the agent who closes her beliefs over conjunction-introduction is likely to take on more false beliefs than the agent who refrains from adding those conjunctions to her stock of beliefs that are almost certainly false (It is important to be clear that I am not saying that agents should avoid belief in all conjunctions, just that one who doesn't adopt the policy of conjoining all of the claims she accepts might avoid a number of false beliefs). One needn't forego most of the true beliefs, since we can still believe those conjunctions that are likely to be true.³⁴ One simply can avoid the conjunctions that are nearly certain to be false if one doesn't close her beliefs under conjunction. And, thus, following an epistemic closure principle is not generally conducive to the achievement of one's epistemic goal when characterized in terms of (G1).

It might be helpful to make the point formally precise. Let us define the conjunctive-closure of a belief set as follows:

DEFINITION. Letting \mathcal{B} be a set of propositions that an agent believes, we define the conjunctive closure of \mathcal{B} as follows: $Conj(\mathcal{B}) = \{p \mid p = \bigwedge S \text{ where } S \subseteq \mathcal{B} \text{ and } S \neq \emptyset\}$.

The definition says that the conjunctive closure of a set contains all and only the propositions that are equivalent to any conjunction of any members of the set. For illustrations sake, let us consider the resulting truth-to-falsity ratio generated if an agent believes each proposition in $Conj(\mathcal{B}_{Arb})$, i.e., if one adopts Douven's arbitrary strategy and then closes her beliefs under conjunction-introduction. There are, of course, two scenarios to consider. There is the highly unlikely scenario where one winds up guessing correctly which ticket will be the winning ticket, and then there is the far more likely scenario that the agent has guessed incorrectly. Of course, in the first scenario, the number of true propositions the agent adds to her stock of beliefs will be equal to the total number of propositions in $Conj(\mathcal{B}_{Arb})$. In the second scenario, we then need to figure out how many of the propositions in the set entail one of the two false propositions. To calculate the expected number of true and false beliefs in $Conj(\mathcal{B}_{Arb})$, we simply need to calculate the total number of propositions in $Conj(\mathcal{B}_{Arb})$

³³See Kyburg (1961, 1970), and Christensen (2004) for standard presentations of the view that belief or rational acceptance is not closed under conjunction-introduction due to the aggregating risk of error.

³⁴Bare in mind that I am not insisting that the agent refrain from adding all conjunctions, especially those that are likely to be true on one's evidence.

and then figure out how many of those propositions logically entail one of the two false propositions. Because the conjunction of each non-empty subset of \mathcal{B}_{Arb} constitutes a distinct proposition, the matter of counting up the number of distinct propositions in $Conj(\mathcal{B}_{Arb})$ is straightforward. There are $2^n - 1$ propositions in $Conj(\mathcal{B}_{Arb})$ (this is just the number of non-empty subsets of the set). Now, how many of these propositions are likely to be true? Well, if $p = \bigwedge S^*$ for some non-empty subset of \mathcal{B}_{Arb} , and p will be true in this case if and only if neither of the two false propositions in \mathcal{B}_{Arb} are in S^* . So, the number of true propositions in $Conj(\mathcal{B}_{Arb})$ is equal to the number of non-empty subsets of \mathcal{B}_{Arb} that do not contain either of the false propositions in \mathcal{B}_{Arb} . There are exactly $2^{n-2} - 1$ such sets. Thus, in the latter case, the resulting ratio of true-to-false beliefs is $\frac{2^{n-2}-1}{2^n-1}$, which is less than $1/4$ for all $n \geq 3$. Thus, unless one gets incredibly lucky, then in a case like the lottery paradox, closing one's beliefs under conjunction-introduction will yield 3 times as many false beliefs as true beliefs.

Informally, what this means is that by closing one's belief set under conjunction-introduction for the sorts of propositions we are considering in the given case, for large enough n , one will almost certainly be adding more than 3 false propositions to one's stock of beliefs for every truth that one's belief set contains. Given, as Douven has rightly suggested earlier, that a belief set whose truth-to-falsity ratio is less than $1/2$ is generally unacceptable, it seems clear that the same could be said for the belief set that one obtains by closure under conjunction. If we are using a naïve counting method of adding up the true and false beliefs in determining how well an epistemic principle serves our epistemic goals, it seems reasonably clear that a closure principle will not come out looking particularly attractive.

These problems for a closure principle are magnified further when we consider closing one's beliefs under a rule like *ex falso quodlibet*. It should be obvious that doing so would be nothing but devastating to the achievement of our epistemic goals. An agent whose beliefs are closed under such a rule will either not add any additional propositions to her stock of beliefs, or else she will add all of them. So, either an epistemic rule that holds we should closed under *ex falso* will either deliver no more true beliefs to one's stock of beliefs for the agent who follows it, or else it will trivialize one's belief set altogether. In either case, one will do no better at the achievement of one's epistemic goal if one follows such a policy. Generally speaking then, the various closure principles that Douven's argument requires are subject to the very same objection that Douven

(2008) tries to make against the inconsistency solutions to the lottery paradox. In other words, if we use our epistemic goals as a criteria to evaluate epistemic principles, then we get just as strong an argument against closure as we would get in favor of a consistency requirement (if we were to assume those closure principles in the argument for a consistency requirement).

Thus, it seems to me that if we are taking seriously the idea that epistemic principles should be evaluated in terms of their conduciveness to the achievement of (G1), we are forced to conclude that principles that entail that we should close our beliefs under conjunction-introduction or ex falso must be rejected. And, thus, we are left with no reason for thinking that an agent who adopts an inconsistent set of beliefs in a case like the lottery paradox is doing worse with regard to the attainment of (G1). I am thus inclined to think that neither a global approach to characterizing one's epistemic goals, nor a local approach as in a principle like (Low) will be plausibly combined with some closure principle to deliver an argument against adopting an inconsistent belief set. Thus, it seems to me that we have run out of options for trying to explain the non-agglomeration of permissions in terms of our epistemic goals.

3.7. Harman's Diachronic Principle

So much then for defending the permissibility solution by appealing directly to our epistemic goals. There are, of course, other strategies and principles that epistemologists have considered basing their defense of the permissibility solution on. One such principle is implicit in Harman's suggestion for reconciling a permissibility principle with a consistency requirement. Here is Harman's explanation for why a high probability principle is compatible with a consistency requirement:³⁵

Although one believes that one ticket will win, one could also infer, for any ticket in the lottery that that ticket won't be the winning ticket. There is no actual contradiction here.

To say one can infer this of any ticket is not to say one can infer it of all.

Given that one has inferred ticket number 1 will not win, then one must suppose the odds against ticket number 2 are no longer 999,999 to 1, but only 999,998

³⁵Harman does not himself endorse anything like the principle that he suggests. Harman is quite clear that he rejects probabilism about degrees of belief, and is generally motivated toward a kind of eliminativism, primarily for cognitive economy reasons. Nevertheless, defenders of permissibility solutions like Douven (2008, 2012) and Ross (2003, 2012) have looked to Harman (1986) for inspiration.

to 1. And one infers ticket number 2 won't win, one must change the odds on ticket number 3 to 999,997 to 1, and so on. If one could get to ticket number 999,999, one would have to suppose the odds were even, 1 to 1, so at that point the hypothesis that this ticket will not win would be no better than the hypothesis that it will win, and one could infer no further. (Harman 1986, p. 71)

In a nutshell, Harman's suggestion is that we could hold that the dynamics of belief revision require updating one's subjective probabilities by conditionalizing on the propositions we add to our stock of beliefs or full acceptances. In other words, when one fully accepts p , one's subjective probability function is upgraded to 1 (representing full acceptance of p), and the rest of one's subjective probabilities are revised accordingly. How exactly are one's subjective probabilities revised? They are revised via a conditionalization principle. Letting p be the proposition one has added to one's stock of beliefs, one's new subjective probability function is determined as follows:

(Update Rule) When S adds p to her stock of beliefs, S 's subjective probabilities are updated as follows: $Pr_{new}(q) = Pr_{old}(q|p)$ for all q .³⁶

Assuming that one adds one proposition to one's stock of beliefs at a time, then one will never add all members of an inconsistent set to one's stock of beliefs if one is guided by a high probability rule (i.e., a high probability is required for it to be permissible for one to add a proposition to one's stock of beliefs). To be formally precise, the high probability principle in this case takes the following form:

(HPP) S is permitted to add p to her stock of beliefs at time t if and only if $Pr_t(p) \geq r$ where $.5 < r < 1$ is some threshold for full belief.³⁷

The reason the combination of (Update Rule) and (HPP) are compatible with a general consistency requirement is that for any (minimally) inconsistent set $\mathbf{S} = \{p_1, \dots, p_n\}$, $Pr(p_i | \bigwedge \mathbf{S} \setminus \{p_i\}) = 0$. This guarantees that as one follows (Update Rule) and begins adding propositions from a (minimally) inconsistent set to one's stock of beliefs, one must eventually reach a stage where all of the remaining members of the set are below the threshold r . Suppose our indexing follows the order in which the agent goes about adding the propositions to her stock of beliefs, either at some

³⁶This is something Douven says explicitly in 'The Sequential Lottery Paradox.'

³⁷Littlejohn (2012, p. 513) suggests a nearby variant on this proposal.

point k where $k < n - 1$, the probability of the remaining members is less than the threshold r , or else (by our observation) when S reaches the stage $n - 1$, $Pr_{t_{n-1}}(p_n) \leq \frac{1}{2}$. In either case, the agent will not be able to add all propositions to her stock of beliefs, while conforming to (Update Rule) and (HPP).

Now, it is reasonable to doubt that such a proposal can (a) be well motivated, or (b) avoid having counter-intuitive implications for justified belief and rational belief revision. Ultimately, I think the proposal has such clearly implausible consequences for rational belief revision and rational decision-making that we have grounds to reject the proposal without having to wade into the muddy waters of how one would motivate such a view. So, we shall turn to the various problems that can be raised for Harman's view. We shall start by considering what Harman's proposal means in the case of the preface paradox.

3.7.1. Harman's Principles and the Preface? How does the combination of (Update Rule) and (HPP) do with regard to the problems the preface exposes for the other principles we considered above? Again, the answer depends in part on what challenge the preface paradox is supposed to pose. If the problem is that one knows the preface proposition to be true (and the principles above fail to respect this fact), then clearly, since Harman's Diachronic solution delivers a consistency requirement, it entails that one is not justified in adding the preface proposition to one's stock of beliefs, at least while one continues to hold all of those beliefs.³⁸ But if one's goal is to defend an account of epistemic justification according to which logical consistency is a necessary condition on belief, then one must deny that one can be justified in believing the preface proposition, at least while one accepts each of the claims made in the book. Thus, to reject an account because it entails that one doesn't know the preface (when one thinks we do) is to reject the goal of defending a consistency requirement altogether, as opposed to objecting to a particular principle for non-agglomeration. This, then, is not a challenge to the permissibility solutions that would undermine their basic principles for explaining non-agglomeration. This is a challenge that would undermine the very idea of a permissibility solution that imposed a consistency requirement, and not an objection to a particular principle for motivating or explaining non-agglomeration.

³⁸One might be permitted to add the preface as long as one subtracts propositions from her stock of beliefs, but one thing (Update Rule) and (HPP) fail to provide is an account of how one contracts one's belief set. Clearly, the account would be incomplete without such principles, but I don't think we need to consider them for our current aims.

However, as we noted above, (Risk-DJ) and the like confront a much more fundamental challenge from the preface paradox. It is highly counter-intuitive to think that we are rationally obliged to contract our belief sets until the point where our belief set is likely to contain only truths. So, it would be problematic if (Update Rule) and (HPP) together entailed:

$$\text{If } Pr(p_1 \& \dots \& p_n) < t \text{ at } t^*, \text{ then not-} Perm(Bel(p_1) \& \dots \& Bel(p_n)).$$

Harman's principle avoids this particular worry. The easiest way to see this is to consider the standard formal presentation of the preface paradox.

(Formal Preface Paradox): Let $\{p_1, \dots, p_n\}$ be a set of propositions such that are k -wise independent of each other for all $k \leq n$. We thus have it that $Pr(p_1 \wedge p_2 \wedge \dots \wedge p_n) = Pr(p_1) \cdot Pr(p_2) \dots \cdot Pr(p_n)$. And, supposing $r < Pr(p_i) = Pr(p_j) < 1$, then for large enough n , $\{p_1, \dots, p_n\}$ can be as likely to contain some falsehood for whatever threshold that one might choose, while each proposition is as near to certain as one cares to make it. That is to say, we can have the following for large enough sets:³⁹

$$Pr(\neg(p_1 \wedge p_2 \wedge \dots \wedge p_n)) = 1 - Pr(p_1) \cdot Pr(p_2) \dots \cdot Pr(p_n) > t < 1 \text{ and } Pr(p_i) > t \text{ for all } i \leq n.$$

This was just the antecedent condition of (Risk-DJ). And, thus, for any such $\{p_1, \dots, p_n\}$, a principle like (Risk-DJ) would entail that one is not permitted to believe each $\{p_1, \dots, p_n\}$ at one time. According to (Risk-DJ), an agent with a large belief set who believed all members of this set would have already done something irrational before considering (FB). One of the virtues of Harman's proposal is that it has no such implications for preface sets of this sort. Given the strong form of probabilistic independence of the propositions in the preface set we are supposing, despite the fact that $\{p_1, \dots, p_n\}$ have a high risk of containing an error, Harman's probabilistic principle allows that one could be permitted to believe all of the members of $\{p_1, \dots, p_n\}$ at one time. After all, conditional on the rest of the preface set being true, each statement in the preface set has the same probability it had before. And, thus, as one adds preface propositions to one's stock of acceptances, the probability of the other propositions remain the same. Hence, Harman's principle is not a

³⁹See Engel (1991) and Olsson (1998) for further discussion of this version of the preface paradox.

risk-avoidance principle, and does not carry the counter-intuitive implications that such principles must carry.

3.7.2. Real Problems for Harman's Principle. But Harman's principles have a variety of problems of their own. It is just that Harman's principles are subject to very different problems than those above. We shall take these in turn.

The first problem with Harman's belief set is that, understood as a diachronic update procedure, it has some wildly implausible consequences for the sorts of propositions that one might come to justifiably believe on the basis of one's evidence. In particular, Douven (2012) shows that by repeated applications of Harman's principle, one can take a slow march toward propositions that are almost certainly false on one's total evidence. Douven presents a new version of the lottery paradox that illustrates why this is the case. To see this, we shall consider Douven's Sequential Lottery.

3.7.3. Douven's Sequential Lottery Example: Douven explains his sequential version of the lottery as follows:

"Let there be a sequence of lotteries, L_1, \dots, L_n , each consisting of m tickets, with $(m - 1)/m > \theta$; the lotteries are to be held consecutively. All lotteries in this sequence are fair, but instead of having a single winner they have a single loser, where for each L_i with $i < n$ the prize is a free ticket for lottery L_{i+1} ; the winners of the final lottery, L_n , receive one million dollars each." (Douven 2012, pp. 56-57)."

In other words, there will be a sequence of lotteries held, where to have a ticket for any lottery other than the first, one will have had to have a winning ticket in all prior lotteries. The problem, Douven observes, is that, based on Harman's suggestion, one can arrive at the justified belief that she will be a million dollar winner in this sequential lottery, i.e., win every lottery, despite the fact that, for a large enough n , it is nearly certain she will lose at some point prior to winning the final lottery. Now, due to the fact that it is nearly certain that one will lose at the outset, one could have also employed (HPP) to infer that one will not be a winner at the very beginning. Whether

one ends up with the conclusion that one will be a winner or not clearly depends on which method of updating one's belief one chooses to employ.

How does one arrive at the judgment one will win? First, based on (HPP), one infers that one will win the first lottery, and adds this proposition to one's stock of beliefs. When one does so, one then conditionalizes on that belief. Post conditionalization, the resulting subjective probability that one will win L_2 is $\frac{m-1}{m}$.⁴⁰ Continuing this update process n - many times, one arrives at the conclusion that one will become a millionaire. And one reaches this conclusion despite the fact that the probability that one would loose would initially be $(\frac{m-1}{m})^n$, which a large enough n makes near zero.

Thus, one can, in this manner, arrive at beliefs that, at the outset, one can recognize to be almost certainly false.

3.7.3.1. Lesson of The Sequential Lottery. The basic lesson of the sequential lottery is that it is highly problematic to add propositions with non-1 probability on one's current evidence to one's stock of beliefs if doing so means revising one's subjective probability function to assign that proposition probability 1. In some ways, this might not seem problematic because one might think that in fully believing p , for all intents and purposes, one is foreclosing on the possibility that one's belief that p is mistaken. And, if there is anything like an epistemic probability function, the difference between $Pr_{new}(p)$ and $Pr_{old}(p)$ is marginal. In fact, $Pr_{new}(p) - Pr_{old}(p) < 1 - t$, and the point of making t high is so that the difference between treating p as highly probable and certain is small enough to make it reasonable to ignore the difference. But, as we know well from countless soritical examples, many small and seemingly insignificant differences can add up to substantial and eminently important differences when compounded. So, while each step along the path of adding new propositions via Harman's procedure may take us seemingly insignificant steps away from a subjective probability function that some way accords with the objective or evidential probabilities, enough small steps away from a reasonable starting point may lead us to accept propositions that are absurd or nearly absurd. So, one problem with Harman's interpretation of (HPP) is that in many ways it isn't conducive to the attainment of our epistemic goals, as it licenses us to come to believe propositions that are almost certainly false by our own lights.

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One might want to respond that when soritical examples go bad, it is sometimes the case that the rule itself is unproblematic, what is problematic is our assumption that we can repeatedly apply the rule. Such observations will be of little help here as the lottery paradox is based on the fact that we have a very large number of propositions, and so need to be able to apply (Update Rule) a large number of times if we are to reach a point where no new lottery propositions can be added to one's stock of beliefs. So, to offer an explanation of non-agglomeration, Harman's (Update Rule) needs to be something we can apply a large number of times.

Another lesson of the sequential-lottery is that Harman's update rule entails that one can add a proposition to one's stock of beliefs, or else that propositions negation. After all, at the initial stage of the update process, t_0 , so prior to adding any propositions to one's stock of beliefs, one might decide to add the proposition that one will not win a million dollars. Depending on how one wants to think about what is permissible at a time, we might then think that even at the initial stage, one is both permitted to believe a proposition and believe its negation. Whether Harman's proposal entails that $Perm(Bel(L_n))$ and $Perm(Bel(\neg L_n))$ at t_0 depends a bit on how we think about permissions. It is true that one is not permitted to simply add L_n to one's stock of beliefs immediately without taking any of the intermediate steps in the first place, but our ordinary way of talking about permissions usually entails that one is permitted to perform some action x even if there are certain other actions one must perform in the meantime to doing x . Hence, we say things like, you are permitted to borrow my lawn mower just as long as you fuel it yourself. So, in this everyday way of talking, if we are thinking of having epistemic justification as having permissions, then Harman's proposal entails that one can be justified in believing a proposition and its negation.

The next question to ask is why is the sequential lottery, and the outcome of the sequential lottery any more problematic than the initial standard lottery. Ultimately, the same issues arise in the case of the standard lottery paradox. The problem in the sequential lottery is that one winds up accepting propositions that on the basis of one's initial evidence has a near zero probability of being true. While the content of the proposition one accepts in the sequential lottery is quite different, the fact remains that Harman's solution is already committed to one being rational in accepting some propositions with a low chance of being true on one's initial evidence. That is because, on Harman's solution, one is justified in believing the conjunction of the lottery propositions that one accepts, and for large enough lotteries, one can accept a vast number of lottery propositions whose

initial joint probability is near zero. To illustrate, suppose that the threshold of (HPP) is .9. Then in a one hundred ticket lottery, one can employ Harman's method to infer that the first 90 tickets will all be losing tickets, and thereby come to accept the conjunction of these propositions. The initial probability that the conjunction of these propositions is true is $1/10$. Also, note that the proposition that it is not the case that the first ninety tickets are all losers has a probability of .9. Thus, by (HPP), one could begin by inferring the negation of the conjunction of these propositions as well. Thus, if the sequential lottery exposes a new problem for Harman's solution over and above problems it faces from a standard lottery paradox, it can't just be the fact that one can come to believe a proposition with an initially low probability, nor that (HPP) licenses one to accept a proposition or its negation, depending on the propositions one chooses to infer using (HPP). Thus, I think that it is fair to say that Harman's solution to both the standard and sequential lotteries stand or fall together.

3.7.3.2. Problem 2 for Harman's Suggestion. In addition to arrive at particular beliefs that are almost certainly false, it also calls out for some explanation. Why should it be the case that when I come to fully accept a proposition p , my subjective probabilities must be revised according to the (Update Rule)? In the case of the lottery paradox, the mere fact that my attitude toward the proposition has changed in no way makes the proposition any more or less likely to be true. For mind-independent propositions of this sort, any rational agent can recognize that adding the proposition to her stock of beliefs will leave the propositions evidential and objective probabilities unaffected. In short, if the rule is suppose to be that agent's actually update their subjective probabilities (which seems clearly to be what Harman has suggested), then how do we justify the idea that it can be rational for an agent to have subjective probabilities that one recognizes to be disproportional to the evidential and/or objective probabilities of the proposition? In light of the lessons taught by the sequential lottery paradox, this question becomes especially pressing, given that via iterated applications of the (Update Rule), the degree to which one's subjective probabilities can end up disproportional to the evidential probability and objective chance of being true can be extremely high. It seems clear that any rule for belief revision that could lead to such a radical departure from having proportional credences can be no guide to rational belief formation.

3.7.3.3. Practical Part of the Objection. This brings us to an obvious practical problem with (HPP) and (Update Rule). Now, in the case of (HPP) and (Update Rule), the divergence between

subjective and evidential probabilities means that we could construct betting situations where an agent who is governed by these two principles is likely to come out at a loss or lose out on expected utility if she acts in accordance with her subjective probabilities.⁴¹ The obvious example is one used by Hawthorne (2004, p. 85) to demonstrate the problem of basing a practical decision on a known or certain belief.⁴² If one treats a lottery proposition as certain, then one would be rational to sell a lottery ticket for even the smallest value, regardless of how large the payoff would be. Obviously, the utility of deciding to keep one's ticket tends to infinity as the payout for having the winner approaches infinity if we hold the size of the lottery fixed. So, we can construct cases where the payout for a winning ticket makes it so that one who treats one's ticket as a certain loser, that is to say, someone who has exercised her permission granted by (HPP) and revised her subjective probabilities in accordance with (Update Rule), would be inclined to sell her ticket at any price, no matter how high the expected utility. Thus, one who is guided by (HPP) and (Update Rule) would be liable to make choices with lower expected utilities than the alternatives. We could go on about the practical irrationality that results from allowing oneself to be guided by (HPP), but I think these considerations are enough to show that an agent should not allow her practical decisions to be guided by (HPP) and (Update Rule).

3.7.3.4. The Problems with (HPP) and (Update Rule) Summarized. Let us sum up the problems with the combination of (HPP) and (Update Rule). There are three main problems with the view that we have noted. First, when combined with (HPP), (Update Rule) allows for an agent to take a slow march from reasonable beliefs to beliefs that are highly implausible. Second, these principles allow for an agent's subjective probabilities to radically diverge from what are commonly recognized as the objective or evidential probabilities of the propositions in question. As a result, these principles allow for one to come to fully believe propositions that are nearly certain to be

⁴¹By subjective probability, I mean the credence or degree of confidence the agent has in the proposition. See Jeffrey (2004, pp. 19-22) for a relevant discussion of subjective and objective probabilities. In the the case of Harman's principles, I don't mean to make too much of the distinction. By evidential or objective probabilities, I simply mean here the probability distribution that treats each lottery ticket as having the same objective chance of winning as every other lottery proposition. (HPP) and (Update Rule) require that it be possible for an agent to have a different degree of confidence than this, since the agent will treat lottery propositions that are fully believed as certain, despite the fact that the evidence has not ruled out the possibility that the proposition is false.

⁴²See McKinnon (2011) for a discussion of Hawthorne's example. Like McKinnon and contra Hawthorne, I don't think the example provides strong evidence in favor of the view that we can only act on the basis of known propositions. I do, however, think it clearly brings out a problem with treating propositions as a given in one's practical reasoning that aren't certain. The remedy for (HPP) is that it builds in facts about an agent's decision-situation. Such principles will be explored in Chapter 5 when we turn to pragmatic encroachment theories of justified belief and their implications for formal coherence constraints on justified belief.

false on one's total evidence. Last, but not least, if one's practical decisions are to be guided by one's subjective probabilities, then an agent who is governed by (HPP) and (Update Rule) will be liable to make wildly self-destructive practical decisions. Together, I think it is fair to say, these problems provide a decisive refutation of these principles.

3.8. Collective Defeat Conditions and Coherentist Explanations

So far we have found serious problems with a number of attempts to explain a consistency requirement. There is one last possibility that we will need to consider, which sets the stage for our investigation into the relationship between substantive coherence and logical consistency, the focus of the next chapter. The basic idea behind the strategy is brought out by problems for a collective defeater principle suggested by Littlejohn:

(High*) PBp if the probability of p on your evidence and what you permissibly believe is sufficiently high. (2012, p. 513)

(High*), unlike (HPP) and (Update Rule), says nothing about the kinematics of belief revision. Nevertheless, the explanation for how this principle rules out inconsistency closely resembles Harman's explanation:

The thought seems to be that it would be impermissible to add a belief to your belief set because the probability that the remaining tickets will lose on your beliefs falls further and further the more beliefs you form. Eventually, it becomes impermissible to add more. (2012, p. 513)

Now, in effect, this principle assumes that one's beliefs might collectively provide defeat for the permissions you would otherwise have to believe some lottery proposition, and it does so just in virtue of the probabilistic negative dependence of the other claims that one accepts. Such a collective defeat principle cries out for some sort of explanation. Littlejohn objects to the proposal, noting

The oddity of this response, however, seems to be that the minimally rational agent knows that the probability of each ticket turning out to be a loser is the same and remains invariant however many lottery beliefs the subject forms. Yet,

(High*) suggests that the reason that you don't have the permission to believe some lottery proposition is that the probability of some ticket turning out to be a loser has dropped below some threshold. (2012, p. 513)

And, Littlejohn's point here seems persuasive to me, unless there is some more general explanation for why the negative dependence between propositions one currently believes and the proposition one is considering adding to one's belief system is relevant to our epistemic evaluation of the agent's belief system as a whole. If a proposition one is considering adding to one's belief system has a high probability, but has a low probability conditional on one's other beliefs, then the proposition in question stands in strong negative dependence relation to the conjunction of the claims that one accepts. The best candidate explanations for why we should accept (High*), or perhaps some analogue to it that gives negative dependence relations such weight, are theories that hold that the global rationality of an agent's belief system depends on the mutual support relations that the beliefs stand in to one another. That is, in effect, what the explanation for non-agglomeration of (High*) seems to assume, and what needs explaining.

While some epistemological theories might give little or no weight to such mutual support relation or explanatory relations, coherentist epistemologists, such as Lehrer (1990, 2000), Bonjour (1985), Lycan (1996, 2012), have famously held that such relations are essential to the evaluation of global rationality of one's beliefs. In a nutshell, one way to motivate a collective defeat condition like (High*) is by way of a coherence theory of epistemic justification. Of course, as we have already seen in our discussion of Lehrer's coherence theory, the assumption that inconsistent claims stand in negative relations of support that preclude coherence is not a claim that should be taken as self-evident. Ultimately, it depends on the precise details of how the coherence relation is formulated. In the next chapter, we shall investigate the formal assumptions that must be made about the substantive coherence relation to obtain a consistency requirement on coherent sets of beliefs.

3.9. Conclusion

We have considered a variety of different approaches to explaining a consistency requirement on belief. What we found is that arguments from our epistemic goals confront one of two problems. Risk-avoidance principles are overly conservative, and forbid us from having rich and complicated

belief systems. And Douven's (2008) argument that inconsistent belief systems will be sub-optimal for achieving our epistemic goals is self-undermining because it requires closure principles that are subject to these very same problems that are supposed to infect inconsistent belief systems. Explanations of a consistency requirement that appeal to the dynamics of updating one's subjective probabilities might afford us with a way to explain non-agglomeration of permissions, but they allow for belief in the patently false, and entail a radical divergence between subjective and evidential probabilities. One strategy that seems to remain is the view that our beliefs can stand in collective defeat relations, but such views require some sort of more general explanation. In the next chapter, we turn to attempts to explicate the coherence relation to see whether coherentists are, in fact, committed to a consistency requirement on coherent beliefs.

CHAPTER 4

Inconsistent Beliefs and Probabilistic Coherentism

4.1. Introduction

In this chapter, we continue our investigation into the claim that logical consistency is a minimal requirement on substantive coherence. We saw in Chapter 2 that Lehrer's formal analysis of the substantive coherence relation allows inconsistent claims to cohere, or at least it fails to provide any obvious explanation for why they cannot. But there have been many other attempts to formally explicate the notion of claims substantively cohering, and perhaps some of these analyses of the coherence relation put the assumption that coherence requires consistency on a more solid foundation. We shall focus on the leading probabilistic measures of substantive coherence that have been proposed, since these analyses of the coherence relation allow the question to be made precise, and the formalism in this case helps to bring out some of the key underlying assumptions necessary for coherence to require consistency. While formal epistemologists have proposed various recipes for constructing an infinite number of distinct measures of coherence, we shall focus our attention on those measures in the literature that have been given a rigorous defense, and garnered a considerable amount of discussion. In this chapter, our main goal will be to determine which of the leading probabilistic measures of coherence impose logical requirements on coherence. In particular, we shall investigate what features lead to a logical consistency requirement on coherence, and shall ask the same questions for Easwaran and Fitelson's dominance avoidance principles introduced in Chapter 1.

4.2. The Other Inconsistency Objection to Coherentism

Before we jump into our inquiry into the nature of coherence and its relation to logical consistency, we need to first note some related work that sets the stage for our investigation and also necessarily complicates our task to some degree. The lottery and preface paradoxes are not the only sort of problems for the thought that coherence requires consistency. William Roche (2013) considers a

problem for coherentism that turns on the simple observation that anyone might come to have some inconsistent beliefs, and yet still be justified in holding some of her beliefs. As Roche notes, if consistency is required for coherence of one's belief system, then this implies that an agent with inconsistent beliefs has an incoherent belief system (Roche 2013, p. 85). Assuming one's system of beliefs must be coherent for the members of that system to be justified, it follows that an agent with inconsistent beliefs cannot be justified in believing any claims whatsoever. This is a patently absurd conclusion that any viable coherentism must avoid. To distinguish these problems, let us call Roche's problem *the problem of inconsistent belief*, since it does not involve the assumption that all of the claims are justified, and let us call the lottery and preface paradoxes *the problems of justified inconsistent belief*, since these are purportedly cases where all of the claims are justified.¹

One thing we shall need to consider is whether the problem of justified inconsistent belief and the problem of inconsistent belief deserve a uniform solution. In their general forms, the problem of justified inconsistent belief and the problem of inconsistent belief look to be intimately connected with one another. After all, they are both based on the assumption that coherence requires consistency, and both problems dissolve if coherentists could deny this premise. While offering a uniform solution would be simple and straightforward – just deny that coherence requires consistency – it would be a mistake. As we shall see from the main example Roche considers, certain cases of inconsistent belief are highly pathological, i.e., involve sets of beliefs that violate Easwaran and Fitelson's strict dominance avoidance principle. And, it seems far less plausible in these pathological cases, unlike cases like the lottery or the preface, that the agent's beliefs are coherent. Roche's example thus raises the question of whether rejecting a consistency requirement on coherence means rejecting all formal requirements on coherence. It is to Roche's example that we shall now turn.

4.2.1. Coherentism & Inconsistency Problems. We shall begin by considering a precise formulation of the problem of inconsistent beliefs, as it has been put forward by Roche (2013, p. 85). Coherentists may seem committed to the soundness of the following argument.

(A) *S*'s belief in *p* is justified only if *S*'s belief system is coherent.

(B) *S*'s belief system is coherent only if *S*'s belief system is consistent.

¹I think it important to emphasize that I do not plan to defend the claim that one can be justified in believing lottery propositions, nor is this assumption required for any of the arguments considered throughout this chapter. We shall simply use lottery examples as ways of investigating the relationship between consistency and certain formal accounts of the coherence relation.

Therefore,

(C) *S*'s belief in *p* is justified only if *S*'s belief system is consistent. (2013, p. 85)

Now, (C) is problematic for a few reasons. As we have noted, many think the preface paradox provides us with grounds to deny (C). But, one doesn't need to accept that one can be justified in believing each member of an inconsistent set of propositions to see a problem with (C).

Roche makes an observation that problematizes (C) without assuming that one can be justified in believing each member of an inconsistent set of propositions. Roche notes that "(C) implies that if *S*'s belief system is inconsistent, then all of *S*'s beliefs are *unjustified*" (p. 85). Roche observes that, by (C), if one happened to believe a necessary falsehood, it would follow from (C) that none of *S*'s beliefs would be justified. The particular example that Roche gives introduces a variety of complications that I think it is best now to avoid. So, instead, consider the following example with the same formal structure.

Tired Logic Student. Suppose *S* believes each member of $\beta_n = \{p_1, \dots, p_{n-1}\}$ where B_n constitutes a highly coherent set of beliefs (however we are to unpack systemic coherence) where all of the beliefs propositions pertain to *S*'s biology coursework. For simplicities sake, we are going to assume that they are highly coherent at least partially in virtue of the fact that they are logically equivalent or nearly logically equivalent.² Now, imagine that *S* has been asked by her logic professor to determine whether or not p_n is a tautology or a logical contradiction. *S* has tried for about an hour to prove that p_n is a contradiction, but failed time and time again. Eventually, she mistakenly decides that if p_n was a contradiction, she would have been able to prove it and so forms the firm conviction that p_n is a tautology and adds that proposition to her stock of beliefs. In fact, it is a logical contradiction.³

²We make this simplification because, on most of the formal characterizations of coherence that we shall consider below, logically equivalent sets maximize coherence, or nearly maximize coherence. So, if we start with a (near) maximally coherent set of propositions, the question is whether adding a single contradiction to the set of propositions yields an incoherent set. The key formal assumption is that $Pr(p_1 \equiv p_2 \equiv \dots \equiv p_n) \approx 1$. It is in keeping with all of the measures of coherence that we are considering below that under the right assumptions, such sets can be maximally or near maximally coherent.

³Roche's example is a tired math professor (2013, 85). In it, Roche gives an example where a math professor claims a highly complicated necessary mathematical falsehood is true and her students come to accept this belief on the basis of the professor's testimony. Cases like this raise the question of the propositional content of the belief in question, whether it engenders actual inconsistency in one's belief system. One might, for instance, deny that all mathematical

Roche observes that (C) entails that none of S 's beliefs are justified, and notes that this is quite implausible. It is one thing to hold that S is not justified in believing p_n , but it is quite another to hold that none of the beliefs in B_n could be justified, especially if they are unrelated perceptual beliefs that are highly coherent and evidentially disconnected from p_n .

Whether the problem is that some beliefs should be justified (as in the Tired Logic Student) or all should be justified (as in the preface), there are two potential responses to the problem. On the one hand, one might deny (A) or else one might deny (B). Roche notes that the key question is whether the coherentist has principled grounds to deny either. Roche considers the probabilistic measure of coherence that he proposes, and observes that in cases like Tired Logic Student, his coherence measure does not provide principled grounds for denying (B). He thus concludes that coherentists ought to reject (A). Ultimately, I think the coherentist must reject (A), though she might also reject (B). To see why, we must first consider Roche's response to his version of the problem and why this is the proper response in some cases, but not others.

4.2.2. Compartmentalized Coherentism. Roche's proposal follows in the spirit of Lycan (1996, 2012) and Olsson (1997) in denying that global coherence is required for one's set of beliefs. The way Lycan formulates his response to this sort of problem is as follows:

"What I have suggested is compartmentalization (Lycan 1996): a belief is justified accordingly as it coheres in a sufficiently large functional subset of the subject's global belief system." (2012, p. 14)

Roche effectively takes up this suggestion, proposing that the coherentists acceptance of (A) be replaced by

(A') S 's belief in p is justified only if S 's p -subset is coherent.⁴

Here S 's p -subset is the cognitive compartment to which S 's belief that p belongs. This, of course, merely provides a schematic answer to the problem, but for our present purposes that is all we really

falsehoods are equivalent to logical contradictions. It is also simplifies things in the present context to consider an example where the problematic proposition clearly isn't justified, so that we don't have to address the issue of whether one can come to have a justified belief in a necessary falsehood, and what the propositional content of that belief would be, and how we would represent an agent's degree of belief in that proposition.

⁴See Roche (2013, p. 87) for discussion of how this principle fits into the more general coherentist theory of epistemic justification.

need. There is a clear and intuitive sense in which propositions can be more or less functionally relevant to each other along a variety of dimensions. In Chapter 5, we consider ways in which propositions may be of pragmatic (ir)relevance to one another. And there is a clear and intuitive sense in which propositions may be evidentially (ir)relevant to each other, which we have already discussed in Chapter 2.⁵ If propositions are of no relevance to each other in the sense that each's truth makes no difference to how likely it is that the others are true, then there is an intuitive sense in which those propositions may not be part of the same functional compartment. We will leave the question of exactly how to formulate the criteria of functional relevance for future research, and simply note that p_n seems to stand apart from the rest of the beliefs in β_n along both evidential and pragmatic dimensions. The key thing to note is that (A') avoids the problem posed by The Tired Logic Student, since p_n is cognitively isolated from the biological beliefs in β_n . Hence, even if the compartment to which p_n belongs is incoherent, it is still possible that β_n remains highly coherent.

One thing we need to consider though is whether the compartmentalization reply to the problem of inconsistent belief is something forced upon those who adopt an analysis of coherence that precludes S 's beliefs from being coherent. As we shall see below, there are some measures that will permit S 's beliefs in this case to be coherent. And, one might then be left with the impression that one is only forced to reject global coherentism if one's analysis of coherence will not permit the tired logic student's beliefs from being coherent. I want to forestall such confusion by noting that there are powerful independent arguments for rejecting global coherentism, so that the abandonment of (A) is necessary, regardless of whether one thinks a set like $\beta_n \cup \{p_n\}$ can be coherent or not. We now turn to Klein and Warfield's argument against global coherentism that shows that a naïve version of global coherentism like that expressed by (A) has wildly implausible consequences.

Klein and Warfield's Problem for Global Coherentism. Klein and Warfield observe a much more general problem for global coherentism, which they understand as the endorsement of a biconditional version of (A). Suppose one accepts:

(A^b) S 's belief in p is justified if and only if S 's belief system is coherent.

⁵In Chapter 2, we noted that propositions can be relevant to each other in the sense that they make each other more or less likely to be true. If the reasonableness of believing p is unaffected by the assumption that q , then q is not epistemically relevant to p .

Klein and Warfield note that “The proposed account implies that either every one of my beliefs is justified or none of them is justified” (1996, p. 121). The argument for this is simple. Suppose one of my beliefs is justified. Then it follows that my belief system is coherent. Hence, by (A^b) , each of my beliefs are justified. The same thought can be run for the assumption that that one of my beliefs is unjustified. The counter-intuitiveness of such an implication is plain. One of the central points of having a notion of epistemic justification is to sort those beliefs that are reasonable and ought to be retained from those that an agent ought not hold.⁶ As Klein and Warfield note, (A^b) collapses the distinction rendering epistemic justification theoretically useless.

Of course, (A^b) , being a biconditional, is a stronger thesis than (A) , but the problem still generalizes. If an agent’s belief system is globally incoherent, then (A) renders all of one’s beliefs unjustified. We, of course, saw at the end of Chapter 1, that there is a clear sense in which a notion of global coherence can be required on a set of beliefs. In particular, if a set of beliefs violates Easwaran and Fitelson’s accuracy-dominance principles, then it is impossible for one’s beliefs to *all* satisfy the basic evidential constraint that one only believe propositions which are supported by one’s total evidence. But, even when one’s beliefs violate accuracy-dominance principles, which the beliefs of our Tired Logic Student do, it ought not follow that *all* of one’s beliefs are unjustified. It should at most follow that there exists some beliefs that are unjustified. So, even if coherence requires accuracy-dominance avoidance principles like those articulated by Easwaran and Fitelson, it ought not to follow from the incoherence of one’s belief system that one has no justified beliefs.

The natural and necessary response to this problem is to adopt the subsystem response that we discussed above. Beliefs that are justified are members of a coherent subsystem, while unjustified beliefs are not. Klein and Warfield (1996, p. 121) worry that there is no definitive way to explain the compartmentalization. While I won’t here attempt to develop a theory of cognitive compartmentalization, I’ve already logged my agreement with Lycan (2012) that Roche’s examples demonstrate the plausibility that subsystems might be isolated in a manner that isolates their

⁶Priest (2001, p. 217) makes a similar point when considering theories of belief revision intended to apply to ordinary agents who have inconsistent beliefs. Priest points out that even if our theory of rational belief holds that inconsistent sets of beliefs are incoherent, we need our theory of belief revision to explain how an agent with inconsistent beliefs ought to proceed in revising her beliefs. Something we should expect from our theory of rational or justified belief is that it help us understand which beliefs ought to be retained, and which need to be abandoned when we discover that our beliefs are globally incoherent. The problem with (A^b) is that it seems to in no way help explain what we ought to do when our beliefs are incoherent.

epistemic statuses, even if one cannot formalized the idea in any obvious way.⁷ The Tired Logic Student seems to give just the sort of case where one's beliefs about biology are evidentially or explanatorily isolated in a manner that screens off any incoherence from infecting or undermining the epistemic status of those beliefs that belong to β_n . By adopting a compartmentalization version of coherentism, one can allow that even if one's beliefs are globally incoherent (perhaps in the sense of violating Easwaran and Fitelson's accuracy-coherence constraints), it doesn't follow that *all* of one's beliefs are unjustified. But it does follow that at least *some* of one's beliefs are unjustified. This is a distinction that we shall need to pay careful attention to as we consider responses to inconsistency problems for coherentism.

There are powerful considerations that push coherentism away from (A), and toward a compartmentalization view. This demonstrates that there is no theoretical advantage to adopting a formalization of coherence that allows the tired logic's students beliefs to be coherent, at least not in the sense that such an analysis is compatible with a plausible form of global coherentism. But now, one might think that these observations push us back in the other direction. If we have reason to reject (A) and rejecting (A) resolves the problem of inconsistent beliefs, then perhaps there is reason to think that rejecting (A) provides a uniform solution to all the inconsistency problems faced by the coherentist, so that there is no need for the coherentist to locate principled grounds for rejecting (B). As plausible as this may seem, I think this line of thinking also represents a mistake. It assumes that all justified beliefs belong to logically consistent compartments. But, this requirement is also too strong. To illustrate, let us consider an agent whose belief system contains an inconsistent compartment.

4.2.3. Inconsistent Compartments. Compartmentalization does not provide a panacea for all of the coherentist's problems with inconsistency. The key examples of justified inconsistent beliefs are cases where the inconsistent set of claims seem to be evidentially connected in ways that

⁷I think the most natural thing to do is to compartmentalize in terms of evidential and/or pragmatic relevance. In the binary case, a proposition is isolated from another, q , just in case the reasonableness of believing p is independent of the reasonableness of believing q . So, if we conditionalize on one belief, this has no effect on the epistemic or pragmatic status of the other, then these propositions are intuitively independent of each other. How we generalize this to sets of propositions will, of course, be fairly complicated. We know that it is possible that p_1 might be evidentially relevant to p_2 , and p_2 might be evidentially relevant to p_3 , even though p_1 is not relevant to p_3 . Fitelson (2012) offers examples of such cases. Should p_1 and p_3 be considered members of the same cognitive compartment in these sorts of cases? We won't be able to arrive at any definitive answer, but one plausible application of the coherence measures we consider is that they are meant to capture how relevant propositions are to one another collectively. This seems to me something worthy of future inquiry, but not something we need to settle now, since the examples we shall focus on will all have fairly obvious answers.

seem to indicate that they cannot be separated into distinct cognitive compartments. In the case of the lottery paradox, all lottery propositions are of relevance to each other, and there seems to be no natural way to divide lottery propositions up into distinct compartments.⁸ Of course, even some proponents of justified inconsistent belief, are ready to deny that we can be justified in believing lottery propositions. So, maybe the lottery paradox is not the best example. The preface has struck some as a more plausible case of justified inconsistent belief, but it must be noted that the standard formulation of the preface seems more amenable to a compartmentalization solution. That is to say, some have thought that the preface belief, a second-order belief about one's book or doxastic state containing errors, belongs to a distinct compartment from the rest of one's first-order beliefs. While this sort of response may be plausible for certain versions of the preface paradox, Easwaran and Fitelson (in press) have presented a version of the paradox that resists a compartmentalization solution. Here is their explanation of the preface paradox:

Homogeneous Preface Paradox. John is an excellent empirical scientist. He has devoted his entire (long and esteemed) scientific career to gathering and assessing the evidence that is relevant to the following first-order, empirical hypothesis: (H) all scientific/empirical books of sufficient complexity contain at least one false claim. By the end of his career, John is ready to publish his masterpiece, which is an exhaustive, encyclopedic, 15-volume (scientific/empirical) book which aims to summarize (all) the evidence that contemporary empirical science takes to be relevant to H. John sits down to write the preface to his masterpiece. Rather than reflecting on his own fallibility, John simply reflects on the contents of (the main text of) his book, which constitutes very strong inductive evidence in favor of H. On this basis, John (inductively) infers H. But, John also believes each of the individual claims asserted in the main text of the book. Thus, because John believes (indeed, knows) that his masterpiece instantiates the antecedent of H, the (total) set of John's (rational/justified) beliefs is inconsistent. (In Press, p. 8)

⁸See Kvanvig (2013, pp. 29-30) for discussion of problems faced by trying to divide the relevant propositions up into different compartments. It is worth noting that Lycan (2012, p. 14) rejects a compartmentalization solution as adequate for the lottery and the preface paradoxes.

In this case, unlike other versions of the preface paradox, the preface proposition is evidentially connected to the body of beliefs in a way that makes it highly implausible to think that the preface proposition belongs to a distinct compartment.⁹ Thus, this version of the preface resists a compartmentalization solution.¹⁰ At least in some contexts then, the coherentist either must deny (B), or else deny that the scientist in the example is making a reasonable inductive inference and thereby arriving at a justified belief. Of the two options, the former seems far more palatable than the latter, and there is thus reason for the coherentist to want to deny that coherence requires consistency.

It is worth flagging that even though coherentists may have reason to deny a logical consistency requirement on coherence, it does not follow from this that the coherentist must or should avoid all logical requirements on the substantive coherence relation. For instance, we might hold that a set of propositions cannot all be coherent (in the substantive sense) if they violate Easwaran and Fitelson's accuracy dominance principles, despite the fact that they can sometimes be coherent and be minimally inconsistent. In what follows, we shall consider what formal constraints on coherence are delivered by the prominent probabilistic analyses of coherence. To start these considerations, we must review the main probabilistic analyses of coherence and the core intuitions on which they are based.

4.3. Probabilistic Measures of Coherence

In recent literature, formal epistemologists have suggested that the intuitive claims made by coherentist epistemologists can be made precise by defining coherence in terms of probabilistic relations that obtain between sets of beliefs or between sets of propositions that are believed. The measures are defined over sets of believed propositions. Since a common claim made by coherentists is that what justifies a set of beliefs is not that they are entailed by or can be inferred from some set of basic beliefs, but that they 'hang together' or stand in certain mutual support relations, a natural idea is

⁹Lycan (2012, p. 14) agrees that compartmentalization cannot be called in to rescue coherentism, and seems to take this position for even standard versions of the preface paradox. I should make it clear that I am not suggesting that compartmentalization should be an accepted solution for standard versions of the paradox, only that it seems like an even less plausible solution to the version just presented.

¹⁰It is worth noting that the example also provides motivation for accepting that some inconsistent sets could be coherent. The set of propositions that Easwaran and Fitelson have us focus on is an example where the propositions provide a great deal of mutual support to one another even if the set is ultimately inconsistent. I thus think the example puts pressure on any completely general account of the nature of the coherence relation to be able to accommodate the fact that some inconsistent sets can be coherent to a high degree.

that the degree to which a set of propositions cohere can be unpacked and made formally precise by characterizing coherence in terms of probabilistic relations that obtain between members of a set. Once claims about the probabilistic relationships that contribute to the coherence or incoherence of a set of propositions are formulated in a precise way, it then becomes possible to investigate what is formally required for a set of propositions to be coherent, i.e., whether coherence requires consistency and/or accuracy-dominance avoidance, etc... .

One thing we should make clear is that Easwaran and Fitelson's dominance avoidance principles are defined over sets of judgments (beliefs and disbeliefs), whereas the coherence measures we shall examine are typically defined over sets of propositions. But we can bridge this gap by focusing on those sets of propositions that would yield accuracy-dominated judgment sets were some individual to believe the propositions. In other words, we will call a set of propositions *weakly (or strongly) dominated* if an agent who believes each of the propositions in the set would have a weakly (or strongly) accuracy-dominated belief set in Easwaran and Fitelson's senses introduced in Chapter 1. Now, the exact nature of the set of beliefs or propositions over which coherence measures are to be defined is, of course, a fairly complicated and controversial matter. For instance, in response to an objection put forward by Akiba (2000), Shogenji (2001, p. 150) suggests that beliefs should be individuated by their sources, not their propositional content. So, according to Shogenji, a set of beliefs may contain multiple distinct token beliefs that share the same propositional content. And, hence, he suggests that coherence measures be defined over tokens of propositions that may contain logically equivalent propositions. Douven and Meijs opt for a very different solution to the sort of considerations raised by Akiba (2000). Douven and Meijs adopt a principle that would restrict the measure to be defined only over sets that "are pairwise logically independent propositions" (2007, p. 418). With the exception of one important issue that we shall note below, it makes very little difference which choice we make with respect to the sorts of consistency and dominance avoidance requirements that a measure will impose. Adopting Shogenji's account will simplify our formal presentation of the proofs throughout, so we shall suppose that coherence is defined over tokens of propositions, and we shall relax the logical independence requirement assumed by Douven and Meijs.¹¹

¹¹The main significance of Douven and Meijs independence restriction is that any measure that respects their condition cannot be defined over sets of propositions containing a logical contradiction, at least not if the underlying logic is classical. One of the interesting questions that we shall investigate is how contradictions impact the coherence of a set of propositions, so it seems potentially illuminating to consider measures that avoid their logical independence assumption.

The structure of the discussion from here is as follows: we shall consider ways in which categorical and gradational coherence are related. Then we will consider probabilistic measures of coherence, and explain the informal intuitions they are intended to capture. We finish the discussion by investigating what formal requirements on coherence are delivered by the various probabilistic analyses of the coherence relation.

4.3.1. Gradational and Categorical Coherence. The general scheme for the formal measures we shall consider is that for a set of propositions S , S coheres to some degree r depending on the probabilistic relations that hold between the members of S . The coherence measures we shall consider map a set of propositions and a probability function on those propositions to some real value. It is convention to represent coherence measures as follows:

$$C(S) = r \text{ (where } S \text{ is a set of propositions and } r \text{ is some real number).}$$

It is important to keep in mind that each such measure is defined relative to a particular probability function on the set of propositions (This will become clear in our definition of particular measures below).

Now, some of the claims we are investigating are about a categorical notion of coherence and its logical properties. The natural question is how do we formulate such questions in a framework involving a gradational notion? And how then do we formalize our target questions about the relationship between coherence and consistency. Sven Hansson suggests the natural answer to the first question:

Given a gradational notion, it is a trivial matter to introduce a categorical notion by just inserting a limit on the scale of degrees of coherence, below which a set is counted as incoherent and above which it is counted as coherent. (2006 p, 94.)

Defining Categorical Coherence in this manner suggest a natural way to make questions about the relationship between coherence and consistency formally precise.

C1: If S is an accuracy-dominated set of beliefs or believed propositions, then $C(S) < t$ (where t is the threshold for coherence simpliciter).

C2: If S is an inconsistent set, then $C(S) < t$ (where t is the threshold for coherence simpliciter).

The question to ask, just as with the reduction of categorical belief to gradiated belief, is what determines the threshold we choose?

All the measures reflect the fact that there are certain probabilistic features that positively contribute to the coherence of a set (i.e., make the set more coherent than it would be otherwise), and other features that negatively contribute to the incoherence of a set (i.e., make the set less coherent). For most measures, there is a value that indicates that overall, the positive coherence making features outweigh the negative coherence making features.¹² It has been commonly assumed that the cutoff for coherence simpliciter is this neutral value (This will become clear in the presentation of the measures below). Those sets that are coherent above the neutral value are coherent, those below incoherent. We shall consider this as a first gloss on coherence simpliciter for all coherence measures that we shall consider. However, we shall consider what positions are available if we allow for another value to represent the threshold for categorical coherence.

4.3.2. Agreement and Support Intuitions. To give a probabilistic analyses of our intuitive notion of coherence, we must start with some core intuitions about what it takes for a set of beliefs to fit together into a coherent package. Generally, there are two sorts of core intuitions with which one might start. On the one hand, we might hold, as do Bovens and Hartmann (2004); Olsson (2002); Glass (2002), that coherence is a matter of the relative agreement that propositions stand in to one another. On these measures, coherence depends on the conditional probabilistic relationships that the propositions stand in to one another. On the other hand, one might hold, as do Shogenji (1999); Fitelson (2003); Douven and Meijs (2007); and Schupbach (2012),¹³ that coherence is a matter of propositions standing in mutual support relations of some kind. These measures depend on both prior and conditional probabilities, and in particular, whether the conditional probabilities are higher than the prior probabilities, and thus depend on whether the propositions are probabilistically relevant to one another.

¹²Roche (2013) makes this assumption, and we shall use this as a starting point for our investigations.

¹³Roche's (2013) coherence measure doesn't fit neatly into either category. The mutual support measures are traditionally understood in terms of propositions being probabilistically dependent on one another, and yet Roche's measure allows probabilistically independent propositions to have a high degree of coherence. When I explain Roche's measure below, it will become clear which intuitions it is meant to capture, and why it occupies a kind of middle position between the two sorts of core intuitions discussed here.

These differences between these core intuitions can be illuminated by a case presented by Bovens and Olsson (2000, pp. 688-689, footnote 1.) and discussed by Glass (2005, pp. 378-379). Consider the following testimony about a roulette wheel.

Scenario 1

Joe: The wheel will come up 49 or 50.

Amy: The wheel will come up 50 or 51.

Now consider a similar case.

Scenario 2

Joe: The wheel will come up within 1 through 70.

Amy: The wheel will come up within 31 through 100.

In which scenario is Amy and Joe's testimony more coherent? The answer one is inclined to offer depends on which of the two core intuitions one is inclined to accept. The fact is that Joe and Amy's testimony increase each other's likelihood of being true in Scenario 1, but decrease each other's likelihood of being true in Scenario 2. Thus, if one accepts that coherence is a matter of mutual probabilistic relevance or dependence, and claims support each other by making each other likely to be true, then the contents of the testimony in Scenario 1 cohere to a higher degree than that of scenario 2.¹⁴ However, if one takes coherence to be a matter of agreement, there is a sense in which the agreement or overlap of Joe and Amy's testimony is greater in the second scenario in that the relative overlap of their claims is greater. Joe and Amy agree on forty percent of cases in the second scenario, and just thirty-three percent of cases in the first scenario. Thus, there is a sense in which their testimony agrees in Scenario 2 more than it does in Scenario 1. To some degree, the question of whether coherence is compatible with inconsistency will depend on which of the core intuitions one is inclined to accept, but the significance of the choice will be easiest to

¹⁴Roche's (2013) measure to be introduced below is an exception here. By treating mutual support as absolute, as opposed to incremental, confirmation, Roche's measure would actually side with the agreement measures on this case. But it seems to me that treating Roche's measure as an agreement measure doesn't necessarily misrepresent Roche's proposal. One might hold that there is an asymmetric notion of agreement. If, for example, we are considering a witness lineup case, and I am convinced that it was either suspect 9 or 10, and someone else says it was suspect 9, then the conditional probabilities of my claim on theirs is 1. And this reflects that they completely agree with my assessment. I, however, do not fully agree with theirs, or at least am not yet ready to fully agree. And the state of play of the conversation is reflected by Roche's analysis of the coherence of our judgments.

see once we have looked at some particular proposals. It is to particular formal coherence measures that we now turn.

4.3.3. Olsson and Glass’s Coherence Measure. We start with Olsson (2002) and Glass’s (2002) coherence measure as it stands apart from the others in a few notable ways. The place to begin to understand any of the measures is to first consider how to make sense of the core intuition when applied to a pair of propositions, and then consider how the analysis of coherence generalizes to sets of any finite cardinality. According to Olsson and Glass’s measure, a pair of propositions $\{H, H'\}$ cohere to the degree to which their probability masses overlap:

$$C_O\{H, H'\} = \frac{Pr(H \wedge H')}{Pr(H \vee H')}.$$

The simple idea here, in Olsson’s words, is that $C_O(S)$ “... measures how much of the total probability mass assigned to the [disjunction of the elements of S] falls into their intersection” (Olsson 2002, p. 249). So, the more the propositions overlap each other in the probability space, the more they agree, and so the more coherent they are.

One of the essential questions is how exactly does one generalize the analysis beyond pairwise coherence? The answer that both Glass and Olsson settle on is the following

$$C_O(S) = \frac{P(\bigwedge_{H \in S} H)}{P(\bigvee_{H \in S} H)}.$$

The method of generalization is another notable feature of this measure in that it only considers probabilistic features of the entire set, taken as a whole, as opposed to considering probabilistic relations that hold within or between proper subsets of a set. The only other measure that has this in common with Olsson’s measure is the one to which we now turn.

4.3.4. Shogenji’s Measure. Shogenji (1999) proposes an analysis of coherence that is strongly rooted in the idea that coherence is a matter of propositions ‘hanging together’ in the sense that they mutually support one another, or are probabilistically dependent on one another. Shogenji explains the core intuition motivating his proposal as follows: “the more coherent two beliefs are, the stronger is the positive impact of the truth of one on the truth of the other.” (p. 339) The

measure should thus increase the coherence of a pair of propositions as the more they positively impact the other's chance of being true. He notes that, given standard definitions of probabilistic independence, one proposition has no impact on the truth of another when $Pr(p \wedge q) = Pr(p) \cdot Pr(q)$. He thus suggests that pair-wise coherence can be defined in terms of the following ratio:

$$C_s(\{H, H'\}) = \frac{Pr(H \wedge H')}{Pr(H) \cdot Pr(H')}.$$

When C_S assigns a pair of propositions a value greater than 1, it follows that each member of the pair has a positive impact on the likelihood that the other is true. When C_S falls between 0 and 1, H and H' have a negative impact of the truth of one proposition on the truth of the other. Shogenji, thus, not implausibly suggests that H and H' cohere if and only if $C_s(\{H, H'\}) > 1$.

The obvious question to ask is how do we generalize the idea that a set of propositions ‘hang together’ in this sense. Shogenji’s proposed generalization is similar to Olsson’s in that it doesn’t take into account the relations of support that hold between subsets of a set of propositions, and instead takes a holistic approach. Formally, the proposal is that coherence of a set of propositions $\{A_1, A_2, \dots, A_n\}$ is defined as follows:

$$C_S(\{A_1, A_2, \dots, A_n\}) = \frac{P(A_1 \wedge A_2 \dots \wedge A_n)}{P(A_1) \bullet P(A_2) \bullet \dots \bullet P(A_n)}.$$

Jonah Schupbach (2011) offers a nice summarization of the generalization: “It thereby provides a measure of the degree to which the members of this information set are statistically dependent on, or relevant to, one another.” (p. 126) And again, a value greater than 1 indicates some degree of positive relevance, less than 1 indicates some degree of negative relevance, and a value of exactly 1 means the propositions are independent of one another. Hence, 1 is the natural neutral value for the measure.

4.3.5. Shogenji and Olsson’s Measures - and Logical Consistency. Before moving forward, it is worth pausing to reflect on an important theoretical upshot of both measures. Both

Shogenji and Olsson's measures entail, not just a logical consistency requirement on categorical coherence, but also satisfy some of the strictest possible consistency requirements that can be defined. Hannson (2006, p. 100) suggests one strict way of defining a consistency requirement as follows:

Gradational Consistency Requirement (GCR): If A is consistent and B inconsistent, then $[C(A) > C(B)]$.¹⁵

Hansson's gradational consistency requirement says that consistent sets are more coherent than any inconsistent set (for those inconsistent sets over which the measures are defined).¹⁶ Both measures satisfy this requirement because both assign all inconsistent sets the maximal degree of incoherence, 0, and all consistent sets of propositions are assigned some value greater than 0. In other words, both measures satisfy (GCR) and the following:

Maximal Consistency Requirement (MCR): If A inconsistent, then $C(A) = r$ where r is a value representing maximal incoherence.

It might thus seem as though we have a straightforward argument that there is a plausible sense in which coherence is intimately tied to a consistency requirement in just the way that mainstream epistemologists have always thought. Inconsistency is the height of incoherence.

As plausible sounding as this thought may seem, there are several highly questionable features of both measures, and these problems are directly tied to the reason that they deliver such strong consistency requirements. These objections relate to the manner in which the measures were generalized to sets of cardinality greater than 2.

4.3.5.1. *Problems with Generalizing.* Due to the similarity of the manner in which Olsson and Shogenji generalize from an analysis of pairwise coherence to sets of any finite cardinality, both are subject to objections pertaining to this method of generalization. In particular, both measures face problems that relate to the fact that these measures do not count relations of overlap or support within proper subsets as features relevant to the coherence of a set of propositions.

¹⁵I have revised the formal representation of the measure of coherence to fit with the rest of the coherence measures we shall consider.

¹⁶Below we shall discuss Roche's observation that there are certain inconsistent sets over which these measures are not defined. It thus isn't quite accurate to say that *all* inconsistent sets are less coherent than any logically consistent set. Some are so incoherent according to these measures their coherence can't be ranked at all.

Fitelson expresses this central complaint about Shogenji's measure as follows:

Since Shogenji's measure is based only on n -wise independence (dependence), in cases where a set is n -wise independent (dependent), but not j -wise (for some $j < n$) independent (dependent), Shogenji's measure does not take into account the 'mixed' nature of the coherence (incoherence) of E (and its subsets), and it judges E as having the same degree of coherence (incoherence) as a fully independent (or fully dependent) set. (2003, p. 197)

Put more formally, a set of propositions $\{p_1, p_2, \dots, p_n\}$ might be such that $\frac{Pr(p_1 \wedge \dots \wedge p_n)}{Pr(p_1) \cdot \dots \cdot Pr(p_n)} = 1$ (and so n -wise independent) despite the fact that $\frac{Pr(p_1 \wedge \dots \wedge p_{n-1})}{Pr(p_1) \cdot \dots \cdot Pr(p_{n-1})} > 1$ or $\frac{Pr(p_1 \wedge \dots \wedge p_{n-1})}{Pr(p_1) \cdot \dots \cdot Pr(p_{n-1})} < 1$. It is plausible to think that if $\frac{Pr(p_1 \wedge \dots \wedge p_{n-1})}{Pr(p_1) \cdot \dots \cdot Pr(p_{n-1})} > 1$, then this should contribute to the coherence of the set to some degree, and yet Shogenji's measure does not allow for such differences to impact the degree to which the set coheres. In short, it only looks at the set as a whole, and does not factor into the account relations of dependence amongst proper subsets of the set. Schupbach (2011, p. 128) calls this *The Depth Problem* for Shogenji's measure.

An intimately related problem, or at least a problem with a common formal diagnosis, Schupbach refers to as *the problem of irrelevant addition*. Schupbach explains "It seems quite clear that whenever one adds an irrelevant proposition to an otherwise coherent information set, the new set (resulting from the addition) must be less coherent than the first." (p. 130). This intuition seems especially plausible if we are understanding coherence as being a matter of mutual probabilistic relevance. A commonly recognized problem with Shogenji's measure is that it does not respect this intuition. If one adds an irrelevant proposition to a set of propositions, then the degree of coherence assigned to a set remains exactly the same. According to Schupbach, "In both of the major problems above [*The Depth Problem* and *The Problem of Irrelevant Addition*], the counterintuitive results given by Shogenji's generalized coherence measure may be seen as a consequence of the fact that it does not dig deeply enough into the probabilistic information about the relevant information sets." (p. 131). He then proposes the solution: "A more intuitive generalization of Shogenji's measure might then take account of the degrees of positive relevance of a set and of all of its subsets (with cardinality $n \geq 2$)" (p. 131).

How does Schubach propose that we do this? Informally, he puts the proposal as follows:

According to this suggestion, one should simply apply Shogenji's measure to all of the subsets (of cardinality $n \geq 2$) of an information set and ultimately calculate the degree of coherence as some weighted average of these results. (p. 131)

Formally, Schubach explains the proposal as follows:

More formally, for any information set $S = \{R_1, R_2, \dots, R_n\}$, let $s(S)$ be the logarithm of Shogenji's generalized coherence measure:

$$s(S) =_{\text{def}} \log \left[\frac{Pr(R_1 \wedge R_2 \wedge \dots \wedge R_n)}{Pr(R_1) \times Pr(R_2) \times \dots \times Pr(R_n)} \right]$$

Then, for any natural number k that satisfies $1 < k \leq n$, we can define the k -wise coherence of an n -membered information set S as the mean values attained by applying the function s to all of the k -membered subsets of S . Let $[S]^k$ denote the set of all subsets of S with cardinality k and recall that there are $m = \binom{n}{k} = \frac{n!}{k!(n-k)!}$ k -membered sets of any n -membered set. (p. 132)

Schubach goes on to define k -wise coherence (really, k -wise Shogenji coherence) as follows:¹⁷

$$C_s^k(S) =_{\text{def}} \frac{\sum_{S' \in [S]^k} s(S')}{|[S]^k|}.$$

He then goes on to define general coherence as follows:

Once we have the degree of k -wise coherence calculated for all k , then the remaining question is how to weight these against each other in order to calculate an overall degree of coherence for S . In principle, any weighting scheme is open to testing. Leaving the specific weighting scheme unspecified, we have: (p. 132)

¹⁷I have simplified Schubach's (2011, p. 132) notation slightly to make it consistent with similar measures defined later on.

DEFINITION 4.1. C : Given a set $S = \{R_1, \dots, R_n\}$ and a weighting system that assigns the vector of positive weights $\langle w_1, w_2, \dots, w_{n-1} \rangle$ to the k -wise degrees of coherence (where $\sum_{i=1}^{n-1} w_i = 1$), the degree of coherence of $S = \{R_1, \dots, R_n\}$ is given by the function¹⁸

$$C(S) =_{def} \sum_{i=1}^{n-1} w_i \times C_s^{i+1}(S).$$

Infinitely many possible weighting schemes emerge. Generally, there is a question of whether greater depth should be given more or less weight. The simplest scheme is one that treats all depths as equal.

$$C_{S^n}(S) =_{def} \frac{\sum_{i=2}^n C_s^i(S)}{(n-1)}$$

For now, we focus on this schema as it is the simplest to work with from a combinatorial perspective, but our investigations will push us to consider alternative weighting schemes eventually.

Next, we turn to the depth problems with Olsson and Glass's measure pertaining to their method of generalization to sets of any finite cardinality. Of the various problems for Olsson's measure related to the method of generalization,¹⁹ the most compelling is the fact that it cannot possibly satisfy a seemingly non-negotiable assumption we all make about the dynamics of increasing the coherence of one's set of beliefs. Klein and Warfield (1994) spell out the basic assumption we all make about methods for increasing coherence as follows:

Suffice it to say that whatever coherence is [and it is not mere logical consistency],

a set of beliefs can be rendered more coherent in two basic ways:

- (a) the *Subtraction Strategy* in which a belief (and perhaps with it many more) is subtracted from a less coherent set, thereby rendering it more coherent;

¹⁸Our presentation of these definitions follow Schupbach's (2011, pp. 132-133) exactly.

¹⁹See Meijs (2006), Boven and Hartmann (2004), Douven and Meijs (2007), Roche (2013) for discussion of problems with Olsson's measure.

(b) the *Addition Strategy* in which one or more beliefs are added to a set **consistent** set of beliefs to render it more coherent. (my bolding) (1994, p. 130)²⁰

The problem is that Olsson's measure forces the rejection of (b). That is to say, on Olsson's measure, one cannot increase the coherence of a set by adding beliefs that reconcile whatever tensions may be responsible for rendering one's beliefs incoherent.

The counter-intuitiveness of failing to satisfy (b) is demonstrated by Boven and Harmann's (2003, pp. 44-45, 50) counter-example to Olsson's measure:²¹

A: My Pet Tweety is a Bird.

B: My Pet Tweety cannot fly.

C: My Pet Tweety is a Penguin.

The intuitive thought is that $\{A, B\}$ do not form a very coherent set. On the other hand, $\{A, B, C\}$ has struck many as a more coherent set. For instance, Boven and Hartmann (2003, p. 50), Meijs (2006, pp. 244-245), and Douven and Meijs (2007, pp. 416-417) have all expressed this judgment. Since Olsson's measure does not allow for a set to become more coherent by the introduction of new information, it cannot capture our comparative nor categorical judgments regarding the coherence of $\{A, B\}$ and $\{A, B, C\}$.

4.3.6. Meijs' Proposal. Meijs (2006) offers the following correction to the measure as a way to accomodate the comparative judgment. Unsurprisingly, Meijs' proposes that we generalize in a way that makes the coherence measure depth sensitive. First, we define the set of subsets of S of cardinality 2 or greater.

DEFINITION. $[S]_1 =_{def} \{S' \mid S' \subset S \ \& \ |S'| \geq 2\}$.

$$C_M(S) =_{def} \text{Straight Average of } \{C_O(S') \mid S' \in [S]_1\}.$$

²⁰I bold the consistency requirement to note that the objection does not presuppose that inconsistent sets can be coherent, or that inconsistent sets can be made more coherent. In other words, the problem does not presuppose any judgment regarding the (in)coherence of inconsistent sets.

²¹This example and variants on it are also discussed in Meijs (2006), and Douven and Meijs (2007).

Meijs' diagnosis of the problems with Olsson's measure is that they stem from the fact that it does not take into consideration the relations of agreement or approximate logical equivalence that obtain between proper subsets of a set. By taking the average of the degrees of overlap of subsets, Meijs's measure takes these properties of the subsets into account, and thereby yields the intuitive result in the Tweety example above, and validates the addition strategy as a legitimate way that one might come to make her belief set more coherent.

4.3.6.1. *Applying Schupbach's Method to Olsson's Measure.* It should be noted that Meijs' method of generalizing Olsson's measure is importantly different from Schupbach's method for generalizing Shogenji's. While Meijs takes the average of Olsson's measure for all subsets of a set, Schupbach first calculates the degree of relevance assigned by Shogenji's measure to all subsets of a certain size, and then averages those values. Up to this point, there has been little discussion of which method for generalization is more plausible. But from a purely formal standpoint, this choice is non-trivial, and may have a significant impact on whether a measure imposes various formal requirements on categorical coherence.

For now, I note that the following provides an alternative method for generalizing Olsson's measure to sets of any finite cardinality. We define k -wise Olsson coherence of S as the average degree of coherence assigned by Olsson's measure to all subsets of S of cardinality k . Formally, we define this:

$$C_O^k(S) = \text{def} \frac{\sum_{S' \in [S]^k} C_O^k(S')}{|[S]^k|}.$$

And then we use this to arrive at a schema to define a general notion of the coherence of S as the weighted average of the k -wise coherences of S as follows:

$$C_{M^w}(S) = \text{def} \sum_{i=1}^{n-1} w_i \times C_O^{i+1}(S).$$

In other words, we keep everything the same as in Schupbach's generalization of Shogenji's measure except that we plug in Olsson's measure as the measure used to define k -wise coherence. This gives us a measure distinct from Meijs's measure, one that may have very different logical properties. As

will become clear, the choice of weighting scheme will make a difference as to whether the measures entail a consistency requirement for categorical coherence.

4.3.7. Depth-Sensitivity and Inconsistency. At this point, we should take stock of an important fact about depth-sensitivity of coherence measures and inconsistent sets. What C_S and C_O capture respectively is that if S is an inconsistent set of cardinality n , then the members of the set cannot be n -wise coherent in the sense that the propositions are positively relevant when considered as a whole, nor do they exhibit a high degree of agreement when considered as a whole. Given that both C_S and C_O only take into account the n -wise relations of dependence and agreement, respectively, it is to be expected that such sets are assigned the maximal degree of incoherence. And in general, whatever one thinks the coherence making features are, if one's formal account of coherence is not depth-sensitive in the sense of taking into account the positive and negative coherence making features of the proper subsets of a set, then it is to be expected that the measure will assign an inconsistent set the value representing maximal incoherence.

This looks like a route through which one might try to get a consistency requirement on coherence. If n -wise (in)coherence-making features are the only features relevant to the assessment of the coherence of a set, then only consistent sets will exhibit coherence-making features. But this path is blocked by the various problems already noted. There seems to be good reason why we should want our measures to be depth sensitive. And, so I think, C_O or C_S both fail to provide a compelling analysis of the coherence relation, especially when we are concerned with inconsistent sets.

In fact, considering why C_O and C_S entail a consistency requirement highlights precisely why we should not hold that all inconsistent sets are equally coherent or incoherent. Once we acknowledge that the relations of support or agreement amongst subsets are to be factored into our calculation of coherence, then these coherence-making features may be exhibited to greater or lesser degrees by the proper subsets of a set. Moreover, we have an explanation for why some inconsistent sets are more coherent than some consistent sets: there may be stronger coherence-making features amongst the subsets of an inconsistent set.

Now, this isn't to say that because measures of coherence should be depth-sensitive it follows that some inconsistent sets are coherent simpliciter. Whether this is possible will depend on how strong and abundant the positive coherence making features of an inconsistent set's subsets can be, and

how we weigh the coherence making features against incoherence-making features. It could turn out that all plausible weighting schemes, will entail that the incoherence-making features of inconsistent sets swamp whatever positive coherence making features are present. Thus far, no one has given an example showing that some measure allows for inconsistent set to be coherent simpliciter. Thus, we shall need to continue on in our investigation of measures that are depth sensitive. We now turn our attention to coherence measures that are fully depth-sensitive.

4.3.8. Coherence as Mutual Support. A number of coherence measures have been put forward as alternative ways to understand coherence as mutual support in terms of Bayesian confirmation measures. Douven and Meijs (2007) note that these measures can be understood as giving a gradational analyses of coherence that follows in the tradition first suggested by C.I. Lewis for defining congruence (a special sort of coherence) of a set as follows:

A set of statements ... will be said to be congruent [i.e., coherent] if and only if they are so related that the antecedent probability of any one of them will be increased if the remainder of the set can be assumed as given premises. (Lewis 1946, p. 338)²²

Formally, this amounts to the claim that:

A set of statements S is *congruent* if and only if $Pr(p|S \setminus \{p\}) > Pr(p)$ for all $p \in S$.

Douven and Meijs (2007) note that this gives a recipe for analyzing a certain kind of categorical coherence, but fails to deliver a gradational analysis of coherence, and it also focuses on a fraction of support relations that can obtain between members of a set of propositions. Douven and Meijs (2007) point out that “It certainly seems preferable to have a measure of coherence that takes into account all the correlations and anti-correlations that may exist between the different non-empty subsets of a set of propositions, that is, it seems preferable to measure the coherence of a set S of propositions by taking some average of $\delta(S)$ ” (p. 411) where $\delta(S)$ is given the following definition:²³

²²See Douven and Meijs (2007, p. 406-410) for an extended discussion of how this notion of congruence fits into a variety of different ways of defining special kinds of coherence.

²³Note that the notation ‘ $[S]$ ’ is used somewhat ambiguously now. The bearers of the coherence making features vary depending on the coherence measure. Some measures identify the coherence-making properties of a set as a function of probabilistic properties of the subsets of the set, as in Shogenji and Olsson’s measures, and some define the coherence-making properties as a function of probabilistic relations between subset pairs, as in the case of this measure. The notation ‘ $[S]$ ’ will be used to refer to the set that contains the bearers of coherence making features, and we shall allow context to disambiguate.

...let m be some measure of confirmation (see below) and let $[S]$ indicate the set of ordered pairs of non-empty non-overlapping subsets of $S = \{R_1, \dots, R_n\}$, that is $[S] = \{\langle S', S^* \rangle \mid S', S^* \subset S \setminus \emptyset \wedge S' \cap S^* = \emptyset\}$... define $[\delta(S)]$ as follows:

$$\delta(S) = \{m(S', S^*) \mid \langle S', S^* \rangle \in [S]\}. \text{ (p. 410)}$$

We should note that confirmation measures are typically defined over propositions, not sets. Letting m be a confirmation measure, we generalize to sets by defining

$$m(S', S^*) =_{def} m(\bigwedge S', \bigwedge S^*).$$

Douven and Meijs note that this provides us with a schema for defining measures of coherence that can be filled out in a variety of ways. There are numerous Bayesian confirmation measures that could be plugged in to this schema to arrive at distinct coherence measures, and we could vary the weighting on the elements of $\delta(S)$ in continuum-many ways. Douven and Meijs provide the following general schema for defining coherence in terms of some confirmation measure m :

DEFINITION 4.2. Given a set $S = \{R_1, \dots, R_n\}$ and an ordering $\langle \hat{S}_1, \dots, \hat{S}_{|[S]|} \rangle$ of members of $[S]$, the degree of m – coherence of S is given by the function (Douven and Meijs 2007, p. 414).²⁴

$$C_m(S) = \frac{\sum_{i=1}^{|[S]|} m(\hat{S}_i)}{|[S]|}.$$

Given the fact that there are any number of confirmation measures, we must make some choices to simplify those measures we shall consider. Without loss of generality, we shall focus on the three confirmation measures that have been claimed to serve the coherentists purposes. We shall consider three coherence measures defined in terms of three different confirmation or support measures (and then variants on those measures).

The first confirmation measure to consider is a so-called incremental confirmation measure. Following Fitelson (1999) and Crupi et al. (2007b, p. 230), we shall say that a confirmation measure, m , is *incremental* (a *measure of relevance*) just in case it satisfies the following three basic desiderata:

²⁴Douven and Meijs here opt for an equal weighting scheme for all subset pairs of S , but we could easily revise to a weighting scheme the favors larger or smaller subset pairs.

$m(h, e) > 0$ if and only if $Pr(h|e) > Pr(h)$.

$m(h, e) < 0$ if and only if $Pr(h|e) < Pr(h)$.

$m(h, e) = 0$ if $Pr(h|e) = Pr(h)$.²⁵

There are a variety of incremental confirmation measures, but we shall focus on the distance measure of confirmation as a representative of such proposals:

$$d(h, e) = Pr(h|e) - Pr(h).$$

The resulting coherence measure has been defended by Douven and Meijs as a plausible candidate for serving the coherentist's purposes on the grounds that this measure yields a coherence ordering on sets that fits with widespread intuitions for a variety of cases.²⁶ Letting m be the distance measure we thus get the first of the three coherence measures based on confirmation that have been defended by at least some parties in the on debate of the nature of coherence:

$$C_d(S) = \frac{\sum_{i=1}^{|S|} d(\hat{S}_i)}{|S|}.$$

Again, one could assign various weights to the degree of confirmation determined by any particular pair, and thereby obtain any number of alternative coherence measures, but, for now, we shall focus on equal weighting schemes for simplicities sake.

As with all of these support measures, it is useful to note that it has a natural neutral point, t , i.e., a point such that if the coherence of S exceeds t , then the positive coherence making features of S outweigh the negative coherence making features of S . On the above measure, a pair of propositions, $\langle p, q \rangle$, mutually support each other to some positive degree when $C_d(\{p, q\}) > 0$, and

²⁵See Crupi et al. (2007a, p. 108), Crupi et al. (2007b, pp. 229-30) for definitions of various confirmation measures satisfying these conditions. Fitelson's criteria is relativized to background knowledge K , which I will treat as implicit throughout our discussion. For extended discussions of the distinction between incremental and absolute confirmation, see Huber (2005), Maher (2005), and Hayek and Joyce (2008). For informative discussion of how our choice of a confirmation measure impacts the ordering of coherence, See Douven and Meijs (2007).

²⁶As Douven and Meijs (2007, p. 411, footnote 14) point out, our criteria for evaluating whether a confirmation measure can serve as a plausible definition of confirmation is distinct from the question of whether the resulting coherence measure is plausible. Ultimately, they rest their defense of the coherence measure defined in terms of the distance measure on the plausibility of its coherence-ordering.

are to some degree negatively relevant to each other when less than $C_d(\{p, q\}) < 0$.²⁷ It is natural to hold that 0 is the neutral point. Of course, we leave it as an open question whether coherentists should identify the threshold for coherence simpliciter with the neutral point, but if so, then it is 0.

The second confirmation measure we shall consider is meant to capture the amount of deductive support that an evidential proposition, e , provides for a hypothesis, h . The confirmation measure was put forward by Fitelson (2004):

$$F(h, e) =_{df} \begin{cases} \frac{P(e|h) - P(e|\neg h)}{P(e|h) + P(e|\neg h)} & \text{if } e \not\models h \text{ and } e \not\models \neg h \\ 1 & \text{if } e \models h \text{ and } e \not\models \perp \\ -1 & \text{if } e \models \neg h \end{cases}$$

There are a few notable features of the measure.²⁸ The most obvious is that unlike the distance measure, $F(h, e)$ takes the maximal value regardless of the prior probability of h if e deductively entails h . So, logically equivalent propositions provide the maximal degree of evidential support to one another on Fitelson's measure, whereas on the distance measure h can only be supported by e at most to degree $1 - Pr(h)$. Fitelson's coherence measure

$$C_F(S) = \frac{\sum_{i=1}^{|[S]|} F(\hat{S}_i)}{|[S]|}$$

can be understood as interpreting coherence as a kind of generalized logical equivalence (Fitelson 2003).

²⁷We should note that Fitelson (2003, p. 195, footnote 2) assumes regularity (that extreme values are only assigned to contradictions and tautologies). It is also worth noting that, as Fitelson points out, the measure can be thought of as capturing a non-standard notion of relevance or dependence. The exceptions are in the extreme deductive cases. Logically tautologies are deemed positively relevant to each other to a maximum degree, and all contradictory propositions are deemed negatively relevant to each other to the maximum degree.

²⁸This is Fitelson's correction of an earlier proposal, which was meant to capture relevance considerations. It is worth noting that the top condition is just Kemeny and Oppenheim's (1952) measure of factual support. One of the key problems with Kemeny and Oppenheim's measure is that it isn't well defined in cases where e is a contradiction, or h is a tautology or contradiction. Thus, a coherence measure defined in terms of their measure wouldn't be defined over a variety of different kinds of inconsistent sets, and the same issue applies to Fitelson's (2003) earlier proposal. Fitelson's (2004) measure thus conservatively extends their measure to all pairs of propositions, and has other virtues discussed below. Also, see Fitelson (2001, pp. 42-43) for discussion of the intuitions that Kemeny and Oppenheim's measure is intended to capture.

The final measure that aims to treat coherence as mutual support is that put forward by William Roche (2013). Roche suggests that the coherentist ought to adopt an absolute, as opposed to incremental, measure of confirmation, and suggests the following:

$$a(h, e) =_{df} \begin{cases} Pr(h|e) & \text{if } e \not\models h \text{ and } e \not\models \perp \\ 1 & \text{if } e \models h \text{ and } e \not\models \perp \\ 0 & \text{if } Pr(h) = 0 \text{ or } Pr(e) = 0 \end{cases}$$

Using this confirmation measure to define coherence, we can think of coherence as mutual absolute confirmation, but it also provides an alternative analysis of coherence as generalized logical equivalence.

$$C_a(S) = \frac{\sum_{i=1}^{|[S]|} a(\hat{S}_i)}{|[S]|}.$$

Unlike for d , sets of logically equivalent propositions are deemed maximally coherent, a feature that Roche sees as a critical virtue that it shares in common with Fitelson's proposal. In cases where h and e aren't equivalent, tautological, or self-contradictory, Fitelson's confirmation measure behaves like a measure of incremental confirmation in the sense that if e doesn't entail h , then e only provides support for h if h is positively dependent on e . And, because of this difference, C_F has some counter-intuitive implications for the way in which it handles subcontraries that Roche's proposal avoids.

In particular, Meijs (2006, pp. 238-240) has shown that Fitelson's coherence measure has the following counter-intuitive feature: it entails that two propositions with near total overlap can be incoherent to some degree, while absolute total overlap guarantees maximal coherence. Meij uses an example of subcontrary propositions to illustrate. For any propositions H_1 and H_2 , if $\top \vdash (H_1 \vee H_2)$ and $Pr(H_1 \equiv H_2) < 1$, then it follows that $C_F(\{H_1, H_2\}) < 0$.²⁹ But if $Pr(H_1 \equiv H_2) = 1$, i.e., the

²⁹The key thing here pointed out by Meijs is that subcontraries are always negatively dependent on each other when they aren't tautologies. Since Fitelson's measure behaves like an incremental confirmation measure for all non-equivalent, non-tautological and consistent propositions, it thus follows that two non-tautological subcontraries must provide factual support against each other, and hence be incoherent to some degree.

propositions are materially equivalent,³⁰ then Fitelson's measure is such that $C_F(\{H_1, H_2\}) = 1$. To put the point informally, on Fitelson's proposal, equivalence yields maximal coherence, while near equivalence can qualify as incoherent. Meijs takes this to show that one has to choose between the desiderata motivating Fitelson's choice of a confirmation measure. Either one needs to adopt a measure meant to capture confirmation as generalized logical entailment, and then define coherence as generalized logical equivalence, or else one needs to adopt a measure of incremental confirmation, and understand coherence as mutual incremental confirmation. Roche's measure thus has some intuitive advantages over Fitelson's measure. Nevertheless, as will become clear, when it comes to the treatment of inconsistent sets, Fitelson's coherence measure has intuitive advantages of its own, and if nothing else, considering his measure will teach us some interesting lessons regarding the possible relationship between coherence and consistency.

4.3.9. Undefined Inconsistency. There is one last thing we need to observe about d and some of the other measures we have considered above, something that was emphasized by Roche (2013): Some measures are not defined over all inconsistent sets. For instance, Roche points out that “If some of the claims in S have a probability of 0, $[C_d(S)]$ is undefined” (2013, p. 68). This is because $d(h, e)$ is undefined for any e where $e \models \perp$, i.e., whenever the propositions we are conditionalizing on have probability 0.³¹ Thus, there is a fairly strong restriction on the sort of inconsistent sets over which $C_d(S)$ can be defined. First, define a minimally inconsistent set as follows:

DEFINITION 4.3. Define S *minimally inconsistent* if and only if S is inconsistent and any proper subset of S is consistent.

Then the only inconsistent sets that C_d can be defined for are minimally inconsistent. The reason being that if S is inconsistent but not minimally inconsistent, then there is some non-empty subset pair of S , $\langle S', S^* \rangle$, such that S^* is inconsistent, and thus $d(\langle S', S^* \rangle)$ is undefined.

³⁰Fitelson (2003, 2004) assumes regularity, which can be easily dropped.

³¹This is at least true according to Kolmogorovian probability theory, since conditional probability $Pr(h|e)$ is undefined when $Pr(e) = 0$. See Fitelson and Hájek (2014) for discussion of the limitations of the standard approach to defining conditional probabilities and an argument against Kolmogorovian orthodoxy. While breaking from orthodoxy would be an improvement in a number of ways, and would afford us a way to define conditional probabilities for some propositions that have probability zero, we will still have to come up with some way to define the confirmation measures over subset pairs where the proposition we are conditionalizing on is a logical contradiction. I think the proposal I present below is the natural way to do it.

Roche mounts a general attack on all measures that fail to be defined over certain kinds of inconsistent sets. Those that so fail to be defined over at least some particular kinds of inconsistent sets include C_O , C_M , C_d , C_S , C_{S^n} .³² Here is a table of kinds of inconsistent propositions that a coherence function may be defined over:

Defined Over at least some?	C_O	C_M, C_{M^w}	C_d	C_S	C_{S^n}	C_F	C_a
Minimally Inconsistent sets	Yes	Yes	Yes	Yes	No	Yes	Yes
Sets that are not minimally inconsistent	Yes	Yes	No	Yes	No	Yes	Yes
Sets, S , containing a contradiction, i.e., $H \wedge \neg H \in S$	Yes	Yes	No	No	No	Yes	Yes
Sets containing multiple contradictions	Yes	No	No	No	No	Yes	Yes
Sets containing only contradictions	No	No	No	No	No	Yes	Yes

As the table indicates, Schupbach's measure is utterly hopeless when it comes to defining the degree of coherence over inconsistent sets.³³ The reason for this is that Schupbach's measure takes the average of the *logarithm* of Shogenji's measure over inconsistent sets. But since $\log(o)$ is either undefined or negatively infinite, the measure will either define all inconsistent sets as being incoherent to an infinite degree, or else be undefined. Since Schupbach's choice to take the logarithm of Shogenji's measure is essential for normalization reasons, I see no way to extend the definition to cover all inconsistent sets in a way that is remotely plausible. We shall thus set Schupbach's proposal aside as it is not a measure that can plausibly inform our understanding of the relationship between consistency and coherence.³⁴

We can set the worry aside for C_O as we have already seen that it has unacceptable implications for how inconsistent sets are treated (i.e., they are all the same for depth-insensitivity reasons). And, we have extended Roche's observation regarding C_O to its alternative generalizations. It is undefined for sets of propositions containing two self-contradictory propositions. Roche (2013, p. 72) considers the obvious reply that in cases where a measure is not defined over an inconsistent

³²Roche (2013, p. 72) notes that, intuitively, not all inconsistent sets are equally coherent or incoherent. He points out that a set containing just two contradictory propositions is less coherent than a set containing many mutually supportive propositions, and one contradictory proposition. He also and observes that C_O , C_d , C_S are undefined for at least some inconsistent sets.

³³The only option available is insisting on an interpretation where inconsistent sets are maximally incoherent. There is seemingly no motivation for this, and we could consider a lottery case where there is the smallest chance that there will be no winner. In such a case, the degree of coherence of the set would be some real number value, and thus a set with a marginally different probability function will be infinitely more incoherent.

³⁴This is not to say that the measure is to be rejected, or theoretically uninteresting. It is just that it won't serve our purposes here. There may well be applications of formal coherence theory that can be restricted to consistent sets of propositions, and for which the measure is theoretically useful.

set of propositions, one might revise so that the measure assigns the set maximal incoherence. Such a proposal would clearly yield a counter-intuitive ranking on the coherence of sets for C_d and C_M , though it seems natural to me in the case of C_O and C_S , as it would mean treating all inconsistent sets in an analogous fashion. It is just that these latter measures have much worse problems pertaining to the way they handle inconsistent sets.

The easiest way to see the problem in the case of C_d is to note that the coherence of a set of generous lottery propositions is well defined (See Chapter 2, Section 3.7 for an explanation of the generous lottery paradox). In fact, as we shall prove below, such a set of propositions can be positively coherent to a relatively high degree on this measure. However, now consider a set of lottery propositions $\{l_1, \dots, l_n\}$ for a generous lottery that will have at least two winning tickets. The probabilities for such a lottery will be very close to the corresponding generous lottery, and yet C_d will be undefined over $\{l_1, \dots, l_n\}$, since $\{l_1, \dots, l_{n-1}\}$ is still inconsistent. Given that a lottery with at least one winning ticket is hardly that different from a lottery with at least two winning tickets, the gap between the way these two cases are treated is highly implausible.

In the case of C_M , there is the jump from allowing a large set with one explicit contradiction to be coherent to some degree (though perhaps not coherent simpliciter), and two contradictions always being maximally incoherent. Given that both can have some coherence-making features amongst the subsets, this would need some explanation. And, that is the other main problem Roche (2013, p.72) notes with stipulating that undefined inconsistent sets are maximally incoherent: We want the measure to explain why a set is coherent to a certain degree, and thus why the undefined set is maximally incoherent. The solution where we merely stipulate this will not offer any such explanation.

A different response to the failure of the definitions to cover all inconsistent sets suggests itself: we could conservatively extend the measures in the manner that Roche's measure of absolute confirmation is an extension of one's conditional probabilities. In the case of C_M , we do this by first extending C_O as follows:

$$C_{O_a}(S) =_{df} \begin{cases} \frac{P(\bigwedge_{H \in S} H)}{P(\bigvee_{H \in S} H)} & \text{if } P(\bigvee_{H \in S} H) > 0 \\ 0 & \text{otherwise} \end{cases}.$$

C_{O_a} is most naturally understood as a measure of n -wise agreement for sets of cardinality n , and philosophically this would just be to insist that a set containing no content that could possibly true, i.e., only contains contradictions, exhibits no n -wise agreement. While I agree with Roche that just stipulating that all sets for which a measure is undefined receive the maximal incoherence value is ad hoc, and explanatorily insufficient, this revision strikes me as following naturally from what the measure intends to capture.

We can then generalize exactly as before using either Meijs or Schupbach's schema. Meijs's schema gives:

$$C_{M_a}(S) =_{def} \text{Straight Average of } \{C_{O_a}(S') \mid S' \subseteq S \text{ \& } |S'| > 2\}.$$

And Schupbach's gives us:

$$C_{M_a^w}(S) =_{def} \frac{\sum_{i=1}^n C_{O_a}^{i+1}(S)}{|[S]^k|}.$$

Whether they entail a consistency requirement remains to be seen, but both proposals will now at least be defined over all inconsistent sets.

As for C_d , one plausible revision is to replace the conditional probabilities with Roche's measure of absolute confirmation. Since Roche's measure of absolute confirmation will assign exactly the same value as the conditional probability function in all cases where the conditional probabilities are well defined, the proposal would conservatively extend the domain of the function C_d to cover absolutely all sets of propositions of cardinality greater than or equal to 2. In the case of the distance measure, it is relatively straightforward to see how this is to be done, and should be clear from the example how the strategy might be generalized to other measures of incremental confirmation.³⁵ We define a new confirmation function as follows:

³⁵It won't generalize for all incremental confirmation measures, since some measures would be dividing by zero in certain cases. For instance, consider Kemeny and Oppenheim (1952) measure of factual support: $k(h, e) = \frac{Pr(e|h) - P(e|\neg h)}{Pr(e|h) + P(e|\neg h)}$. Instead of revising in the manner Fitelson (2004) proposed, we might try replacing conditional probabilities with Roche's measure of absolute confirmation. That is, we define a new measure of factual support as follows: $k_a(h, e) = \frac{a(e|h) - a(e|\neg h)}{a(e|h) + a(e|\neg h)}$. This measure will not be well defined in cases where e is a contradiction. Hence, this strategy cannot be universally applied. Fitelson's proposed generalization seems far more plausible for this reason.

$$d_a(h, e) = a(h, e) - Pr(h).$$

It is worth noting that $d_a(h, e) = d(h, e)$ for all pairs within the domain of $d(h, e)$. Hence, when we define a coherence measure $C_{da}(S)$ as per our recipe above, we get a measure that agrees with C_d about the coherence of all sets S over which C_d is a well-defined function.³⁶ In the case of a lottery with two winning propositions, the coherence measure will now be well defined, and will be treated similar to a one ticket winning lottery, and this strikes me as a considerable improvement.

Moreover, there is something natural about the revision to this confirmation measure. The distance function measures the degree of positive or negative change when we conditionalize on evidence. But we might just as intuitively think of this confirmation measure as measuring the distance between the absolute confirmation provided by some evidence and the probability of the hypothesis prior to considering the support provided by that evidence. Thus, this proposed solution also enjoys some intuitive appeal. But it is not without some counter-intuitive costs as well, especially when we consider how $C_{da}(S)$ treats certain sets of inconsistent of propositions.

4.3.10. Contradictions as Coherence Makers. The most significant problem with this proposed revision is that this introduces a new and highly counter-intuitive means to increasing the coherence of a set of propositions. In some cases, the measure says that we can increase the coherence of a set of propositions by adding logical contradictions. Here is an example that illustrates the point. We start with a very incoherent set of propositions: $\{H, \neg H\}$. For any probability function where $0 < Pr(H) < 1$, we have it that $C_d(\{H, \neg H\}) = C_{da}(\{H, \neg H\}) = -.5$.³⁷ For simplicities sake, let us assume $Pr(H) = Pr(\neg H) = .5$. But, now consider the set $\{H, \neg H, H \wedge \neg H\}$. While this set is undefined for C_d , it is defined for C_{da} , and the value is surprising:

FACT 4.4. *If $Pr(H) = .5$, then $C_{da}(\{H, \neg H, H \wedge \neg H\}) = -.25$.*

³⁶Meijs's measure, C_M , can be revised in a manner not entirely disanalogous to this, but the limitations of C_M are not going to be of great relevance to the questions we shall be considering, since only the most perversely inconsistent sets are undefined for C_M .

³⁷This fact is obvious enough to not require much comment. The fact is that $Pr(H|\neg H) - Pr(H) + Pr(\neg H|H) - Pr(\neg H) = -(Pr(H) + Pr(\neg H)) = -1$. We take the average of the two pairs, and thus get $-1/2$.

PROOF. To calculate $C_{d_a}(\{H, \neg H, H \wedge \neg H\})$, we simply need to take the average of the pairs of non-empty disjoint subset pairs of $\{H, \neg H, H \wedge \neg H\}$. Roche (2013) has observed that there will be $3^n - 2^{n+1} + 1$ such pairs for any set of size n .³⁸ Thus, there are 12 such pairs that are averaged in this case. We break up the pairs, $\langle S, S' \rangle$, into two groups based on their values when plugged into d_a :

$a(\bigwedge S \bigwedge S') - Pr(\bigwedge S) = -5$	$a(\bigwedge S \bigwedge S') - Pr(\bigwedge S) = 0$
$\langle \{H\}, \{\neg H\} \rangle$	$\langle \{\neg H, H\}, \{H \wedge \neg H\} \rangle$
$\langle \{\neg H\}, \{H\} \rangle$	$\langle \{\neg H, H\}, \{H \wedge \neg H\} \rangle$
$\langle \{\neg H\}, \{H, H \wedge \neg H\} \rangle$	$\langle \{\neg H, H \wedge \neg H\}, \{H\} \rangle$
$\langle \{H\}, \{\neg H, H \wedge \neg H\} \rangle$	$\langle \{H, H \wedge \neg H\}, \{\neg H\} \rangle$
$\langle \{H\}, \{H \wedge \neg H\} \rangle$	$\langle \{H \wedge \neg H\}, \{H\} \rangle$
$\langle \{\neg H\}, \{H \wedge \neg H\} \rangle$	$\langle \{H \wedge \neg H\}, \{\neg H\} \rangle$

Since half of the pairs are assigned -5 when plugged into d_a , and half are assigned 0 , we get an average of $-.25$.³⁹ \square

This isn't just an issue that arises for this particular way of defining incremental confirmation. It is a problem that one faces if (i) one interprets coherence as a function of incremental confirmation relations, and (ii) holds that coherence measures should be defined over all inconsistent sets, including those containing logical contradictions. The reason (i) gives rise to a problem is that incremental confirmation measures assign the neutral value when $Pr(h) = Pr(h|e)$, which necessarily holds when h is a contradiction (assuming e 's probability isn't also 0). Thus, contradictions cannot be confirmed nor disconfirmed for all incremental confirmation measures. One will thus be able to

³⁸Let us briefly note why this calculation is correct. First, let $S(n, 3)$ be the number of ways of partitioning a set of size n into 3 non-empty subsets, and $S(n, 2)$ be the number of ways of partitioning a set of size n into 2 non-empty subsets. These are standardly referred to as Stirling set numbers, or Stirling numbers of the second kind. See Weisstein ("Stirling Numbers of the Second Kind") for references, further explanation of Stirling numbers, and this particular choice of notation. One way to see that Roche's calculation is correct is to note that the number of non-empty subset pairs of a set of size n where the order of the partitioned sets matter is equal to $6 \cdot S(n, 3) + 2 \cdot S(n, 2)$. $6 \cdot S(n, 3)$ is the number of subset pairs, $\langle S', S'' \rangle$, such that $S' \cup S'' \neq S$. The way to think of this calculation is that we need to count up the number of partitions where one set represents the elements not contained in the binary subset pair, and we need two other sets representing each of the subsets in the binary subset pair. There are six permutations for any choice of a 3 subset partition, and hence $6 \cdot S(n, 3)$ gives the right value. It is easy to see that $2 \cdot S(n, 2)$ is the number of subset pairs where $S' \cup S'' = S$. Weisstein explains, and it is easy to verify, that $S(n, 3) = \frac{1}{6}(3^n - 3 \cdot 2^n + 3)$ and $S(n, 2) = 2^{n-1} - 1$. So, then the number of non-empty binary subset pairs of S is just $(3^n - 3 \cdot 2^n + 3) + 2^n - 2 = 3^n - 2^{n+1} + 1$.

³⁹The calculation holds for all probability functions, the assumption of the particular probability merely simplifies the choice of how we group the sets, and was made for ease of presentation.

bring certain incoherent sets closer to the neutral value by introducing a logical contradiction to the set.

Epistemologists who favor interpreting the coherence relation as a function of incremental confirmation are thus forced to bite one bullet or another. Either she must reject the idea that coherence is defined over all inconsistent sets, or else accept that one can sometimes make a set more coherent by adding contradictions. For our purposes, we need not consider measures that require that we take the first option. It may well be that there are important applications for coherence measures that can only be defined over consistent sets, but such measures don't seem to offer us much opportunity to learn or say much about the coherence of inconsistent sets. It is also worth noting that if people do regularly hold inconsistent beliefs, we might still want measures that can be applied to such inconsistent beliefs, and have a theory for such measures. Clearly, the alternative is also rather counter-intuitive. But as we have we have already noted, one need not adopt an incremental measure of confirmation as the basis for one's interpretation of coherence as a matter of mutual support. One could adopt either Fitelson or Roche's approach, since both entail that adding a contradiction can only change the coherence of a set by making it less coherent.

4.3.11. Taking Stock. Before we investigate which probabilistic accounts of the coherence relation entail a consistency requirement, we ought to take stock of what we have learned about the measures thus far. First, there are only two measures that entail that inconsistent sets are always less coherent than consistent sets, namely, C_S and C_O . Both do so only in virtue of being depth insensitive in ways that make them highly implausible. Thus, from here we can set these measures aside as informing our understanding of the relationship between consistency and coherence (we have said all there is to say about their relationship anyhow: inconsistent sets are all the same: maximally incoherent). We must set aside Schupbach's proposal as well, since it isn't defined over inconsistent sets, or again is, but in a highly implausible way. This leaves C_M , C_{MW} , C_F , C_d and C_a . On all of these measures, some inconsistent sets will be more coherent than some consistent sets,⁴⁰ and as we discussed above, this seems exactly as we should expect.

⁴⁰The reason is obvious, and thus I omit a proof. In each case, a pair of consistent propositions can be as near to maximally incoherent as one cares to make them (though won't ever be maximally incoherent). On all of the measures, some inconsistent sets can have subsets that exhibit coherence-making features and thus aren't maximally incoherent.

In pursuing the question of whether certain accounts of the coherence relation entail a consistency requirement, we start by considering Roche's example. In the process, we shall also consider how these measures relate to Easwaran and Fitelson's accuracy-dominance avoidance principles. Even if coherence doesn't entail a consistency requirement, it may well entail accuracy-dominance avoidance principles. We now turn to both issues, starting with the case of the Tired Logic Student.

4.4. Tired Logic Student and the Coherence Measures

The natural place to start is with the case of the Tired Logic Student as it represents the most extreme sort of inconsistent belief set that we will need to consider. If a coherence measure can allow for such a set to be coherent simpliciter, then it imposes none of the formal coherence constraints on justified belief that we outlined in Chapter 1, i.e., a set wouldn't need to be consistent, nor avoid accuracy-dominance principles. The key formal question to consider about the Tired Logic Student case is whether or not the various coherence measures we have considered will allow for a set $TLS = \{p_1, \dots, p_n\}$ to be coherent when p_n is a self-contradictory proposition.

4.4.1. Roche's Measure. Roche (2013) considers this question and proves the following theorem:

Theorem 4.1. *For any n , $C_a(TLS) < \frac{1}{3}$.*

REMARK. See the appendix for a formal sketch of Roche's proof. Informally, the basic idea behind the proof is that at least $2/3^{rds}$ of the subset pairs of TLS contain the contradiction in their union, i.e., $2/3^{rds}$ of every $\langle S', S'' \rangle \in [TLS]$ is such that $p_n \in S'$ or $p_n \in S''$. When this is the case, $a(S', S'') = 0$, i.e., no absolute confirmation is provided. The measure averages the values assigned to all of the pairs, and so this guarantees that the average must be less than $1/3$. What this shows is that no set containing an explicit contradiction can be coherent simpliciter in the sense of falling above the neutral value of .5.

Roche doesn't consider the Tired Logic Student case for the other measures. This is quite reasonable in the case of Douven and Meijs's C_d , given the fact that C_d is not defined over TLS . However, $C_{da}(TLS)$ is defined over TLS , and the value C_{da} assigns to TLS depends on the probability of

the members of $\{p_1, \dots, p_{n-1}\}$. It is thus an open question as to whether $C_{d_a}(TLS)$ can exceed the neutral value for C_{d_a} . The Following lemma is needed to answer that question:

LEMMA. *For any choice of the probability of p_1 (note that p_1 through p_{n-1} are logically equivalent and thus have the same probability), $C_a(TLS) \geq C_{d_a}(TLS)$.*

REMARK. See appendix for formal proof. The reason why is simply that $d_a(h, e) = a(h, e) - Pr(h) \leq a(h, e)$. Hence, the values of the subset pairs to be averaged by C_{d_a} will each be less than or equal to the values to be averaged by C_a .

This Lemma establishes that $C_{d_a}(TLS)$ cannot exceed $1/3$. Now, we can prove an analogous limit theorem for $C_{d_a}(TLS)$.

Theorem 4.2. $\lim_{Pr(p_1) \rightarrow 0} C_{d_a}(TLS) = C_a(TLS)$.

REMARK. Informally, the reason for this is that as $Pr(p_1)$ tends to zero, each of the values to be averaged by the measures are identical for subset pairs containing the contradiction. For those subset pairs free of the contradiction, the value assigned by $d_a(\cdot)$ approaches 1, which is the value assigned by $a(\cdot)$.

There are two immediate lessons to be taken away from this example. The first is that if we think of coherence as mutual support, and support is to be understood in terms of our revised distance measure, then coherence does tolerate inconsistency in one sense: Inconsistent sets can be coherent simpliciter if that is a matter of being more coherent than incoherent (recall that the neutral value for C_{d_a} is 0). In other words, the positive mutual support relations for such inconsistent sets can outweigh the mutual undermining relations for large enough sized sets. It is also worth noting that TLS is a strict-accuracy dominated set, and so even a strict accuracy dominated set can be coherent simpliciter on this measure. However, the example also demonstrates that there is a limit on how coherent a set containing a contradiction can be, and the limit falls well below the maximum value that can be assigned by the measure. If one is inclined to insist that coherence measures should not allow for strictly dominated sets to be coherent, then one could insist that the relevant threshold for categorical coherence is greater than $1/3$.⁴¹

⁴¹Note that we have not proved that this is the limit for all strictly dominated sets, but have just proved that sets containing explicit contradictions cannot be coherence to a degree above $1/3$.

4.4.2. Meijs's Measure. Now, Meijs's measure, unlike the rest, does not have any obvious neutral point. If we were forced to choose a minimum degree of coherence that could serve as coherent simpliciter, it seems to me it would have to be .5. The intuitive reading of the proposal would be that a set whose value falls below .5 would exhibit more internal disagreement than agreement. And above .5 would be the converse. And, in this case, we have the following:

Theorem 4.3. $\lim_{n \rightarrow \infty} C_{M_a}(TLS) = 1/2$.

REMARK. The proof depends on observation that the situation is such that each subset of TLS is supposed to have complete overlap if it does not contain a contradiction, but has no overlap if it contains the contradiction. So, the degree of coherence assigned by C_{M_a} to TLS is just the number of subsets with at least two elements that don't contain the contradictory proposition divided by the total number of subsets of TLS (where the relevant subsets are those that have more than 1 element from TLS). This ratio approaches $1/2$ as n converges to infinity.

The function is strictly increasing in this case, so the upshot is that on this particular choice of a scheme to generalize Olsson's measure, we have it that contradictions cannot be coherent simpliciter at least if the threshold is at least $1/2$. Moreover, the result holds for the measure obtained by plugging C_{O_a} into Schupbach's alternative weighting scheme:

Theorem 4.4. $\lim_{n \rightarrow \infty} C_{M_a^W}(TLS) = 1/2$ and $C_{M_a^W}(TLS) < 1/2$.

REMARK. The proof in this case is substantially more complicated, given the combinatorial complexity introduced by Schupbach's scheme. There is no simple way to explain informally why this holds other than to say that each i -wise degree of logical equivalence is offset by each $n - i$ -wise degree of logical equivalence. See the Appendix for the formal results.

The important thing to observe is that whether we choose Schupbach or Meijs's method for revising Olsson's measure, we wind up with the same result.

4.4.3. Fitelson's Measure. The last measure to consider is C_F . Recall that the neutral point for Fitelson's measure is 0. The following theorem establishes that no set containing a contradiction can be coherent to a degree above the neutral point.

Theorem 4.5. $C_F(TLS) < -1/3$.

REMARK. The proof turns on the fact that $2/3^{rds}$ of the pairs in $[TLS]$ contain the contradiction, p_n , in one of the sets in the pair. All such subset pairs are assigned a value of -1 , and hence when the factual support between all of the subset pairs are averaged, the value is less than $-1/3$.

Like the other measures based on the idea that coherence is a kind of generalized logical equivalence, we get that sets containing contradictions have stronger incoherence making features than they have coherence making features.

4.4.4. Tired Logic Student - Taking Stock. At this point, we should pause momentarily to note something that isn't all that surprising. Only one measure out of all of those we have considered allows for a set containing a contradiction to be coherent simpliciter, and that is the coherence measure, C_{da} , which treats coherence as mutual incremental confirmation. The main reason the distance measure has this entailment while others do not is the way that the distance measure d_a treats those subset pairs that contain the contradiction. In the case of all of the other measures, if a proper subset or subset pair (depending on which is apt for exhibiting (in)coherence-making features) contains a contradiction, then that subset or subset pair exhibits the relevant incoherence-making feature to the maximal degree. However, in the case of the distance measure, a subset pair containing a contradiction need not exhibit an incoherence making feature to any degree, and if it does, it may due so to some fairly low degree (The maximal amount of disconfirmation e can supply for h is $Pr(h)$, so if h has a low initial probability, then e disconfirms h to a small degree). Thus, while all of the other measures hold that the incoherence making features are maximally strong and as abundant as there are subsets or subset pairs containing the contradiction, C_d holds that the incoherence making features of a set containing a contradiction need neither be strong nor abundant.

Now, what exactly should we conclude from these results so far? Should we conclude that the breakdown is simply this: if one is inclined to understand coherence as a kind of generalized logical agreement, then one is committed to the impossibility of contradictions being a part of a coherent set, but one avoids such a commitment if one thinks of coherence as mutual incremental confirmation? Things are not nearly so simple. The result that the measures of generalized logical agreement do not allow contradictions to be part of a coherent set depends on our choice to take the straight average of the coherence-making features for all of our various measures.

When spelling out different weighting schemes, Schupbach (2011, pp. 132-133) points out that a scheme might give more or less weight to the values assigned to smaller or larger subsets (in the case of his measure, which averages values assigned to proper subsets, not subset pairs). He calls those that give more weight to smaller subsets *depth increasing*, and those that give more weight to larger sets *depth decreasing*. The same terminology can be transferred over to subset pairs if we focus on the size of the union of the pairs as opposed to the size of the set (Henceforth, when I refer to the size of a subset pair $\langle S', S'' \rangle$ I will mean the cardinality of their union $S' \cup S''$). If we adopted a more depth increasing approach, i.e., a weighting scheme that assigns greater weight to the smaller subsets or subset pairs then we could arrive at a drastically different result, and make it so that any of the coherence measures that treat coherence as a matter of generalized logical equivalence would also allow for sets containing a contradiction to be near to maximally coherent.

The first step to see how increasing depth sensitivity increases the maximum value of TLS is to consider the measure obtained by plugging C_{O_a} into Schupbach's depth increasing weighting scheme⁴²

$$C_{M_a^{DI}}(S) =_{def} \sum_{i=1}^{n-1} \frac{2(n-i)}{n(n-1)} C_{O_a}^{i+1}(S).$$

This measure is similar to $C_{M_a^w}$, except that greater weight is assigned to smaller subsets in an linearly increasing fashion. And, it is easy to verify that

Theorem 4.6. $\lim_{n \rightarrow \infty} C_{M_a^{DI}}(TLS) = 2/3$, and for all n , $C_{M_a^{DI}} < (TLS)^{2/3}$

If the threshold for the measure is $1/2$, then it follows that we now have a generalized logical coherence measure that allows for a set containing a contradiction to be coherent simpliciter. If one wants to insist that the threshold must be higher, we can obtain a corresponding result for any such threshold. As the weighting scheme is tilted more in favor of smaller sets, the maximum value that can be assigned to TLS approaches 1. The reason is that the average value assigned to the smaller sets or subset pairs is already near the maximal value for this and all of the measures under consideration. It is just that in the straight-average schemes, these high values are offset by low

⁴²See Schupbach (2011, p. 133) for the weighting scheme when applied to Shogenji's measure. The measure is linearly increasing, and we could easily increase the rate of increase to make the measure as tolerant of contradictions or inconsistency as we care to.

values assigned to larger subsets or subset pairs. But if we adjust the weighting scheme to favor smaller sets, we can revise any coherence measure to make it tolerant of some self-contradictory beliefs.

Theorem 4.7. *For any $\delta > 0$, the measures C_{M_a} , $C_{M_a^w}$, C_{d_a} , C_a and C_F there is a depth increasing weighting scheme that assigns TLS a coherence value, t , such that $t > 1 - \delta$.*

REMARK. The easiest way to prove this is to consider an extreme case where we give tremendous weight to the binary subsets or binary subset pairs, i.e., $|S'| = 2$ or $\langle S', S'' \rangle$ where S' and S'' are singleton sets. We simply design the weighting scheme in such a way that each such binary subset or subset pair is given the same weight, so that the average of values assigned to these subsets or subset pairs is r percent responsible for determining the degree of coherence of the set. Since the average value assigned to the binary subset pairs for all measures will converge to the maximum value as the size of TLS grows, the coherence value assigned to TLS will converge to the maximal value as r converges to 1, i.e., as the binary subset or subset pairs are given increasingly more weight.

The upshot is that whether an analysis of coherence as generalized logical equivalence allows sets containing a contradiction to be coherent depends on our choice of a weighting scheme. In principle, our analysis of the coherence relation can go either way with regard to imposing a no-contradictions requirement. Ultimately, no particular philosophical argument has been presented for one weighting scheme over the alternatives. Generally, the only virtue considered is how easy the weighting scheme is to work with from a purely combinatorial standpoint. But the upshot of the foregoing is that we now have at least one kind of argument against certain weighting schemes: If one wants to hold that a strictly dominated set cannot be coherent, then we now have a compelling argument to rule out certain depth-increasing weighting schemes.

We might hope that an analogous point could be made regarding our measure of coherence in terms of incremental confirmation C_{d_a} . That is, if we adopted a different weighting scheme, perhaps C_{d_a} could be adapted to entail that all sets containing a contradiction are incoherent. However, it turns out that there is only one weighting scheme that would rule out the possibility of a set containing a contradiction from being more coherent than incoherent, and that is a scheme where all subset

pairs that fail to partition TLS are given no weight at all.⁴³ This, of course, is just to say that the only weighting scheme that would rule out the possibility that a set containing a contradiction could be coherent is a scheme that makes the measure depth insensitive in just the way that C_O and C_S were depth insensitive. Thus, it is the case that taking an incremental approach in terms of a confirmation measure like d_a would commit one to holding either that contradictions can be members of sets where the coherence-making features outweigh the incoherence making features, or else that coherence is not depth sensitive.

Whether this constitutes a reductio of measures along the lines of C_{d_a} is far from obvious. It is also worth emphasizing that coherentists who adopt C_{d_a} are not thereby committing to the rejection of Easwaran and Fitelson's accuracy-dominance avoidance principles. If they were, then I think we would have a reductio of the position. But, coherentists adopting a plausible weighting scheme for C_{d_a} , i.e., one that is depth sensitive, are merely committing to accuracy dominance norms not following directly from their analysis of the coherence relation. They may accept epistemic principles in addition to a substantive coherence norm of some kind, and it may well be that they can derive Easwaran and Fitelson's norm from some other set of principles. After all, the basic evidential constraint that one's beliefs should be supported by the total evidence available seems plausible even on a coherentist picture of epistemic justification. Easwaran and Fitelson's dominance avoidance principles seem to be independently well motivated, and don't cry out for an explanation the way that a consistency requirement does.

At this point, I think we have a relatively clear understanding of the coherentist's options with respect to sets containing a contradiction. On plausible analyses of the coherence relation in terms of generalized logical equivalence, the coherentist has the option to go either way. On the plausible analysis of coherence as mutual incremental confirmation characterized in terms of C_{d_a} , the coherentist is committed to accepting that some sets containing a contradiction can be coherent beyond the neutral point. Next we consider the coherentist's options in less pathological cases of inconsistent propositions.

⁴³The reason for this is implicit in what has already been said. Subsets containing a contradiction can all exhibit almost no incoherence-making features on this measure. Those, not containing it, can exhibit coherence-making features to nearly the maximal degree. Hence, the coherence making features can out weigh the incoherence making features if they are given any weight at all.

4.5. Minimally Inconsistent but Coherent

The next issue to consider is whether minimally inconsistent sets can be coherent simpliciter on our various measures. For a number of the measures, the easiest way to answer this question will be to consider some set of lottery propositions. In the cases where a subset pair exhibits a coherence making relationship when the subsets are probabilistically dependent on each other, it will be useful to consider a revised version of the lottery paradox, similar to the one we considered in Chapter 2. In the other cases, only the conditional probabilities are relevant (for instance, C_a only depends on the conditional probabilities), and in those cases the conditional probabilities of the paradox we shall consider coincide with the conditional probabilities of a standard lottery. Thus, we consider the degree of coherence that can be assigned to sets of propositions that have probabilities corresponding to the following lottery.

4.5.1. The Extra Generous Lottery. We shall call the new lottery paradox an *extra generous lottery paradox* because it is just like the generous lottery from Chapter 2, but rather than suppose that there is just a small chance that the administrators of the lottery will decide that all of the tickets are winning tickets, we assume that the odds are extremely high. The point of this is to ensure that the lottery propositions start out with a very low probability and are thus probabilistically dependent on one another to a very high degree (this will be true for virtually any incremental confirmation measure). Again, the lottery is run so that a ticket is drawn to be the winning ticket. The authorities running the lottery announce that this ticket has been drawn and is guaranteed to be a winning ticket. This initial drawing has propositions corresponding to a standard lottery:

l_i : The i^{th} ticket was not drawn in the initial lottery drawing.

These propositions form a set

$$L^n = \{l_1, \dots, l_n\}$$

that constitute a standard lottery case, with probabilities associated with the standard lottery paradox. But then, the authorities announce that they are feeling extremely generous and want to allow for the possibility that all tickets will be declared winning tickets. Some randomizing

mechanism is chosen to determine whether all tickets in the lottery will be declared winners. The randomizing mechanism is designed so that the probability that all tickets will be declared winners is $1 - r$. Thus, r is the probability that the only winning ticket is the one that was initially drawn. We thus have the proposition:

NG : Not all tickets are declared winners.

We have stipulated that $Pr(NG) = r$. We then obtain our extra generous lottery propositions as follows:

e_i : i^{th} ticket is not a winner.

Note that for e_i we have the following equivalence:

$$e_i = l_i \wedge NG.$$

That is to say, the i^{th} ticket loses only if not all tickets are declared winners, and the i^{th} ticket was not initially drawn. Since l_i and NG are independent propositions, we have it that

$$Pr(e_i) = Pr(l_i) \cdot Pr(NG).$$

Finally, the target inconsistent set is:

$$L_E^n = \{e_1, e_2, \dots, e_n\}.$$

The question we shall pursue next is whether a member of this set can be coherent simpliciter on the measures we have considered, starting with Roche's measure. The relevant probability function will be the same as the one explained in Chapter 2 except that we will be concerned with cases where the prior probability that all of the lottery propositions are false will approach 1, i.e., where $Pr(NG) \approx 0$.

4.5.2. Roche's Measure. Roche (2013) has observed that his measure does not permit inconsistent sets to be coherent for cases like our *Tired Logic Student*, but has not explored whether his measure will allow for other kinds of inconsistent sets to be coherent. The following theorem answers this question in a definitively negative way:

Theorem 4.8. *If S is inconsistent ($\Pr(\bigwedge S) = 0$), then $C_a(S) < \frac{1}{2}$.*

It is important to note the generality of this result. It demonstrates that no inconsistent sets whatsoever can be coherent beyond the neutral point on Roche's measure. Of course, we haven't yet established that $1/2$ is the least upper bound. In answer to that question, we have the following theorem:

Theorem 4.9. $\lim_{n \rightarrow \infty} C_a(L_E^n) = 1/2$.

Lotteries thus maximize the degree of coherence that can be assigned to inconsistent sets on Roche's measure. Note that while Roche's measure does not allow for inconsistent sets to be coherent simpliciter, it does respect the fact that certain kinds of inconsistency are epistemically worse than other kinds. Minimally inconsistent sets, while they cannot be coherent simpliciter, can be coherent to a degree higher than any set containing an explicit contradiction. And, as we noted earlier, if we adjust the weighting scheme so that it is depth increasing, we can increase how coherent a set containing a contradiction can be. I would even conjecture that we can adjust the weighting scheme, so that a minimally inconsistent set can be coherent simpliciter, while a set containing a logical contradiction cannot. In particular, if we adjust the weighting scheme to be depth increasing so that the supremum of the values that can be assigned to a set containing a contradiction is $1/2$, then one would expect that the value that could be assigned to a minimally inconsistent set would rise to some degree as well. Given that the supremum is already $1/2$, it seems as though minimally inconsistent sets should be able to exceed $1/2$. Given the combinatorial complexities that arise when we adjust the weighting schemes, especially in the case of the measures that average values of subset pairs, I won't here be able to prove the conjecture to be true, and shall leave this as a question to be pursued in later research.

For now, we must make due with the observation that coherentists adopting Roche's measure with a straight average weighting scheme will not allow for minimally inconsistent sets to be coherent

simpliciter. But with an adjusted weighting scheme, Roche's measure can allow for minimally inconsistent sets to be coherent simpliciter, which follows from exactly the same considerations that made Theorem 4.7 true.

4.5.3. Douven and Meijs's Coherence Measure. How about Douven and Meijs's measure? The answer follows from our earlier observations about the relationship between C_{d_a} and C_a : $C_{d_a}(S) \leq C_a(S)$.

LEMMA 4.5. $\lim_{r \rightarrow 0} C_d(L_B^n) = C_a(L_B^n)$. (Note that we are looking at r 's convergence to 0, not n 's convergence to infinity).

Theorem 4.10. The following sequence $\lim_{r \rightarrow 0} C_d C_d((L_B^2)), \lim_{r \rightarrow 0} C_d((L_B^3)), \dots$, converges to $1/2$.

REMARK. The proof follows straight forwardly from Lemma 5 and Theorem 4.9. See the appendix for the precise details.

Note that while C_{d_a} assigns sets containing a contradiction a value above the neutral point, the maximum degree of coherence that can be assigned to a contradiction is below the value assigned to a minimally inconsistent set. C_{d_a} thus reflects the fact that violation of certain kinds of inconsistency are epistemically worse than others. As a consequence, there is a coherence threshold t such that minimally inconsistent sets can be coherent above t , while sets containing an explicit contradiction cannot be. The coherentist simply needs to hold that the threshold for coherence simpliciter, t , is such that $1/3 < t < 1/2$. This doesn't ensure the all kinds of sets violating Easwaran and Fitelson's accuracy-dominance avoidance principles all have maximum coherence values falling below the threshold for a minimally inconsistent set, but it does at least show that the measures treat the most pathological cases of inconsistency to be less coherent.

The main upshot we are left with is that of all of the measures, C_{d_a} is the friendliest to the idea that highly pathological sets can be coherent to some degree. And, that there is a high threshold that both minimally inconsistent sets and sets containing a contradiction can reach. That the threshold, $1/3$ is high is captured by the fact that if $Pr(H) > 2/3$ and $Pr(H') > 2/3$, then $C_F(\{H, H'\}) < 1/3$. Just how friendly the measure is to accuracy-dominated sets is something that we shall have to leave as a question for future research.⁴⁴

⁴⁴I wish it were possible to prove the more general result in the manner I prove it for Fitelson's measure below. Unfortunately, the challenge in being able to determine whether this measure will impose some limit on sets that are

4.5.4. Meijs's Measure. Like Roche's measure, the coherence of a bonus lottery is the same as the coherence of a standard lottery, i.e., it is independent of our choice of r . And like Roche's measure, we get convergence to $1/2$.

Theorem 4.11. $\lim_{n \rightarrow \infty} C_{M_a}(L_B^n) = 1/2$ and $C_{M_a}(L_B^n) < 1/2$.

Recall that the maximum value that can be assigned to a set containing a logical contradiction by $C_{M_a}(L_B^n)$ is also $1/2$. Thus, unlike the measures we have considered so far, there is no distinction drawn between the coherence of a set of lottery proposition, and a set containing a logical contradiction. The result holds for the generalization of Olsson's measure using Schupbach's alternative weighting scheme.

Theorem 4.12. $\lim_{n \rightarrow \infty} C_{M_a^w}(L_B^n) = 1/2$ and $C_{M_a^w}(L_B^n) < 1/2$.

Thus, on both this account of coherence as logical equivalence and on Roche's proposal we have it that sets of lottery propositions can be no more coherent than incoherent. There is one last measure of coherence as generalized logical equivalence to which we now turn.

4.5.5. Fitelson's Measure. Fitelson's measure stands apart from the rest in that the only restriction it places on inconsistent sets is that they cannot be coherent to the maximal degree.

Theorem 4.13. *The following sequence $\lim_{r \rightarrow 0} C_F((L_B^2))$, $\lim_{r \rightarrow 0} C_F((L_B^3))$, ..., converges to 1.*

And so Fitelson's measure is the only one that, as formulated with a straight average weighting scheme, allows for inconsistent sets to be coherent simpliciter, while holding that all sets containing a contradiction are incoherent to some degree. This measure holds that certain kinds of inconsistency are epistemically worse than other kinds of inconsistency. Additionally, because of the intuitive way in which Fitelson's measure deals with inconsistent subset pairs (It assigns them a value indicating

accuracy dominated either in the weak or strong sense arises for the reasons we discussed above regarding the fact that contradictions can be coherence makers on this measure. For all of the other measures, when a subset or subset pair is inconsistent, that subset or subset pair exhibits an incoherence making feature to the maximal degree, which can be shown to offset coherence making features that may be exhibited by other complementary subsets or subset pairs. In my other proofs, I have employed the strategy of carving up the space of subsets or subset pairs to bring out the fact that such offsets are inevitable. Because of the issues with d_a 's treatment of contradictions, I see no analogous way to carve up the space of subset pairs. Thus, until alternative methods are known to handle such cases, I think these more modest claims must suffice.

that they exhibit incoherence making relations to a maximal degree), unlike C_{da} , we can prove a more general result:

Theorem 4.14. *If S is strictly-dominated, then $C_F(S) < 0$.*

Consequently, a coherentist who adopts Fitelson's analysis of the coherence relation can reasonably hold that (i) it is possible for certain inconsistent sets to be near to maximally coherent, and (ii) sets that violate Easwaran and Fitelson's strict dominance avoidance principles cannot be more coherent than they are incoherent. This, I think, is a rather intuitive balance, but still says nothing about what the measure holds for sets that respect Easwaran and Fitelson's weak accuracy-dominance principles.⁴⁵ It would seem highly intuitive if the measures put forward to understand the substantive coherence relation related to Easwaran and Fitelson's dominance avoidance principles in such a way that substantive coherence ruled out coherent sets of propositions that are weakly or strictly dominated. Alas, the next result shows that if the threshold for coherence simpliciter is the neutral value, then it is not the case that Fitelson's measure entails such a relationship. We can show this with the following example.

4.5.5.1. *Party Example.* Let us begin with an informal description of the example. Let us suppose there are seven friends that are almost inseparable. Ronald, Mack, Charlie, Dennis and Dee spend all of their free time partying together. The gang's other two friends, Frank and Artemis, had a bitter falling out and will never be found together. Frank and Artemis have made an arrangement where they will take turns hanging out with their friends, dividing their time with the gang evenly. Also, it is worth noting that Frank or Artemis will stay home if it is not their turn to hang with the gang. One evening you decide that you would like to find the group. You narrow down their possible destinations to m different party locations (each being as likely as the next). Now, as you inquire into the groups location, testimony from various people forces you to consider the coherence of the following seven propositions:

H_0 : Ronald goes to party 1

H_1 : Mack goes to party 1.

⁴⁵It also says nothing about their requirement that belief sets be representable. Given that the coherence measures depend on combinatorial and logical properties of sets of propositions, and the dominance avoidance principles supervene on combinatorial and logical properties of sets of properties, it is much easier to study the relationship between the dominance avoidance principles than representation principles discussed in Chapter 1.

H_2 : Charlie goes to party 1.

H_3 Dennis goes to party 1.

H_4 Dee goes to party 1.

H_5 : Frank goes to party 1, and Artemis doesn't.

H_6 : Artemis goes to party 1, and Frank doesn't.

Do these propositions cohere with each other. We know that, as a belief set (i.e., if some agent were to believe all of those propositions), they form a weakly dominated set, since $\{H_5, H_6\}$ is at least half inaccurate at all worlds, and is entirely inaccurate at any world where everyone didn't go to the party. Nevertheless, the probability function for the above scenario is plausibly such that, according to C_F , the set $S = \{H_0, H_1, H_2, H_3, H_4, H_5, H_6\}$ coheres to a degree above the neutral value 0. It is straightforward to confirm that, given our assumptions about the groups behavior, the following facts about the probabilities for any subset pairs $\langle S', S^* \rangle$ in $[S]$ where either H_5 or H_6 is not in $S^* \cup S'$. The first probabilistic fact to note about the case is that we have a ceiling on the initial probability for each proposition is less than $1/m$, and by the rule that the probability of a conjunction is no greater than the probability of its conjuncts, it follows that:

$$(A): Pr(\bigwedge S') < 1/m.$$

The next thing to note is that on the assumption that if any one of the propositions in S are true, exactly 6 out of 7 must be true. And, if H_5 or H_6 is not in $S^* \cup S'$, then given our background assumptions,

$$(B): Pr(\bigwedge S^* | \bigwedge S') \geq 1/2.^{46}$$

And last, on the assumption that $m > 2$, it immediately follows that

$$(C): Pr(\bigwedge S^* | \neg \bigwedge S') < 1/m.^{47}$$

From these facts, we have it that

⁴⁶Given our description of the scenario, it is easy to see that $\bigwedge S'$ entails $\bigwedge S^*$ when neither H_5 nor H_6 are in $S^* \cup S'$, and that this is true if we add exactly one of them to S' . In the case where one of H_5 and H_6 is in S^* (and the other is not in $S^* \cup S'$), there is a 50% chance that it is Frank's turn, and a 50% chance that it is Artemis's turn. $S \bigwedge'$ entails that the gang is at the party 1, so conditional on the assumption that the gang is at party 1, it follows that there is a fifty percent chance that $\bigwedge S^*$ is true. Hence, this fact holds.

⁴⁷This follows from (A) and (B) on the assumption that $m > 2$, from the general rule that if $Pr(H|E) > Pr(H)$, then $Pr(H|\neg E) < Pr(H)$.

If either H_5 or H_6 is not in $S^* \cup S'$, then $F\langle S', S^* \rangle \geq \frac{1/2 - 1/m}{1/2 + 1/m}$.

Now, we thus get it that when the antecedent holds, $F\langle S', S^* \rangle$ converges to 1 as m tends toward ∞ . And we know the following about the factual support measure employed in the measurement of coherence:

$$F\langle S', S^* \rangle = -1 \text{ if both } H_5 \text{ and } H_6 \text{ are in } S^* \cup S'.$$

In general, if the union of a subset pair forms a logically inconsistent set, then the factual support measure assigns the value representing maximal disconfirmation to the subset pair. Putting these observations together, we get:

$$\text{FACT 4.6. } \lim_{m \rightarrow \infty} C_F(\{H_0, H_1, H_2, H_3, H_4, H_5, H_6\}) = \frac{18}{602}.$$

The upshot of Fact 6 is that if we think of the neutral value, 0, as the cutoff for coherence simpliciter, then it follows that it is possible for a weakly-dominated set to be coherent simpliciter. And this raises a natural worry. Given that this measure is meant to formally capture a notion of coherence as a kind of generalized logical equivalence, it seems counter-intuitive to think that an accuracy-dominated set, can be logically equivalent to a high enough degree to qualify as coherent simpliciter. This result might then seem to reveal a failure on the part of the measure to reflect its underlying philosophical motivations.

Ultimately, I don't think this line of argument is sound. The following theorem shows that C_F can be brought into line with the intuition that weakly dominated sets cannot be coherent simpliciter. The fact is that while accuracy-dominated sets can be coherent to a degree above the neutral value, there is a relatively low ceiling on exactly how coherent such a set can be.

Theorem 4.15. *For any set S , if S is weakly-dominated, then $C_F(S) < 1/9$.*

Consequently, if we hold that the threshold of coherence is above $1/9$, it follows that no weakly dominated set can be coherent simpliciter. On some measures, there may well be reason to think

that the neutral value is the most plausible threshold for coherence simpliciter.⁴⁸ I see no reason to hold to this position with regard to C_F .

I can imagine one objecting at this point that the problem the party example shows is not that the measure allows a weakly dominated set to be coherent simpliciter, but rather that it fails to deliver the result that a weakly dominated set is highly incoherent. Even if we require a higher threshold for coherence greater than $1/9$, there is still something counter-intuitive about the level of coherence to which some weakly dominated sets can rise, or so goes the objection.

While the initial plausibility of this objection is plain, the appropriate response is not unlike the response we considered to the claim that all inconsistent sets are less coherent than their consistent counterparts, or to the claim that all inconsistent sets are highly incoherent. That a set is weakly dominated entails that it exhibits some incoherence making features, just as a set being inconsistent guarantees that it has at least some incoherence making features.⁴⁹ And in the case of a weakly dominated set, the incoherence making features are substantial on C_F . This is reflected in the fact that the positive degree of coherence is nowhere near the maximal value of the measure.⁵⁰ Nevertheless, the consistent subset pairs of an inconsistent or even weakly dominated set might exhibit coherence making features to a high degree. And so it is certainly true that even a weakly dominated set can have at least some coherence making features. And the coherence making features of the set in question are actually quite strong in the party example for each subset pair that does not contain both H_5 and H_6 . In fact, in that example, as long as $S' \cup S^*$ doesn't contain both H_5 and H_6 , then either $\bigwedge S'$ entails $\bigwedge S^*$ or $\bigwedge S^*$ entails $\bigwedge S'$. Given that the measure in question is meant to treat coherence as a kind of generalized logical equivalence, all such pairs thus provide a high degree of logical support to one another, and have a relatively high degree of logical equivalence. And, the party example is described so that if at least one of claims in the

⁴⁸That the neutral value must be the threshold for coherence simpliciter makes some sense with regard to a measure like C_d , since a set of claims whose conjunction is highly probable may not cohere to a degree much above the neutral point. For instance, if $Pr(H_1 \wedge \dots \wedge H_n) > t$, then it is easily shown that $C_d(\{H_1, \dots, H_n\}) < 1 - t$. So, if a proponent of C_d wants to hold that having a high prior probability isn't a barrier to being coherent simpliciter, then the proponent of C_d will have to hold that the threshold is equal to or below the neutral value. However, no such issues arise for C_F , which is demonstrated by the fact that logically equivalent claims can be maximally coherent even if their probability is near 1.

⁴⁹Even for measures that treat coherence as some average of the amount of incremental confirmation subset pairs provide for each other, all subset pairs that partition that set provide no positive confirmation for one another. For all of the other measures we have considered, the subset pairs that partition the set exhibit some incoherence making feature to the maximal degree, e.g., $a(S', S'') = 0$ and $F(S', S'') = -1$ if $\bigwedge(S' \cup S'') = \perp$.

⁵⁰In our proof of **Theorem 4.15**, we demonstrate that at least $4/9^{ths}$ of all subset pairs exhibit the incoherence making feature to the maximal degree for C_F .

set is true, then it follows that all but one of the claims in the set must be true, even though the prior probability of each member of the set may be near zero. So, overall, there is a strong degree of mutual support and logical equivalence exhibited by members of the set in question. Thus, the weakly dominated set in question happens to exhibit many coherence making features that many non-dominated sets may not exhibit.

In short, the status of being maximally incoherent, or nearly so should be reserved for those sets of claims that exhibit no coherence making features, and exhibit incoherence making features to a high degree. And as we have observed, this is simply not true of the party example. For this reason, I think it would be rather counter-intuitive if all weakly dominated sets were near to maximally incoherent. I thus do not think we should be moved to reject C_F on the basis of this objection.

On the contrary, the coherence measure C_F offers a rather plausible position that coherentists might adopt who are sympathetic to the idea that some amount of inconsistency is compatible with coherence. What we have shown is that the coherence measure C_F validates the following principles relating logical constraints on belief sets and coherence:

- (C3) Inconsistent sets of propositions can be coherent to a near maximal degree.
- (C4) There is a degree of coherence out of reach to any strictly or weakly dominated set of propositions.

(C3) follows from Theorem 4.13, which shows that for any threshold we might choose, there is a generous lottery example of an inconsistent set of propositions with a degree of coherence above the threshold. But, as we have noted, that some inconsistency might be compatible with incoherence should not entail that certain kinds of pathologically inconsistent sets cannot be coherent to a high degree. And, Theorems 4.14 and 4.15 entail (C4).

Overall, C_F offers a very plausible picture of the logical constraints on substantive coherence for anyone who is sympathetic to the idea that (substantive) coherence and inconsistency can sometimes be compatible. And, this may well provide some reason to prefer C_F over rival accounts of the coherence relation. On the other hand, as we have discussed above, there are certain features of Fitelson's measure, in particular the way the measure handles subcontraries, that may provide grounds to rejecting C_F as a plausible analysis of the coherence relation. We thus shouldn't rest

the idea that coherence and inconsistency are sometimes compatible on the results for C_F alone. So, let us try to summarize the lessons that can be drawn from our formal investigation into how the measures handle lottery situations.

4.5.6. Taking Stock - Coherence Measures and Lotteries. So what lessons are to be learned from the way that the coherence measures handle lottery and generous lottery situations? The first thing to note is that, at least on an equal weighting scheme (that is to say, when the (in)coherence-making features of each subset or subset pair is given equal weight), not all measures allow for some inconsistent sets of propositions to be categorically coherent. In fact, no coherence measure that (a) has an equal weighting scheme, and (b) is based on the idea that coherence is a matter of agreement⁵¹ allows for a set of lottery propositions to be coherent above the neutral value. So, there are certainly some accounts of the coherence relation that entail a consistency requirement on categorical coherence. However, just as in the case of belief sets containing explicit contradictions, any coherence measure under consideration can be revised in order to allow some minimally inconsistent sets to be coherent (relative to any threshold t below the maximum value) by adopting a depth increasing weighting scheme for reasons analogous to those that make Theorem 4.7 true.⁵² Thus, it is not the case that understanding coherence as a matter of agreement requires accepting a consistency requirement on categorical coherence.

The question of a consistency requirement on categorical coherence for measures based on the idea of coherence as agreement comes down to our choice of weighting scheme. Since the choice of a weighting scheme has been given very little attention,⁵³ the question of how different weighting schemes will impact the intuitiveness of different coherence measures is an uncharted domain. Given that virtually nothing is known about how the choice of different weighting schemes impacts the intuitiveness of the orderings yielded by our various coherence measures, it is not at all clear whether there is any good reason to prefer a depth decreasing or equal weighting scheme (one of

⁵¹Recall that there is a notion of relative agreement that Roche's measure can be thought to capture, and so it can be understood as falling within this category.

⁵²It is trivial to extend the result of Theorem 4.7 to also cover minimally inconsistent sets. The smaller the subsets or subset pairs are that we consider, the higher the coherence-making properties on average will be. Thus, if we adjust the weighting scheme to favor the smaller subsets of a set, the higher the coherence value an inconsistent set can be assigned on any of the measures under consideration. Given the trivial nature of extending the result, we shall omit the formal proof.

⁵³The only place where this issue has been discussed in any detail is Schupbach (2011). And even then, Schupbach mostly considers the question in the context of the one coherence measure we have set aside, since it cannot be defined over any inconsistent sets of propositions, or else treats all inconsistent sets the same.

which is required for a consistency requirement). For now, what we can confidently say is that proponents of coherence measures that treat coherence as agreement or mutual overlap have been given no argument against accounts of coherence that are tolerant of some inconsistency. And, thanks to the results above, we now know what shape an argument for a consistency requirement on categorical coherence (when coherence is understood as mutual agreement) would have to take: it would have to be an argument against a depth increasing scheme for weighing the coherence-making features of a set of believed propositions.

The other major lesson that can be drawn from the formal results regarding lottery propositions is that depth sensitive coherence measures (at least those defined over inconsistent sets) that treat coherence as a matter of mutual support or mutual probabilistic relevance are all tolerant of some inconsistency to a relatively high degree.⁵⁴ C_F allows for some minimally inconsistent sets to be coherent to a nearly maximal degree, and C_d allows for the degree of coherence to be well above the neutral value. And, from the proofs of Theorems 4.10 and 4.13, it is clear that some tolerance for inconsistency will hold for most coherence measure defined in terms of incremental confirmation. Since there are examples like the extra generous lottery where every proper subset pair (every subset pair $\langle S', S'' \rangle$ of S where $S' \cup S'' \subset S$) is positively dependent on one another (i.e., $Pr(\bigwedge S' | \bigwedge S'') > Pr(\bigwedge S')$), the only exceptions will be coherence measures that assign a massive amount of weight to n -wise coherence-making features (where n is the cardinality of S). In other words, in order for a coherence measure understood in terms of mutual probabilistic relevance to entail a consistency requirement, it must do so for reasons analogous to why the depth insensitive measures C_S and C_O entail a consistency requirement: it must consider n -wise coherence to be drastically more important than all other k -wise relations of support (where $k < n$) in determining the overall coherence of a set of propositions. To motivate a consistency requirement on coherence understood in terms of mutual probabilistic relevance, the proponent of a consistency requirement faces an even taller task than she does in motivating a consistency requirement on a coherence measure that treats coherence as a matter of agreement. And, thus, I think it even more plausible that a proponent of coherence as mutual probabilistic support could reject a consistency requirement on categorical coherence.

⁵⁴Again, since Roche's measure only depends on conditional probabilities, I am including it amongst the agreement measures.

I, thus, think our investigations have demonstrated that none of the intuitions about the coherence relation or coherence-making features forces a coherentist to accept a consistency requirement. Until some theoretical argument can be given for why coherentists should accept an equal weight or depth increasing weighting scheme, it seems to me that coherentists have no good reason to think substantive coherence requires a consistency requirement for any theory of the nature of the coherence relation. Perhaps such arguments can be given, but I think any such argument will have to be given in terms of a particular choice of a coherence measure. The impact of choosing a depth increasing versus equal weighting scheme will vary a great deal depending on the exact nature of the measure.

The last thing to note is that even if one wants to insist that any plausible account of the coherence relation will impose certain logical requirements on categorical coherence, either in terms of a consistency requirement or an accuracy-dominance avoidance requirement, our proceeding reflections were still not in vain. The results from this chapter should then aid the coherentist in her choice of a coherence measure. For example, if one thinks coherence requires consistency, then the results from the proceeding sections provide us with grounds to rule out depth increasing weighting schemes, and/or to prefer an account of coherence as mutual agreement over coherence as a matter of probabilistic relevance. So even if one approaches the debate over the relation of consistency and coherence in a dogmatic fashion, and thinks that the relation of coherence to consistency is a desiderata, as opposed to an issue up for debate, the formal investigation above should help narrow down the available choices of coherence measures. At the same time, until some reason is given for thinking we should prefer some particular weighting scheme over the alternatives, or accept that the coherence-making features of proper-subsets of all inconsistent sets must be outweighed by the incoherence-making features, the claim that coherence requires consistency ought to be appreciated for what it is, namely, dogma.

4.6. Conclusion

In this chapter we have covered a lot of territory, so let's briefly recapitulate the key steps that we have taken. We first distinguished two different kinds of inconsistency problems, cases where one is purportedly justified in believing all members of an inconsistent set of propositions, and cases where one is justified in believing some propositions, despite having inconsistent beliefs. We

noted that there are strong independent motivations for adopting a compartmentalized coherentism as opposed to a global coherentism, and noted how compartmentalization will help address the latter sort of problem, though not the former. In particular, we noted that there are some cases like the lottery and preface paradoxes where a compartmentalization approach will not answer inconsistency problems faced by coherentism, since there is no plausible sense in which the beliefs can be cognitively isolated from one another.

From there we went on to explore whether certain probabilistic accounts of coherence impose consistency and dominance avoidance requirements on categorical coherence. We first noted that the only measures that impose the strictest consistency criteria, namely that all inconsistent sets be maximally incoherent, are depth insensitive coherence measures, and we noted a variety of counter-intuitive implications that come with a measure being depth insensitive. In light of these reflections, we noted that there is a reason why all inconsistent sets ought not be treated as equally incoherent: their proper subsets may sometimes exhibit coherence-making features to differing degrees, and these coherence-coherence making features may well be quite abundant. So, at the very least, our reflections led us to observe that there is good reason why all coherentists ought to avoid the strictest consistency requirement.

Next, we went on to consider Roche's worry that many of the coherence measures in the literature fail to be defined over inconsistent sets of propositions. We eventually determined that the definition of many, though not all, coherence measures can be revised so as to be defined over most, if not all, inconsistent sets of propositions. We then set out to explore some of the obvious challenges in extending certain coherence measures, especially those that treat coherence as matter of mutual probabilistic relevance. In particular, we noted that we were forced to make a difficult choice. Either mutual relevance must be defined in a manner that allows it to be possible that we can increase coherence by adding contradictions, or else contradictory propositions must be treated in the manner of Fitelson's proposed factual support measure F . And we noted the theoretical costs that come with each choice.

At this point, we introduced all of our various coherence measures and found a few important trends. One important feature of most measures is that they treat cases like the tired logic students differently from the way that they treat a generous lottery example. That is to say, the maximum degree of coherence possible for a set containing a contradiction is less than the maximal degree

of coherence that is possible for a minimally inconsistent set. The exceptions to this rule are the various ways of revising Olsson's measure to make it depth sensitive. Consequently, there are a variety of accounts of the coherence relation that treat different kinds of inconsistency differently, and mostly these differences are what our intuitions would tell us to expect: dominated belief sets can be less coherent than minimally inconsistent belief sets. Ultimately, we were able to prove that in terms of basic intuitions, C_F provides a highly intuitive balance between tolerating some kinds of inconsistency (minimal inconsistency does not stand in the way of categorical coherence), while prohibiting other kinds (there is a very low ceiling of the degree of coherence that dominated sets are able to attain).

Additionally, our investigation led us to the inescapable conclusion that whether coherence is compatible with a consistency requirement comes down to a choice of the weighting scheme we choose. And, really, this shouldn't be surprising. Given that an inconsistent set can have some coherence-making features and some incoherence-making features, the question of a consistency requirement boils down to how much weight to give to the coherence-making features, and how much weight to give to the incoherence-making features. The end result of this investigation seems to shift the burden to those who claim that coherentism is committed to a consistency requirement. Until some reason can be given for choosing between weighting schemes, the claim that coherence requires consistency has the status of dogma, i.e., it is claim with nothing but appeals to authority to justify it.

CHAPTER 5

Pragmatic Coherence and Pragmatic Closure

5.1. Introduction

In the previous chapters, we have focused primarily on what tolerating some inconsistency in our beliefs means for various kinds of epistemic coherence constraints on justified belief. And, up to this point, I have argued that, from a purely epistemic point of view, inconsistent belief can sometimes be completely copacetic. There is no argument from our epistemic goal against tolerating some inconsistency that does either undermine itself or else have wildly implausible skeptical implications. Logic can still play a fairly robust constraint role on belief, either through logical constraints on degrees of belief, as argued by Christensen (2004), or else via accuracy-dominance avoidance principles that are derivable from our basic evidential norms, as argued by Easwaran and Fitelson (in press). Furthermore, we can still allow for apparently *sound* (not merely valid) arguments to be our guide when extending our beliefs. And, we have seen that there are various ways of understanding the substantive coherence relation that do not conflict with the tolerance of some inconsistency. Thus, allowing for some inconsistent beliefs to be justified doesn't force us to choose a side in the debate between foundationalists and coherentists. So, from a purely epistemic point of view, I see no good theoretical motivation for denying that in some cases we can have justified inconsistent belief.

We conclude our study of formal coherence constraints on belief and their relation to the inconsistency paradoxes by turning to a very different sort of normative challenge to justified inconsistent beliefs. Some have worried that, even if inconsistent beliefs engender no epistemic incoherence, they nevertheless engender a kind of pragmatic incoherence. In particular, Ross and Schroeder (2014) have claimed that if belief has an important pragmatic role to play in the sense that an agent who believes a proposition, p , will be disposed to treat p as true in her practical deliberations, then it follows that an agent will not be permitted to hold inconsistent beliefs (at least not for small sets of

closely related propositions).¹ They thus contend that there is an avenue through certain pragmatic encroachment views of justified belief to the conclusion that an agent cannot be justified in having an inconsistent set of beliefs, at least not for a small set of intimately related beliefs. In this chapter, we shall examine Ross and Schroeder's claim, and consider whether pragmatic encroachment theory brings with it any special motivations for formal coherence constraints on justified belief.

We shall pursue this question by focusing most of our attention on one particular formulation of a pragmatic encroachment theory, namely the version of pragmatic encroachment that has been defended by Fantl and McGrath (2002, 2009). We shall focus on their proposal for a few reasons.² First, Fantl and McGrath have provided one of the most detailed characterizations of the pragmatic encroachment thesis, and in doing so, they have provided a precise formalization of pragmatic conditions on justified belief that they intend to endorse. With their formalization of the pragmatic conditions on belief in hand, we can consider the logical properties of their pragmatic conditions (e.g., are these pragmatic conditions closed under logical consequence? Can they be satisfied by a logically inconsistent set of propositions? etc...) to see what if any logical constraints on belief follow from their pragmatic conditions. Second, Fantl and McGrath's view has been subjected to a variety of objections pertaining to engendering failure of deductive cogency constraints. For instance, Stanley and Hawthorne (2008) object on the basis that Fantl and McGrath's proposal engenders epistemic closure failure. Brown (2013) and Ross and Schroeder (2014) go so far as to argue that Fantl and McGrath's pragmatic conditions may allow for justified belief in logical contradictions.³ But these claims are controversial as they conflict with Fantl and McGrath's 'proof' that their pragmatic conditions enjoy certain closure properties, namely that they are closed under modus ponens. So, in pursuing this question, we will also come to better understand the theoretical benefits and costs of adopting a pragmatic view of belief of the sort defended by Fantl and McGrath. Third, and most importantly, I think we shall find that Fantl and McGrath's pragmatic conditions

¹Ross and Schroeder argue this point with the aim of establishing that their particular explanation of pragmatic encroachment enjoys a number of theoretical advantages over rival rival explanations of pragmatic encroachment, and one of the main advantages they claim for their view is that their view explains why we think we have a rational obligation to avoid inconsistency.

²Weatherson's (2005) pragmatic account of belief is closely related to Fantl and McGrath's (2009) pragmatic theory of justification. Weatherson's view is also subject to some of the objections that Ross and Schroeder (2014) make to Fantl and McGrath's (2009). Ultimately, I think Weatherson's view provides less compelling replies, and for this reason choose to focus most of our attention on a theory of justification that I take to be more promising, namely Fantl and McGrath's (2009).

³Ross and Schroeder (2014) put this objection in an unrestricted form. Brown (2013) merely argues that this follows on one interpretation of their proposal.

ultimately enjoy a highly plausible answer to the question of when inconsistency within one's beliefs can be tolerated from a purely pragmatic perspective. I shall argue that when Fantl and McGrath's pragmatic conditions are properly understood, they provide a rather plausible account of when, from a pragmatic point of view, we can and cannot afford to tolerate inconsistency within our beliefs.

5.2. The Pragmatic Role of Categorical Belief

5.2.1. What is Categorical Belief? In trying to understand pragmatism about epistemically justified belief, the obvious place to start is with how the pragmatic encroachment theorist answers the question: Why have categorical belief around in our theoretical tool-kit? We know, after all, that if we follow Christensen (2005, Ch. 2) and opt for an eliminativist attitude toward categorical belief, then we can formulate the principles of practical rationality in terms of constraints on an agent's degrees of belief. This allows for worries of inconsistent belief to be set aside. So what is the explanatory contribution of categorical belief to our theory of rational belief and rational agency? Weintraub aptly explains one of the pragmatic encroachment theorist's main motivations for thinking categorical belief is a useful thing to employ in our theory of rational agency.

Decision theory, it is often said, is an idealization. Even if it is not strictly true, its characterization of agents in terms of subjective probabilities and utilities is akin to the (false) Newtonian assumption that the planets are point-masses; indeed to the very supposition - incorporated in many scientific theories - that various properties (mass, electric charge, etc.) can be characterized in terms of real numbers. But the analogy is not apposite. The false assumptions we make about physical bodies yield predictions which are approximately true. We won't get very far, on the other hand, if we classify the planets in terms of the qualitative term 'has a large mass'.

When it comes to predictions of actions, the situation is reversed. We are seldom able to ascribe real-valued degrees of belief to agents in a predictively useful way. Of course, we can always fit utilities and probabilities to an individual action. But for another action, we will typically have to invoke substantially different values. So we cannot assume that degrees of belief remain stable, and neither can we predict how they change. But if we cannot determine degrees of

belief independently of the actions to which they give rise, they are useless for predicting behaviour, even if they can be ascribed consistently with it. Apart from fair lotteries, we do much better to invoke qualitative information about belief. Weintraub (2001, p. 443).

Weintraub goes on to conclude that both degrees of belief and categorical belief have a place in our theory of practical rationality. Although Weintraub does not explicitly endorse a pragmatic encroachment theory of epistemic justification, it seems to me that her argument against reductivism fits extremely well with Fantl and McGrath's account of the relationship between categorical belief and degrees of belief.

So what is categorical belief? Amongst pragmatic encroachment theorists, there are two competing answers that have been put forward.⁴ On the one hand, Fantl and McGrath, at least in some places, seem to hold that categorical belief is a matter of having a degree of belief sufficient to be moved to act in accordance with one's belief. More precisely, they say,

(PT): 'And a plausible thesis about the relation between credence and outright belief is that you have an outright belief that p only if your credence in p is high enough so that it wouldn't stand in the way of your being moved to act on the basis of p .' (2009, p. 44)

Ross and Schroeder (2014) refer to this sort of view as *pragmatic credal reductivism*. According to Fantl and McGrath, an agent has a categorical belief in p only when her degree of belief rises to a level where she can treat p as a reason for action when her decisions depend on whether p is true or false in a decision-context. It is worth emphasizing from the start that what counts as *high enough* is going to depend on the agent's decision-context. In so-called low-stakes contexts where nothing of great importance is at stake when acting on the basis that p , a lower credence may suffice than

⁴There is a nearby issue taken up by Dustin Locke (2013, p.4 footnote 6) regarding the norms of premising, i.e., when it is permissible to use claims as premises in practical deliberation. Locke argues that the norms of premising must be divorced from the norms of belief in the sense that one might be justified in believing p , despite the fact that one is not justified in treating p as a premise in one's practical deliberations. If he is right, then there is no need to accept that there are deductive cogency constraints on categorical belief that follow from pragmatic constraints on justified belief. So, our inquiry will be into whether pragmatic encroachment theories are committed for formal coherence constraints on justified belief. One issue worthy of further investigation is how considerations regarding pragmatic conditions on belief might apply to formal constraints on sets of propositions that we are permitted to treat as premises in practical deliberation. It seems plausible that our investigations in this chapter could help to illuminate the answer to that question.

what is required in a high stakes context.⁵ And it is in this sense that pragmatic factors play a role in determining whether an agent's evidence is sufficient for knowledge-level justification.⁶

Ross and Schroeder defend an alternative account of categorical belief that is meant to explain roughly the same pragmatic conditions on knowledge or justified belief, but in a way that avoids any kind of credal reductivism. They call their view the *reasoning dispositional account of belief*. This view boils down to the thesis that

(RDAB) believing that p defeasibly disposes the believer to treat p as true in her reasoning.⁷

What is it to treat p as true? Ross and Schroeder explain:

In the context of practical reasoning, we may say that an agent treats a given proposition p as true just in case she evaluates her alternatives by the same procedure by which she would evaluate them conditional on p . (2014, p. 264)

As Ross and Schroeder concede, there are places where Fantl and McGrath sound as if they mean to endorse a view roughly equivalent to this proposal. Throughout Fantl and McGrath's work, especially *Knowledge in an Uncertain World*, they speak of an agent believing a proposition just

⁵Dustin Locke, Ross and Schroeder may object to this version of the view. One of the critical things Ross and Schroeder observe is that one might have a high enough credence so that one would be rational to act as if the claim was true, but one may nevertheless attend to some of the contrary-possibilities in her practical deliberation. They contend that this would not then be a case where one has the full belief in question even though she has an adequately high credence. I think they are right and for this reason don't think the most plausible characterization of Fantl and McGrath's view is a view of pragmatic credal reductivism. Instead, Fantl and McGrath might be thought of as laying out the necessary epistemic conditions for when a belief that p is justified. Dustin Locke (2015) provides an intimately related pragmatic analysis of belief that avoids the problems raised by Ross and Schroeder.

Fantl and McGrath are sensitive to the fact that quite often a proposition might not be relevant to one's current decision-context. In such contexts, one doesn't act on the basis of p , but not because one's degree of belief is too low. On their account, one can fail to have sufficient confidence in the truth of a proposition when one wouldn't be moved to act on the basis of p if p were relevant to one's practical deliberations at a time. And, one's confidence can be sufficient because one would act on the basis of p were p of practical relevance.

⁶It should be noted that Fantl and McGrath (2009) clearly accept that a non-factive notion of justification can be sufficient for knowledge. See Littlejohn (2010) for discussion and criticism of their arguments on this point. See Fantl and McGrath (2012) for further discussion.

⁷The disposition is defeasible because in certain extremely high-stakes contexts, perhaps contexts where everything one values is on the line, one isn't willing to treat the proposition as true (Ross and Schroeder 2014, p. 267). As Maher (1986) and Kaplan (1996, 103–5) point out, if epistemic certainty isn't required, then there will be contexts in which one wouldn't be rational to act as if a proposition is true, no matter how good one's fallible evidence was. Thus, the disposition must be indefeasible, otherwise one would count as having almost no beliefs at all. See Roger Clarke (2013, pp. 15–16) for a clear explanation for why anything short of epistemic certainty would allow for certain contexts where one is not rational to act as if one's beliefs are true.

in case she is ready to put the proposition to work in her practical deliberations.⁸ And as we shall see below, the pragmatic conditions on belief that Fantl and McGrath endorse entail that to be justified in believing a proposition, it must be rationally permissible for one to deliberate as one would were she to have conditionalized on p . Exactly which position Fantl and McGrath should be understood as defending is not of critical importance. What is important is what, if any, formal coherence requirements on belief follow from these two ways of analyzing belief.

For that purpose, let us briefly consider what the views are supposed to have in common, and the ways in which they come apart, starting with how they come apart. The first and perhaps most significant difference between the two views is that there are clearly situations where one might be justified in acting as if p is true, have a credence in p high enough so that one would be rational to act in precisely the manner that would be rational conditional on p , and yet where one fails on the reasoning dispositions view to count as believing p . The basic sort of cases that Ross and Schroeder describe are instances where, despite the high degree of belief S has in p , S attends to various not- p possibilities when deliberating about what to do. Here is an informal example along the lines of a formal example described by Ross and Schroeder:⁹

(Hiking Case) Mrs. Takeahike is considering whether or not to pack her rain gear for a hike she will be going on. Her rain gear is heavy and so she would much prefer to leave it behind. She has read several weather reports and comes to have a very high degree of belief that it will not rain on her hike. Let us stipulate that she comes to rationally believe there is a 95% chance it will not rain (this is the likelihood cited by the national weather service, and many other highly reliable sources). She starts considering the possibilities, and thinks to herself: there is a non-trivial chance that it will rain. If it does rain, the temperature will make a big difference about how safe and uncomfortable I will be. She thinks to herself, even if the 5% chance turns out to be true, and it does rain, there is a 95% chance

⁸Ross and Schroeder make this clear in footnote 8, where they observe that Fantl and McGrath sometimes speak of belief as follows: if you believe that p then you must be “prepared to put p to work as a basis for what you do, believe, etc.” The footnote is on (R&S, 2012, p. 268). Their quote is from Fantl and McGrath (2009, p. 143).

⁹Ross and Schroeder (2014, p. 265) provide a highly formal case where an agent considers the various utilities for a bunch of mutually exclusive ways that the target proposition p could be false, and arrives at the calculation that it would be rational to act in a manner that would be rational conditional on p . For ease of presentation, I provide a less formal case that has the same basic upshot.

that it will be warm enough so that if it does rain, I will still be safe and reasonably comfortable. She thus decides to leave the rain gear at home.

We can stipulate that, in this case, Takeahike's degree of belief and evidence would have made it rational for her to not bring her rain gear even if it was going to be very cold. In other words, we can stipulate that it is reasonable for her to act as if it isn't going to rain. Does she believe it won't rain in this case? According to the pragmatic credal reductivist view, at least other things being equal, since her degree of belief will be above the threshold determined by her practical situation, she will count as believing that it won't rain. On the reasoning disposition view, however, she may not. The procedure she uses to make her decision is not the one she would employ on the assumption that it is not going to rain, since on that assumption, she would not need to go through and consider the various temperatures she might face that day if it does rain, and she wouldn't need to engage in the sort of probabilistic reasoning that she employs. Her evaluation of the alternatives is not the same as it would be conditional on p , and hence she doesn't treat the proposition as true in her deliberations. And, this is the main difference. No matter how high one's credence may be, Ross and Schroeder note that one might be inclined to attend to contrary-possibilities, reason in a probabilistic manner about what to do, and thus fail to believe on the reasoning disposition account. This difference may make all the difference as to whether pragmatic encroachment entails a logical consistency requirement on justified belief. Before turning to consider the implications of this difference, let us turn to the main feature that is supposed to unite the two accounts.

According to Ross and Schroeder, the reasoning disposition account can be understood as entailing pragmatic encroachment in the sense that knowledge that p requires being rational to act as if p is true (2014, pp. 271-271).¹⁰ The key premises in their argument is that one is justified in having an occurrent belief that p in a decision-context only if one is rational to act as if p is true in that decision-context, and justified occurrent belief is required for knowledge. Thus, justified occurrent belief is likewise meant to be subject to an analogous pragmatic constraint. Now, in what follows, we shall consider how the pragmatic encroachment theorist might derive a logical

¹⁰This contention has been disputed by (Lutz 2014, p. 1720). Lutz notes that the account may be interpreted as entailing that one knows p only if one is rational to act as if p , but notes that this is not forced on us by the reasoning disposition account. For our purposes, it will be useful to suppose, at least for arguments sake, that the reasoning disposition account entails the pragmatic constraints on belief that we consider in the next section. If the reasoning disposition account undermines the case for pragmatic encroachment as Lutz contends, then I think this will only serve to make it less plausible that a logical consistency requirement follows from the reasoning disposition account of justified belief. But more on that below.

consistency requirement on belief either directly from the pragmatic conditions on belief, or else from them in conjunction with the claim that one must be disposed to treat p as true.

Our first task for evaluating the case for a consistency requirement that comes from a pragmatic view of belief is to consider whether these pragmatic conditions are closed under logical entailment. If they are, then pragmatic encroachment theorists of both stripes would be committed to a logical consistency requirement on justified belief. After that, we shall broaden our perspective to consider whether being disposed to treat p as true, provides stronger motivations for a logical consistency requirement as Ross and Schroeder contend.

5.2.2. Pragmatic Constraints on Epistemic Justification. Given that, on the proposed views, when one categorically believes a proposition to be true in a decision-context, she is inclined to act accordingly, there are several basic pragmatic constraints on justified belief that we can formalize. The principle that first suggests itself is the following:¹¹

The Justification-Action Principle (JA): S is justified in believing that p only if
 S is rational to act as if p . (Fantl and McGrath 2002, p. 78)

This principle entails that if there is some action, A , that would be rational for S to perform on the assumption that p is true, but S 's current epistemic position is such that she wouldn't be rational to perform A , then S must not be justified in believing p to be true.¹²

Let us see how this principle works by considering an idealized decision-context. Let us suppose an agent S is faced with the choice between two actions, A and B , whose utility is related to the truth of p as follows:

D1	p	not- p
A	10	-10
B	9	9

¹¹Ross and Schroeder endorse a condition of this sort on knowledge, and can be reasonably thought to accept it for justified occurrent beliefs. I will go on to discuss ways of strengthening this condition so that it ranges over rational preferences. Ross and Schroeder do not say one way or another whether they would accept the more general condition. We shall thus simply consider what the implications are if one does accept the more general condition without imposing this assumption on Ross and Schroeder.

¹²On Ross and Schroeder's proposal, a version of (JA) would seemingly hold for occurrent beliefs, though perhaps not dispositional beliefs.

What the above matrix tells us is that if p is true, then S prefers A over B , and if p is false, she should be indifferent between the choices.

According to the justification-action principle, if the epistemic probability of p is too low for S to be rational to do A rather than B , then S is not justified in believing p to be true.¹³ It is straightforward to calculate the expected utilities of choosing A and B in terms of the agent's epistemic probabilities. In this case, we have the following expected utilities:

$$\text{Expected Utility of doing A} = Pr(p) \cdot 10 - Pr(\neg p) \cdot 10 = 10 \cdot (2 \cdot Pr(p) - 1).$$

$$\text{Expected Utility of doing B} = 9.$$

And, it follows that the expected utility of doing A exceeds that of doing B just in case $Pr(p) > .95$. Thus, in this decision-context, in order for it to be rational for S to act as if p is true, i.e., to choose to do A , S 's rational degree of belief in p must be at least .95 (by rational degree of belief, I mean the degree of belief the agent should assign given her epistemic position with respect to p). Thus, in this context, we get that if the agent's evidence makes it rational for her to have a degree of belief above .95, then it follows that she would be rational to act as if p is true. It would then follow that S meets at least this particular pragmatic condition on justified belief. One may still need to meet other pragmatic and/or epistemic conditions for one's belief to be justified in this decision-context. As we shall see, this is not the only pragmatic condition on justified belief that Fantl and McGrath endorse. In fact, this condition is strictly weaker than some of the other pragmatic conditions on justified belief that we shall consider.

So how could we strengthen Fantl and McGrath's pragmatic conditions on epistemic justification? In order to appreciate the motivation for the stronger pragmatic constraints on justified belief that Fantl and McGrath defend, it is useful to note that (JA) expresses the validity of a certain pragmatic argument form, namely:

(JA1) S is justified in believing p .

¹³Fantl and McGrath appeal to epistemic probabilities, presumably because the normative constraint is supposed to say when an agent should believe a proposition to be true, i.e., when she should assign the proposition a high enough probability to be able to use it as the basis of action. One's degree of belief can, of course, be irrational, i.e., not proportioned to the strength of one's evidential position. And so, an agent may have a degree of belief that exceeds the threshold, but for epistemically irrational reasons. For instance, if she really wants p to be true and has a degree of belief above the threshold, it shouldn't follow that she is justified. This problem is avoided by formulating the principle in terms of epistemic probabilities.

(JA2) *S* is justified in believing that if *p*, then *A* is the best thing she can do.

Therefore,

(JA3) *S* is rational to do *A*.¹⁴

Fantl and McGrath suggest that arguments of this sort seem to offer a valid way of reasoning about what an agent ought to do, given what she justifiably believes, and they note how counter-intuitive it be for someone to assert (JA1) and (JA2), but then to go on to deny (JA3).

Fantl and McGrath go on to observe that pragmatic arguments like the one above don't just seem valid when we restrict our attention to actions an agent might choose to perform. They note that such arguments seem plausible when we consider agent's judgments of their preferences. That is to say, Fantl and McGrath (2002, p. 76) claim that the following argument form also appears valid:

(J1) *S* is justified in believing *p*.

(J2) *S* is rational to prefer *A* to *B*, given *p*.

Therefore,

(J3) *S* is rational to prefer *A* to *B*, in fact.

To see the intuitive appeal of holding that such arguments are valid, suppose someone were to say the following.

C1: Sam is justified in believing that he will be having steak at the party this evening.

C2: Sam is rational to prefer having red wine to white with dinner, given that she is having steak this evening.

C3: But, Sam is not rational to prefer having red to white wine with dinner.

Just as in the case of actions, Fantl and McGrath note that such judgments seem rather incoherent. Fantl and McGrath thus suggest that there is strong intuitive support for accepting the corresponding principle:

¹⁴Fantl and McGrath (2002, pp. 74-76) note that a commitment to a knowledge-action principle would mean committing to a version of this argument form where justification is replaced by knowledge. So, I am here simply extrapolating from their observation. They endorse a generalized preference version of this argument form that I discuss below.

(JP1) If S is (epistemically) justified in believing A , then for all states of affairs, A and B , S is rational to prefer A to B , given p only if S is rational to prefer A to B , in fact.
(2002, p. 77)

This principle expresses a commitment to the validity of the argument form in question, but Fantl and McGrath think this principle is only half of the picture. They maintain that the following argument form enjoys just as much intuitive support as that captured by JP1:

(J4) S is justified in believing p .

(J5) S is rational to prefer A to B .

Therefore,

(J6) S is rational to prefer A to B , given p .

Fantl and McGrath offer a defense of this argument via a reductio (See Section 5.7 below for references and discussion), which we shall consider in due time. For argument's sake, let us now grant that the argument form enjoys some prima facie intuitive appeal. According to Fantl and McGrath, if one accepts (JP1), one has equally strong reason to accept:

(JP2) If S is (epistemically) justified in believing A , then for all states of affairs, A and B , S is rational to prefer A to B , in fact only if S is rational to prefer A to B , given p .

Together (JP1) and (JP2) entail that one is justified in believing p just in case the rational preference ranking one is rational to have on the assumption that p is true corresponds to the rational preference ranking one has in fact. Fantl and McGrath express this via the following principle:

(JP) If S is justified in believing p , then S is rational to prefer as if p . (2002, p. 77)

Informally, to rationally prefer as if p is to have preferences that are preserved on the assumption that p is true. It will be useful to consider how we can unpack this principle in formal terms.

5.3. Formalization of the Pragmatic Conditions on Belief

For the sake of understanding the relationship between Fantl and McGrath's pragmatic conditions on belief and logical consistency and closure principles, it will be useful to consider how Fantl and

McGrath formalize their pragmatic conditions. They offer the following two definitions as a way of formally understanding the consequent of (JP).

(D1) S is rational to prefer X to Y , given $p =_{def} S$ is rational to prefer $X \& p$ to $Y \& p$. (p. 91)

(D2) S is rational to prefer as if $p =_{def}$ for any states of affairs X and Y , S is rational to prefer X to Y , given p , iff S is rational to prefer X to Y , in fact. (p. 91)

In effect, (D1) is meant to unpack what it means for S to be rational to prefer one state of affairs to another, on the assumption that p is true. It is useful to note that in the formal framework Fantl and McGrath appeal to as a means to formally explicate the nature of an agent's rational preference ranking, one is rational to prefer $X \& p$ to $Y \& p$ just in case one is rational to prefer X to Y upon updating one's subjective probability function by conditionalizing on p .¹⁵ In other words, (D1) entails that one is rational to prefer X to Y , given p just in case one would be rational to prefer X to Y , were one to assign p a probability of 1 and update one's preferences accordingly. So, intuitively, one is rational to prefer as if p if and only if from a practical standpoint, one could treat p as certain without changing one's practical judgments. Keeping this understanding of (D1) in mind, (D2) then can be understood as holding that when one is rational to prefer as if p is true, all of one's preferences would be preserved were one to update her subjective probabilities by conditionalizing on p . So, more informally, Fantl and McGrath's pragmatic conditions entail that to be justified in believing p , one must be able to treat p as an assumption for all intents and purposes.

5.3.1. Preferences as a function of desire and degrees of belief. It will be useful to pause momentarily to consider how an agent's preferences can be understood in terms of the relationship between her desires and subjective probabilities or degrees of belief. In the *Logic of Decision*, Jeffrey (1990) suggests that one's preference ranking over propositions (or states of affairs) can be interpreted in terms of a desirability function, $Des()$, which is defined over all propositions in one's language with the exception of the impossible proposition \perp .¹⁶ Each proposition is assigned some

¹⁵Jeffrey (1990, p. 90) states this assumption explicitly. We shall show below that it need not be taken as an assumption, but rather is derivable from Jeffrey's axiomization of the desirability function used to formulate an agent's preference ranking.

¹⁶Fantl and McGrath point to Jeffrey (1990) as a means to understanding choices of actions as just part of a broader theory of an agent's rational preference ranking of propositions. They never explicitly endorse Jeffrey's axiomatization of a desirability function, but they do make many assumptions that are derivable from that axiomatization.

real value by an agent's desirability function, and the agent's preferences are ordered accordingly. An agent's rational preference ranking is given by the ordering on the values assigned by the desirability function. So an agent S prefers p to q just in case $Des(p) > Des(q)$. The formal constraint on an agent's degrees of belief or subjective probabilities is that they should respect the probability axioms presented in Chapter 1. The only additional axiom required to formulate the relationship between an agent's desires and subjective probabilities is the following:

(*Des Axiom*) If $Pr(X \wedge Y) = 0$ and $Pr(X \vee Y) \neq 0$

$$Des(X \vee Y) = \frac{Pr(X) \cdot Des(X) + Pr(Y) \cdot Des(Y)}{Pr(X) + Pr(Y)} \quad (\text{Jeffrey 1990, p. x80})$$

This axiom requires that the desirability of any proposition is the weighted average of the desirability of all of the mutually exclusive ways that the proposition could be made true.¹⁷

To see how this works, let us consider another toy example. Suppose part of S 's desirability function is represented by the following matrix.

D2	p
q	10
$\neg q$	5

According to *Des Axiom*, the desirability of p will be the weighted average of the desirability of $p \& q$ and $p \& \neg q$ as these represent two mutually exclusive and, in this simplified setting, exhaustive ways (in the sense that $(p \& q) \vee (p \& \neg q) = p$) that p could be true. If, for instance, $Pr(q) = Pr(\neg q)$, each way that p could be true would then be given equal weight, and so the desirability of p would be the straight average of $Des(p \& q)$ and $Des(p \& \neg q)$: $Des(p) = .75$. If, on the other hand, the conjunction of p and q is twice as likely as the conjunction of p and $\neg q$, i.e., $Pr(p \& q) = 2 \cdot Pr(p \& \neg q)$, then $Des(p) = 8\frac{1}{3}$. More generally, the closer $Pr(q)$ is to 1, the closer $Des(p)$ is to 10.

¹⁷If we think of propositions as sets of possible worlds, one can think of each singleton set containing a particular possible world as being propositions with exactly one weigh of being made true. Since there is only one way to be true, as one's subjective probabilities are revised, the desirability function will continue to assign singleton sets the same value, unless the world contained in the singleton has no chance of being true on the updated probabilities. Thus, on Jeffrey's proposal, since there is just one way in which such a singleton set can be realized (the world in the singleton can be the actual world), each such singleton's desirability remains fixed if it isn't ruled out as impossible.

This gives us a static picture of how the desirability of p depends on S 's degree of belief in q , and fits with Jeffrey's assumption that if one were to conditionalize on q , then one's new desirability function $Des(\cdot|q)$ would be such that $Des(p|q) = Des(p \& q)$.¹⁸ So, were one to become certain that q was true, according to the above matrix, it would follow that $Des(p) = 10$. This is why it is reasonable to think of the claim that one is rational to prefer X to Y , given p can be equated to the claim that one is rational to prefer $X \& p$ to $Y \& p$. And, formally this helps to explain why, one can think of (JP) as a condition that states that one is justified in believing p just in case one's rational preferences would be preserved were one to update one's degrees of belief by conditionalizing on p .

Another important feature of this formalization we should note is that there is good reason why we exclude the impossible proposition from our preference-ranking. As Jeffrey explains,

It is not that $Des [p \wedge \neg p]$ has the value 0; rather $Des [p \wedge \neg p]$ has no value at all, not even 0. We leave the impossible proposition out of the preference ranking [(i.e., any proposition logically equivalent to $p \& \neg p$)]. Jeffrey (1990, p. 78).

Given that the *Des* Axiom holds that the desirability of each proposition is the weighted average of the mutually exclusive and exhaustive ways that the proposition could be true, and since there are no ways that the impossible proposition could be true, there is no reasonable way to compute the desirability of the impossible proposition. This may seem like a trivial point, but the implications Fantl and McGrath's proposal has for epistemic closure and logical consistency principles turn on how we handle impossible propositions.

The last feature of this proposal we should note is that Jeffrey's approach to defining rational preferences over all states of affairs can also be used to formalize Fantl and McGrath's justification action principle. In fact, as Fantl and McGrath (2002, footnote 7) point out, in Jeffrey's explication of the decision-theoretic framework, actions can be treated as a special kind of state of affairs, where we can think of our ranking of action as the ranking of the state of affairs brought about

¹⁸See Jeffrey (1990, p. 90) for his initial presentation of this assumption. For further discussion of the kinematics of the desirability function's change in cases where one revises one's degrees of belief but doesn't do so by conditionalizing on any particular proposition (that is, when one may increase or decrease her belief in some propositions, but not by coming to hold them as certainly true or false), see Jeffrey (1990, pp. 164-177).

It is also worth noting that this assumption is actually derivable from Jeffrey's definition of conditional probability and his axiomatization of *Des*. Also see Jeffrey (2004, Ch. 3) for a detailed presentation for a generalized theory of conditionalization, for instance, an account of how to revise one's subjective probabilities on the basis of probabilistic observation reports (reports of the form: there is a x percent chance that p is true).

or proposition made true by the agent S performing the corresponding action.¹⁹ Thus, in this formal framework, if an agent is justified in preferring as if p , the agent must be rational to act as if p . Thus (JP) entails (JA). Consequently, meeting the formal conditions specified by (JP) can be taken as sufficient for satisfying the formal conditions specified by (JA).²⁰ This then covers the basic pragmatic conditions that help us to understand when we are rational to act on the basis of a proposition p . Of course, nothing in the pragmatic conditions that we have considered thus far tell when an agent's epistemic position or evidence is sufficient to justify her in believing a proposition. All we have considered so far are constraints on justified belief. But if Fantl and McGrath are right about the formal properties that their pragmatic conditions on belief enjoy, a logical consistency requirement on justified categorical belief immediately follows from these necessary conditions.

5.4. Closure Under Modus Ponens

Fantl and McGrath have worried about whether their pragmatic conditions on justified belief are closed under modus ponens since those argument forms we considered above appear to assume that knowledge or justification is closed under modus ponens.²¹ If their pragmatic conditions are closed under modus ponens and conjunction introduction as well, then it follows that their pragmatic conditions entail a logical consistency requirement on justified belief.

The first step to seeing why Fantl and McGrath's pragmatic conditions, as officially formulated, entail a logical consistency requirement is to note their proof that the condition on belief specified by D1 and D2 is closed under modus ponens. Fantl and McGrath (2002) purport to provide a proof

¹⁹See Jeffrey (1990, p. 83-84) for his explanation of this point.

²⁰We are making the relationship explicit because it seems to be the key thing Hawthorne and Stanley (2008) have overlooked in the argument to be presented below.

²¹It is far from clear to me that this is the right way to think of the argument forms in Section 5.2.2 above. Fantl and McGrath's proof of closure assumes that the conditional employed in the argument form is a material conditional, and they aim to establish that their pragmatic conditions are closed under modus ponens when the conditional is so understood. As we shall see below, their proposal faces the dilemma that their proof of closure is sound on pain of their view entailing infallibilism. They might worry then that the argument form in question is shown to be invalid and so there exist counter-examples to the principles that are meant to motivate their pragmatic conditions on belief. The obvious reply available to them is that the argument in question involves a very different kind of conditional. It may well be that the argument is valid when the conditional is properly understood. What will be clear below is closure of modus ponens for the strengthened conditional must not entail closure under logical consequence. We shall leave the question of the proper formulation of the conditional for later research, and simply note that the results to come need not be taken to refute their formulation of pragmatic encroachment, it just undermines the interpretation of the principles (JP) and (JA) as involving a material conditional as they interpreted in their (2002) paper.

I think their later work regarding the purpose of categorical belief and justified categorical belief provides a more general picture regarding the pragmatic role of categorical belief and this general picture provides motivations for their pragmatic conditions on justified belief that would survive counter-examples to knowledge or justification's closure under modus ponens.

of this fact in appendix B of ‘Evidence, Pragmatics and Justification.’ In the process, they also appear to prove that their pragmatic conditions are closed under single premise entailment (though they don’t make this explicit). Fantl and McGrath say the following:²²

Our condition [JP] on justification is supported by intuitions about the closure of knowledge under modus ponens. It is incumbent upon us, then, to show that our condition itself has this closure property. For suppose our condition isn’t closed under modus ponens. The worry would arise whether the truth of our account would undermine its support. Although the failure of closure for a necessary condition on justification doesn’t entail the failure of closure for justification (and therefore knowledge), all the same, it would be cause for concern. We would need to appeal to further elements of the concept of justification in order to show how justification could have the closure property even though a necessary condition of it didn’t.

We therefore seek to show that our condition is closed under modus ponens, or in other words, that the following argument is valid:

(1''') S is rational to prefer as if p.

(2''') S is rational to prefer as if $(p \supset q)$.

Therefore,

(3''') S is rational to prefer as if q. (2002, pp. 90-91)

Fantl and McGrath go on to give a formal proof that their pragmatic conditions are closed under modus ponens.²³ Let $Pref(p)$ formally stand for the statement that S is rational to prefer as if p . Fantl and McGrath show that the following theorem is provable for their pragmatic conditions on belief.

THEOREM 5.1. *If $Pref(p)$ and $Pref(p \supset q)$, then $Pref(q)$.*

While I have relegated most of the proofs we have considered to appendices, this is one proof that we shall need to carefully consider. The formal coherence constraints on justified belief imposed

²²See Warfield’s ‘When Closure Does and Does not Fail’ for a discussion of why a failure of a necessary condition on knowledge to be closed under logical entailment need not result in knowledge failing to be closed under logical entailment.

²³It has been claimed by Stanley and Hawthorne that their pragmatic conditions fail to be closed under conjunction-introduction.

by Fantl and McGrath's pragmatic conditions turn on whether their proof is sound, which we now consider. We can streamline their proof if we note that D1 and D2 together entail D3:

D3: S is rational to prefer as if p if and only if for any states of affairs X and Y ,
 S is rational to prefer $X \& p$ to $Y \& p$ iff S is rational to prefer X to Y , in fact. (p.
 91)

Proof. D3 follows from D1 and D2 by simply substituting S is rational
 to prefer $X \& p$ to $Y \& p$ for S is rational to prefer X to Y , given p in D2.
 D1 says we can do this because the thing we substituting is equivalent
 by definition.

It is also worth noting that their proof is a biconditional proof. For the sake of our considerations, it will only be necessary to consider one direction of the proof and to note that the other direction is essentially the same argument taken in reverse.

Proof. The proof then proceeds as follows simply by observing that if S is rational to prefer as if r , then we can add r as a conjunct to both sides or eliminate r from both sides of any rational preference claim about S . With this assumption by Fantl and McGrath made explicit, the proof can be presented as follows:

- | | |
|---|--------------|
| 1. S is rational to prefer as if p . | Assumption |
| 2. S is rational to prefer as if $p \supset q$. | Assumption |
| 3. Assume S is rational to prefer A to B . | Assumption |
| 4. S is rational to prefer $A \& p$ to $B \& p$. | 1, 2 and D3. |
| 5. S is rational to prefer $A \& p \& (p \supset q)$ to $B \& p \& (p \supset q)$ | 2, 3 and D3 |
| 6. S is rational to prefer $A \& p \& q \& (p \supset q)$ to $B \& p \& q \& (p \supset q)$ | 5 and SLE. |
| 7. S is rational to prefer $A \& p \& q$ to $B \& p \& q$. | 2, 6 and D3. |
| 8. S is rational to prefer $A \& q$ to $B \& q$. | 1, 7 and D3 |
| 9. S is rational to prefer A to B , given q . | 8 and D1. |

In order to complete the proof, we would need to also show that if we assumed the conclusion, then we could derive our third assumption. I won't duplicate the proof as it would proceed in exactly reverse order. From 1, 2 and 9, we could move to 8, and then 7 and so on until we derive our third

assumption in an exactly analogous manner. In light of this proof, Fantl and McGrath conclude that this pragmatic condition on belief is closed under modus ponens.

5.4.1. Generalizing Their Proof: Pragmatic Closure Under Multi-Premise Entailment. Now, we are one small step away from proving that Fantl and McGrath’s pragmatic conditions are closed under multi-premise entailment. All that we would need to show is that their pragmatic conditions are also closed under conjunction-elimination, since together closure under conjunction-elimination and modus ponens will be sufficient for any condition to be closed under multi-premise closure. Christensen (2004) makes a similar observation for attempting to impose limited closure principles on belief (as opposed to conditions on belief):

If an ideally rational agent believes both P and $((P \supset Q))$, she believes Q . Suppose we tried to advance a limited closure principle as follows: if Q is entailed by any pair of an ideally rational agent’s beliefs, then the agent believes Q . But it would also amount to imposing an unlimited closure requirement. For any two beliefs will entail their conjunction; and, once that is admitted as a belief, it may in turn be conjoined with a third belief, etc., until the agent is required to believe any proposition that is entailed by any finite number of her beliefs. This is, of course, incompatible with the threshold account of rational binary belief, as the lottery cases demonstrate. (p. 26)

Now, Fantl and McGrath aren’t arguing that *belief* is closed under modus ponens, just that a necessary condition on belief is closed under modus ponens, namely *the property of being rational to prefer as if a proposition is true*. And they are not claiming that belief is closed under entailment for any two propositions (regardless of the arguments form). Nevertheless, the point generalizes for this property. In this case, we will need to show that Fantl and McGrath’s pragmatic conditions are closed under conjunction-elimination. Ultimately, it is straightforward to show that D3 delivers closure under conjunction-elimination via a reductio proof:

THEOREM 5.2. *If $Pref(p \& q)$, then $Pref(p)$ and $Pref(q)$.*

PROOF. Assume S is rational to prefer as if $p \& q$. Now, for a reductio, suppose S is not rational to prefer as if p . There must be some A and B such that it is not the case that S is rational to prefer A to B if and only if S is rational to prefer $A \& p$ to $B \& p$. There are two cases to consider.

Case 1: S is rational to prefer $A \& p$ to $B \& p$ but S is not rational to prefer A to B . By addition via D3, it follows that S is rational to prefer $A \& p \& (p \& q)$ to $A \& p \& (p \& q)$. By equivalence, S is rational to prefer $A \& p \& q$ to $B \& p \& q$, which via D3 means S is rational to prefer A to B . Contradiction!

Case 2: S is rational to prefer A to B , but S is not rational to prefer $A \& p$ to $B \& p$. By D3, S is rational to prefer $A \& p$ to $B \& p$ if and only if S is rational to prefer as if $A \& p \& (p \& q)$ to $B \& p \& (p \& q)$. By equivalence, S is rational to prefer $A \& p$ to $B \& p$ if and only if S is rational to prefer as if $A \& p \& q$ to $B \& p \& q$. By D3, S is rational to prefer A to B if and only if S is rational to prefer as if $A \& p \& q$ to $B \& p \& q$. It follows that S is rational to prefer $A \& p \& q$ to $B \& p \& q$, and thus that S is rational to prefer $A \& p$ to $B \& p$. Contradiction!

□

The key thing to note about the proof of Theorem 5.2 is that it only assumes principles and inferences that are necessary for Fantl and McGrath's proof that their pragmatic conditions are closed under modus ponens. As we shall see, this means that if Fantl and McGrath's pragmatic conditions are closed under modus ponens, then they are closed under multi-premise entailment. The proof of this depends on the following Lemma:

LEMMA 5.3. *Let R be any property of propositions. If R is closed under modus ponens, conjunction-elimination and substitutivity of logically equivalent propositions, then it follows that R is closed under multi-premise entailment.*

PROOF. Let $R(p)$ for all $p \in X$ and suppose $X \vdash q$. Assuming compactness, there is a set X' such that X' is finite and $X' \vdash q$. Let us enumerate the members of X' so that $\{p_1, \dots, p_n\} = X'$. We can reason as follows:

- (1) $R(p_1), \dots, R(p_n)$. (by hypothesis).
- (2) $R(p_1 \supset p_1 \wedge p_2)$ (the fact that p_2 is logically equivalent to $p_2 \wedge (p_1 \supset p_1 \wedge p_2)$ and conjunction-elimination).

- (3) $R(p_1 \wedge p_2)$ (by closure under modus ponens and premises 1 and 2).
- (4) $R(p_3 \supset p_1 \wedge p_2 \wedge p_3)$ (by the fact that $p_1 \wedge p_2$ is equivalent to $p_1 \wedge p_2 \wedge (p_3 \supset (p_1 \wedge p_2 \wedge p_3))$).
-
- (5) Thus, $R(p_1 \wedge \dots \wedge p_n)$.
- (6) $(p_1 \wedge \dots \wedge p_n)$ is equivalent to $(p_1 \wedge \dots \wedge p_n) \wedge q$, since q is entailed by X' .
- (7) So, $R(p_1 \wedge \dots \wedge p_n \wedge q)$.
- (8) By n -many applications of conjunction elimination, we eventually obtain that $R(q)$.

□

Now, it immediately follows from this that the only way Fantl and McGrath's pragmatic conditions are satisfied for an inconsistent set of sentences is if their pragmatic conditions are trivial. This is delivered by the following theorem:

THEOREM 5.4. *For any property P , if N is a necessary condition of P and N is closed under multi-premise entailment, then P only holds of an inconsistent set of propositions if N holds of all propositions whatsoever.*

PROOF. This follows trivially. If P holds of an inconsistent set of claims then so does N . Since an inconsistent set logically entails every proposition, it follows that N holds of every proposition by the closure of N . □

Now, we know that Fantl and McGrath's pragmatic condition will not hold of every proposition for an agent that has any practical interests whatsoever (an agent has practical interests if there is any proposition p such that if p is true A is preferable to B , but if p is false then A isn't preferable to B). Suppose I prefer chocolate to vanilla ice cream, unless I am having a Sundae. In which case, I prefer Vanilla. Then either the proposition I am having a sundae or its negation fails to satisfy their pragmatic conditions for me. I am either not rational to prefer as if I am having a sundae, or else I am not rational to prefer as if I am not having a sundae. So, Fantl and McGrath's pragmatic condition will only hold of absolutely all propositions if S is an agent who doesn't care about anything whatsoever, and so doesn't care about being right or wrong about anything whatsoever. Consequently, for ordinary human agents engaged in any activity where she has some preferences over the outcomes, Fantl and McGrath's pragmatic condition does not hold of all propositions.

By Theorems 5.1-5.3 above, it follows that Fantl and McGrath's pragmatic conditions do rule out JIB. On their proposal, it is impossible for an agent to be epistemically justified in believing each of a logically inconsistent set of propositions. Consequently, it looks as though their proposal must deliver a consistency constraint on epistemically justified beliefs in the form of a pragmatic defeater condition on epistemic justification.

5.4.2. Taking Stock. Given the level of formalization of the last section, it is worth pausing to take stock of the philosophical road we have taken so far in this chapter. We started out considering whether a pragmatic analysis of the normative role of belief or epistemic justification would furnish us with a pragmatic defeater condition on epistemic justification that would preclude justified inconsistent beliefs altogether. The answer we seem to have arrived at is that if Fantl and McGrath's proof that their pragmatic conditions are closed under modus ponens is sound, then the results can be generalized and their pragmatic conditions on belief rule out all inconsistency. As formulated, Fantl and McGrath's pragmatic conditions do seem to entail that it is never acceptable for an agent to believe a logically inconsistent set of claims. For many epistemologists who want to locate a fallibilist position that is not only compatible with but entails an unrestricted consistency requirement on categorical belief, a pragmatic analysis of belief or epistemic justification appears to deliver.

Alas, appearances can be deceiving. There are a couple of facts that should make us suspicious of the claim that a pragmatic condition on belief can deliver so much. The first is that Fantl and McGrath's opponents have contended that their proposal has the opposite problem, namely that it tolerates too much inconsistency. In fact, Ross and Schroeder (2014) and Brown (2013) have recently suggested that (JP) is subject to the problem that it cannot rule out justified belief in contradictory propositions (i.e., $B(p)$ and $B(\neg p)$). The question both papers raise is why can't one prefer as if p and not- p in cases where p is of no practical relevance whatsoever (That is to say, if one's preferences would be exactly the same if the proposition were true or false). If the formal arguments above are correct, somehow Fantl and McGrath's pragmatic analysis rules this out. But it is hard to see how one could fail to be rational to prefer as if p when p is of no practical relevance (intuitively, a proposition being practically irrelevant entails that the truth or falsity of a proposition makes no difference to one's rational preference ranking).

The other reason to be skeptical is that it is not at all clear how such a view could possibly avoid inconsistency in cases of lotteries. Perhaps it might avoid justified inconsistent beliefs in certain kinds of lotteries, namely those where the lottery propositions are collectively relevant to an agent's practical deliberations. But in general, when lottery propositions have a near one probability, how do Fantl and McGrath's pragmatic conditions rule out justifiably believing such propositions without collapsing into infallibilism? Alas, the answer is that their pragmatic conditions entail infallibilism.

5.5. Infallibilism Objection

5.5.1. Informal Presentation of the Problem. Up until the consideration of pragmatic analysis of justified belief, we have seen that only one approach to epistemic justification delivers a full blown logical consistency constraint, views according to which full belief in a proposition p is identified with being subjectively certain about p , i.e., assigning p probability 1. The pragmatic conditions on belief appear to deliver a very different route to a consistency requirement, but the obvious worry to have is that they only entail a consistency requirement in virtue of imposing infallibilist standards on epistemically justified belief. Subjective certainty is sufficient to be able to treat a proposition as true in one's practical deliberations, and so, as Fantl and McGrath note, their pragmatic conditions on belief are entailed by infallibilism. But, they deny that those pragmatic conditions entail infallibilism. Alas, as officially formulated, Fantl and McGrath's pragmatic conditions do have infallibilist implications. As we shall see, the problem is that as defined, one can be rational to prefer as if p is true only if one is epistemically certain that p is true. I will provide a careful formal argument for why their pragmatic conditions, as officially formulated, entail infallibilism, but let us begin with an informal presentation of the problem.

5.6. Pragmatic Argument for Infallibilism

The easiest way to appreciate why their pragmatic conditions entail infallibilism is to consider an example. Suppose that I am at the grocery store picking up items for a potluck dinner. My friend calls me to let me know that reports have come out regarding a wine shortage in my area. Let us suppose my friend is generally reliable, that his source of the information is reliable, and that this information makes it reasonable for me to be nearly, though not completely, certain that

wine will be unavailable at the dinner. I have agreed to pick up certain items regardless of what alcohol will be available, though there are some that I should only pickup if wine is available. Then this is a situation where the following proposition that I will not be having wine has a high epistemic probability on the available evidence, and where we might stipulate that it is rational for me to act as if I will not be having wine (We stipulate that I will not purchase the items that we would want with wine). Supposing I believe I will not be having wine, and that belief is true and ungettiered, one might think then this is just the sort of case where a pragmatic encroachment theory of justification would allow that I know or am epistemically justified in believing that I am not having wine. But, now, suppose for illustration sake that I prefer the state of affairs where I have red wine to the state of affairs where I have white. These states of affairs are highly improbable, given my friend's testimony and other evidence, but not entirely ruled out. Thus, they have some very low epistemic probability. On Jeffrey's framework (and any plausible formalization of Fantl and McGrath's pragmatic conditions), this means they would still certainly fall somewhere on my rational preference ranking even if we just look at my preferences at this particular moment in time. Moreover, these preferences are relevant to my rational deliberations in this context. Thus, this assumption is decision-theoretically coherent (i.e., is possible by the lights of standard decision-theory). But now notice that with this assumption, the following argument should be valid if I know or am justified in believing that I will not have wine.

IF-Argument.

(IF1) I am rational to prefer the state of affairs where I have red wine with dinner to that where I have white.

(IF2) I am justified in believing that I will not be having wine.

Thus,

(IF3) I am rational to prefer that I have red wine with dinner to the state of affairs where I have white, given that I will have neither.

By Fantl and McGrath's proposal, (IF3) is shorthand for:

(IF3') I am rational to prefer having red wine and not having wine to having white wine and not having wine.

Once put into this form, it is easy to see that (IF3) is absurd, and is so for a multitude of reasons. First and foremost, the propositions being compared are both impossible, and so it makes no sense to say I prefer one over the other. How do we rank the preferability of a logical contradiction? As we noted above, the impossible proposition is left off of the preference ranking altogether.

What this means is that if either A or B is an impossible proposition (i.e., a proposition logically equivalent to $p \wedge \neg p$ for some p), then it is not true that S is rational to prefer A to B . And so (IF3) is necessarily false by the lights of the framework Fantl and McGrath point to as the way to formalize their proposal. Since (IF3) is absurd and the argument from (IF1) and (IF2) is an instance of (JP2), the above argument shows that (IF1) and (IF2) cannot both be true at the same time if (JP2) holds. Since, in the above case, (IF1) is built in to the description of the case (I do prefer red to white wine), it must be true. Consequently, we can conclude that on Fantl and McGrath's proposal, I must not be justified in believing nor know that I will not be having wine in the above scenario.

Let us pause for a moment to ask, is this a bad thing? Initially, it might not seem so. One might be inclined toward the following flatfooted response. Whether I will be having wine or not is of practical interest to me, given that I have preferences over the kind of wine I might be having. Those rational preferences do not make sense if I take it as given that I will not be having wine.²⁴ If I am justified in believing something, then it must be reasonable for me to take it as given when engaging in practical deliberation, including my preferring. In this case, I cannot take it as given, and so I am not justified in believing it. Fantl and McGrath's pragmatic conditions have exactly the right sort of implications for the above case.²⁵

There are several problems with the flatfooted response. First, the argument has skeptical implications for anyone who has a general disposition to prefer red to white wine.²⁶ Suppose one has a general disposition in the sense that whenever having wine is epistemically possible, she prefers

²⁴One might want to insist that rational preferences in these cases should be given a counter-factual reading. I think this is a mistake. I am certainly not denying that one may also have preferences that are dispositional in nature, and hold counter-factually. However, in the formal framework appealed to, an agent can have preferences over propositions that have very low probabilities. As long as this is the case, the agent might have well defined desires over these propositions. Even if such propositions are always removed from the agent's preference ranking, we need some guarantee that the removal procedure doesn't invalidate the argument for the closure of their pragmatic conditions under modus ponens.

²⁵This is analogous to Dodd's (2011) flatfooted acceptance of infallibilism on the grounds that it sits best with our assertive practices.

²⁶This is not something that would bother Dodd (2011), but is an implication I find highly implausible.

red to white wine and this fact is reflected in her rational preference ranking in all contexts (We assume it is an indefeasible disposition for simplicities sake, but nothing hinges on this). Then the only way that the above argument for thinking that an agent S is not justified in believing she will not have wine could fail (assuming JP2) is if S is epistemically certain that she will not be having wine. If the proposition that she will not have wine has probability zero for S , then the proposition that she is having red wine (and the same for white) will not be included in S 's rational preference ranking (at least not in Jeffrey's framework). Consequently, anyone with such dispositional preferences will only be in position to justifiably believe or know that they are having wine when that belief is epistemically certain. One almost never has such evidence for contingent propositions of this sort, and so an agent with such a disposition will never have fallible knowledge that she will not be having wine.

Second, there is nothing special about the proposition in question to think this sort of skeptical argument is limited to some small subset of contingent propositions. Any proposition whose negation is of any practical relevance at all, i.e., is more or less preferable than some proposition, will be subject to an IF-argument exactly like that above. In other words, suppose not- p is practically relevant to S (which implies that p has probability less than 1). Without loss of generality, suppose S is rational to prefer not- p to some proposition A . Then by (JP2), we can reason as follows:

(IF4) S is rational to prefer not- p to A .

(IF5) S is not rational to prefer not- p to A , given p .

Therefore,

(IF6) S is not justified in believing that p .

Again, if (JP2) is valid then this argument must be as well (IF4 and IF6 entail not-IF5 according to JP2). This doesn't yet establish that Fantl and McGrath's pragmatic view entails infallibilism, but it gets us very close to that conclusion. It establishes that, on their view, infallibilism holds for propositions whose negations have any practical relevance for an agent whatsoever. In one small step, we can take the argument to the stronger conclusion, that infallible knowledge is simply unavailable for agent's with any rational preferences (i.e., if there are any propositions A and B such that S prefers A to B) if (JP2) is valid.

The easiest way to make the step is to note that there is only one way that an argument like that above cannot be constructed for some proposition p whose negation is epistemically possible (i.e., $Pr(\neg p) > 0$), and that is if for every proposition, A , one is rational to be indifferent between not- p and A . Formally, this is equivalent to the proposition that S is rational to be *completely indifferent* about $\neg p$.

(CIP): $Des(\neg p) = Des(A)$ for all A (where $Des(\cdot)$ is the desirability function that determines the preference-or-indifference relation for S).

Now, the only way that (CIP) is true is if one is rational to be completely indifferent about all propositions whatsoever. This is because, according to (CIP), for any A and B , $Des(A) = Des(\neg p)$ and $Des(B) = Des(\neg p)$. But from this it immediately follows that $Des(A) = Des(B)$ for all A and B . Thus, the only way (CIP) holds is if S is not rational to have any preferences whatsoever. Since we can run the argument for infallibilism if (CIP) fails to hold and (CIP) holds only if S has no preferences whatsoever, it thus follows that, on Fantl and McGrath's proposal, fallible knowledge is impossible for an agent with any rational preferences whatsoever.

The only possibility left over is that one can have fallible knowledge in cases where one has no rational preferences. It should be obvious that this possibility is of no help or interest to any proponent of pragmatic encroachment. Most obviously, such a possibility in no way helps with understanding the way in which evidential standards seem to adjust between high stakes and low stakes situations. If there are any stakes, be they low or high, for some agent, S , then the agent must have rational preferences, which by the above considerations has shown to entail on Fantl and McGrath's proposal that knowledge requires epistemic certainty. The only distinction the proposal is capable of making between situations is *some-stakes* versus *no-stakes* situations. In *some-stakes* situations, Fantl and McGrath's pragmatic conditions imply that the evidential threshold for knowledge is epistemic certainty. In *no-stakes* situations, it is a mystery as to what the evidential standards on knowledge are.²⁷

5.6.1. Charting The Possible Responses to the Pragmatic Argument for Infallibilism. There are three possible responses one might make to the pragmatic argument for infallibilism.

²⁷This is to generalize Brown's (2013) observation that a principle like (JP) offers no explanation of the epistemic standards governing a belief in practically irrelevant propositions.

Two involve accepting that fallibilism is not compatible with pragmatic principles like JP. In other words, fallibilism and pragmatism cannot both be right. Proponents of infallibilism, I am sure, would be quite happy to accept that belief plays an important pragmatic role, and would be right in pointing out that only their view can be combined with the strong pragmatic principles that Fantl and McGrath have defended (at least as officially formulated).²⁸ The widely recognized problem for infallibilists is that few of our beliefs obtain the degree of epistemic justification required by infallibilism. If no one is justified in fully believing most propositions by the infallibilist's lights, it is difficult to see how the infallibilist's view provides much of a pragmatic role for knowledge or justified beliefs to play. The other option in this vein would be to take the above lessons to show that, since fallibilism is clearly right, belief cannot play the pragmatic role designated by Fantl and McGrath, at least not if that means accepting pragmatic principles like JP. While I have no intention to defend pragmatic encroachment views against these responses, it is clear that the second possibility means giving up on the idea that one can derive formal constraints on justified belief from pragmatic constraints on belief that take the form of JP. The third option would be to hold that the spirit of Fantl and McGrath's proposal is correct: belief does play an important pragmatic role. It is just that, in defending their pragmatic conditions on belief, Fantl and McGrath have gone astray. It seems clear to me that the third option is the right one to consider, as Fantl and McGrath surely didn't intend for their pragmatic conditions to fail to be satisfied in cases like the wine-example above, and one might think weaker pragmatic principles might avoid these problems, and may still entail a consistency requirement.

5.7. Fallibilist Version of their Pragmatic Conditions

At this point, we are faced with a choice. There are two obvious routes we might take to try to define weaker pragmatic conditions on belief: weaker in the sense that they allow for outright belief in situations where an agent is less than fully certain. The first way would be to retreat to a condition that only quantifies over practical actions. After all, the pragmatic argument for infallibilism turned essentially on the fact that the rational preference principle required preservation of one's

²⁸Dodd (2011) "Against Fallibilism" provides the basic strategy one might use to draw this inference. Dodd considers standard intuitions that motivate infallibilism, and contends that the right response is to simply accept that infallibilism is the best way to make sense of those intuitions. Anyone sympathetic to infallibilism might be inclined to draw a similar conclusion here, namely, one might be inclined to accept that our intuitions about the pragmatic role of justified belief are best explained by infallibilism.

rational preference ranking over all propositions (including those inconsistent with the proposition in question). Recall that in the informal presentation of the argument for infallibilism, the example I give is one where I would be rational to act as if I would not be having wine, but as we saw, not one where I would be rational to prefer as if I would not be having wine. If one simply requires that conditionalizing on a proposition preserve one's ranking on which actions (which is a special subclass of propositions in Jeffrey's framework) are rational to perform, then the argument we considered above would clearly be unsound. Thus, a credence falling short of full certainty is compatible with (JA). One solution would be to just retreat to (JA).

For now, I want to note that retreating to (JA) is not the only option for defining a pragmatic condition on belief (as we shall see, it comes at a cost). As far as I can see, the infallibilist problem for Fantl and McGrath's rational preference condition on belief was not that the preference based arguments that they initially consider are invalid, namely (JP1). The problem comes about in the move from (JP1) to (JP2). It will be useful here to pause briefly on the argument Fantl and McGrath give for (JP2). What (JP1) says is that if an agent is justified in believing p and rational to prefer A to B , given p , then the agent must be rational to prefer A to B , all things considered. They offer the following argument for (JP2):

Now for the final strengthening. As it stands, the consequent of our principle is simply a conditional, rather than a biconditional. That is, our principle leaves open whether being justified in believing that p ensures that for states of affairs A and B , if S is rational to prefer A to B in fact, then S is rational to prefer A to B , given p . We now argue this issue should be closed. Assume you're justified in believing that the train goes to Foxboro (p). And assume that you are, in fact, rational to prefer boarding (B) the train to waiting (W) for the next train. Could it turn out that you are not rational to prefer B to W , given p ? No. There are two ways you might fail to be rational to prefer B to W , given p . First, you could be rational to be indifferent between B and W , given p . In this case, since you would be justified in believing that p , then it seems that you would be rational to be indifferent between B and W in fact, which, by hypothesis, you are not. Second, you could be rational to prefer W to B , given p . Again, since you would be justified in believing that p , it seems that you would be rational to prefer W

to B in fact, which, by hypothesis, you are not. Thus, we can strengthen our principle by making its consequent a biconditional: for any states of affairs A and B , S is rational to prefer A to B , given p , iff S is rational to prefer A to B , in fact. When you satisfy this condition with respect to p , we will say that you are rational to prefer as if p . (2002, p. 76)

From this argument, they conclude that we should accept the biconditional (JP). Now, there are two things to notice about the argument. The first is that in this case, the B and W that they choose are propositions corresponding to actions, and as we noted above, there is no problem with a biconditional form of such a condition holding for actions. In short, there is no problem with accepting their argument for actions. But, one cannot generalize from a proper subset of propositions (in this case, the set corresponding to actions) to all propositions (including those that do not correspond to actions).

But this doesn't get at the heart of the problem. The second, and key thing to notice is that the B and W are presumably consistent with p , since p is a proposition whose truth does (at least on reasonable assumptions) not depend on either B or W . For propositions where either observation holds, there is no problem with the sort of argument that Fantl and McGrath offer in defense of (JP2). However, for any p with probability less than 1, there will be A and B such that S , as a matter of logical necessity, cannot rationally prefer A to B , given p . The first sort of case is just the one we considered above. We might choose A or B that are inconsistent with p . In such a case, given p , S has no preference over A and B because these correspond to the impossible proposition on the assumption that p . So, whatever one's all things considered preferences between A and B are, they cannot be preserved when one conditionalizes on p . So, one limitation on the argument should be that the sort of argument holds only for propositions A and B that are compatible with p .

One might think that in cases where p is consistent with A and B , Fantl and McGrath's argument is sound, but this is not quite right either. The other possibility is that while A and B are propositions distinct from one another, the distinction between them collapses on the assumption that p . For instance, I might prefer $A \& p$ to A . To put the idea intuitively, a proposition A might be desirable to some degree whether p is true, but might be even more so on the assumption that p is true. Here is one such example.

(Poker Example) Suppose one is playing poker and one is currently behind in a hand. If a jack comes on the river, one is at worse going to split the pot. Thus, a jack coming on the river is desirable. However, the hands are such that if the jack of hearts comes, then one makes a royal straight, and will take the hand outright.

Letting A be the proposition that the next card is a jack, and p be the proposition that the next card is a heart, we have it that the $Des(A \& p) > Des(A)$. Such judgments are clearly important to being able to understand an agent's rational preference ranking. But, now, if we let X be $A \& p$ and Y be A , then I cannot rationally prefer X to Y , given p (given their logical equivalence on the assumption that p , i.e., $X \& p$ is equivalent to $Y \& p$). Some rational preferences cannot and should not be preserved on the assumption that p is true. Again, it should be clear that the sort of argument that Fantl and McGrath give ought not to hold in this case, and from an intuitive standpoint, wasn't ever meant to hold for these sorts of propositions.

As I noted, one way to avoid this problem is to have a principle that only quantifies over actions. But the other option, would simply be to constrain the principle so that when considering whether an agent S can prefer as if a proposition p is true, we must hold that their rational preference ranking over propositions be preserved for those propositions, A and B , that meet two conditions:

- (i) A and B are both still possible on the assumption that p , and
- (ii) A and B continue to represent distinct states of affairs, i.e., one could be true without the other being true, on the assumption that p is true.

Formally, this suggestion is made precise via the following definition:

DEFINITION 5.5. S is rational to fallibly prefer as if p if and only if $p \neq \perp$ and $\forall A, B (Antec(p, A, B) \supset Conseq(p, A, B))$

$$Antec(p, A, B) =_{def} Pr(A \wedge p) > 0 \text{ and } Pr(B \wedge p) > 0 \text{ and } A \wedge p \neq B \wedge p.$$

$$Conseq(p, A, B) =_{def} Des(A) > Des(B) \text{ iff } Des(A \wedge p) > Des(B \wedge p).$$

Informally, the antecedent condition ensures that A and B are distinct and possible on the assumption that p . The consequent condition says that the rational preference ranking is preserved. In effect, together what this says is that S is rational to fallibly prefer as if p just in case S 's rational

preference ranking over all of the states of affairs that, on the assumption that p , are still possible and aren't collapsed into logical equivalence by the assumption that p , remains the same conditional on the assumption that p . Formally, we shall represent the claim that S is rational to fallibly prefer as if p like this: $Pref_f(p)$.²⁹ Let us define the corresponding justification-preference principle as follows:

(JFP) S is justified in believing p only if S is rational to fallibly prefer as if p .

One way to think about the rational preference condition is that it is supposed to be satisfied for those propositions that one can put to work in one's practical deliberations. In short, it holds for those propositions that one can treat as a given without then being able to reason to practically irrational decisions (More on this in Section 5.9). I see no reason to think the preference condition above cannot serve this purpose as long as we are considering whether one is justified in believing propositions that are logically compatible with the actions from which one must choose. Now, the impossible proposition stands apart from most other propositions that one might treat as a given in one's practical deliberations in that it, as a matter of logical necessity, forecloses on all propositions whatsoever.

Next, I want to turn to a couple of objections that have been posed against Fantl and McGrath's pragmatic conditions on belief, which help to demonstrate why, if the pragmatic conditions in question are supposed to be part of a sufficiency condition for being able to use propositions as premises in one's practical reasoning, (JFP) and (JA) have very different consequences for the sorts of practical arguments that are valid. As will become clear, a rational preference condition avoids problems, or at least has formal closure properties, that a rational action principle does not.

5.8. Purported Counterexample to Pragmatic Adjunction

One of the main theoretical differences between a pragmatic condition on belief that quantifies over actions and one that quantifies over preferences is sort of closure behavior that those conditions exhibit. The differences can be brought into focus by considering a purported counter-example

²⁹One caveat is that we need to explain the condition that p not equal the impossible proposition. The reason for this, quite simply, is that given that impossible propositions are incompatible with all states of affairs, the antecedent condition never holds and thus the rational preference condition is vacuously satisfied for it. Thus, without it, one would always be rational to prefer as if \perp . Ultimately, I don't think this choice makes a huge difference.

to the multi-premise closure of Fantl and McGrath's pragmatic conditions that was presented by Hawthorne and Stanley (2008). Here is what Hawthorne and Stanley say:

A further concern about the theoretical framework in which Fantl and McGrath are operating is that it makes trouble for multi-premise closure, which is the principle that one can generate knowledge by deduction from sets of premises that one knows. Suppose I am faced with a choice between *A* and *B*. *B* does not risk a fine. If law *x* is in force then if I do *A*, I pay a 10 pound fine to authority *w*, and if law *y* is in force then if I do *A* I pay a 10 pound fine to authority *v*. I do not prefer to do *A* conditional on having to pay both fines. Suppose I am .9 confident that law *x* is in force and .9 confident that law *y* is in force but only .8 confident that both are in force. I prefer doing *A* to doing *B*, since the risk of two fines is sufficiently low. It is not at all hard to find utility assignments such that it is true that I prefer that I do *A* to that I do *B* iff I prefer that I do *A* to that I do *B* conditional on law *x* being in force, and I prefer that I do *A* to that I do *B* iff I prefer that I do *A* to that I do *B* conditional on law *y* being in force, but I do not prefer that I do *A* to that I do *B* iff I prefer that I do *A* to that I do *B* conditional on law *x* being in force and law *y* being in force. Suppose further that credence .8 is sufficient for full belief, and one has deduced the conjunction that *x* is in force and law *y* is in force from the conjuncts. Fantl and McGrath's framework militates in favor of the conclusion that in this setting (supposing nothing else is pertinent), one knows that law *x* is in force, one knows that law *y* is in force, but does not know that law *x* and law *y* are in force, and this despite firmly believing that *p*&*q* on the basis of the belief in the conjuncts. We are aware that many contemporary epistemologists are more than ready to jettison multi-premise closure as an unrealistic fantasy. In our view this may have far-reaching consequences that have not been properly appreciated. For now, at least, let us underscore the conditional claim that if one holds multi-premise closure in high esteem, one ought to be wary of Fantl and McGrath's theoretical framework. (2008, pp. 576-577)

The intended counter-example begins with the observation that there are utility functions where, all things considered, it is rational for S to do A rather than B , and propositions p and q such that the following holds:

(HS1) It is rational for S to prefer $A \& p$ to $B \& p$.

(HS2) It is rational for S to prefer $A \& q$ to $B \& q$.

But where

(HS3) It is rational for S to prefer $B \& p \& q$ to $A \& p \& q$.

If this is a context where A and B exhaust the actions that S is in position to choose between, then this is a case where S is rational to act as if p and act as if q , but not rational to act as if $p \& q$. Stanley and Hawthorne go on to observe that all other non-pragmatic conditions on knowledge can be satisfied. One might believe p , q , and $p \& q$, and each of these propositions might be true and have an epistemic probability and rational credence as near to (but strictly less than one) as one would care to make it. Since Fantl and McGrath's proposal is motivated by the assumption that one can know propositions that are less than fully certain, the threshold for justified belief ought to be less than 1, Hawthorne and Stanley conclude that the agent in such a situation would know p , know q but fail to have epistemic justification for $p \& q$ and thus fail to know $p \& q$. If they were right, this would be a counter-example to closure of both justification and knowledge based on the closure-failure of Fantl and McGrath's pragmatic conditions. They take this as grounds to reject Fantl and McGrath's analysis of knowledge and justified belief.

5.8.1. Reply on Behalf of Fantl and McGrath. I don't agree with Hawthorne and Stanley that closure failure would provide us with a compelling reason to dismiss Fantl and McGrath's proposal. Nevertheless, there is a problem with Hawthorne and Stanley's counter-example to closure that is worth noting. While the example is one where it is rational for S to act as if p and rational to act as if q , but where S is not rational to act as if $p \& q$, it is not a case where the agent is rational to prefer as if p . This can easily be made plain. Consider again (HS2) and (HS3). Now, let $X = B \& q$ and $Y = A \& q$. S is rational to prefer X to Y , given p but S is not rational to prefer X to Y , in fact. It follows from our fallible preference principles that S is not rational in preferring as if p , even on our amended fallible preference condition. And, of course, we could give the analogous argument for q . Thus, the pragmatic conditions on epistemic justification are not,

contrary to Hawthorne and Stanley's conclusion, satisfied by the premises in the counter-example that they give. So the example does not establish that the relation *S is rational to fallibly prefer as if* x is non-adjunctive.

This is not to say that our newly defined pragmatic conditions on belief are fully adjunctive. As will become clear below, there are limits to the adjunctiveness of these pragmatic conditions, as there must be in order to stop these pragmatic constraints from entailing infallibilism. However, the limits on the adjunctiveness of $Pref_f$ are actually quite weak. The fact is that $Pref_f$ is adjunctive right up to the point of incompatibility. That is to say, the following theorem holds for $Pref_f$:

THEOREM 5.6. *If $Pref_f(p)$ and $Pref_f(q)$ and $Pr(p \wedge q) > 0$, then $Pref_f(p \wedge q)$.*

PROOF. We shall prove this by reductio.

- (1) Suppose the antecedent holds and $\neg Pref_f(p \wedge q)$.
- (2) Then $\exists A, B$ s.t. $Antec(p \wedge q, A, B)$ and $\neg Conseq(p \wedge q, A, B)$ [Just by definition of $\neg Pref_f(p \wedge q)$].
- (3) $Antec(p, A, B)$ [If A and B are distinct and still possible on the assumption that $p \wedge q$ is true, then they are distinct and possible on the assumption that p is true.].
- (4) $Antec(q, A \wedge p, B \wedge p)$ [Same as 3].
- (5) $Des(A \wedge p) > Des(B \wedge p)$ iff $Des(A) > Des(B)$ [by $Pref_f(p)$ and 4].
- (6) $Des(A \wedge q \wedge p) > Des(B \wedge q \wedge p)$ iff $Des(A \wedge p) > Des(B \wedge p)$ [By $Pref_f(q)$, 2 and 5].
- (7) $Des(A \wedge q \wedge p) > Des(B \wedge q \wedge p)$ iff $Des(A) > Des(B)$ [By 5 and 6].
- (8) $Conseq(p \wedge q, A, B)$ [By 7 and definition].
- (9) $\neg Conseq(p \wedge q, A, B)$ and $Conseq(p \wedge q, A, B)$ [By 2 and 8].

□

This theorem ensures that the pragmatic condition on epistemic justification is adjunctive in a fairly strong sense, at least if pragmatic conditions on belief are given by (JFP) rather than (JA). It ensures that if propositions p and q are compatible with one another and S is rational to fallibly prefer as if each is true, then S is rational to fallibly prefer as if their conjunction, $p \wedge q$, is true.

And it is worth emphasizing that the adjunctiveness is not limited to pairs of propositions, but holds over sets of logically consistent propositions.

THEOREM 5.7. *If S is rational to fallibly prefer as if each member of $\{p_1, \dots, p_n\}$ is true and $Pr(\bigwedge\{p_1, \dots, p_n\}) > 0$, then S is rational to fallibly prefer as if $\bigwedge\{p_1, \dots, p_n\}$ is true.*

PROOF. The theorem follows trivially from the fact that we can apply Theorem 5.6 $n - 1$ times to the set $\{p_1, \dots, p_n\}$. □

This ensures that if an agent, S , is epistemically justified in believing a set of propositions and they are collectively relevant to S , then S can, in some sense, put those beliefs to work in practical deliberation by preferring as if they are all true.

As shall become clear below, the fact that the fallible preference condition is adjunctive, even to a limited extent, will entail that (JFP) rules out certain kinds of justified inconsistent belief. (JFP) will thus be the more attractive account of pragmatic encroachment theory to anyone who finds a logical consistency requirement to have a lot of *prima facie* plausibility. Before we consider the implications for certain logical consistency principles that follow from the above results, we should ask the obvious question about closure failure: Are there any special reasons internal to pragmatic encroachment theory for thinking that closure failure or the failure of adjunction would be problematic? Considering this question will help us to see what is at stake in the choice between (JA) and (JFP). In order to pursue this question, let us try to figure out what would be wrong with pragmatic encroachment principles if they generated closure failure in the sort of case that Hawthorne and Stanley consider.

5.9. Pragmatic Non-Adjunction, Practical Irrationality and The Preface Paradox

One natural worry is implicit in Ross and Schroeder's discussion for why their own view is supposed to entail a logical consistency requirement on justified belief. While discussing their reasoning dispositions account of belief and logical consistency requirements on belief, Ross and Schroeder make the following observation about the pragmatic dangers of holding an inconsistent set of beliefs:

Once again, while PCR is hard to reconcile with Consistency [where ‘Consistency’ names a consistency requirement on belief], the reasoning disposition account can help to explain why Consistency is true. For according to the reasoning disposition account, someone who believes that p is disposed to treat p as true in any reasoning in which it is relevant whether p . And so if this account of belief is true, there would be an inherent danger in believing jointly inconsistent propositions. For if one is ever in a situation where all these propositions are relevant to one’s reasoning, then one will be at risk of treating each of them as true in one’s reasoning. And this would prevent one from reasoning coherently, and hence from arriving at a good deliberative conclusion. This risk would be particularly great if one had inconsistent beliefs among a small set of closely related propositions, for such propositions are particularly likely to be jointly relevant in one’s reasoning. (2014, p. 284)

Ross and Schroeder’s concern about pragmatically incoherent reasoning from inconsistent beliefs is intended to show that their own dispositional account of belief rules out justified inconsistent beliefs. Nevertheless, their observations raise an interesting worry for cases of pragmatic non-adjunction like the example given by Hawthorne and Stanley. In particular, there is a worry that one might be able to reason to a practically irrational conclusion by reasoning from justified beliefs to some judgment about what one’s rational preferences should be. Suppose one is in exactly the sort of situation that Stanley and Hawthorne describe. In the example, one is rational to prefer A to B , all things considered, and one is rational in having this preference conditional on p and conditional on q . But suppose this agent is rational to prefer B to A , conditional on the assumption that $p \& q$. The subject might then try to reason as follows.

P1: I am justified in categorically believing p and believing q .

P2: I know that B is preferable to A , given $p \& q$.

P3: Therefore, I am rational to prefer B to A .

If the agent in this sort of case were to reason in this manner, she would wind up making a practically irrational judgment about what to do or prefer. This is a kind of practical incoherence that a pragmatic condition on belief must avoid.

But do pragmatic encroachment theorists have a principled reason to deny the legitimacy of such reasoning? The answer is clearly yes, but the proper diagnosis of why this sort of reasoning is unsound depends on which pragmatic principle is assumed: (JA) or (JFP). Let us take each option in turn.

First, suppose the pragmatic role of justified belief is to be at least partially characterized by (JA) and not (JFP). A proponent of (JA) is committed to the validity of the following reasoning:

P1': I am justified in categorically believing $p \& q$.

P2: I know that B is preferable to A , given $p \& q$.

P3: Therefore, I am rational to prefer B to A .

But if epistemic justification is non-adjunctive, then one cannot go from P1 to P1'. By hypothesis, one isn't rational to act as if $p \& q$ is true, and so one isn't justified in believing $p \& q$. Thus, it is a mistake to conjoin p and q in one's reasoning about what to do. The problem then with the argument from P1 and P2 to P3 is that the reasoning assumes that one can infer P1' from P1, but this sort of inference is invalid. So, in a nutshell, the grounds for rejecting the argument from P1 and P2 to P3 is that the reasoning is invalid.

Denying the validity of this sort of reasoning may seem as counter-intuitive as denying epistemic closure principles. But it doesn't seem any more so to me. Considering an example will be illuminating. Christensen (2005, pp. 50-55) discusses a preface case where the question of perfect accuracy is relevant to one's practical decision making. Simplifying the details a bit, here is the sort of example that Christensen considers:³⁰

The Society for Historical Exactitude Example (SHEE): Suppose that Professor Smart has written a history book, and has investigated each of the claims in the book thoroughly, so that she has a very high degree of evidential support for each claim, p_1 through p_n , made in the book. Now suppose that the Society for Historical Exactitude has set up a major cash prize for any book that is found to be completely error-free. No one has ever won the prize for obvious reasons:

Given the level of detailed information contained in any book eligible for the prize,

³⁰We are condensing Christensen's extended discussion into a relatively simple example, so that we can bring out the differences between (JA) and (JFP). The only crucial feature of the example is that the propositions in the text are collectively relevant to the practical decision an agent is making.

there is barely a non-zero chance that anyone will win the prize. Over the years, the prize has grown to be quite massive. Now let us suppose that Professor Smart needs to purchase a new car. If the Professor's book is completely error-free, then she will be able to afford a much more expensive vehicle, and it will be best for her to purchase an expensive vehicle. Let us set out the relevant propositions:

(Expensive) Professor Smart purchases an expensive vehicle.

(Cheap) Professor Smart purchases a cheap vehicle.

The Professor's situation is such that it is rational to prefer (Expensive) to (Cheap) if she is going to win the monetary prize, which she will if the conjunction of all of the propositions in her book is true. Otherwise, let us stipulate that she is rational to prefer (Cheap) to (Expensive). And let us also stipulate that the odds of her winning the monetary prize are so low that the practically rational decision would be for Professor Smart to purchase the cheap vehicle.

This is a situation where each of the propositions are collectively relevant in the sense that what is practically rational turns on whether all of her beliefs are true, and this is a situation where it is clear that the agent would not be rational to act on the assumption that the conjunction of p_1 through p_n is true. She may well be rational to act as if each claim is individually true, since we can imagine that if she became certain that any individual claim in the book was true, she would still be rational to doubt that her book will win the prize (the possibility of error amongst the other claims in the book would still remain incredibly high), and thus she should still purchase the cheap car. In other words, the example is one where the agent is rational to act as if each proposition in the book is true, but not rational to act as if the conjunction of all of the propositions in the book is true. According to the principle (JA), Professor Smart's individual beliefs all satisfy the pragmatic conditions on belief, she is rational to act as if p_i is true for $1 \leq i \leq n$, and, assuming other evidential conditions are met, she is justified in believing each of the propositions in the book. It is just that she isn't justified in believing or acting upon the assumption that $p_1 \wedge \dots \wedge p_n$ is true. In short, (JA) allows that being rational in acting as if each member of the set of claims are true need not require that one to be willing to act as though one is infallible on the matter in question, and that all of one's beliefs are true.

(JA) is compatible with holding that even in this decision-situation, she is justified in believing all of the claims in her book. And (JA) affords the pragmatic encroachment theorist with an explanation for why it is unacceptable to reason from P1 and P2 to P3, and why it is unacceptable for Professor Smart to reason from the claims made in the body of her book to the conclusion that she should buy an expensive car. She is not justified in believing the conjunction of those claims, nor can she treat the conjunction as true in her practical deliberations in this particular context. This all seems quite plausible, but there are some theoretical costs.

Most notably, in any practical decision-situation where we have a number of propositions that we take ourselves to be justified in believing, we cannot simply consider what is rational to do on the assumption that all of our beliefs are accurate. If (JA) is the principle that describes the pragmatic role of belief, then we always must ask ourselves whether we are also justified in believing the conjunction of those propositions, and whether it would be rational to act on the assumption that they are all true. All things consider, I don't think the model of belief provided by (JFP) is any simpler, but it does not require this particular step. We should turn now to (JFP) and consider what response it affords the pragmatic encroachment theorist to the examples we have considered in this section.

Next, let us return to the argument from P1-P3. Someone who accepts (JFP) cannot deny the validity of the argument form, since the validity of this argument form follows from (JFP) and Theorem 5.6 above.³¹ Thus, in situations where P2 is true and P3 is false (recall that the imagined situation is one where the judgment expressed by P3 is practically irrational), (JFP) requires that we deny the truth of P1. This follows straightforwardly from Theorem 5.6 and (JFP). How is this response to be applied in the case of (SHEE)? We know that Professor Smart would not be rational in preferring as if $\bigwedge\{p_1, \dots, p_n\}$ is true (She is not rational to buy an expensive car), so, assuming the book is consistent, Theorem 5.7 and (JFP) together entail that Professor Smart must not be justified in believing all of the members of her book afterall.³² It is possible that she is justified in believing some members of her book, but this will depend on the details of her epistemic probability

³¹There will be counterexamples to the adjunctiveness of the fallible preference condition in cases where the propositions are incompatible with one another, but P2 will be false in those cases. Thus, there can be no counterexample to the specified argument form in question if (JFP) is assumed.

³²The assumption of consistency is merely a simplifying assumption. We shall show below that one cannot be justified in believing an inconsistent set of propositions when those propositions are collectively relevant if (JFP) is true. Those results make it clear that we could drop the consistency assumption, but to take up those details at this point would be unnecessarily distracting.

function. At the very least, it seems unavoidable that there will be cases where (JFP) entails that Professor Smart is not justified in believing any of the propositions stated in her book: Consider a decision-context where all of the propositions in the book have the same degree of evidential support, and where some practical decision she is making depends on whether or not her book is perfectly accurate. Parity reasoning seems to force us to conclude that if any one proposition in the book is justified, then any other must be as well, but (JFP) entails that she is not justified in believing all of the claims put forward in her book.³³ Thus, she must not be justified in believing any of them.

At this point, it might seem as though this gives pragmatic encroachment theorists with an obvious reason to prefer (JA) to (JFP). We have already rejected various substantive epistemic principles on the grounds that they entail skepticism for preface cases, and (JFP) has skeptical implications for at least some preface examples. While I think an outright rejection of (JFP) in virtue of these observations would be a bit hasty, it is worth noting that if pragmatic encroachment theory is to be understood in terms of (JA) and not a principle like (JFP), then it is obvious that pragmatic encroachment theory provides no substantive motivations for accepting any kind of closure or consistency principles whatsoever. So, if the analysis of the role of justified belief ends at (JA), then we can stop now and conclude that pragmatic encroachment theory offers no more reason to be wary of inconsistency than any other theory.

But as I have said, I think rejecting (JFP) for how it handles a particular version of the preface case is a bit hasty. There are several significant differences between the skeptical implications of (JFP) and other principles that we have rejected. For example, the principles that we considered in Chapter 3 put forward by Kroedel (2012, 2013a, 2013b) to explain a consistency requirement didn't just entail that there are certain decision-situations where one cannot be justified in believing a particular set of propositions. The problem with Kroedel's principle was that it entailed that one could never be permitted to believe a large set of propositions that are informationally rich in any situation whatsoever because doing so would be inherently too risky. A relevant fact about Fantl and McGrath's pragmatic conditions on belief is that they are thought to be highly shifty (one's beliefs may satisfy the pragmatic conditions on belief in one context, but the epistemic status

³³Hawthorne (2004) holds that the salience of parity reasoning generally explains why we tend to think an inconsistent set of claims cannot be justifiably believed. (JFP) doesn't entail that parity considerations will always be relevant, especially in preface situations, but in conjunction with a principle we introduce in the next section, it is clear that parity considerations will rule out justified belief in all practically relevant lotteries.

of one's beliefs may change when the stakes change).³⁴ This is sometimes thought to be a vice of the theory, but in cases like (SHEE) it might be thought a virtue. One thing the pragmatic encroachment theorist can point out is that they do not deny that Professor Smart was justified in believing the claims when she was in the process of writing her book, when she was submitting her book, and on most occasions. The only situations where she loses her justification for believing all of the propositions in her book are decision-situations where the practical rationality of her choices depend on whether the claims in her book are perfectly accurate. Given how remarkably rare such situations actually are, it is rarely ever the case that (JFP) will entail that an agent loses her justification in preface-like situations. Thus, the pragmatic encroachment theorist might hold that while (JFP) has skeptical implications for some decision-situations, these are not so widespread to really threaten skepticism in a manner that is all that analogous to the other principles we have considered.

Perhaps one will find this defense of (JFP) unsatisfying, and hold that only (JA) is the plausible principle. As we have noted, if they do, then no special deductive cogency constraints are motivated by pragmatic credal reductivism. At the very least, it is worth considering what sorts of inconsistency can and can't be tolerated if (JFP) is assumed. It is to this question that we next turn.

5.10. Pragmatic Irrelevance and Justified Inconsistent Belief

So far we have noted that Fantl and McGrath's pragmatic conditions on belief rule out inconsistency by entailing infallibilism. While there is no reason to think (JFP) entails infallibilism, there is reason to doubt that it rules out justified inconsistent belief. In this section, we consider a couple of examples that definitively show that (JFP) must be compatible with at least some justified inconsistent belief. The first example was provided by Ross and Schroeder to show that a principle like (JA) doesn't even rule out justified belief in contradictory propositions. They observe that

³⁴Ross and Schroeder (2014, pp. 277-280) object to Fantl and McGrath's proposal precisely because changes in stakes without changes in one's evidence can impact which beliefs propositions one is justified in believing. Neta (2007) makes a related objection based on the fact that believing different propositions may contain different risks, and thus impose different standards for belief for an agent in a single context. Thus, it isn't even clear that all of one's beliefs would be subject to the same evidential standards in this context. The principle (JJ) that we consider below helps at least answer Neta's concern, but also makes epistemic standards somewhat less shifty.

there exist some propositions that are utterly irrelevant to one's practical deliberations. They give the following example:³⁵

m: The width of Salvador Dali's mustache when he painted *The Persistence of Memory* was twice the length of Cleopatra's nose when she met Mark Antony.
(2014, p. 280)

They go on to explain that:

For most of us, most of the time, our credence in *m* makes no difference to how we should act. And so, regardless of what our credence in *m* may be, it will be rational for us to act as if *m*, in the sense of doing what would be rationally optimal conditional on *m*. (p. 280)

The key thing to note is that the very same observations apply to the negation of *m*. And, just as *m* is irrelevant to how we should act, it also seems that it is irrelevant to what we should prefer. In short, there seem to be no rational preferences that depend on whether *m* is true or false, and so both *m* and *m*'s negation vacuously satisfy (JFP). The basic issue then is that neither (JA) nor (JFP) rule out belief in contradictory propositions when those propositions are utterly irrelevant in one's practical situation.³⁶

If pragmatic credal reductivism failed to rule out justified belief in contradictory propositions, then it would fail to impose even the most relaxed deductive coherence constraints on justified belief that we introduced in Chapter 1. If Ross and Schroeder are right, then pragmatic credal reductivism is in conflict with Easwaran and Fitelson's witness-set avoidance principles, and the basic evidential condition on justified belief. This example would provide the basis for a reductio of pragmatic credal reductivism. However, these problems are avoided if we accept the following principle put forward by Fantl and McGrath to explain how the pragmatic conditions on belief relate to the evidential conditions on justified belief:

³⁵This example is analogous to examples given by Brown (2013, p. 10). She notes that the proposition that Mars has two moons is practically irrelevant to most of us. And she draws similar conclusions for one interpretation of Fantl and McGrath's (2009) proposal.

³⁶Ross and Schroeder (2014, p. 280) take this example to show that pragmatic credal reductivism fails to require that one's belief be sufficiently supported over its negation by one's total evidence. We shall later show that Fantl and McGrath (2009) have suggested offered a straightforward answer to this problem in terms of threshold principle.

(JJ) You are justified in believing p iff p is warranted enough to justify you in φ -ing, for any φ . (2009, p. 123)

In this principle, φ is supposed to range over all actions, intentions and beliefs, and other propositional attitudes that can be practically rational/irrational. Now, there are two key aspects of (JJ) that ensure that it does not permit one to justifiably believe a proposition without a high degree of evidential support.³⁷ First, Fantl and McGrath note that a proposition, p , might lack sufficient warrant for justifying one in φ -ing, even though p is not relevant to φ . Of course, p might not be able to justify one in φ -ing because it fails to be relevant, but also because it fails to be sufficiently warranted. In general, they hold that if some proposition q is such that S is not rational to act or prefer as if q , and q enjoys at least as much warrant for S as p , then p must suffer from the same evidential weakness. And this means that p is not warranted enough to justify all φ , in particular, whatever φ stands in our way of being able to act or prefer as if q is true. In other words, (JJ) can be construed as entailing the following thesis:³⁸

At Least As Much Warrant Principle (ALMWP): If S is not justified in believing q , and q is at least as warranted as p , then S is not justified in believing p .

If the degree to which a proposition is warranted corresponds to the proposition's epistemic probability, then this delivers a kind of threshold requirement on justified belief. On a probabilistic interpretation of the degree to which a proposition is warranted, it follows that no epistemically unjustified proposition is more probable than some epistemically justified proposition. In other words, (JJ) motivates a single premise closure principle along the lines of the principle we considered in Chapter 1. More to our present point, as long as the threshold for being able to act or prefer as if some proposition p is true is above .5 in a given decision-situation, then m and m 's negation cannot both be sufficiently warranted in that situation. Hence, (JJ) does rule out justified belief in contradictory propositions in all but the most unusual decision-situations (The only exception might be a no-stakes situation where one has absolutely no rational preferences whatsoever).

³⁷Fantl and McGrath do point out various options for how (JJ) could be interpreted, and Brown (2013) objects to (JJ) for imposing excessively strict evidential standards if we interpret the principle the way I will below. The alternative to interpreting (JJ) in the manner I do below fails to resolve the objection that pragmatic credal reductionism allows for justified belief in contradictory propositions. Ultimately, I think addressing her argument would take us too far afield, but I do think her arguments force upon us an interpretation of (JJ) in the manner I accept below.

³⁸See Fantl and McGrath (2009, pp. 63-68) and Brown (2013) for an explanation of why Fantl and McGrath's evidential conditions can be understood as entailing the following sort of principle.

So, (JJ) will rule out justified belief in contradictory propositions, and it will entail Easwaran and Fitelson's dominance avoidance principles, but a threshold condition like (JJ) will obviously not rule out all justified inconsistent belief. Moreover, the combination of (JJ) and (JFP) will together still allow for at least some cases of justified inconsistent belief. The easiest way to see this is to consider a set of lottery propositions that are both individually and collectively irrelevant to one's preferences.³⁹

Alien Lottery Example (ALE) Suppose aliens somewhere in a galaxy far, far away are holding a standard n ticket lottery. The aliens hold the lottery for no reason whatsoever, no one pays any attention to it. The winner is randomly selected, and never announced.

Suppose we come to learn of this alien civilization and their lottery. It seems reasonable to suppose that this is a lottery such that for virtually all people in all contexts, the lottery propositions for this lottery are of no practical significance. So, there is no reason why one couldn't be rational in preferring as if any of them are true. And, whatever threshold is determined by one's decision-situation and (JJ), the alien lottery could have been large enough so that each lottery proposition is sufficiently probable to satisfy (JJ). Consequently, pragmatic credal reductivism must allow for some inconsistent justified belief.

5.10.1. Practically Relevant Sets of Lottery Propositions. Whether the pragmatic conditions on belief are cashed out in terms of (JFP) or (JA), Pragmatic Credal Reductivism tolerates at least some justified inconsistent beliefs. In particular, we have shown that one can be justified in believing a logically inconsistent set of claims when the propositions in question are of no practical significance. But does this mean that (JFP) imposes no stronger logical consistency constraints on justified belief than any other threshold view? The answer is implicit in our discussion of (SHEE). (JFP) is compatible with one being justified in believing all preface propositions, including the judgment that at least some of the claims put forward in one's book are false, in situations where

³⁹If one is inclined to hold that lottery propositions can't be justified for independent reasons, then we could have put the example in terms of a pragmatically insignificant preface case. In fact, I think the point should hold in any ordinary preface case where one's rational preferences don't turn on whether one's book is perfectly accurate. We shall stick to lottery cases, since the formal points are a bit simpler this way, and it is worth noting that nothing in the pragmatic encroachment theory even rules out justified belief in lottery cases.

the question of perfect accuracy is not of practical relevance. But one cannot be justified in believing all preface propositions when the question of perfect accuracy is of practical relevance. (JFP) and (JJ) have similar implications for a set of lottery propositions.

First, let us define what we mean when we speak of a set of propositions being collectively relevant to one's practical deliberations.

A set of propositions $\{p_1, \dots, p_n\}$ are collectively relevant to S if and only if there exists some propositions A and B such that S is rational to prefer A to B on the assumption that $p_1 \wedge \dots \wedge p_n$ is true, but S is rational to prefer B to A on the assumption that $p_1 \wedge \dots \wedge p_n$ is false.

In standard lottery situations where the lottery propositions are of some practical relevance, all maximally consistent subsets of lottery propositions are collectively relevant in the above sense. And it follows from (JJ) and (JFP) that one cannot be justified in believing all lottery propositions in such cases.

To see why, consider a standard n ticket lottery situation where one is given the opportunity to purchase a lottery ticket, let us say, ticket number 1. Let us suppose that there is exactly one winning ticket, which happens to be worth one million dollars. Each lottery proposition, l_i , says the i^{th} ticket is a losing ticket. Can one be rational to prefer as if each lottery proposition is true? If the price of the lottery ticket exceeds the worth of the winning ticket, then the answer is clearly yes, since purchasing the ticket is then an irrational thing to do regardless of the outcome of the lottery. But then this is really a practically irrelevant lottery since there is no practical choice to be made that depends on the truth of the propositions in question. So, let us suppose that the value of the winning ticket exceeds the purchase price. Can one be justified in believing all lottery propositions in this case?

It might seem as though the answer depends on the size of the lottery and/or the exact price of the ticket, but it actually follows from (JFP) that the situation is structured so that no matter how we fill in those details of the case, one cannot be justified in believing all lottery propositions. The argument for this claim must be broken up into two cases: the case where one is rational to purchase the ticket, and the case where one isn't. The first case is easy to dispense with. It follows from both (JA) and (JFP) that one cannot be justified in believing that ticket 1 is a losing ticket.

If ticket 1 is a losing ticket, it has no value, and so the best option on that assumption is to not purchase it. So, if it is rational to purchase the ticket, then it is not rational to act or prefer as if l_1 is true.

Now, one might think that one can be rational to act or prefer as if each lottery proposition is true in the latter case. If l_1 is true, one is rational to not purchase the ticket, and we are supposing in the second case that one is rational not to purchase the ticket. So, it looks like we are rational to prefer as if l_1 is true. This may well be the case, but the same considerations do not hold for the rest of the lottery propositions. We know that the set of propositions $\{l_2, \dots, l_n\}$ are collectively relevant in the latter case. If $l_2 \wedge \dots \wedge l_n$ is true, then ticket 1 must be a winning ticket, and so one ought to purchase ticket 1 on the assumption that $l_2 \wedge \dots \wedge l_n$ is true. By hypothesis, one ought not to purchase the ticket, so one is not rational to prefer as if $l_2 \wedge \dots \wedge l_n$ is true. By Theorem 5.7, it follows that one must not be rational to prefer as if each member of $\{l_2, \dots, l_n\}$ is true, and so (JFP) entails that one cannot be justified in believing each member of $\{l_2, \dots, l_n\}$. Given that (JJ) allows us to engage in parity reasoning (each lottery proposition suffers from the same epistemic weakness), it follows that one must not be justified in believing any lottery propositions in this case. So, regardless of the probability of l_i or the ratio of purchase price to the value of the winning ticket, (JFP) and (JJ) together entail that one cannot be justified in believing any lottery propositions whatsoever.

The upshot of all of this is that Fantl and McGrath's pragmatic conditions on belief, when properly understood, do in fact rule out certain kinds of inconsistent belief, and do so in a way that depends crucially on whether one would be rational to conjoin one's beliefs when making practical decisions. And it seems to me that the way in which their pragmatic conditions rule out inconsistent belief provides some explanation for why so many epistemologists have found a consistency requirement on belief so intuitive. To put the point in slogan form: inconsistent belief is acceptable only when the beliefs in question are collectively irrelevant. The explanation that (JFP) offers for why an inconsistency requirement seems to enjoy so much *prima facie* plausibility comes down to the fact that when we are considering a set of propositions in a decision-situation, we take those propositions to all be relevant to our practical or theoretical deliberations at that point in time. And according to (JFP), if they are collectively relevant in the sense we have defined above, it is likely to follow that one is obliged to avoid inconsistent beliefs. Hence, when examining most simple cases of an agent

trying to reason from an inconsistent set of propositions to some practical or theoretical judgment, (JFP) and (JJ) accord with the commonly expressed intuition that the agent in question has at least some unjustified beliefs, and that an agent need not consider her subjective probabilities to figure this out.

Out of all of the substantive theories that we have considered, pragmatic encroachment theories seem to have the best claim to have substantive grounds for placing additional limits on how much logical inconsistency can be rationally acceptable. At the same time, the pragmatic conditions on belief allow for justified inconsistent belief in exactly the sorts of cases where it seems most plausible: namely, most ordinary preface cases. It thus seems to me that (JFP) is tolerant of about as much inconsistency as is normally thought reasonable (The most controversial case being (SHEE)). We should close our discussion of pragmatic theories by considering Ross and Schroeder's claim that their dispositional account of belief provides a more robust and plausible consistency requirement on justified belief than Fantl and McGrath's pragmatic credal reductivism.⁴⁰

5.11. Reasoning Dispositions Account of Belief and Deductive Cogency

The last question that we will need to consider is simply whether a reasoning dispositions account of belief places more extensive limits on the amount of inconsistency that can be tolerated than the combination of (JFP) and (JJ). Ross and Schroeder have claimed that their reasoning dispositions account does place more extensive limits on how much inconsistency can be tolerated than pragmatic credal reductivism, and they have claimed that this gives their analysis of belief a significant theoretical advantage over rival pragmatic encroachment views (amongst other theoretical advantages that are beyond the scope our research here). So, let us review how a reasoning disposition account is supposed to rule out justified inconsistent belief.

Recall again the central principle of the reasoning disposition account:

(RDAB) Believing that p defeasibly disposes the believer to treat p as true in her reasoning.

⁴⁰We should keep in mind that Ross and Schroeder do not consider (JJ) when characterizing pragmatic credal reductivism, and as we noted above, that does make a significant difference.

What is it to treat as true? Presumably, it is to act as if p is true and to take p as a given in one's practical deliberations. Now, one thing to consider is whether a consistency requirement immediately follows from the fact that the state in question is a defeasible disposition. Weintraub has indicated the basic problem with trying to give an argument based on the fact that belief is a defeasible disposition when replying to Stalnaker's argument for the adjunctiveness of belief. Stalnaker (1984) analyzes belief as a disposition to act as if a proposition is true.⁴¹ The critical observation Weintraub makes is that defeasible dispositions to act as if P is true and to act as if Q is true are not closed under conjunction-introduction. She points out that, "An action can tend to be successful (i.e., be more often successful than not) in P -situations and in Q -situations while tending to be unsuccessful in $P \wedge Q$ -situations." Weintraub gives an example of a situation where a certain treatment to a hospital patient is effective in $4/5$ P -situations and $4/5$ Q -situations, but devastating in all $P \wedge Q$ -situations.

As a general point, this shows that defeasible dispositions, in this case a disposition to act as if (which is distinct from the disposition to treat as true), need not be closed under conjunction-introduction. So, one might wonder, if defeasible dispositions are not generally closed under conjunction-introduction, then why can't one have logically inconsistent defeasible dispositions? For instance, why can't one be defeasibly disposed to act as if each member of a set of lottery propositions are true. I see no reason to think that one couldn't have such defeasible dispositions. Consider again a practically relevant set of lottery propositions where one would be rational not to purchase any lottery tickets – assume the expected utility of purchasing a ticket is negative. In such a case, one is disposed in the vast majority of situations to act as if each individual lottery proposition is true, i.e., as if each lottery ticket is a loser, but is not rational to act as if $l_2 \wedge \dots \wedge l_n$ is true. So, it seems like there could be at least certain lottery situations where one can be defeasibly disposed to act as if each lottery proposition is true without being disposed to act as if a conjunction of a bunch of them are true. In short, one can have inconsistent dispositions without there being any conflicts between the various defeasible dispositions one has. Hence, if believing p is a matter of being disposed to act as if p is true, I see no reason why this analysis immediately rules out justified inconsistent beliefs. And, it should be obvious that the very same kinds of considerations would transfer over to preface paradox cases as well.

⁴¹She is responding to Stalnaker's (1984, p. 90) argument that belief must be closed under conjunction-introduction.

Now, analyzing belief as a defeasible disposition to act as if p is true is different from analyzing belief as a disposition to treat p as true. Still one might wonder why one couldn't be disposed to treat each lottery or preface proposition as true, given that doing so wouldn't lead one into any kind of practical incoherence.⁴² After all, one can act as if each proposition is true in the vast majority of situations! One possible worry is that it is more appropriate to reason probabilistically in lottery situations, but often an agent might not be aware of exactly how large the lottery is, and thus not in position to reason probabilistically about what one should do.

Another answer that springs to mind is that if one treats a set of propositions as true, then one is inclined to employ those propositions together in deductive arguments regarding what one ought to do. In that case, being disposed to treat a set of inconsistent propositions as true would lead to practically irrational decisions. However, if that is the reason for accepting a logical consistency requirement, then it seems to me that the analysis of belief as a disposition to treat as true simply builds in the assumption of multi-premise epistemic closure and consistency principles. The proposed analysis of belief wouldn't then give us some substantive reason for accepting a consistency or closure principle, the dispositional account would simply include, as a matter of assumption, consistency and closure principles as part of the definition of what it is to believe a proposition.⁴³ Thus, this reply offers no argument for a consistency requirement on justified belief, and merely offers a way such principles could be incorporated into an account of justified belief.

Ultimately, Ross and Schroeder don't see a consistency constraint on belief coming merely from the analysis of belief as a defeasible disposition, but rather from the appropriate standards we should use to determine whether a belief is rational (or justified). What does it take for a belief to be rational on the reasoning dispositions account? Ross and Schroeder go on to explain that the standards of justification for beliefs are pragmatic on their view. First, they note that 'On this account, belief is a kind of heuristic. And heuristics are justified pragmatically, in terms of the ends they serve.' (2014, p. 274) They hold that dispositional belief serves two distinct ends:

⁴²Again, if one is not inclined to think that lottery propositions can all be justified, the same points can be made about preface situations.

⁴³This is akin to our diagnosis of Douven's (2008) argument for a consistency requirement on justified belief that was based on the assumption of closure of categorical belief under conjunction-introduction. See Chapter 3, Section 6 for our earlier discussion.

(i) ‘...one of the ends of belief is to allow the believer to arrive at good deliberative conclusions – which, in the case of practical reasoning, amounts to choosing courses of action with high expected utilities.’

(ii) ‘...the second end of belief is to minimize the cognitive costs of reasoning.’

(2014, p. 274)

It is clear that pragmatic credal reductivism is meant to allow belief to serve the first end. What really sets the dispositional account of belief apart from pragmatic credal reductivism is the this second end, as the dispositional account prevent agents from having to try to reason, to use their terms, “in an ideal Bayesian manner” (p. 274). Ross and Schroeder suggest that a natural requirement on holding a justified set of beliefs is the following condition:

Procedural Rationality Condition: A set of beliefs is rational only to the extent that it is licensed by rules or procedures that strike an optimal balance between minimizing expected cognitive costs and maximizing the expected value of the agent’s deliberative conclusions. (p. 274)

Unfortunately, Ross and Schroeder fail to provide any particulars regarding the rules or procedures that they think would naturally fit with the reasoning disposition account. Nevertheless, Ross and Schroeder maintain that the reasoning disposition account of belief along with the procedural rationality condition will support the following consistency requirement on belief:

Consistency: There is rational pressure to avoid logically inconsistent beliefs. In particular, where A is a small set of related propositions that are jointly inconsistent, it is rationally impermissible to simultaneously believe every proposition in A . (p. 283)

There are two different issues to consider with respect to this consistency principle. One is whether their account of rational belief will really entail Consistency. And, then there is a question of whether Consistency is really any stronger than the limited consistency conditions entailed by the combination of (JFP) and (JJ).

So why think the reasoning disposition account will entail Consistency? Ross and Schroeder argue as follows,

For according to the reasoning disposition account, someone who believes that p is disposed to treat p as true in any reasoning in which it is relevant whether p . And so if this account of belief is true, there would be an inherent danger in believing jointly inconsistent propositions. For if one is ever in a situation where all these propositions are relevant to one's reasoning, then one will be at risk of treating each of them as true in one's reasoning. And this would prevent one from reasoning coherently, and hence from arriving at a good deliberative conclusion. This risk would be particularly great if one had inconsistent beliefs among a small set of closely related propositions, for such propositions are particularly likely to be jointly relevant in one's reasoning. (p. 284)

They go on to contend that belief forming procedures and rules could prevent us from incoherent practical reasoning from small sets of inconsistent beliefs in one of two ways:

They could either prevent the formation of inconsistent beliefs in the first place (or at least prevent the formation of inconsistent beliefs among small sets of closely related propositions), or else they could allow such beliefs to be formed but prevent them from being jointly operative. (p. 285)

So why think that the procedures should prevent the formation of inconsistent beliefs as opposed to merely preventing their joint operation? Ross and Schroeder's answer is that allowing inconsistent belief would just be too cognitively expensive. As they put it,

But preventing the joint *formation* of such inconsistent beliefs would be much less cognitively costly than preventing their joint *operation*. For preventing the formation of jointly inconsistent beliefs would require only that a check for consistency be performed prior to forming any new belief. However, preventing the operation of jointly inconsistent beliefs would require that checks for consistency be performed far more frequently, prior to the employment of any given belief. (p. 285)

They thus conclude that the rules and procedures mentioned in the Procedural Rationality Condition must entail a consistency requirement, at least over small sets of closely related beliefs.

There are two things about this discussion that strike me as problematic. The first is simply that without any details about how the cognitive rules and procedures are supposed to function, it is very difficult to see how one could think through the cognitive cost-benefit analysis of the two options to figure out which is more costly in cases of small sets of inconsistent propositions.⁴⁴ What seems absolutely clear to me is that these rules and procedures would not rule out inconsistent belief in cases like the preface paradox. Whatever the cognitive rules and procedures are, they must be compatible with writing books that are rich in informational content. And, unless the procedures are infallible, for large enough books, there is virtually no chance that one's beliefs are perfectly accurate. Adding the proposition that one's book contains at least some errors, and thereby introducing some inconsistency to one's belief system, seems to pose no risk of engendering incoherent practical judgments, and there seems to be virtually no risk that one's beliefs will lead one to engage in practically incoherent reasoning even in cases where the propositions are collectively relevant to some decision one is making. That is to say, even if the account allows for inconsistent belief in a case like (SHEE), whatever the cognitive rules are, it seems obvious that any remotely rational person would understand that it is inappropriate to conclude that they are going to win a major prize, and it would thus be inappropriate to buy an expensive car. Somehow, the cognitive rules and procedures prevent the joint operation of all of one's beliefs expressed in one's book, and whatever the rules are, they don't seem to be very cognitively expensive. Of course, preface situations do not involve small sets of propositions, but why would it be any more costly to prevent the joint operation of inconsistent beliefs in cases where the relevant set of propositions is small as opposed to preface situations where they are large. Perhaps there are reasons, but until they are given, I simply don't find it obvious that the cognitive cost-benefit analysis would favor prevention of inconsistency over prevention of joint operation of inconsistent beliefs.

The other problem with the discussion, an issue I find more pressing, is that the Consistency principle is just not a very strong principle, and thus isn't really controversial. As we saw in the last section, (JFP) rules out inconsistent belief in large sets of propositions in certain decision-situations, in particular, whenever the propositions are collectively relevant. The kind of consistency constraints

⁴⁴As we saw in Chapter 1, Section 2, a number of philosophers have attempted to formulate rules that would rule out inconsistency, but couldn't do so without collapsing their view into a version of infallibilism. Until procedural rules are put forward that avoid the skeptical collapse, I think it is reasonable to at least remain agnostic about the possibility of formulating any such rules.

that (JFP) imposes are at least novel and interesting.⁴⁵ Ross and Schroeder's Consistency, however, only rules out small sets of closely related propositions. For this reason, Consistency is a principle that follows on pretty much every account of justified belief that we have considered. And, the fact of the matter is that this sort of principle is entailed by the combination of (JFP) and (JJ). Recall our toy decision-context from Section 2.2 above. In that case, in order for an agent to be rational to act as if p was true, the epistemic probability of p needed to be greater than .95. From (JJ) it follows that the threshold in that context would be at least .95, and so the smallest inconsistent set of propositions that were all sufficiently probable would contain at least 20 propositions. We don't have any general theory about what the threshold will be in most decision-situations, but most who have considered the matter think that it would be reasonably high.⁴⁶ And as long as that is the case, one could not be justified in believing a small set of closely related propositions if they are inconsistent on Fantl and McGrath's proposal. Thus, the restrictions on inconsistent justified belief imposed by the reasoning disposition account do not give the reasoning disposition account any advantage over pragmatic credal reductivism.

5.12. Conclusion

What have we learned from considering pragmatic encroachment theories? Perhaps the most notable thing is that Fantl and McGrath's pragmatic conditions only rule out justified inconsistent belief in all contexts if their conditions are interpreted along the lines of (JP). In that case, pragmatic encroachment theory has highly implausible skeptical implications. However, we also learned that on the least radical revision of their pragmatic conditions on belief, (JFP), pragmatic encroachment theory actually strikes a plausible balance in allowing inconsistent justified belief in most preface cases, while ruling out justified belief in most lottery propositions. The exceptions include preface cases where the propositions in question are collectively relevant to some practical decision one is making in a particular decision context (i.e., where the agent should do something different if her beliefs are perfectly accurate), and sets of lottery propositions that have absolutely no practical relevance whatsoever. Both exceptions do strike me as somewhat counterintuitive, but both also seem to fit reasonably well with the basic assumptions of pragmatic encroachment theory:

⁴⁵Any theory of epistemic justification that imposes an n -wise consistency principle will entail Consistency.

⁴⁶In fact, one of the most plausible worries about pragmatic encroachment theory put forward by Brown (2013) is that the standards would typically be very high, higher than is intuitively plausible.

The point of having categorical beliefs is to use those beliefs in one's practical deliberations. If that is a correct view of the point of belief, then we might expect for our ordinary intuitions to not apply to cases where beliefs in the target propositions, by hypothesis, have no role to play (In the lottery case, the beliefs can't be used to reason to any practical conclusion because nothing depends on the outcome). As for (SHEE), we have already noted how the shiftiness of pragmatic encroachment theory mitigates the skeptical consequences of that example to a considerable degree. Of course, if one insists that (SHEE) is a counterexample to (JFP), then we get a pragmatic encroachment theory in the form of (JA) that is no less tolerant of inconsistent belief than any threshold account.

The overarching lesson I think we can take away from our investigation is that even the best candidate theories for motivating deductive cogency requirements on categorical belief provide no compelling reason to rule out justified inconsistent belief. Lehrer's coherence theory, a theory built on the idea of ruling out justified inconsistent belief, seems to only offer an explanation of why inconsistency must be avoided in a small set of cases with arbitrary assumptions built into them. Permissibility theories seem to offer nothing but roads to a consistency requirement that eventually lead to radical skepticism. For coherence theories, the best motivation for a consistency requirement came from our proofs that there are certain probabilistic interpretations of the coherence relation that are incompatible with justified inconsistent belief. But those interpretations were no better motivated than nearby analyses of the coherence relation that are highly tolerant of inconsistency. Last, but not least, we found that pragmatic theories could rule out a certain amount of inconsistency, but at the end of the day, also had to allow for justified inconsistent belief in a great many cases. Consequently, insofar as there are formal coherence requirements on justified belief, I think they must be requirements that take the form of the accuracy-coherence norms defended by Easwaran and Fitelson that we considered in Chapter 1.

Appendix

We now verify the main technical results from Chapter 4. Most of the proofs depend on the logical relationship between elements that are in the union of a subset pair of a set S . Throughout, it will be convenient to have an expression for saying when a set of propositions are in the union of a subset pair. So, when I say, ‘a subset pair, $\langle S', S'' \rangle$, contains S^* ,’ I will mean that $S^* \subseteq S' \cup S''$. When I speak of subset pairs containing a proposition, p , I will mean $p \in S' \cup S''$. I will also write ‘ $|S|$ ’ to refer to the cardinality of S . Using this shorthand and the notation clearly stated in Chapter 4, we now verify the main technical results of Chapter 4.

Proof of Theorem 4.1.

Theorem 4.1. *For any finite n where $|TLS| = n$, $C_a(TLS) < \frac{1}{3}$.*

PROOF. Roche’s proof proceeds from a few basic observations. The first is that if $\langle S', S'' \rangle$ is inconsistent, i.e., $S' \cup S''$ is inconsistent, then $a(\langle S', S'' \rangle) = 0$. So, any $\langle S', S'' \rangle \in [TLS]$ that contains the contradiction, p_n , is such that $a(\langle S', S'' \rangle) = 0$. The second is that for any set S , if the cardinality of S is n (henceforth written, $|S| = n$), then there are $3^n - 2^{n+1} + 1$ subset pairs in $[S]$.⁴⁷ The number of pairs not containing p_n is the number of subset pairs of the set $TLS - \{p_n\}$, which is $3^{n-1} - 2^n + 1$. If all of the propositions in $TLS - \{p_n\}$ are logically equivalent, then they are assigned the maximum value 1 by $a(\cdot)$. Thus, supposing they are logically equivalent, $C_a(TLS)$ is the average of the values

$$C_a(TLS) = \frac{3^{n-1} - 2^n + 1}{3^n - 2^{n+1} + 1}$$

⁴⁷See Roche (2013, p. 86) for his version of the proof, which I have simplified to some degree.

Roche observes that $\frac{3^{n-1} - 2^n + 1}{3^n - 2^{n+1} + 1}$ is less than $1/3$ and quickly converges to $1/3$ as n goes to ∞ . Since supposing the members of $TLS - \{p_n\}$ are logically equivalent maximizes $C_a(TLS)$, it follows that $C_a(TLS) < \frac{1}{3}$.

□

Proof of Theorem 4.2.

Theorem 4.2. $\lim_{Pr(p_1) \rightarrow 0} C_{d_a}(TLS) = C_a(TLS)$.

First note that for any choice of the probability of p_1 (note that p_1 through p_{n-1} are logically equivalent and thus have the same probability), $C_a(TLS) \geq C_{d_a}(TLS)$. The reason why is simply that $d_a(h, e) = a(h, e) - Pr(h) \leq a(h, e)$. Hence, the values of the subset pairs to be averaged by C_{d_a} will each be less than or equal to the values to be averaged by C_a . Now, by definition

$$\begin{aligned} C_{d_a}(TLS) &= \frac{\sum_{\langle S', S'' \rangle \in [TLS]} a(\langle S', S'' \rangle)}{|[TLS]|} - \frac{\sum_{\langle S', S'' \rangle \in [TLS]} Pr(\bigwedge S')}{|[TLS]|} \\ &= C_a(TLS) - \frac{\sum_{\langle S', S'' \rangle \in [TLS]} Pr(\bigwedge S')}{|[TLS]|}, \text{ by definition of } C_d(\cdot). \end{aligned}$$

Given that all of the claims in TLS are logically equivalent except p_n , we have it either $Pr(\bigwedge S') = 0$ if $p_n \in S$ or else $Pr(\bigwedge S') = Pr(p_1)$. Thus, we have it that

$$C_{d_a}(TLS) = C_a(TLS) - Pr(p_1) \cdot \frac{|[TLS - \{p_n\}]|}{|[TLS]|}$$

And, it is obvious that

$$\lim_{Pr(p_1) \rightarrow 0} C_a(TLS) - Pr(p_1) \cdot \frac{|[TLS - \{p_n\}]|}{|[TLS]|} = C_a(TLS).$$

Thus,

$$\lim_{Pr(p_1) \rightarrow 0} C_{d_a}(TLS) = C_a(TLS)$$

Proof of Theorem 4.3.

Theorem 4.3. $\lim_{n \rightarrow \infty} C_{M_a}(TLS) = 1/2$.

PROOF. By definition, we have it that

$$C_{M_a}(TLS) = \frac{\sum_{S' \in [TLS]_1} C_{O_a}(S')}{|[TLS]_1|}$$

The proof depends on the following observation: First, for all $S' \in [TLS]_1$, $C_{O_a}(S') = 1$ if $p_n \notin S'$ and $C_{O_a}(S') = 0$ if $p_n \in S'$.

and thus calculating the coherence of TLS reduces to a problem of calculating the ratio of sets $S' \in [TLS]_1$ such that p_n is not in S' to the total number of sets in $[TLS]_1$. We thus have it that $C_{M_a}(TLS) = \frac{2^{n-1} - n}{2^n - (n+1)}$, and it is easily shown that this ratio is less than $1/2$ and converges to $1/2$ as n tends towards ∞ .

□

Proof of Theorem 4.4.

Theorem 4.4. $\lim_{n \rightarrow \infty} C_{M_a^W}(TLS) = 1/2$ and $C_{M_a^W}(TLS) < 1/2$.

PROOF. Let n be the cardinality of TLS . Then by definition, we have:

$$C_{M_a^W}(TLS) = \frac{\sum_{i=1}^{n-1} C_{O_a}^{i+1}(TLS)}{n-1}$$

And, by definition, we have it that

$$C_{O_a}^{i+1}(TLS) = \frac{\sum_{S' \in [TLS]_1} C_{O_a}(S')}{\binom{n}{i+1}}$$

Again, for all for $S' \in [TLS]^K$, i.e., subsets of cardinality k , $C_{O_a}(S') = 0$ when $p_n \in S'$ and $C_{O_a}(S') = 1$ when $p_n \notin S'$. Thus, $\sum_{S' \in [TLS]_1^{i+1}} C_{O_a}(S')$ is equal to the number of sets in $[TLS]^{i+1}$ such that do not contain p_n . There will be $\binom{n-1}{i+1}$ such sets. It thus follows that It is straightforward to verify that

$$C_{O_a}^{i+1}(TLS) = \frac{\binom{n-1}{i+1}}{\binom{n}{i+1}} = \frac{n-1-i}{n}$$

$$\frac{\sum_{i=1}^{n-1} C_{O_a}^{i+1}(TLS)}{n-1} = \frac{\sum_{i=1}^{n-1} \frac{n-1-i}{n}}{n-1} = \frac{1+2+\dots+n-2}{(n-1) \cdot n} = \frac{(n-2) \cdot (n-1)}{n \cdot (n-1) \cdot 2} = \frac{n-2}{2 \cdot n}$$

And, $\frac{n-2}{2 \cdot n}$ is less than $1/2$ for $n \geq 2$ and clearly converges to $1/2$. Hence, $\lim_{n \rightarrow \infty} C_{M_a^W}(TLS) = 1/2$, and $C_{M_a^W}(TLS) < 1/2$. \square

Proof of Theorem 4.5.

Theorem 4.5. $C_F(TLS) < -1/3$.

PROOF. The proof requires only one new observations and then we can employ the facts used from above. First, if $p_n \in S'$ or $p_n \in S^*$, then $F(S', S^*) = -1$ (it is a general feature of the F that if $S' \cup S''$ is inconsistent, then the degree of negative deductive support between S' and S'' is maximal). If $p_n \notin S'$ and $p_n \notin S^*$, then $F(S', S^*) = 1$ (since the members of the set are logically equivalent and logically equivalent sets support each other maximally on F . In our proof of Theorem 4.1, we showed that more than $2/3^{rds}$ of the pairs in $[TLS]$ contain p_n , so $2/3^{rds}$ of the values we are averaging are -1. and the remainder of the values are value 1, which means the average of those values must be less than $-1/3$. \square

Proof of Theorem 4.6.

Theorem 4.6. $\lim_{n \rightarrow \infty} C_{M_a^{DI}}(TLS) = 2/3$, and for all n , $C_{M_a^{DI}} < (TLS)^{2/3}$

PROOF. By definition,

$$C_{M_a^{DI}}(TLS) = \sum_{i=1}^{n-1} \frac{2(n-i)}{n(n-1)} C_{O_a}^{i+1}(TLS).$$

□

And in our proof of theorem 4.4., we showed that

$$C_{O_a}^{i+1}(TLS) = \frac{n-1-i}{n}$$

Thus, via substitution, we have

$$\begin{aligned} C_{M_a^{DI}}(TLS) &= \sum_{i=1}^{n-1} \frac{2(n-i)}{n(n-1)} \cdot \frac{n-1-i}{n} \\ &= \frac{2}{n^2 \cdot (n-1)} \cdot \sum_{i=1}^{n-1} (n-i) \cdot (n-1-i) \end{aligned}$$

.

And, it can be shown that

$$\sum_{i=1}^{n-1} (n-i) \cdot (n-1-i) = \frac{1}{3} n \cdot (n-1) \cdot (n-2)$$

It is worth noting that this equality was arrived at using wolfram alpha. Via substitution, we thus have

$$C_{M_a^{DI}}(TLS) = \frac{2 \cdot n \cdot (n-1) \cdot (n-2)}{3 \cdot n^2 \cdot (n-1)}.$$

And as n tends to ∞ , it is clear that $\frac{2 \cdot n \cdot (n-1) \cdot (n-2)}{3 \cdot n^2 \cdot (n-1)}$ tends to $2/3$, and is less than $2/3$ for $n \geq 2$.

Proof of Theorem 4.7.

Theorem 4.7. *For any $0 < \delta$, the measures C_{M_a} , $C_{M_a^w}$, C_{d_a} , C_a and C_F there is a depth increasing analogue to the measure that assigns TLS a coherence value, t , such that $t > 1 - \delta$.*

PROOF. I'm going to provide a basic sketch of why this claim must be true, rather than constructing an exact weighting scheme that demonstrates that the theorem holds. Let $m()$ be a measure of the coherence making relations that hold amongst the subsets or subset pairs of a set. In all case, the measure assigns values to subsets that range between 1 and -1 or else 0 and 1.

As the cardinality of TLS goes to infinity, the average of the values assigned to binary subsets or binary subset pairs converges to 1 (subsets whose cardinality is 2, or else subset pairs whose union is cardinality 2). There are $\binom{n-1}{2} \cdot 2$ subset pairs free of the contradiction (the number of such subsets containing exactly 2 elements, times the number of ways to partition them into 2 sets), and $2 \cdot (n-1)$ subset pairs containing the contradiction. All of those free of the contradiction can receive a value near the maximal value (which is 1 in all cases) of the relevant coherence measures. So, the average value assigned can be approximately $\frac{\binom{n-1}{2} - n - 1}{\binom{n}{2}}$, which is equal to $\frac{n-2}{n} - \frac{n-1}{\binom{n}{2}}$. As n go to infinity, $\frac{n-1}{\binom{n}{2}}$ converges to 0 and $\frac{n-2}{n}$ converges to 1. An analogous point clearly holds for binary subsets and for the measures assigning a coherence-making value to subsets, rather than subset pairs (The number of subsets is the number of subset pairs divided by 2, since order of the pairs doesn't matter in the case of a subsets).

We can take a weighted average of the values assigned to the subset or subset pairs of $[TLS]$ (recall that $[S]$ is an ordering on the disjoint, non-empty subset pairs of S , or else non-empty subsets of cardinality greater than or equal to 2) to arrive at analogous coherence measures. Let w_h be the uniform weight assigned to each binary pair, or subset pair. Now, there will be $\binom{n}{2} \cdot 2$ such binary subset pairs, and $\binom{n}{2}$ binary subsets. As $w_h \cdot \binom{n}{2} \cdot 2$ or $w_h \cdot \binom{n}{2}$ approaches 1 (the total weights must sum to 1), the contribution to coherence provided by all of the other values assigned to the non-binary subsets or subset pairs converges to 0 (In the case of $C_{M_a^w}$, we just let w_h go to 1). So, as $w_h \cdot \binom{n}{2} \cdot 2$ converges to 1, the value assigned by the coherence measure will converge to the average value assigned to the binary subsets or subset pairs. Since these values converge to the maximum value, we thus can arrive at a scheme assigning a near maximum degree of coherence to TLS as the cardinality of TLS goes to infinity. \square

Important Facts about the Relevant Probability Function for the Extra Generous Lottery. To emphasize, there are three key facts about the lottery propositions that will be relevant to considering whether they can cohere or not. The first is to note that any conjunction of lottery propositions will start with a low probability if r is low:

Fact 1: If $S' \subseteq L_E^n$ and S' is non-empty, then $Pr(S') \leq r$.

This follows from the fact that each generous lottery proposition entails NG , and $Pr(NG) = r$. Next, the conditional probability of any generous lottery proposition on another generous lottery proposition is equivalent to the probability of one standard lottery proposition conditional on any other:

Fact 2: $Pr(e_i|e_j) = Pr(l_i|l_j)$.

The argument for this is simply that the following equalities hold by definition or the assumption of independence of NG and the truth of the standard lottery propositions. $Pr(e_i|e_j) = \frac{Pr(l_i \wedge l_j \wedge NG)}{Pr(l_j \wedge NG)} = \frac{Pr(l_i \wedge l_j) \cdot Pr(NG)}{Pr(l_j) \cdot Pr(NG)} = \frac{Pr(l_i \wedge l_j)}{Pr(l_j)} = Pr(l_i|l_j)$.

The same arguments will generalize over all disjoint non-empty subset pairs of $[L_E^n]$:

Fact 3: If $\langle S', S'' \rangle \in [L_E^n]$, $Pr(\bigwedge S' | \neg \bigwedge S'') = \frac{n - (|S'| + |S''|)}{n - |S''|}$.

These are the key facts that we shall appeal to in our proofs below. Now let us continue proving the theorems from this chapter.

Proof of Theorem 4.8.

Theorem 4.8. *If S is inconsistent ($Pr(\bigwedge S) = 0$), then $C_a(S) < \frac{1}{2}$.*

The proof proceeds via a few simple observations. First, assuming S is inconsistent, any pair, $\langle S', S'' \rangle$, that partitions S will be such that $a(\langle S', S'' \rangle) = 0$. Thus, we divide the pairs in $[S]$ into those that partition S and those that don't.

DEFINITION 5.8. $[S]^P =_{def} \{ \langle S', S'' \rangle | \langle S', S'' \rangle \in [S] \text{ and } S' \cup S'' = S \}$. ('P' because these are the pairs that *partition* S)

DEFINITION 5.9. $[S]^{NP} =_{def} \{ \langle S', S'' \rangle \mid \langle S', S'' \rangle \in [S] \text{ and } S' \cup S'' \neq S \}$. ('NP' because these are the pairs that do *not partition* S).

Now, C_a is the average of the values assigned to all of the subset pairs in $[S]$. Since those that partition S are all assigned 0, we have it that:

$$C_a(S) = \frac{\sum_{\langle S', S'' \rangle \in [S]^{NP}} a(\langle S', S'' \rangle)}{|[S]|}$$

Next, we define a bijection from $[S]^{NP}$ to $[S]^{NP}$ as follows:

DEFINITION. $f : [S]^{NP} \rightarrow [S]^{NP}$ such that $f(\langle S', S'' \rangle) = \langle S - S' \cup S'', S'' \rangle$.

The reason we defined the function is that f takes each element in $[S]^{NP}$ to a unique element in $[S]^{NP}$ such that the sum of the values assigned to both by $a(\cdot)$ is less than or equal to 1. In other words, we chose f so that the following holds:

$$(5.12.1) \quad a(\langle S', S'' \rangle) + a(f(\langle S', S'' \rangle)) \leq 1$$

The proof of 5.12.1 goes as follows. First, note that if $Pr(\bigwedge S'') = 0$, then $a(\langle S', S'' \rangle) + a(f(\langle S', S'' \rangle)) = 0$. Otherwise,

$$a(\langle S', S'' \rangle) + a(f(\langle S', S'' \rangle)) = \frac{Pr(\bigwedge S' \wedge \bigwedge S'') + Pr((\bigwedge S - S' \cup S'') \wedge \bigwedge S'')}{Pr(\bigwedge S'')}.$$

Let A be $(S' \cup S'')$ and B be $(S - S' \cup S'') \cup S''$. Then

$$a(\langle S', S'' \rangle) + a(f(\langle S', S'' \rangle)) = \frac{Pr(\bigwedge A) + Pr(\bigwedge B)}{Pr(\bigwedge S'')}.$$

Since, $A \cup B = S$, and S is inconsistent, $\bigwedge A$ and $\bigwedge B$ are inconsistent with each other. Thus, $Pr(\bigwedge A) + Pr(\bigwedge B) = Pr(\bigwedge A \vee \bigwedge B)$. Since $\bigwedge A$ and $\bigwedge B$ both entail $\bigwedge S''$, $Pr(\bigwedge A \vee \bigwedge B) \leq Pr(\bigwedge S'')$. Thus,

$$a(\langle S', S'' \rangle) + a(f(\langle S', S'' \rangle)) \leq \frac{Pr(\bigwedge S'')}{Pr(\bigwedge S'')}.$$

Thus, 5.12.1 must be true.

Next we observe that since f is a bijection, the following summations are equal:

$$\begin{aligned} C_a(S) &= \frac{\sum_{\langle S', S'' \rangle \in [S]^{NP}} a(\langle S', S'' \rangle)}{|[S]|} = \frac{\sum_{\langle S', S'' \rangle \in [S]^{NP}} a(f(\langle S', S'' \rangle))}{|[S]|} \\ &= \frac{\sum_{\langle S', S'' \rangle \in [S]^{NP}} a(\langle S', S'' \rangle) + a(f(\langle S', S'' \rangle))}{2 \cdot |[S]|} \end{aligned}$$

From 5.12.1, we know that for each pair $\langle S', S'' \rangle \in [S]^{NP}$, $[a(\langle S', S'' \rangle) + a(f(\langle S', S'' \rangle))] \leq 1$, so it follows that

$$\sum_{\langle S', S'' \rangle \in [S]^{NP}} a(\langle S', S'' \rangle) + a(f(\langle S', S'' \rangle)) \leq |[S]^{NP}|$$

Thus, we have it that if S is inconsistent, then it follows that

$$C_a(S) \leq \frac{|[S]^{NP}|}{|[S]| \cdot 2} < \frac{1}{2}.$$

Proof of Theorem 4.9.

Theorem 4.9. $\lim_{n \rightarrow \infty} C_a(L_E^n) = 1/2$.

The key thing to note is in our standard lottery case and in the extra generous lottery, the following is true.

$$(5.12.2) \quad \text{For each pair } \langle S', S'' \rangle \in [S]^{NP}, [a(\langle S', S'' \rangle) + a(f(\langle S', S'' \rangle))] = 1$$

First, note that

$$[a(\langle S', S'' \rangle) + a(f(\langle S', S'' \rangle))] = Pr(\bigwedge S' | \bigwedge S'') + Pr\left((\bigwedge (S - S' \cup S'')) | \bigwedge S''\right), \text{ by definition of } f.$$

Perhaps the easiest way to see that 5.12.2 is true is to consider what $\bigwedge S'$, $\bigwedge S''$ and $\bigwedge(S - S' \cup S'')$ says. Each is a set of lottery propositions, so let us suppose $S' = \{e_1, \dots, e_j\}$, $S'' = \{e_{j+1}, \dots, e_k\}$, and $(S - S' \cup S'') = \{e_{k+1}, \dots, e_n\}$. The conjunctive contents of each set say that NG is true, and that the initial ticket drawn was amongst the respective lottery propositions, e.g., $\bigwedge S''$ says that NG is true and the lottery ticket that was initially drawn was not amongst tickets $j + 1$ through k . On the assumption that $\bigwedge S''$, NG is true and the initial ticket must have been drawn from either tickets 1 through j , or else from tickets $k+1$ through n . Thus, $Pr((\bigwedge(S - S' \cup S'') \vee (\bigwedge S')) | \bigwedge S'') = 1$. Since $\bigwedge S'$ and $\bigwedge(S - S' \cup S'')$ are mutually exclusive, it follows that

$$Pr((\bigwedge(S - S' \cup S'') \vee (\bigwedge S')) | \bigwedge S'') = Pr(\bigwedge S' | \bigwedge S'') + Pr((\bigwedge(S - S' \cup S'') | \bigwedge S'')$$

Thus, 5.12.2 is true. And from 5.12.2 it immediately follows that

$$\sum_{\langle S', S'' \rangle \in [S]^{NP}} a(\langle S', S'' \rangle) + a(f(\langle S', S'' \rangle)) = |[S]^{NP}|$$

Since we demonstrated in the proof of Theorem 4.8 that

$$= \frac{\sum_{\langle S', S'' \rangle \in [S]^{NP}} a(\langle S', S'' \rangle) + a(f(\langle S', S'' \rangle))}{2 \cdot |[S]|}$$

We have it that

$$(5.12.3) \quad C_a(S) = \frac{|[S]^{NP}|}{2 \cdot |[S]|}$$

To complete the proof, we just need to show that as n approaches ∞ , $\frac{|[S]^{NP}|}{|[S]|}$ converges to 1. This is easily done. We have already observed that there are $3^n - 2^{n+1} + 1$ subset pairs in $[S]$. Since $[S]^{NP} = [S] - [S]^P$, to determine $[S]^{NP}$, we simply need to subtract the number of subset pairs

that partition S . Stirling numbers of the second kind are the number of ways to partition a set of size n is $2^{n-1} - 1$.⁴⁸ We multiply these by 2 since order of the partition matters, we get that

$$(5.12.4) \quad \frac{1}{2} \cdot \frac{|[S]^{NP}|}{|[S]|} = \frac{1}{2} \cdot \frac{[(3^n - 2^{n+1} + 1) - (2^n - 2)]}{3^n - 2^{n+1} + 1}$$

It is easily shown that

$$(5.12.5) \quad \lim_{n \rightarrow \infty} \frac{1}{2} \cdot \frac{[(3^n - 2^{n+1} + 1) - (2^n - 2)]}{3^n - 2^{n+1} + 1} = \frac{1}{2}.$$

This is fairly obvious once this is simplified to

$$\frac{1}{2} \cdot \left(1 - \frac{[2^n - 2]}{3^n - 2^{n+1} + 1} \right),$$

since it is obvious that $\frac{[2^n - 2]}{3^n - 2^{n+1} + 1}$ converges to 0. And, from 5.12.3, 5.12.4 and 5.12.5, it follows that $\lim_{n \rightarrow \infty} C_a(L_E^n) = 1/2$.

Proof of Theorem 4.10.

LEMMA. $\lim_{r \rightarrow 0} C_d(L_B^n) = C_a(L_B^n)$.

PROOF.

$$\begin{aligned} C_d(L_B^n) &= \frac{\sum_{\langle S', S'' \rangle \in [S]} d(\langle S', S_* \rangle)}{|[S]|} \\ &= \frac{\sum_{\langle S', S'' \rangle \in [S]} a(\langle S', S_* \rangle) - Pr(\bigwedge S')}{|[S]|} \\ &= \frac{\sum_{\langle S', S'' \rangle \in [S]} a(\langle S', S_* \rangle) - r \cdot \left[\frac{|S| - |S'|}{|S|} \right]}{|[S]|} \end{aligned}$$

⁴⁸See Weisstein ("Stirling Numbers of the Second Kind") for further discussion and additional references.

$$\begin{aligned}
&= \frac{\sum_{\langle S', S'' \rangle \in [S]} a(\langle S', S^* \rangle) - r \cdot \left[\frac{|S| - |S'|}{|S|} \right]}{|[S]|} \\
&= C_a(L_B^n) - r \cdot \frac{\sum_{\langle S', S'' \rangle \in [S]} \left[\frac{|S| - |S'|}{|S|} \right]}{|[S]|}
\end{aligned}$$

As r converges to 0, the above clearly converges to $C_a(L_B^n)$. Thus, the Lemma must hold. \square

Theorem 4.10. *The following sequence $\lim_{r \rightarrow 0} C_d((L_B^2))$, $\lim_{r \rightarrow 0} C_d((L_B^3))$, ..., converges to $1/2$.*

PROOF. This follows from the Lemma above and Theorem 4.9 \square

Theorem 4.11. *$\lim_{n \rightarrow \infty} C_{M_a}(L_B^n) = 1/2$ and $C_{M_a}(L_B^n) < 1/2$.*

PROOF. By definition, we have that

$$C_M(L_B^n) = \frac{\sum_{S' \in [L_B^n]_1} C_{O_a}(S')}{2^n - 1},$$

Now, First, we need the following fact

$$(5.12.6) \quad \text{For all } S' \in [L_E^n], C_{O_a}(S') = \frac{Pr(\bigwedge S')}{Pr(\bigvee S')} = \frac{n - |S'|}{n}$$

Why is (5.12.6) true? If $S' \in [L_E^n]$, we can order each set of lottery propositions: $\{e_1^{S'}, \dots, e_{|S'|}^{S'}\}$. Now, $\bigwedge S'$ is equivalent to $e_1^{S'} \wedge \dots \wedge e_{|S'|}^{S'}$, which by definition is equivalent to the conjunction of the corresponding standard lottery propositions all conjoined with NG . Thus, $\bigwedge S$ is equivalent to $NG \wedge l_1^{S'} \wedge \dots \wedge l_{|S'|}^{S'}$. We have an analogous equivalence for $\bigvee S'$: $\bigvee S'$ is equivalent to $NG \wedge (l_1^{S'} \vee \dots \vee l_{|S'|}^{S'})$. Since NG is independent of the initial drawing by the description of the case, we get the following two equalities:

$$(5.12.7) \quad Pr(NG \wedge l_1^{S'} \wedge \dots \wedge l_{|S'|}^{S'}) = Pr(NG) \cdot Pr(l_1^{S'} \wedge \dots \wedge l_{|S'|}^{S'})$$

And,

$$(5.12.8) \quad Pr(NG \wedge (l_1^{S'} \vee \dots \vee l_{|S'|}^{S'})) = Pr(NG) \cdot Pr(l_1 \vee \dots \vee l_{|S'|}^{S'})$$

And, from standard lotteries it is obvious that

$$Pr(l_1^{S'} \wedge \dots \wedge l_{|S'|}^{S'}) = \frac{n - |S'|}{n},$$

since this is just the odds the standard lottery ticket chosen does not correspond to the ticket mentioned by any of the lottery propositions in S' . And, $Pr(l_1 \vee \dots \vee l_{|S'|}^{S'}) = 1$, since the disjunction of any two standard lottery propositions has a probability of 1 (after all, one won't be chosen as the winner). Hence by substitution of equivalents in 5.12.7, 5.12.8, and simplification, we get that $C_{O_a}(S') = \frac{Pr(\bigwedge S')}{Pr(\bigvee S')} = \frac{n - |S'|}{n}$.

Next, we break up the subsets belonging to $[L_E^n]$ into three groups. Those subsets that contain $n - 1$ members of L_E^n , those subsets containing fewer than $n - 1$ members of L_E^n , and finally L_E^n itself. Formally, these are:

$$[L_B^n]^{=n-1} = \{S' \mid S' \in [L_B^n] \ \& \ |S'| = n - 1\}$$

$$[L_B^n]^{<n-1} = \{S' \mid S' \in [L_B^n] \ \& \ |S'| < n - 1\}.$$

$$\{L_B^n\}$$

Now, we know $C_{O_a}(L_B^n) = 0$, and by 5.12.6, if $S' \in [L_B^n]^{=n-1}$, then $C_{O_a}(S') = \frac{n - |S'|}{n} = \frac{1}{n}$. There are exactly n members of $[L_B^n]^{=n-1}$, since $\binom{n}{n-1} = n$. Thus,

$$(5.12.9) \quad \sum_{S' \in [L_B^n]^{=n-1}} C_{O_a}(S') + C_{O_a}(L_B^n) = 1$$

Since $[L_B^n] = [L_B^n]^{<n-1} \cup [L_B^n]^{=n-1} \cup \{L_B^n\}$ we have the following equivalence:

$$(5.12.10) \quad C_{M_a}(L_B^n) = \frac{\sum_{S' \in [L_B^n]_1} C_{O_a}(S')}{2^n - 1} = \frac{\sum_{S' \in [L_B^n]_1^{=n-1}} C_{O_a}(S') + \sum_{S' \in [L_B^n]_1^{<n-1}} C_{O_a}(S')}{2^n - 1}$$

By 5.12.9, we have

$$(5.12.11) \quad C_{M_a}(L_B^n) = \frac{1 + \sum_{S' \in [L_B^n]_1^{<n-1}} C_{O_a}(S')}{2^n - 1}$$

Next, we define a bijection from $[L_B^n]^{<n-1}$ to $[L_B^n]^{<n-1}$ as follows:

$g : [L_B^n]^{<n-1} \rightarrow [L_B^n]^{<n-1}$ such that $f(S') = S - S'$.

Since for all $S' \in [L_B^n]^{<n-1}$, $S' \cup g(S') = S$ and $S' \cap g(S') = \emptyset$, we have it that $|S'| + |g(S')| = n$.

By 5.12.6,

$$(5.12.12) \quad C_{O_a}(S') + C_{O_a}(g(S')) = \frac{|S'| + |g(S')|}{n} = 1$$

Since g is a bijection, we have

$$\begin{aligned} \sum_{S' \in [L_B^n]_1^{<n-1}} C_{O_a}(S') &= \sum_{S' \in [L_B^n]_1^{<n-1}} C_{O_a}(g(S')) = \frac{\sum_{S' \in [L_B^n]_1^{<n-1}} C_{O_a}(S') + C_{O_a}(g(S'))}{2} \\ &= \frac{\sum_{S' \in [L_B^n]_1^{<n-1}} 1}{2}, \text{ by 5.12.12.} \end{aligned}$$

$$= \frac{|[L_B^n]_1^{<n-1}|}{2}, \text{ since we are adding 1 for each of the subsets in } [L_B^n]^{<n-1}.$$

Now, to calculate $|[L_B^n]^{<n-1}|$ we simply need to determine how many non-empty subsets, S' , of $[L_B^n]$ there are such that $2 \leq |S'| \leq n-2$. There are of course 2^n subsets of L_B^n . These include all of the subsets of cardinality 0, 1, $n-1$ and n , which we need to subtract from 2^n . Thus,

$$(5.12.13) \quad \begin{aligned} |[L_B^n]^{<n-1}| &= 2^n - \left(\binom{n}{0} + \binom{n}{1} + \binom{n}{n-1} + \binom{n}{n} \right) \\ &= 2^n - (2 \cdot n + 2) \end{aligned}$$

And, thus by 5.12.11 and 5.12.13, it follows that

$$C_{M_a}(L_B^n) = \frac{2^n - 2n - 1}{(2^n - 1) \cdot 2}$$

And, it is easily shown that $\frac{2^n - 2n - 1}{(2^n - 1) \cdot 2} < \frac{1}{2}$ and this value converges to $\frac{1}{2}$ as n approaches infinity. \square

Proof of Theorem 4.12.

Theorem 4.12. $\lim_{n \rightarrow \infty} C_{M_a^w}(L_B^n) = 1/2$ and $C_{M_a^w}(L_B^n) < 1/2$.

PROOF. By definition, we start with

$$C_{M_a^w}(L_B^n) = \frac{\sum_{i=1}^{n-1} C_{O_a}^{i+1}(L_B^n)}{n-1}$$

In our previous proof, we established that for $S' \in L_B^n$, $C_{O_a}(S') = \frac{n - |S'|}{n}$. Since $C_{O_a}^{i+1}(S')$ is equal to the average of the values assigned by C_{O_a} to all sets of cardinality $i+1$, we have it that

$$\frac{\sum_{i=1}^{n-1} C_{O_a}^{i+1}(L_B^n)}{n-1} = \frac{\sum_{i=1}^{n-1} \frac{n - (i+1)}{n}}{n-1} = \frac{\frac{1}{n} + \frac{2}{n} + \dots + \frac{n-2}{n}}{n-1} = \frac{1 + 2 + \dots + (n-2)}{n \cdot (n-1)}$$

$$= \frac{(n-2) \cdot (n-1)}{n \cdot (n-1) \cdot 2}$$

And, it is obvious that, for $n \geq 2$, $\frac{(n-2) \cdot (n-1)}{n \cdot (n-1) \cdot 2} < 1/2$ and that $\lim_{n \rightarrow \infty} \frac{(n-2) \cdot (n-1)}{n \cdot (n-1) \cdot 2} = 1/2$.

□

Proof of Theorem 4.13.

Theorem 4.13. *The following sequence $\lim_{r \rightarrow 0} C_F((L_B^2))$, $\lim_{r \rightarrow 0} C_F((L_B^3))$, ..., converges to 1.*

PROOF. The proof is similar to our proof of Theorem 4.9. In that proof, we showed that as the cardinality of L_B^n goes to ∞ , then ratio of subset pairs in $[L_B^n]^{NP}$ to the ratio of subset pairs in $[L_B^n]$ goes to 1. Thus, the average value assigned to subset pairs $[L_B^n]$ converges to the average value assigned to the subset pairs in $[L_B^n]^{NP}$ as the sequence of limits progresses. Thus, to establish Theorem 4.13, all we need to prove is the following:

$$(5.12.14) \quad \text{If } \langle S_j, S_k \rangle \in [L_B^n]^{NP}, \text{ then } \lim_{r \rightarrow 0} F(S_j, S_k) = 1$$

Let us suppose $\langle S_j, S_k \rangle \in [L_B^n]^{NP}$. Then by definition, we have it that

$$F(S_j, S_k) = \frac{Pr(\bigwedge S_k | \bigwedge S_j) - Pr(\bigwedge S_k | \neg \bigwedge S_j)}{Pr(\bigwedge S_k | \bigwedge S_j) + Pr(\bigwedge S_k | \neg \bigwedge S_j)}$$

The key observation to the proof is that

$$(5.12.15) \quad \text{For any } \langle S_j, S_k \rangle \in [L_B^n]^{NP}, \lim_{r \rightarrow 0} Pr(\bigwedge S_k | \neg \bigwedge S_j) = 0$$

By definition of conditional probabilities,

$$(5.12.16) \quad Pr(\bigwedge S_k | \neg \bigwedge S_j) = \frac{Pr(\bigwedge S_k \wedge \neg \bigwedge S_j)}{Pr(\neg \bigwedge S_j)}$$

The easiest way to see this is to note that $\bigwedge S_k$ is true only if NG is true, so

$$(5.12.17) \quad Pr(\bigwedge S_k \wedge \neg \bigwedge S_j) \leq Pr(NG)$$

And $\neg \bigwedge S_j$ is true if NG is false, so

$$(5.12.18) \quad Pr(\neg \bigwedge S_j) \leq Pr(\neg NG)$$

Together, (5.12.16) and (5.12.18) entail that

$$Pr(\bigwedge S_k | \neg \bigwedge S_j) \leq \frac{Pr(NG)}{Pr(\neg NG)} = \frac{r}{1-r}$$

Clearly, $\lim_{r \rightarrow 0} \frac{r}{1-r} = 0$, and so $\lim_{r \rightarrow 0} Pr(\bigwedge S_k | \neg \bigwedge S_j) = 0$. Thus, (5.12.15) must be true.

The last thing to note is that $P(\bigwedge S_k | \bigwedge S_j)$ is completely independent of r . Again, both $\bigwedge S_k$ and $\bigwedge S_j$ are true just in case NG is true and their respective tickets were not initially chosen. So, on the assumption that $\bigwedge S_j$ is true, the probability that $\bigwedge S_k$ is true comes down to the chance that the ticket initially drawn was not in those corresponding to the lottery propositions in S_k . In fact, it is easy to verify that

$$(Pr(\bigwedge S_k | \bigwedge S_j)) = \frac{|S| - |S_j \cup S_k|}{|S| - |S_j|}$$

Thus, by (5.12.14) and (5.12.15), we have it that

$$\text{If } \langle S_j, S_k \rangle \in [L_B^n]^{NP}, \text{ then } \lim_{r \rightarrow 0} F(S_j, S_k) = \frac{Pr(\bigwedge S_k | \bigwedge S_j) - 0}{Pr(\bigwedge S_k | \bigwedge S_j) + 0} = 1$$

Since, as we noted at the outset, the average value assigned by F to the pairs in $[L_B^n]$ converges to the average value assigned to the pairs in $[L_B^n]^{NP}$ as n goes to ∞ , Theorem 4.13 is established. \square

Proof of theorem 4.14.

Theorem 4.14. *If S is strictly-dominated, then $C_F(S) < 0$.*

PROOF. The proof is analogous to our method for proving Theorem 4.8. The first thing to note is that if S is strictly dominated, there is some subset S^w of S that is mostly false at all possible worlds, i.e., more than half of the claims in S^w are false at all possible worlds (This is one of the equivalences that Easwaran and Fitelson prove that we explained in Chapter 1). Thus, if a set S' contains at least half of S^w , then $S' \models \perp$. Next, let $[S]^P$ and $[S]^{NP}$ be the sets of subsets that do and don't partition S , respectively. Now, we can define a bijection from $[S]^{NP}$ to $[S]^{NP}$ as follows:

$$\langle S', S'' \rangle^{wit} = \langle S - S' \cup S'', S'' \rangle.$$

$\langle S', S'' \rangle^{wit}$ is such that either $\langle S', S'' \rangle$ or $\langle S', S'' \rangle^{wit}$ contains at least half of S^w . Thus, either the union of the sets in $\langle S', S'' \rangle$ or $\langle S', S'' \rangle^{wit}$ entail \perp . Since $F(X, Y) = -1$ if $X \cup Y \models \perp$, it follows that either $\langle S', S'' \rangle^w$ or $\langle S', S'' \rangle$ is assigned -1 by F . Thus, the average of these pairs must be less than or equal to 0. Since $\langle S', S'' \rangle^w$ is a bijection, it follows that

$$\sum_{\langle S', S'' \rangle \in [S]^{NP}} F(\langle S', S'' \rangle) = \frac{\sum_{\langle S', S'' \rangle \in [S]^{NP}} F(\langle S', S'' \rangle) + F(\langle S', S'' \rangle^{wit})}{2}.$$

Thus, the average value assigned to the pairs in $[S]^{NP}$ is less than 0. The only other pairs to consider are those such that $S' \cup S'' = S$, and these will get value -1 . Consequently, the average of all the values assigned by F to the pairs in $[S]$ will be less than 0, and $C_F(S)$ is just the average value assigned to all such pairs. \square

Proof of Fact 6. Let us calculate the coherence of propositions in the party example. From our observations about the relevant probability function in party example, we know the following two claims to be true:

- (O1) If either H_5 or H_6 is not in $S^* \cup S'$, then $F\langle S', S^* \rangle \geq \frac{1/2 - 1/m}{1/2 + 1/m}$.
- (O2) If both H_5 and H_6 is not in $S^* \cup S'$, then $F\langle S', S^* \rangle = -1$.

As m tends to finity, (O1) implies

(O1') If either H_5 or H_6 is not in $S^* \cup S'$ and as m converges on ∞ , then $F\langle S', S^* \rangle \approx 1$.

So, our calculation of the coherence of $C_F(\{H_0, H_1, H_2, H_3, H_4, H_5, H_6\})$ comes down to figuring out the ratio of subset pairs satisfying the antecedent of (O1) and the number of subset pairs satisfying the antecedent of (O2). Letting $[S]^* = \{\langle S', S'' \rangle \in [S] \mid \{H_5, H_6\} \subseteq S' \cup S''\}$, we have it that as m converges on ∞ ,

$$C_F(\{H_0, H_1, H_2, H_3, H_4, H_5, H_6\}) \approx \frac{[S] - 2 \cdot [S]^*}{[S]}$$

In the proof of Theorem 4.15, we explore in detail how to calculate this ratio. Plugging our values into the formula derived below, we arrive at

$$C_F(\{H_0, H_1, H_2, H_3, H_4, H_5, H_6\}) \approx \frac{[S] - 2 \cdot [S]^*}{[S]} = \frac{120}{1932}.$$

Proof of Theorem 4.15.

Theorem 4.15. *If S is weakly-dominated, then $C_F(S) < 1/9$.*

PROOF. First, let us begin with a brief explanation of why the theorem is true. If S is weakly dominated then there is a witness set $S^w \subseteq S$ where at least half of S^w is false at all possible worlds. This means that for any subset of $S' \subseteq S^w$, if $|S'| > \frac{|S^w|}{2}$, then $\bigwedge S' \models \perp$. In other words, if a set, S' , contains more than half of S^w , then at any world w least one of the members of S' is false at w . Hence, the conjunction of that set cannot be satisfied at any world. Now, if the union of a binary pair in $[S]$ is inconsistent set, i.e., $\bigwedge (S_1 \cup S_2) \models \perp$, then $F(S_1, S_2) = -1$. Thus, the following is a fact:

FACT 5.10. Supposing S^w is a witnessing set for S , if $\langle S', S'' \rangle \in [S]$ is such that $(S_1 \cup S_2) \cap S^w > \frac{|S^w|}{2}$, then $F(\langle S', S'' \rangle) = -1$.

So, we can break up $[S]$ into two sets of subset pairs using the following definition:

DEFINITION 5.11. $[S]^{w>1/2} = \{\langle S', S'' \rangle \mid \langle S', S'' \rangle \in [S] \text{ \& } (S_1 \cup S_2) \cap S^w > \frac{|S^w|}{2}\}.$

DEFINITION 5.12. $[S]^{w \leq 1/2} = \{\langle S', S'' \rangle \mid \langle S', S'' \rangle \in [S] - [S]^{w > 1/2}\}.$

Since C_F is average of the values assigned to the subset pairs in $[S]$ by F and the maximum value that can be assigned to pairs in $[S]^{w \leq 1/2}$ is 1, while the value assigned to all members of $[S]^{w > 1/2}$ is -1 , it follows that for any weakly dominated set S ,

$$(5.12.19) \quad C_F(S) \leq \frac{|[S]^{w \leq 1/2}| - |[S]^{w > 1/2}|}{|[S]|}$$

The reason the theorem is true boils down to the fact that either $[S]$ is strictly dominated and we can appeal to our last theorem, or else:

$$(5.12.20) \quad \frac{|[S]^{w \leq 1/2}| - |[S]^{w > 1/2}|}{|[S]|} < 1/9$$

In order prove the theorem, we shall need to consider three different cases depending on the cardinality of S^w . The cases are:

Case 1: $|S^w|$ is odd.

Case 2: $|S^w|$ is even.

Case 1: $|S^w|$ is odd.

This case simply requires us to observe that if at least half of S^w is false at all possible worlds, and $|S^w|$ is odd, then more than half of $|S^w|$ is false at all possible worlds. Hence, S would then be strictly-dominated as well as weakly dominated. It then follows from our last theorem that $C_F(S) < 0$.

Case 2: $|S^w|$ is even.

We need to calculate:

$$(5.12.21) \quad \frac{|[S]^{w \leq 1/2}| - |[S]^{w > 1/2}|}{|[S]|}$$

To simplify the presentation of our calculations, it will be useful to define a few formal functions, and make a few basic formal observations. First, we have already noted that $|[S]|$ is a function of n , namely, $f(n) = 3^n - 2^{n+1} + 1$. Similarly, if $|S| = n$ and $|S^w| = k$, then the number of subset pairs in $[S]^{w > 1/2}$ will be a function of n and k . Thus, we can let $g(n, k) = |[S]^{w > 1/2}|$. Given that $[S] = [S]^{w \leq 1/2} \cup [S]^{w > 1/2}$ and $[S]^{w \leq 1/2} \cap [S]^{w > 1/2} = \emptyset$, it follows that $|[S]^{w \leq 1/2}| - |[S]^{w > 1/2}| = f(n) - 2 \cdot g(n, k)$. Thus, what we need to prove is that for all $n \geq 2$ and $k \leq n$ and k is even,

$$(5.12.22) \quad \frac{f(n) - 2 \cdot g(n, k)}{f(n)} \leq 1/9.$$

To simplify things further, let us note that

$$\frac{f(n) - 2 \cdot g(n, k)}{f(n)} \leq 1/9 \text{ if and only if } \frac{g(n, k)}{f(n)} \geq 4/9$$

So it will suffice to prove $\frac{g(n, k)}{f(n)} \geq 4/9$ for all k is an even positive integers.

We will prove this in two steps, first by proving that it is true when k is 2. Then we shall prove that $g(n, k)$ is a strictly increasing function with respect to k over the even integers, which will establish that it is true for all of the even integers.

Step 1: We prove 5.12.22 for $k = 2$.

Assuming that $|S^w| = 2$, we can let $S^w = \{H, H'\}$. And we note that, in this case, every subset pair $\langle S', S'' \rangle \in [S]^{w > 1/2}$ is such that $\{H, H'\} \subseteq S' \cup S''$. So, to calculate the number of subset pairs, $\langle S', S'' \rangle \in [S]^{w > 1/2}$, we can think of this as a two step choice problem where we first select the elements of $S - S^w$ that go into S' and S'' , and then the second step involves choosing where to place H and H' (each has to go in one of the two sets). We can start by calculating the number of subset pairs where S' and S'' both contain at least one element from $S - S^w$. We know that

there are $3^{n-2} - 2^{n-1} + 1$ subset pairs in $[S - S^w]$, and four choices for where to place H and H' . Thus, the number of pairs $\langle S', S'' \rangle \in [S]^{w>1/2}$ where S' and S'' each contain at least one element from $S - S^w$ is given by the formula: $4 \cdot (3^{n-2} - 2^{n-1} + 1)$.

We now need to calculate the number of pairs where either S' or S'' contain no members from $S - S^w$. Let's start with the case where $S'' \cap (S - S^w) = \emptyset$ and $S' \cap (S - S^w) \neq \emptyset$. Now, there are $2^{n-2} - 1$ non-empty subsets of $S - S^w$, and three choices for where to place H and H' (we can't place both in S' , since this would make S'' empty). So, there are $3 \cdot (2^{n-2} - 1)$ subset pairs in $[S]^{w>1/2}$ where S'' contains no members from $S - S^w$ and S' contains at least one member from $S - S^w$. There are the same number of subset pairs where S' contains no members of $S - S^w$ and S'' contains at least one member from $S - S^w$. So, there are $6 \cdot (2^{n-2} - 1)$ subset pairs in $[S]^{w>1/2}$, where either S' or S'' contains no member of $S - S^w$, but where the other contains at least one element in $S - S^w$. Finally, there are two subset pairs where neither S' nor S'' contain any elements from $S - S^w$, namely $\langle \{H\}, \{H'\} \rangle$ and $\langle \{H'\}, \{H\} \rangle$. Thus, we have it that

$$g(n, 2) = 4 \cdot (3^{n-2} - 2^{n-1} + 1) + 6 \cdot (2^{n-2} - 1) + 2.$$

This function can be simplified to:

$$g(n, 2) = 4 \cdot 3^{n-2} - 2^{n-2}$$

So, in order for 5.12.22 to hold in the case where $k = 2$, it must be the case that

$$\frac{g(n, 2)}{f(n)} = \frac{4 \cdot 3^{n-2} - 2^{n-1}}{(3^n - 2^{n+1}) + 1} \geq 4/9.$$

That this claim holds for $n \geq 2$ is easily to prove, though I will omit the proof here. Thus, we can conclude that 5.12.22 holds when $k = 2$.

Step 2: We prove $g(n, k)$ is strictly increasing with respect to k over the positive even integers.

The proof will be complete if we can show that $g(n, k)$ is a strictly increasing function with respect to k over the positive even integers. This will establish that for all $k+2 \leq n$, $g(n, k) < g(n, k+2)$ and $\frac{f(n) - 2 \cdot g(n, k)}{f(n)} < \frac{f(n) - 2 \cdot g(n, 2)}{f(n)} \leq 1/9$. And, given that we have proven the inequality

for $k = 2$ and k is odd, this will entail that there can be no n and k such that $\frac{|[S]^{w \leq 1/2}| - |[S]^{w > 1/2}|}{|[S]|} \geq 1/9$.

To prove $g(n, k)$ is strictly increasing with respect to k over the positive even integers, let us start with any $|S^w| = k \leq n - 2$. Since $k \leq n - 2$ there are at least two elements, a_1 and a_2 , in S that are not in S^w . So now let $S^a = S^w \cup \{a_1, a_2\}$. Now, $g(n, k + 2)$ is just the cardinality of the following set:

DEFINITION 5.13. $[S]^{a > 1/2} = \{\langle S', S'' \rangle | \langle S', S'' \rangle \in [S] \text{ \& } (S_1 \cup S_2) \cap S^a > \frac{k+2}{2}\}$.

To prove $g(n, k + 2) > g(n, k)$, we just need to show that $|[S]^{a > 1/2}| > |[S]^{w > 1/2}|$. We shall do this by comparing their relative complements. So, we shall prove that

$$|[S]^{a > 1/2} - [S]^{w > 1/2}| > |[S]^{w > 1/2} - [S]^{a > 1/2}|, \text{ which is sufficient to show that } |[S]^{a > 1/2}| > |[S]^{w > 1/2}|.$$

First, we observe:

$$[S]^{w > 1/2} - [S]^{a > 1/2} = \{\langle S', S'' \rangle | \langle S', S'' \rangle \in [S] \text{ \& } |(S_1 \cup S_2) \cap S^w| = \frac{k}{2} + 1 \text{ \& } \{a_1, a_2\} \cap (S_1 \cup S_2) = \emptyset\}.$$

Why does this give the correct entry conditions for this relative complement? First, if $(S_1 \cup S_2) \cap S^w \geq \frac{k}{2} + 2$, then it would immediately follow that $\langle S', S'' \rangle \in [S]^{a > 1/2}$. Second, if $\langle S', S'' \rangle \in [S]^{w > 1/2}$, then $(S_1 \cup S_2) \cap S^w \geq \frac{k}{2} + 1$ by definition of $[S]^{w > 1/2}$. So, to be in this relative complement a subset pair must be such that $|(S_1 \cup S_2) \cap S^w| = \frac{k}{2} + 1$. Now, if either a_1 or a_2 are in $(S_1 \cup S_2)$, then $(S_1 \cup S_2) \cap S^w \geq \frac{k}{2} + 2$, which would again mean that $\langle S', S'' \rangle \in [S]^{a > 1/2}$. Hence, these are the correct entry conditions for a subset belonging to this relative complement. A similar argument can be given for why we have it that (I leave the argument implicit, it isn't difficult to workout):

$$[S]^{a > 1/2} - [S]^{w > 1/2} = \{\langle S', S'' \rangle | \langle S', S'' \rangle \in [S] \text{ \& } |(S_1 \cup S_2) \cap S^w| = \frac{k}{2} \text{ \& } \{a_1, a_2\} \subseteq S_1 \cup S_2\}.$$

With these two identities in hand, we can now do some combinatorics to determine which relative complement has more members. To start with, let us work through one way to calculate the pairs

in $[S]^{w>1/2} - [S]^{a>1/2}$ whose union contain exactly j elements from $S - S^a$. One way to do this calculation is to first figure out how many subsets of S there are that contain exactly j elements from $S - S^a$ and exactly $k/2 + 1$ elements from S^w . Then, for each set, S' , meeting these conditions, we need to calculate the number of ways to break S' up into two non-empty disjoint pairs, $\langle S_1, S_2 \rangle$, such that $(S_1 \cup S_2) = S'$. We get the total number of pairs in $[S]^{w>1/2} - [S]^{a>1/2}$ whose union contain exactly j elements from $S - S^a$ by multiplying these values.

The first part of the calculation is straight forward. There are $\binom{n - (k + 2)}{j}$ subsets of $S - S^a$ with exactly j elements, and there are $\binom{k}{k/2 + 1}$ subsets of S^w with exactly $k/2 + 1$ elements. So, in total, there are $\binom{n - (k + 2)}{j} \cdot \binom{k}{k/2 + 1}$ subsets that contain exactly j elements from $S - S^a$ and $k/2 + 1$ elements from S^w . Now, for each such subset, S' , how many ordered pairs, $\langle S_1, S_2 \rangle$, are there where $S_1 \cup S_2 = S'$? That is, how many ways are there to partition any such S' into two non-empty subsets where the order of the subsets matters? First, note that each such subset, S' , has $j + k/2 + 1$. So, we need to calculate the number of ways that a set of $j + k/2 + 1$ can be partitioned into two disjoint non-empty sets where the order of the sets matters. To calculate this, we start by noting that Stirling numbers of the second kind are “The number of ways of partitioning a set of n_1 elements into m_1 nonempty sets,”⁴⁹ where order doesn’t matter, which we shall denote ‘ $S(n_1, m_1)$ ’.⁵⁰ Since order of our partitions matter, we multiply by 2. Hence for each S' where S' contains exactly j elements from $S - S^a$ and $k/2 + 1$ elements from S^w and no other elements, there are $2 \cdot S(j + k/2 + 1, 2)$ ordered pairss, $\langle S_1, S_2 \rangle$, where $S_1 \cup S_2 = S'$. Thus, the number of pairs in $[S]^{w>1/2} - [S]^{a>1/2}$ whose union contain exactly j elements from $S - S^a$ is given by the function:

$$\binom{n - (k + 2)}{j} \cdot \binom{k}{k/2 + 1} \cdot 2 \cdot S(j + k/2 + 1, 2)$$

Now, the total number of pairs in $[S]^{w>1/2} - [S]^{a>1/2}$ is just the sum of all these values for $j = 0$ to $j = n - (k + 2)$. That is to say,

$$|[S]^{w>1/2} - [S]^{a>1/2}| = \sum_{j=0}^{n-(k+2)} \binom{n - (k + 2)}{j} \cdot \binom{k}{k/2 + 1} \cdot 2 \cdot S(j + k/2 + 1, 2).$$

⁴⁹Weisstein (online)

⁵⁰There are many conventions for denoting Stirling numbers of the second kind. Wolfram indicates that this notation was adopted by (Riordan 1980, Roman 1984).

Now, the calculations of the number of pairs in $[S]^{a>1/2} - [S]^{w>1/2}$ proceeds in an analogous fashion. It is easy to work out for reasons analogous to those given above that the number of pairs in $[S]^{a>1/2} - [S]^{w>1/2}$ whose union contain exactly j elements from $S - S^a$ is given by the function:

$$\binom{n - (k + 2)}{j} \cdot \binom{k}{k/2} \cdot 2 \cdot S(j + k/2 + 2, 2).$$

The only value that may not be obvious from our considerations above is the fact that we have $S(j + k/2 + 2, 2)$ as opposed to $S(j + k/2 + 1, 2)$ in the above formula. There are two key things to keep in mind when determining the values to be plugged into the function for Stirling numbers when calculating the number of pairs in $[S]^{a>1/2} - [S]^{w>1/2}$. Any pair in $[S]^{a>1/2} - [S]^{w>1/2}$ will have both a_1 and a_2 in their union, whereas any pair in $[S]^{w>1/2} - [S]^{a>1/2}$ will have neither in their union, but there will be one less element from S^w in the union of a pair in $[S]^{a>1/2} - [S]^{w>1/2}$ than there is in a pair in $[S]^{w>1/2} - [S]^{a>1/2}$. Thus, we are partitioning a set with exactly one extra element when considering pairs in $[S]^{a>1/2} - [S]^{w>1/2}$. As for the total number of pairs in $[S]^{a>1/2} - [S]^{w>1/2}$, this is given by the sum:

$$|[S]^{a>1/2} - [S]^{w>1/2}| = \sum_{j=0}^{n-(k+2)} \binom{n - (k + 2)}{j} \cdot \binom{k}{k/2} \cdot 2 \cdot S(j + k/2 + 2, 2).$$

To finish our proof that $|[S]^{a>1/2} - [S]^{w>1/2}| > |[S]^{w>1/2} - [S]^{a>1/2}|$, we simply need to note that it is obvious that

$$\binom{k}{k/2} \cdot 2 \cdot S(j + k/2 + 2, 2) > \binom{k}{k/2 + 1} \cdot 2 \cdot S(j + k/2 + 1, 2).$$

For any even k , it is straight forward to show that $\binom{k}{k/2} > \binom{k}{k/2 + 1}$ (proof omitted) and that Stirling numbers of the second kind are strictly increasing with respect to the first value.⁵¹ Thus, the combinatorics confirm that $|[S]^{a>1/2} - [S]^{w>1/2}| > |[S]^{w>1/2} - [S]^{a>1/2}|$, and so $g(n, k)$ is a strictly increasing function with res

⁵¹As a matter of fact, as Weisstein ("Stirling Numbers of the Second Kind") notes, $S(n_1, 2) = 2^{n_1} - 1$. And, of course, $2^{n_1+1} - 1 > 2^{n_1} - 1$.

pect to k over the positive even integers. We have thus shown that 5.12.22 is for any even k , thus,

$$(5.12.23) \quad \frac{|[S]^{w \leq 1/2}| - |[S]^{w > 1/2}|}{|[S]|} < 1/9.$$

And, this shows that our theorem must hold for all weakly dominated sets of beliefs. □

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