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# Analysis of Welfare and Wealth Inequality in a Money Search Model

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# **Analysis of Welfare and Wealth Inequality in a Money Search Model**

Ismahile Ba Boukari, Ph.D.

University of Connecticut, 2014

## **ABSTRACT**

This thesis presents three related models to examine the welfare and wealth distributional effects in a monetary search model with non-degenerate distribution of money. The first chapter presents a procedure for solving an extension of Molico's (2006) model that relaxes the assumption of full bargaining power to buyers. The procedure is used to investigate the effects of a change in the buyer's bargaining power when money supply is fixed. The results suggest that when the buyer's bargaining power is very high, decreasing it increases the average welfare while the opposite is true when it is relatively low. The results also suggest that wealth inequality increases when the bargaining weight of buyers decreases. The second chapter extends the environment of the first chapter to allow changes in the supply of money in order to investigate the effects of an expansionary monetary policy implemented through a lump-sum money transfer. The results show that an increase in the growth rate of money lowers wealth inequality when the bargaining power of the buyer is low or when it is high but the money growth rate is low. However, when the buyers have high bargaining power an increase in money supply can increase inequality at high money growth rates. An increase in money supply increases the average welfare when the money growth rate is low, but high money growth rates leads to a reduction in average welfare. The third chapter checks the robustness of the results of the second Chapter by allowing agents to meet other potential trading partners during the bargaining process, and also by computing the welfare cost of inflation as an additional welfare measure. The results indicate that expansionary monetary policies are not welfare improving. In particular the results show that an increase in money supply leads to a decrease in the average welfare and an increase in the welfare cost of inflation. However, money creation can reduce the wealth inequality when the money growth rate is high, although the opposite holds when the money growth rate is low.

# Analysis of Welfare and Wealth Inequality in a Money Search Model

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A Dissertation

Submitted in Partial Fulfillment of the  
Requirements for the Degree of Doctor of Philosophy  
at the  
University of Connecticut

2014

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Ismahile Ba Boukari

2014

# APPROVAL PAGE

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# Chapter 1

## The Effects of Bargaining Power on Welfare and Wealth Inequality in a Money Search Model

### 1.1 Introduction

The main goal of this chapter is to improve the existing solution procedure developed in Molico (1997, 2006)<sup>1</sup> in order to expand the realm of possible applications of money search models. The paper also investigates the effect of the buyer's relative bargaining power on aggregate welfare and wealth inequality. The generalized Nash bargaining is the most common procedure used to determine the outcome of bargaining in a random match between buyer and seller. As pointed out by Muthoo (1999), the relative bargaining power of the players in bargaining games is a very important determinant of their share of the generated surplus.

While empirical observation shows that fiat money is widely used in economic transactions, modeling fiat money is not an easy task. It is difficult to develop a model in which agents endogenously hold fiat money in equilibrium because fiat money earns no interest and provides no utility. Consequently most researchers have resorted to shortcuts. Traditionally there are two main shortcuts for introducing money as a medium of exchange in a model.

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<sup>1</sup>Molico (2006) is the published article based on Molico (1997) which is Molico's Ph.D. dissertation. Therefore any comment on Molico (1997) also applies to Molico (2006) and vice versa.

The first shortcut is the money-in-the-utility function approach as in Sidrauski (1967). This approach considers the real money balance as an argument of the consumer's utility function. The intuition is that money as a medium of exchange facilitates transactions by reducing the time needed to perform the economic transactions and therefore increases the consumer's utility. The second approach is the cash-in-advance model as in Lucas (1980). In this approach, agents can trade certain goods only by using cash, therefore making them hold money in equilibrium.

These two approaches can be considered reduced form models and are not well suited for analyzing the effects of monetary policy because they are subject to the well-known Lucas critique. A better approach is to model explicitly the frictions that give rise to the use of money as a medium of exchange. One strand of the literature that explicitly accounts for these frictions is the money search model approach. This approach is an expansion of the idea formulated by Jevons (1875). According to Jevons (1875) money as a medium of exchange helps to ease the double coincidence of wants problem that arises in a barter economy. In the money search model, the double coincidence of wants can be modeled using a mechanism of random bilateral matching of specialized agents. Early contributors to this literature include Jones (1976), Diamond (1984), and Kiyotaki and Wright (1989, 1991, 1993).

Subsequent researchers have successfully built on the model developed by Kiyotaki and Wright (1993) by among other things relaxing the strong assumptions on the indivisibility of goods and money as well as the unit inventory restrictions on goods and money. However, removing the indivisibility restriction on money leads to a non-degenerate distribution of money which is difficult to track analytically. Hence, most authors imposed additional assumptions to obtain a degenerate distribution of money while still maintaining the divisibility of money. Therefore, these authors cannot study the distributional effect of policy, because a non-degenerate distribution of money across individuals in equilibrium is essential in studying the distributional effect of monetary policy.

Molico (1997) obtains a non-degenerate distribution of money by tracking the distribution

of money numerically and therefore he is able to study the distributional effects of monetary policy. However, Molico's solution procedure is restricted to a case where the buyer has full bargaining power. As pointed out by Muthoo (1999), it is highly unlikely that in a real-life bargaining process that one agent will always make the offer.

Molico's results have also been criticized by Deviatov and Wallace (2001) on the ground that the take-it-or-leave-it assumption can sometimes lead to an inefficient division of the matching surplus. Therefore there is a need to relax the take-it-or-leave-it assumption in Molico (1997, 2006).

Despite being highly cited in the literature, no successful attempt has been made in generalizing Molico's solution procedure so that buyers can have a relative bargaining power less than one. This can be explained mainly by the numerical difficulties in solving the generalized Nash bargaining problem. My contribution is to propose an extension of Molico's solution procedure that overcomes these difficulties.

As noticed by Berentsen (2000), Molico's model has some interesting features, especially when one wants to account for the distributional effect of monetary policies. The model is a search theoretical model with divisible money and divisible goods where agents can hold any non-negative amount of money. In addition, the model yields a non-degenerate distribution of money and prices. Other search models of money lack one or more features included in Molico (2006). For instance, the two most prominent models of divisible money and divisible goods used in the monetary search field are models developed by Shi (1997) and Lagos and Wright (2005).

Both models use tricks to deal with the intractability of agents' money holding induced by the divisibility of money. In Shi (1997), agents are large households and the members of each household combine their resources and share consumption. Thanks to the law of large numbers, families are protected against the uncertainty of a random match and they hold the same amount of money from one period to the next. Therefore, the distribution of money holding becomes degenerate in equilibrium if each large family starts with the same

amount of money.

In Lagos and Wright (2005), there are two markets in each period. The first market is the decentralized market, called the day market, where agents trade their specialized goods. The second market is a centralized market, called the night market, where agents trade for a general good. One of the functions of the night market is to allow the agents to rebalance their money holding to be at the same level as at the beginning of the period, so that the money distribution becomes degenerate in equilibrium.

Other models can yield non-degenerate distributions of money inventories but they have to impose other restrictive assumptions. One example, provided by Deviatov and Wallace (2001), is able to analytically track the money holdings of agents by imposing two restrictive assumptions. In particular, the model assumes that money is indivisible and agents cannot hold more than two units of money. Zhou (1999) also presents a model where the money distribution is non-degenerate and where the agents can hold any non-negative amount of money. However, this model deals only with indivisible money.

Dressler (2011) uses a computational approach like in Molico (2006). This model yields a non-degenerate money holding while maintaining the divisibility of both money and goods. However, in order to reduce numerical complexity, it replaces the standard decentralized market with a centralized market where agents are price takers. However, as Williamson and Wright (2010) pointed out, using the competitive pricing mechanism eliminates the possibility of obtaining a non-degenerate endogenous distribution of prices.

In a recent paper, Menzio et al. (2013) presents a model that delivers a nondegenerate distribution of money holdings as the model in Molico (2006). But unlike the model in Molico (2006) that uses a numerical method to track the distribution of agents' money holding, the model in Menzio et al. (2013) can be solved analytically. Although the authors derive important theoretical results, such as existence and uniqueness of the monetary steady state, they acknowledge that they have yet to develop a computational algorithm that will yield quantitative results necessary for analyzing the effect of monetary policies.

The numerical procedure used in this paper is based on a six-step procedure developed by Molico (2006). Relaxing the assumption on the bargaining power of the buyers introduces a computational complexity in solving the Nash bargaining problem. To deal with this optimization problem, I use a hybrid optimization procedure which combines a global optimization procedure, simulated annealing method, with a local optimization procedure, the Nelder-Mead method.

The results of the simulations show that in equilibrium, when the buyers' bargaining power is very high, decreasing their bargaining weight increases the average welfare of the agents while of opposite result is obtained when the buyers' bargaining power is relatively low. The results also show that wealth inequality, measured by the Gini coefficient, rises when the bargaining weight of the buyers decreases.

One paper that also analyzes the impact of relative bargaining power on welfare is Lagos and Wright (2005). The authors found that an increase in the relative bargaining power of a seller leads to an increase in the welfare cost of inflation in the United States. Dutu (2006) came to the same conclusion for the cases of Australia and New Zealand. Here, I do not check on the welfare impact of inflation.

The remainder of the chapter proceeds as follows. The next section describes the economic environment while Section 1.3 presents the value function. Section 1.4 lays out the Nash bargaining problem while Section 1.5 defines the equilibrium. Section 1.6 describes the computational procedure. Section 1.7 presents the results of the simulations, and Section 1.8 concludes the chapter.

## **1.2 Economic environment**

The environment is that of Molico (2006). Time is discrete. There is a continuum of infinitely lived agents with unit mass. The agents have a positive discount rate  $r > 0$ . Every agent specializes in the production of one type of good and the consumption of another type of

good. The goods are perfectly divisible and non-storable. I assume that an agent does not consume his own production good.

When an agent produces a quantity  $q$  of his production good he suffers a disutility  $C(q)$  which is a twice continuously differentiable, increasing and convex function. Likewise, a consumption of  $q$  units of his consumption good give the agent a utility  $U(q)$  which is a twice continuously differentiable, increasing and concave function.

In particular, following Molico (2006), I adopt the following specific functional forms for the utility and disutility functions:  $U(q) = 100 \log(1 + q)$  and  $C(q) = \left( \frac{1}{1 - q} - 1 \right)$  where  $0 \leq q \leq 1$ .

In addition to consumption goods, there is a perfectly divisible and storable good called fiat money which does not enter into the utility function of any agent nor facilitate production. The role of fiat money is to facilitate trading. Each agent can hold any non-negative amount of money. The money is supplied only by the government. In this model the government does not have any other function besides providing a stock of fiat money at the beginning of period zero.

Since agents cannot consume their production good they must trade in order to consume. I assume that agents meet pairwise at random according to a Poisson process with arrival rate  $\alpha$ . I assume that in a random matching there is no double coincidence of wants because in this paper I only focus on stationary monetary equilibrium. In a random match between two agents there is probability  $\sigma$  of a single coincidence of wants where a match trade can occur.

Another feature of this economic environment is the fact that agents are anonymous and there is no public record-keeping therefore making each agent's trading history private information. This feature makes it impossible to use credit in trading. Since each agent's trading history is unobservable and there is no double coincidence of wants, trade must involve the use of money.

I also assume that two agents involved in the trading process have a full knowledge of each



other's current money holdings and that the pricing mechanism is derived from a generalized Nash bargaining procedure.

Specifically, in time period  $t$ , in a single coincidence of wants random pairwise match, one of the agents is chosen randomly to propose the pair  $(q, d)$ , where  $q$  is the quantity consumption good produced by the seller for the buyer and  $d$  is the quantity of money transferred from the buyer to the seller. With probability  $\theta$ , the buyer is the proposer and with probability  $1 - \theta$ , the seller is the proposer.  $\theta \in [0, 1]$  is thus interpreted as the relative bargaining power of a buyer.

When the proposal is made by one agent, the other can accept or reject the proposal. If a proposal is accepted then the trade occurs and the agents in that pair wait until the next period. If the proposal is rejected, agents can either end the bargaining process by walking away from the bargaining table or can decide to continue with the bargaining process, in which case they wait for a small period of time  $\delta$  (with  $\delta \rightarrow 0$ ) at the end of which one of them is chosen again randomly to propose a new pair  $(q, d)$ .

The process continues until the end of the period. According to Trejos and Wright (1995), no agent in a trading match ever walks away from the bargaining table in equilibrium for this type of model. Furthermore, all proposals are accepted in the first round. Although no proposal is ever rejected, it is the risk of rejection and the delaying of settlement that drives the result.

### 1.3 Value function

In this paper I focus on stationary monetary equilibria. So let  $d(b, s)$  be the quantity of money given by the buyer, with money holding  $b$ , to the seller, with money holding  $s$ , in exchange for the production of the quantity  $q(b, s)$ .

Also, let  $F(y)$  be the measure of agents with money holding  $m$  such that  $m \leq y$  in equilibrium.

Let  $V(m)$  denote the equilibrium value function of an agent who enters a new period with  $m$  amount of money. Then  $V(m)$  is described by the following Bellman equation:

$$\begin{aligned}
V(m) = \frac{1}{1+r} \Big\{ & \alpha\sigma \int_0^\infty [U(q(m, m_s)) + V(m - d(m, m_s))] dF(m_s) \\
& + \alpha\sigma \int_0^\infty [-C(q(m_b, m)) + V(m + d(m_b, m))] dF(m_b) \\
& + (1 - 2\alpha\sigma)V(m) \Big\}. \tag{1.1}
\end{aligned}$$

In Equation (1.1) the three terms of the expression inside the curly braces are respectively the expected values associated with buying, selling and not trading (either because the agent was not matched or because in a match neither agent likes the other agent production good) in equilibrium in a given time period.

An alternative expression of the Bellman equation found by rearranging equation (1.1) seems to improve the robustness of the numerical procedure. The alternative Bellman equation is the following:

$$\begin{aligned}
V(m) = \frac{1}{1+r} \Big\{ & \alpha\sigma \int_0^\infty [U(q) + V(m - d(m, m_s)) - V(m)] dF(m_s) \\
& + \alpha\sigma \int_0^\infty [-C(q) + V(m + d(m_b, m)) - V(m)] dF(m_b) \\
& + V(m) \Big\}. \tag{1.2}
\end{aligned}$$

In Equation (1.2) the first two terms of the expression inside the curly braces are respectively the expected increases in the value associated with buying and selling in a single coincidence of wants while the third term is just the value function before trading.

It is important to notice that in Equations (1.1) and (1.2) the quantities of consumption good and money exchanged are derived from the solution of the generalized Nash bargaining problem.

## 1.4 Generalized Nash bargaining

In a single coincidence of wants, the terms of trade  $(q, d)$  are determined by the Nash Bargaining process. Following Trejos and Wright (1995), since by assumption agents cannot meet other potential partners during a bargaining process, I will set the agents' threat point to zero. Therefore, the terms of trade  $(q, d)$  solve the following problem:

$$\max_{q,d} \left[ U(q) + V(m_b - d) \right]^\theta \left[ -C(q) + V(m_s + d) \right]^{(1-\theta)} \quad (1.3)$$

subject to

$$0 \leq q \leq 1 \quad (1.4)$$

$$0 \leq d \leq m_b \quad (1.5)$$

$$U(q) + V(m_b - d) \geq V(m_b) \quad (1.6)$$

$$-C(q) + V(m_s + d) \geq V(m_s) \quad (1.7)$$

$$m_s + d \leq \overline{m}. \quad (1.8)$$

Equation (1.4) expresses the constraint imposed on the quantity of good exchanged in order to get a non-negative disutility for the seller based on the specific functional form used in this chapter. Equation(1.5) takes into account the buyer's budget constraint. Equations (1.6) and (1.7) are the participation constraints of respectively the buyer and the seller. Equation (1.8) imposes a constraint on the money holding of a seller to ensure that seller's money holding does not exceed the upper bound  $\overline{m}$  of the domain  $[0, \overline{m}]$  which is used for performing the interpolation of the value function of agents.

The solution of the generalized Nash bargaining is used to compute the new money holding after trade  $m'$  of an agent with money  $m$  when the agent is either a buyer or a seller (otherwise the money holding of the agent is unchanged). Therefore the law of motion of an

agent's money holding is given by the following equation:

$$m' = \begin{cases} m - d & \text{with probability } \alpha\sigma \text{ (agent was a buyer)} \\ m + d & \text{with probability } \alpha\sigma \text{ (agent was a seller)} \\ m & \text{with probability } 1 - 2\alpha\sigma \text{ (agent was not involved in bargaining).} \end{cases} \quad (1.9)$$

## 1.5 Equilibrium definition

An equilibrium consists of a distribution of money holdings  $F(m)$ , terms of trade  $q(m_b, m_s)$  and  $d(m_b, m_s)$ , a value function  $V(m)$  such that given the aggregate money holdings  $M$ :

1.  $F$  is an invariant distribution given  $d$ ;
2.  $V$  satisfies the Bellman equation (1.2) given  $(q, d)$  and  $F$
3.  $(q, d)$  solves the generalized Nash bargaining problem equations (1.3)-(1.8) given  $V$ .

The focus of this chapter is a monetary equilibrium in which there exists at least one matching trade where the buyer and the seller exchange strictly positive quantities of good,  $q$ , and money,  $d$ .

However, I cannot provide a proof of the existence nor the uniqueness of such equilibrium due to the complexity of the model. Therefore I will make the assumption that the monetary equilibrium exists. In addition in this model the money holdings of agents is analytically intractable, therefore I use a numerical procedure to solve for the steady-state monetary equilibria.

## 1.6 Computational procedure

The computational algorithm used to solve for the stationary distribution of money holding is based on a six-step procedure developed by Molico (2006). First, given the average money holding in the economy, define a grid (of thirty points for example) on the space of money

holdings,  $\{0, m_1, m_2, m_3, \dots, \bar{m}\}$ , such that the mass of agents with money holding greater than  $\bar{m}$  is not significant. Second, guess a value function at grid points defined in step 1, and use numerical interpolation to find the value at all money holdings in  $[0, \bar{m}]$ . Third, draw a large sample (of size 10,000 for example) of money holdings from a guessed distribution which has the same mean as the mean of the money holding defined in step 1. Fourth, update the value function at the grid points of money holding using the Bellman equation. Fifth, update the money holdings in the sample of money holdings tracking the distribution of money holdings by simulating the random pairwise matching procedure of each individual money holding in the sample. Sixth, estimate the probability density function of the new distribution of money holding based on the updated sample of money holdings. Step 4 to step 6 are repeated until the convergence of the value function at the grid points and the convergence of the sample distribution of money holdings are achieved.

To implement the solution procedure, I need to take care of some important numerical details. Chief among them is the numerical procedure for solving the bargaining problem required in steps 4 and 5. When the buyer has all the bargaining power, the bargaining problem is reduced to solving a two-variable constrained optimization problem. In this case the objective function is just the trade surplus of the buyer while the two main constraints are the incentive compatibility constraints of the buyer and the seller.

The problem can be further reduced by noticing that the incentive compatibility constraint of the buyer is redundant because the buyer makes a take-it-or-leave-it offer to the seller. For the same reason the incentive compatibility of the seller will hold with equality. Using this simplified version, Molico (2006) applied the substitution method to transform the problem into a single-variable constrained optimization problem, solving for example for  $d$ , and using the incentive compatibility problem of the seller to recover the value of the second variable, for example  $q$ .

However, when the buyer does not have full bargaining power the objective function of the optimization is a product of the surpluses of the buyer and seller raised to their

respective relative bargaining power. Now there are four main constraints: the two non-negativity constraints for the surpluses of buyer and seller and the two incentive compatibility constraints of buyer and seller. Here there is no simplification available to reduce the problem to a single optimization problem.

We have to tackle the two-variable constrained optimization directly. The objective function is extremely difficult to maximize. The main difficulty is that the objective function is not always defined for all values of  $(q, d)$  in the optimization search domain defined by some additional constraints. This situation arises from the fact that some pairs  $(q, d)$  lead to a negative surplus for a buyer or a seller. Since in computing the objective function we need to raise the value of the surplus of buyer and seller to a fractional power, we end up with an undefined value when the surplus of at least one of the partners is negative. It is clear that this situation arises because of the violation of the non-negativity constraint of the agents' trade surpluses.

Another difficulty is related to a proper handling of the constraints, mainly the incentive compatibility constraints of the agents. One popular way of handling constraints is to apply the penalty method which consists of adding to the objective function a term that depends on the degree of violation of the constraints. The drawback of this method is that the solution of the optimization may violate one or more of the constraints.

Therefore a solution to remedy those difficulties will require an appropriate handling of the non-negativity constraints of the trade surpluses and all other constraints.

The suggestion is to convert the constrained maximization problem into an unconstrained maximization problem. In particular, for a given  $(q, d)$ , set the objective function to the calculated generalized Nash product of agents' surpluses when all constraints are satisfied. When some are not satisfied set the value of the objective function to a strictly negative value with large absolute value (for example  $-100$ ). This procedure corresponds to penalizing any  $(q, d)$  that violates any constraints. This way of handling the constraints in effect makes the unconstrained maximization problem extremely difficult to solve with conventional gradient-

based optimization methods like the quasi-Newton method. This is due to the fact that the numerically-computed gradient will not always carry meaningful information when there is an abrupt change in the objective function at a given pair  $(q, d)$  that violates the constraints. Therefore, the best course of action is to use gradient-free numerical optimization procedures.

In this paper, I use a hybrid optimization procedure: a global optimization procedure based on a simulated annealing method (see Corona et al. (1987) and Creutz (1983)) as the main optimization procedure followed by a local optimization procedure based on the Nelder-Mead method (see Yang et al. (2005)) to eventually improve the accuracy of the solution.<sup>2</sup> In summary, to solve the optimization problem, first use the simulated annealing starting with a guess of  $(q, d) = (0, 0)$ , then use the simulated annealing solution as a first guess for the Nelder-Mead simplex algorithm.

Note that although I choose to use the simulated annealing as a global optimization procedure, other heuristic optimization procedures such as the genetic algorithm or the particle swarm optimization method can be useful alternatives. These methods are often collectively called nature-inspired optimization procedures because they mimic some behavior found in nature. For instance, the process of slow cooling of melted metal in metallurgy (simulated annealing), the principle of natural evolution by the means of selection, crossover, and mutation (genetic algorithm) or the dynamics of the collective behavior of a flock of birds or school of fish (particle swarm optimization). A survey of these methods can be found in Rao (2009) and Gili and Winker (2008).

In step 4 we also need to specify how the value function will be updated. In this chapter I will use an update equation based on Equation (1.2). In particular, if at iteration  $k$  we

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<sup>2</sup>Simulated annealing has been successfully used in economics. For example, Goffe et al. (1994) uses simulated annealing to solve four econometrics problems and report overall better results when compared to some other optimization algorithms like conjugate gradient, quasi-Newton or simplex methods. Goffe et al. (1994) uses a Fortran implementation of the version of the simulated annealing algorithm developed by Corona et al. (1987). The main drawback of simulated annealing is that it is quite time-consuming for the computer.

denote the current value function by  $V^k$ , then the updated value function  $V^{k+1}$  is given by:

$$\begin{aligned}
V^{k+1}(m) = \frac{1}{1+r} & \left\{ \alpha\sigma \int_0^\infty [U(q) + V^k(m - d(m, m_s)) - V^k(m)] dF(m_s) \right. \\
& + \alpha\sigma \int_0^\infty [-C(q) + V^k(m + d(m_b, m)) - V^k(m)] dF(m_b) \\
& \left. + V^k(m) \right\}.
\end{aligned} \tag{1.10}$$

Another important implementation detail is related to the appropriate procedure for estimating the distribution of money holdings in step 6. Following Molico (2006) I use a nonparametric kernel density method.

Let  $\{m_1, m_2, \dots, m_n\}$  be a random sample of  $n$  independent observations from a probability distribution with unknown density function  $f(m)$ . A kernel density estimator for  $f$  is defined as:

$$\bar{f}(m) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{m - m_i}{h}\right), \tag{1.11}$$

where  $K$  is the kernel function and  $h$  is the smoothing bandwidth. In this paper I use the Gaussian kernel  $K(u) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}u^2)$ .

The estimator based on (1.11) with a Gaussian kernel can lead to large biases at the boundary if the support of the density is not the whole real line as in this essay which is concerned with non-negative money holdings. To deal with a possible increased boundary bias one can choose to use the technique of the negative reflection (Silverman 1987) which gives the following density estimator:

$$\tilde{f}(m) = \frac{1}{nh} \sum_{i=1}^n \left[ K\left(\frac{m - m_i}{h}\right) - K\left(\frac{m + m_i}{h}\right) \right], \tag{1.12}$$

where  $m \geq 0$ .

However, as pointed out by Silverman (1987) the area under the estimated density will no longer integrate to one. A solution is to renormalize the estimates using the following



equation:

$$\hat{f}(m) = \frac{\tilde{f}(m)}{\int_0^\infty [\tilde{f}(m)] dF(m)} \quad (1.13)$$

The main difficulty in using the kernel density estimator is related to the choice of the smoothing bandwidth  $h$ . A possible approach for estimating the bandwidth is least squares cross-validation as described in Silverman (1987) and Li and Racine (2007). However, this procedure is computationally very expensive and, as pointed out by Silverman (1987), it also has the tendency to break down when the number of repeated observations in the sample of data is higher than some threshold. Therefore I simply use Silverman's rule-of-thumb to select the bandwidth  $h$  as follows:

$$h = 0.9An^{-1/5} \quad (1.14)$$

where  $A = \text{minimum (sample standard deviation, (sample inter quartile range) )}/1.34$  and  $n$  is the of number of observations in the sample.

Other implementation details are related to procedures for performing numerical integration, numerical interpolation and for generating numbers from the empirical distribution of money holdings. In this paper I use the Gauss quadrature method (see Yang et al. (2005) and Press et al. (2007)) to compute numerical integrations. To perform interpolation, I rely on a shape preserving interpolation procedure developed by Schumaker (1983) (see also Judd (1998) for a description of the same procedure). To generate random variables from the estimated kernel density I use the generalization of the sample method described by Silverman (1987) and Hörman and Leydold (2000). In essence this method consists in drawing an observation from the sample of money holdings and adding some noise, and returning the sum as a random number from the distribution of money holdings. This procedure allows me to obtain values of money holdings beyond values that are already present in the sample of money holdings.

Table 1.1: Replication of the economic effects of changes in the interest rate published in Molico (2006), page 713.

Discount rate	0.01	0.005	0.001
Average price ( $E(p)$ )	7.509	3.794	0.769
Coefficient variation (price)	0.599	0.442	0.325
Average quantity of good ( $E(q)$ )	0.827	0.844	0.855
Welfare ( $W$ )	5.321	5.437	5.510

Notes:  $\alpha = 1$ ,  $\sigma = 0.1$  and  $M = 100$ .

Table 1.2: Molico's original results of the economic effects of changes in the interest rate.

Discount Rate ( $r$ )	0.01	0.005	0.001
$E(p)$	7.61	3.88	0.78
Coef. Var. of $p$	0.60	0.47	0.34
$E(q)$	0.825	0.842	0.854
$W$	5.32	5.43	5.51

$M = 100$ ,  $x = 0.1$ .

Source: Molico (2006), page 713.

To test the preceding numerical method, I try to replicate the results in Molico (2006) related to the economic effects of interest rates in an economic environment similar to the one described here but where the buyer has all the bargaining power. The results of my numerical simulations, using equation (1.1), are summarized in Table 1.1. They are in general close to Molico's results shown in Table 1.2 where  $x$  is the probability of a single coincidence of wants.

## 1.7 Simulation results

In this section I explore the impact of a change in the buyer's bargaining power on welfare and the distribution of money holdings. The analysis is extended to the price and quantity of consumption goods exchanged, the average quantity of money exchanged, and the income

velocity of money.

To simulate the model I need to specify the values of the arrival rate  $\alpha$ , and the probability of single coincidence of wants  $\sigma$ . I use  $\alpha = 1$  and  $\sigma = 0.1$ . Those values are taken from Molico (2006). I also need to set the value of the total money holdings in economy  $M$ . I normalize the money stock by setting  $M = 1$ .

Following Molico (2006) I use the following ex ante welfare measure:

$$\text{Welfare} = \int_0^\infty rV(m) dF(m). \quad (1.15)$$

To compute the average price of consumption goods, I need to find the prices of goods exchanged based on the terms of exchange. If the terms of exchange  $(q, d)$  contain strictly positive quantities of good and money exchanged, then the price of the consumption good is just the ratio of the quantity of money to the quantity of consumption good exchanged. Otherwise, that is when  $(q, d) = (0, 0)$ , the price of the consumption good will be set to zero.

To assess wealth inequality I use both the Gini coefficient and the Lorenz curve. I also report results for an alternative measure of wealth inequality, the standard deviation of money holdings, used by Molico (2006).

Next, I will present and discuss the mains findings of the simulations results, based on Equation (1.2), shown in Table 1.3 and the illustrated in the figures below.

First, the average price and the dispersion of prices (as measured by the standard deviation) are, in general, increasing in the sellers' bargaining weight. Second, at low levels, an increase in the sellers' bargaining weight increases welfare, but at high levels the result is reversed. Third, wealth inequality, as measured by the Gini coefficient, is increasing in the sellers' bargaining weight. Fourth, the income velocity of money is also increasing in the sellers' bargaining weight.

To understand the intuition behind some of the results, it is important to take into account the following two features of the model. First, in this model agents can only partially

Table 1.3: Effects of change in bargaining bargaining power

Buyer's bargaining power	1	0.8	0.5	0.3
Standard deviation (money)	0.553	0.604	0.720	1.038
Gini coefficient	0.298	0.336	0.384	0.519
Average price	0.075	0.147	0.464	0.864
Coefficient variation (prices)	0.584	0.500	0.699	1.022
Average quantity (good)	0.828	0.834	0.777	0.694
Real balance(M/P)	13.330	6.822	2.153	1.157
Annual velocity money	0.062	0.122	0.371	0.633
Welfare	5.327	5.443	5.229	4.765

Notes:  $\alpha = 1$ ,  $\sigma = 0.1$ ,  $r = 0.01$  and  $M = 1$ .

self-insure against idiosyncratic risk related to random trading opportunities by hoarding money. Second, whenever the seller has some positive bargaining power, there will be a holdup problem. The severity of the holdup problem increases with the seller's bargaining weight. When agents decide to carry money, they are making an investment because they are forgoing current consumption in favor of future consumption. However, when sellers have positive bargaining weight, they share the benefit of the buyers' investment in money. This generates the holdup problem since buyers may reduce the amount of money they carry which can impact the quantity of goods and money exchanged in bilateral trades.

The intuition behind the first result, illustrated in Figure 1.1, is the following. For given money holdings of a buyer and a seller, as the bargaining weight of the seller increases and thus his share of the surplus of the trade, he will be able produce a smaller quantity of the good in exchange of the same amount of money. Therefore, the average price of goods exchanged will be increasing in the seller's bargaining weight. The result concerning the dispersion of prices can be explained by the fact that as the bargaining power of the sellers increases, the decrease in the valuation of money by the poor is smaller than that of rich agents, therefore the gap between the price asked by poor and rich sellers increases leading to an increased dispersion of prices. However, a closer look at the coefficient of variation

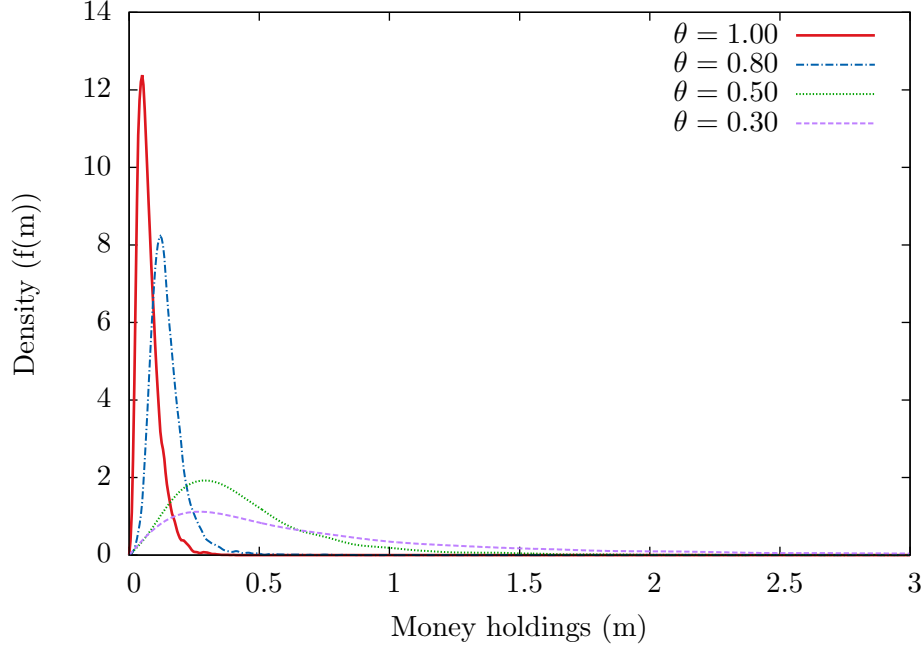


Figure 1.1: Probability density of prices for various buyer's bargaining power

of prices shows that the coefficient of variation of prices is decreasing in sellers' bargaining power when the sellers' bargaining power is very low and increasing in their bargaining power when it is high. This means that when the bargaining power of sellers is low the increase in the dispersion of prices (measured by the standard deviation) is smaller than the increase in the average price. The reverse is true when the sellers' bargaining power is high.

The result concerning the effect of bargaining power on welfare can be explained through its differential effect on poor and rich agents. When the sellers' bargaining weight is very low, increasing their bargaining power increases the value function of the poor agents and reduces that of rich agents (See Figure 1.2). This is due to the fact that poor sellers with additional bargaining power can get more money during a trading match, therefore they have higher chances of consuming more in future meetings since as buyers, poor agents' incentive compatibility constraint will likely be binding. This means that for them the positive self-insurance value of money is higher than the holdup effect, and their valuation of money increases. However, for rich agents, the holdup effect dominates the self-insurance effect so their valuation of money decreases. So, when the sellers' bargaining power is low, an

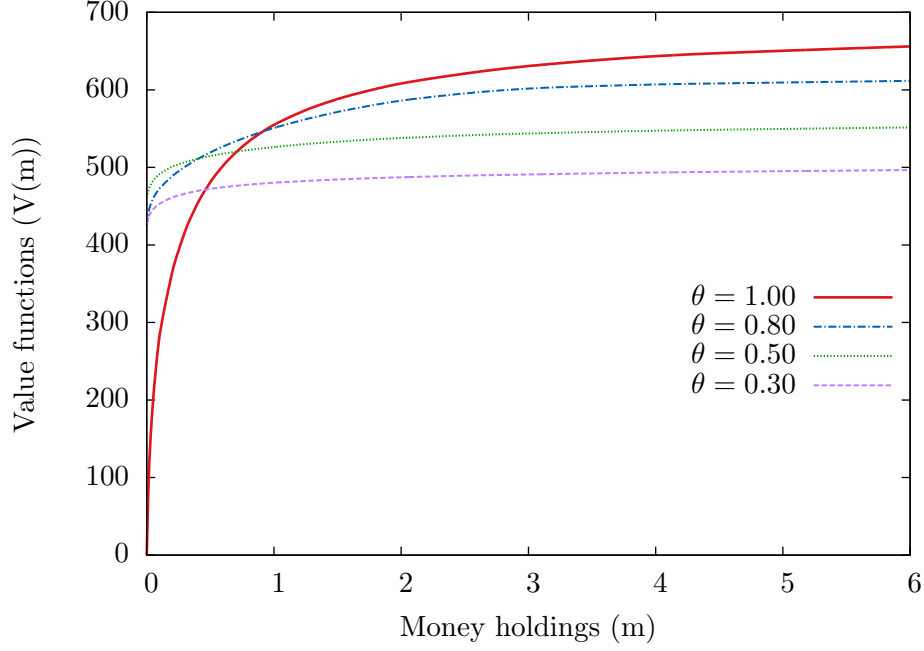


Figure 1.2: Value functions for various buyer's bargaining power

increase in their bargaining power increases the value function of poor agents by more than the decrease in the value function of rich agents. Thus, the aggregate welfare, measured as the average of one period return of agents' investment in money, increases. In contrast, when the bargaining power of the sellers is high, the holdup effect dominates the self-insurance effect for poor and rich agents and thus leads to a decrease in welfare.

The intuition behind the results on wealth inequality, illustrated in Figures 1.3 and 1.4, is as follows. As mentioned before, an increase in the bargaining power of the sellers' has a different effect on poor and rich agents' valuation of money. As sellers gain more and more bargaining power, rich sellers ask for more and more money than poor sellers in the order to produce the same quantity. Therefore, the wealth gap between poor and rich agents widens.

The intuition behind the result concerning the velocity of money is the following. As, the bargaining weight of the sellers increases, they will require more and more money in exchange for the same quantity of good to produce per trade. Therefore, the quantity of money exchanged in each match increases which means that the velocity of money increases since by definition, the income velocity of money is the ratio of the value of goods exchanged

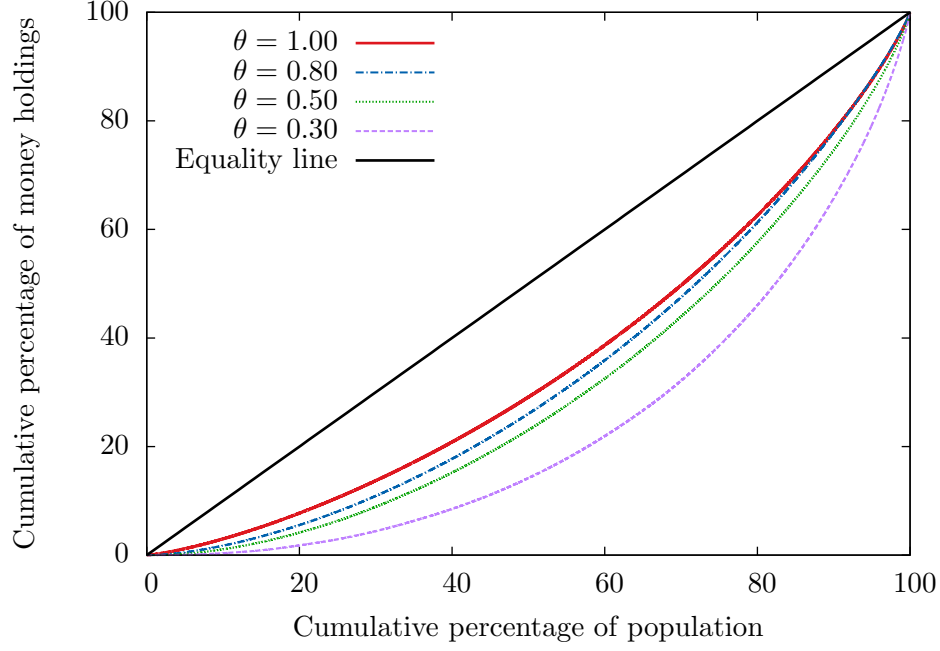


Figure 1.3: Lorenz curves for various buyer's bargaining power

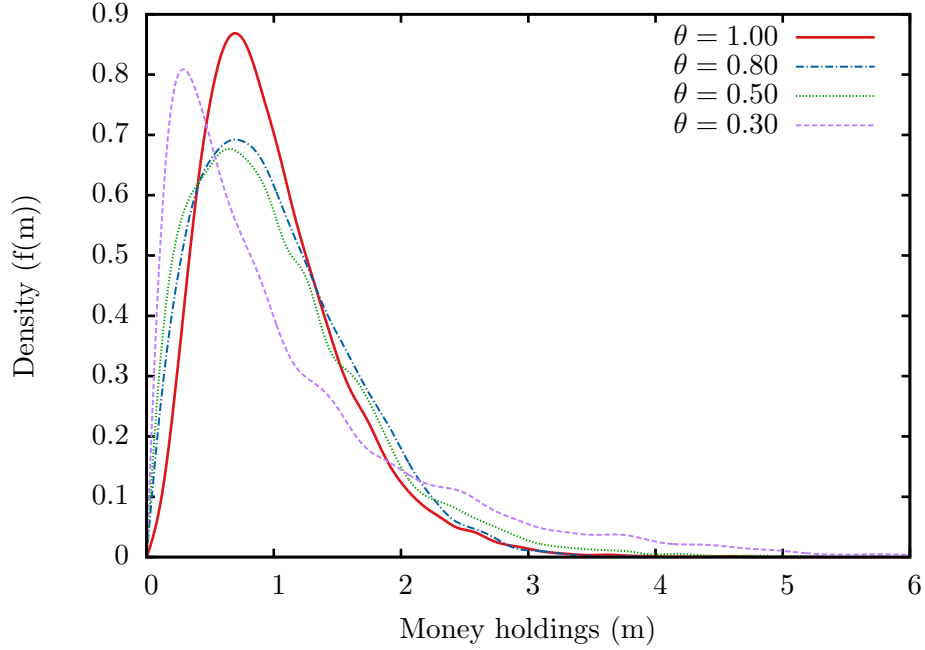


Figure 1.4: Probability density of money holdings under for various buyer's bargaining power to the aggregate amount of money in the economy. Since the stock of money is constant by assumption, the income velocity of money increases with the average quantity of money exchanged.

## 1.8 Conclusion

In this chapter I proposed a solution procedure to solve a monetary search model when the buyers do not have full bargaining power in random match of agents. The proposed procedure extends the solution procedure developed in Molico (2006) for the case where the buyers have full bargaining power.

The procedure is first used to replicate Molico's (2006) results for the case where the stock of money is constant. Next, the procedure is used to investigate the economic effect of changes in the buyers' bargaining power. The results of simulations show that when there is decrease in the buyers' bargaining power there is an increase in the Gini coefficient and thus an increase in the wealth inequality. I also find that when buyers' bargaining power is very high, a reduction in their bargaining power improves the average welfare in the economy while the opposite result is true when their bargaining power is already low.



# Chapter 2

## The Effects of Monetary Growth on Welfare and Wealth Inequality in a Money Search Model without Exogenous Breakdown

### 2.1 Introduction

The goal of this essay is to assess the effect of money growth on aggregate welfare and wealth inequality by extending the model in Molico (1997, 2006) to account for settings in which the buyers do not necessarily have all the bargaining power. Molico (1997, 2006) develops a search model of money with divisible goods and money and non-degenerated distribution of money holding where agents can have unbounded money holdings. The model uses a bargaining protocol where agents cannot meet other potential trading partners during the bargaining process. Following Trejos and Wright (1995), I call it bargaining without exogenous breakdown. Using numerical simulation, Molico (1997, 2006) finds that lump-sum creation of money has a beneficial effect on welfare. However, the finding is based on the assumption that in a random matching pair the buyer makes a take-it-or-leave offer to the seller. This assumption implies that the buyer takes all the surplus resulting from a trade and leaves the seller with zero surplus.

Deviatov and Wallace (2001) criticize this way of dividing the gain of the trade as being non-optimal. The authors suggest that the result of the beneficial effect of money creation in Molico (1997) may be driven by the extreme trade surplus sharing procedure. Deviatov and Wallace (2001) use an economic environment similar to that in Molico (1997) but in which money is indivisible and agents can hold only up to two units of money. It develops a trading protocol and lump-sum money trading along with some agent-individual ex-post rationality constraints to avoid a non-optimal sharing of trade surplus following random meetings. The model is solved using an analytical procedure and the results show that expansionary monetary policy is beneficial.

In a subsequent paper, Deviatov (2006) extends the model in Deviatov and Wallace (2001) by allowing the use of a lottery trade in a random meeting between buyers and sellers in order to alleviate the assumption of the indivisibility of money while keeping the upper bound of an agent's money holding to two units of money. The author uses numerical computation to solve the model, and the results show that money creation is beneficial when sellers' participation constraints are binding which is equivalent to buyers making take-it-or-leave-it offers in the model developed by Molico (2006). Deviatov (2006) then concludes that the trade surplus sharing rule adopted by Molico (1997) is in some sense optimal contrary to the conjecture in Deviatov and Wallace (2001). The conflicting results show the need to check the robustness of the findings in Molico (1997, 2006) by examining a case where the buyers do not have all the bargaining power.

The papers by Molico (1997, 2006), Deviatov and Wallace (2001), and Deviatov (2006) contribute to a strand of literature that argues that expansionary monetary policy can be welfare enhancing as a counterpoint to Friedman (1969) who argues in favor of a contractionary monetary policy. According to Friedman (1969), since the cost of producing money is almost zero, then the optimal policy would be to set the rate of return on money equal to the rate of return of other assets. This is achieved by setting the nominal interest rate to zero which requires an inflation rate that is the negative of the real interest rate. So, the optimal

monetary policy would be a deflationary one. This monetary policy can be implemented through a lump-sum tax on the stock of agents' money holdings.

Friedman's proposal has been found to be optimal in most monetary models, for instance in a money-in-the-utility function model by Benhabib and Bull (1983) and in a cash-in-advance model by Grandmont and Younes (1973). In addition, Lagos (2010) shows that Friedman's proposal can also be implemented in a search theoretical model of money. In contrast, a few models find that the Friedman proposal is not the best monetary policy. For example, Levine (1991), and Kehoe et al. (1992) develop heterogeneous agents models in which expansionary monetary policies are optimal policies. Kehoe et al. (1992) is an extended version of Levine (1991). Both papers present an endowment economy with infinitely-lived agents who, subject to preference shocks, randomly alternate between being buyers or sellers. They find that an expansionary monetary policy through a lump-sum transfer to agents is welfare improving. This is due to the fact that an equal transfer in effect redistributes real balances from the relatively rich to the relatively poor agents which in turn leads to an increase in the number of trades. However, as pointed out by Kehoe et al. (1992), the model leads to an equilibrium where the money holdings degenerate in equilibrium, in particular at the end of each period sellers hold all the money stock.

In this paper I study the economic effect of an expansionary monetary policy in a search model where buyers do not necessarily have full bargaining power. In the model, agents are not allowed to meet other potential partners while trading. The expansionary monetary policy is implemented through a lump-sum transfer to each agent in each period after the closing of the goods market. Due to the complexity of the model, the steady state results were obtained through computer simulations. The solution procedure used in this paper is an adaptation of the procedure developed in Chapter 1.

The pattern of the impact of money creation on the average welfare depends on the bargaining power of the buyer. When the bargaining power of the buyer is high, low rates of money supply increase welfare and high rates reduce welfare. However, when the bargaining

power of the buyers is low, a faster rate of money creation can increase the average welfare even at moderate rates. These results show money creation can be welfare improving even when buyers do not have full bargaining power.

The results also reveal that when buyers have low bargaining power, a faster rate of monetary expansion always leads to a decline in wealth inequality. However, when buyers' bargaining power is high, at low rates an increase in the growth rate of money supply reduces wealth inequality but at high rates, an increase in the rate of monetary expansion increases wealth inequality.

The rest of this paper is organized as follows. Section 2.2 describes the environment. Section 2.3 deals with the value function. In Section 2.4, I formulate the Nash bargaining problem. Section 2.5 defines the steady state equilibrium. In Section 2.6, I present the computational procedure. Section 2.7 is devoted to the presentation and the discussion of the results, and Section 2.8 concludes the paper.

## 2.2 Economic environment

The economic environment in this chapter is the same as the one described in Chapter 1 with one major difference: the stock of money is no longer constant. As in Chapter 1, I assume that in a random match with a single coincidence of wants, the partners in a current bargaining pair cannot meet other potential partners during the period of delay between two successive offer proposals. Following Trejos and Wright (1995), I call this type of bargaining a bargaining without exogenous breakdown.

The government changes the money supply by making lump-sum transfer  $\tau M_t$  to individuals, where  $M_t$  is the total per capita money holdings (or equivalently the aggregate money holdings since the total measure of agent is one) at the beginning of period  $t$ . Therefore, if  $M_{t+1}$  denotes the per capita money holding at the beginning of period  $t + 1$ , then

$$M_{t+1} = \int_0^\infty (m_t + \tau M_t) dF_t(m_t) = (1 + \tau)M_t \quad (2.1)$$

where  $F_t(m_t)$  is the distribution of money holdings  $m_t$  at the end of the period  $t$  and before the government lump-sum transfer to agents. We also have that for all  $t$ :

$$M_t = \int_0^\infty (m_t) dF_t(m_t). \quad (2.2)$$

The timing is the following. At the beginning of period  $t$ , the goods market opens and individuals trade using the money holdings carried from the previous period. After the closing of the goods market, the government proceeds with the monetary transfer to all the individuals.

In this chapter I focus on the time invariant state equilibrium. When the growth rate of the money supply is different from one ( $\tau \neq 1$ ) then the aggregate money stock will either grow or shrink continually over time, making it difficult to obtain stationary equilibrium values for nominal variables such as the individual money holdings,  $(m_t)$ , or the quantity of money exchanged during trading,  $(d_t)$ . Therefore to obtain a time invariant stationary equilibrium I normalize nominal variables of period  $t$  by  $M_t$  which is the total money stock at the beginning of the period. So, if  $y_t$  denotes a nominal variable and  $\tilde{y}_t$  denotes its corresponding normalized version then  $\tilde{y}_t$  is defined as:

$$\tilde{y}_t \equiv \frac{y_t}{M_t} \quad (2.3)$$

Now let  $m_t^*$  be the money holding of an agent at the closing of the goods market in period  $t$  and before the lump-sum transfer. Let  $\tilde{m}_{t+1}$  denotes the quantity of money that agent will carry to the beginning of the next period  $t + 1$ . We have that:

$$m_{t+1} = m_t^* + \tau M_t. \quad (2.4)$$

Dividing both sides of equation (2.4) by  $M_{t+1}$  we get:

$$\tilde{m}_{t+1} = \frac{m_t^*}{M_{t+1}} + \frac{\tau M_t}{M_{t+1}}. \quad (2.5)$$

Now, using equation 2.1 we get:

$$\tilde{m}_{t+1} = \frac{\tilde{m}_t^* + \tau}{1 + \tau}. \quad (2.6)$$

The actual value of  $\tilde{m}_t^*$  will depend in part on whether in a period  $t$  the agent had the opportunity to meet a partner or not, and whether the agent was a buyer or a seller in case the agent had the opportunity to trade.

We can now use equation (2.6) to write the law of motion of an agent's money holding in the stationary equilibrium as:

$$\tilde{m}' = \begin{cases} \frac{\tilde{m} - \tilde{d} + \tau}{1 + \tau} & \text{with probability } \alpha\sigma \text{ (agent was a buyer)} \\ \frac{\tilde{m} + \tilde{d} + \tau}{1 + \tau} & \text{with probability } \alpha\sigma \text{ (agent was a seller)} \\ \frac{\tilde{m} + \tau}{1 + \tau} & \text{with probability } 1 - 2\alpha\sigma \text{ (agent was not involved in bargaining)} \end{cases} \quad (2.7)$$

where  $\tilde{m}'$  is the agent's beginning of next period normalized money holding,  $\alpha$  is the probability of meeting someone in given period, and  $\sigma$  is the probability of single coincidence of wants in a random match.

## 2.3 Value function

Let  $V(\tilde{m})$  be the steady state value function of an agent who enters a new period with a normalized amount of money  $\tilde{m}$ . Let  $d(\tilde{b}, \tilde{s})$  be the quantity of money given by the buyer, with normalized money holding  $\tilde{b}$ , to the seller, with normalized money holding  $\tilde{s}$ , in exchange for the production of the quantity  $q(\tilde{b}, \tilde{s})$  in equilibrium. Also, let  $F(\tilde{y})$  be the measure of agents with normalized money holdings  $\tilde{m}$  such that  $\tilde{m} \leq \tilde{y}$  in equilibrium. Then the

agent's steady state value function  $V(\tilde{m})$  must satisfy the following Bellman equation:

$$\begin{aligned}
V(\tilde{m}) = \frac{1}{1+r} \Big\{ & \alpha\sigma \int_0^\infty \left[ U(q(\tilde{m}, \tilde{m}_s)) + V\left(\frac{\tilde{m} - \tilde{d}(\tilde{m}, \tilde{m}_s) + \tau}{1+\tau}\right) - V\left(\frac{\tilde{m} + \tau}{1+\tau}\right) \right] dF(\tilde{m}_s) \\
& + \alpha\sigma \int_0^\infty \left[ -C(q(\tilde{m}_b, \tilde{m})) + V\left(\frac{\tilde{m} + \tilde{d}(\tilde{m}_b, \tilde{m}) + \tau}{1+\tau}\right) - V\left(\frac{\tilde{m} + \tau}{1+\tau}\right) \right] dF(\tilde{m}_b) \\
& + V\left(\frac{\tilde{m} + \tau}{1+\tau}\right) \Big\}. \tag{2.8}
\end{aligned}$$

In Equation (2.8) the first term of the expression inside the curly braces is the expected increase in the value function of the agent when he is a buyer and the second term is the expected increase in his value function when is he is a seller. The third term is just the agent's continuation value.

## 2.4 Generalized Nash bargaining

I use the Generalized Nash bargaining procedure to determine the terms of trade  $(q, \tilde{d})$ . Following Trejos and Wright (1995), since by assumption agents cannot meet potential partners during a bargaining process, I will set their threat points to zero. Therefore, the terms of trade  $(q, \tilde{d})$  between a buyer with money holding  $\tilde{m}_b$  and a seller with money holding  $\tilde{m}_s$  solve the following problem:

$$\max_{q, \tilde{d}} \left[ U(q) + V\left(\frac{\tilde{m}_b - \tilde{d} + \tau}{1+\tau}\right) \right]^\theta \left[ -C(q) + V\left(\frac{\tilde{m}_s + \tilde{d} + \tau}{1+\tau}\right) \right]^{(1-\theta)} \tag{2.9}$$

subject to

$$0 \leq q \leq 1 \tag{2.10}$$

$$0 \leq \tilde{d} \leq \tilde{m}_b \tag{2.11}$$

$$U(q) + V\left(\frac{\tilde{m}_b - \tilde{d} + \tau}{1+\tau}\right) \geq V\left(\frac{\tilde{m}_b + \tau}{1+\tau}\right) \tag{2.12}$$

$$-C(q) + V\left(\frac{\tilde{m}_s + \tilde{d} + \tau}{1+\tau}\right) \geq V\left(\frac{\tilde{m}_s + \tau}{1+\tau}\right) \tag{2.13}$$

$$\frac{\tilde{m}_s + \tilde{d} + \tau}{1+\tau} \leq \tilde{m} \tag{2.14}$$

where  $\tilde{m}$  is an upper bound of money holding such that the mass of agents with money holdings greater than  $\tilde{m}$  is negligible and  $\theta$  is the buyer's bargaining power.

Equation (2.10) expresses the constraint imposed on the quantity of good exchanged in order to get a non-negative disutility for the seller based on the specific functional form used in this chapter. Equation (2.11) takes into account the buyer's budget constraint. Equations (2.12) and (2.13) are respectively the participation constraints of the buyer and the seller. Equation (2.14) imposes a constraint on the money holding of a seller to ensure that seller's money holding does not exceed the upper bound of the domain  $[0, \tilde{m}]$ .

## 2.5 Equilibrium definition

An equilibrium in this economy consists of a distribution of money holdings  $F(\tilde{m})$ , terms of trade  $q(\tilde{m}_b, \tilde{m}_s)$  and  $\tilde{d}(\tilde{m}_b, \tilde{m}_s)$ , a value function  $V(\tilde{m})$  such that given the constant growth rate  $\tau$  the following requirements are met:

1.  $F$  is an invariant distribution given  $\tilde{d}$ ;
2.  $V$  satisfies the Bellman equation (2.8) given  $(q, \tilde{d})$  and  $F$ ;
3.  $(q, \tilde{d})$  solves the generalized Nash bargaining problem equations 2.9–2.14 given  $V$ .

In this chapter I focus on a monetary equilibrium, that is, an equilibrium in which there exists at least one matching trade where the buyer and the seller exchange strictly positive quantities of good  $q$  and money  $\tilde{d}$ .

## 2.6 Computational procedure

To solve the model, I use a modified version of the solution procedure presented in Chapter 1 to account for the change in money stock from one period to another. For example, in step 4 of the numerical procedure described in Chapter 1, I use the following new update equation



for the value function from iteration  $k$  to the next iteration  $k + 1$ :

$$\begin{aligned}
V^{k+1}(\tilde{m}) = & \frac{1}{1+r} \left\{ \alpha \sigma \int_0^\infty \left[ U(q(\tilde{m}, \tilde{m}_s)) + V^k \left( \frac{\tilde{m} - \tilde{d}(\tilde{m}, \tilde{m}_s) + \tau}{1 + \tau} \right) - V^k \left( \frac{\tilde{m} + \tau}{1 + \tau} \right) \right] dF(\tilde{m}_s) \right. \\
& + \alpha \sigma \int_0^\infty \left[ -C(q(\tilde{m}_b, \tilde{m})) + V^k \left( \frac{\tilde{m} + \tilde{d}(\tilde{m}_b, \tilde{m}) + \tau}{1 + \tau} \right) - V^k \left( \frac{\tilde{m} + \tau}{1 + \tau} \right) \right] dF(\tilde{m}_b) \\
& \left. + V^k \left( \frac{\tilde{m} + \tau}{1 + \tau} \right) \right\}.
\end{aligned} \tag{2.15}$$

In addition, I use Equations 2.9–2.14 to determine the terms of trade and Equation 2.6 to update the money holdings of the agents.

Table 2.1: Replication of effects of monetary expansion published in Molico (2006), page 719

Money growth rate (%)	0	0.5	1	2	3	5	10	20
Standard deviation of money (%)	36.753	39.213	43.114	49.049	52.421	56.860	61.416	62.737
Coefficient variation (prices)	0.396	0.246	0.239	0.254	0.271	0.292	0.324	0.352
Average quantity (good)	0.848	0.851	0.848	0.842	0.835	0.823	0.795	0.752
Real balance(M/P)	32.895	12.924	8.719	5.567	4.203	2.949	1.867	1.250
Annual velocity money	0.026	0.066	0.097	0.151	0.199	0.281	0.432	0.617
Welfare	13.664	13.854	13.855	13.827	13.787	13.698	13.482	13.120

Notes:  $\alpha = 1$ ,  $\sigma = 0.25$ ,  $r = 0.01$  and  $M = 100$ .

Table 2.2: Molico’s original results of the effects of monetary expansion

$\mu$	0%	0.5%	1%	2%	3%	5%	10%	20%
Std. $\hat{m}$ (%)	42.3	40.6	44.2	49.4	53.2	57.0	61.8	62.0
M/E(p)	31.8	12.85	8.60	5.47	4.15	2.93	1.83	1.18
Coef. Var. p	0.46	0.25	0.24	0.25	0.27	0.30	0.33	0.36
E(q)	0.843	0.851	0.848	0.840	0.830	0.821	0.794	0.756
W	13.57	13.86	13.87	13.84	13.79	13.68	13.42	12.95
Annual vel	0.026	0.066	0.098	0.153	0.198	0.280	0.433	0.638

$x = 0.25$ ,  $r = 0.01$ .

Source: Molico (2006), page 719.

I tested the modified solution procedure on the Molico’s model where buyers have all the bargaining power. The results, presented in Table 2.1 are in general in line with Molico’s

results shown in Table 2.2, where  $\mu$  is the growth rate of money,  $x$  is the probability of a single coincidence of wants, and  $r$  is the discount rate.

## 2.7 Simulation results

In this section I present the simulation results of the economic effects of changes in the growth rate of money using the parameters values from Molico (2006). To account for the non-monotonic effects of bargaining power on the outcome of the simulations I run two sets of simulations: one set for the case where the buyers' bargaining power is high ( $\theta = 0.8$ ) and another set for the case where the buyers' bargaining power is low ( $\theta = 0.4$ ). Tables 2.3 and 2.4 summarize the results of the different simulations.

In equilibrium, an increase in the rate of money growth always raises the average real price and decreases the dispersion of prices when the bargaining power of the buyer is low. However, when the bargaining power of the buyer is high, low rates of money creation decrease the price dispersion while higher rates increase the price dispersion.

Also, in general, a faster growth rate of money supply leads to a decline in the average quantity of good consumption exchanged in equilibrium. However, when buyers have low bargaining power, low growth rates of money will have limited impact on the quantity of consumption good exchanged.

When buyers have low bargaining power, a faster rate of monetary expansion always leads to a decline in wealth inequality. However, when buyer's bargaining power is high, at low rates an increase in the money supply growth rate reduces inequality but a high rates, an increase in the rate of monetary expansion increases wealth inequality.

In addition, the average real money balance is always decreasing in the money growth rate while the annual velocity of money is always increasing in the money growth rate.

The pattern of the impact of money creation on the average welfare depends on the bargaining power of the buyer. When the bargaining power of the buyer is low, low rates of

Table 2.3: Effect of monetary expansion when the buyer's bargaining power is 0.80

Money growth rate(%)	0	2	5	10	20
Standard deviation (money)	0.509	0.495	0.564	0.610	0.620
Gini coefficient	0.291	0.276	0.313	0.336	0.342
Average price	0.068	0.183	0.340	0.536	0.801
Coefficient variation (prices)	0.544	0.255	0.293	0.329	0.351
Average quantity (good)	0.846	0.842	0.823	0.796	0.752
Real balance(M/P)	14.635	5.451	2.945	1.864	1.249
Annual velocity money	0.057	0.155	0.281	0.433	0.618
Welfare	13.730	13.826	13.698	13.482	13.120

Notes:  $\alpha = 1$ ,  $\sigma = 0.25$ ,  $r = 0.01$  and  $M = 1$ .

money supply increase welfare and high rates reduce welfare while at high rates, the results are reversed. However, when the bargaining power of the buyers is low, a faster rate of money creation can increase the average welfare even at moderate rates.

To get an insight into these results, we need to take into account two effects of a lump-sum transfer policy in addition to the two effects of the bargaining power mentioned in Chapter 1.

First, a lump-sum transfer has a redistributive effect by working as a subsidy for the poor (agents with money holding less than the average money holding) and as a tax for the rich (agents with money holding more than the average money holding). Second, the lump-sum transfer, by inducing inflation, has a negative real balance effect which reduces the average real money balance of agents.

The intuition behind the effect of money creation on wealth inequality is the following. On the one hand, the redistributive effect of a lump-sum money transfer reduces wealth inequality, but on the other hand, the real balance effect of money creation can increase inequality. This negative effect comes from the fact that the real balance effect increases the opportunity cost of holding money, therefore poor agents in particular will have to transfer a more significant portion of their wealth to rich agents to induce them to produce as the growth rate of money increases. This leads to an increase in both the average real price and

Table 2.4: Effect of monetary expansion when the buyer's bargaining power is 0.40

Money growth rate(%)	0	2	5	10	20
Standard deviation (money)	0.818	0.806	0.774	0.716	0.654
Gini coefficient	0.425	0.424	0.412	0.388	0.358
Average price	0.579	0.643	0.700	0.774	0.901
Coefficient variation (prices)	0.836	0.725	0.639	0.522	0.421
Average quantity (good)	0.751	0.754	0.751	0.744	0.726
Real balance(M/P)	1.726	1.556	1.428	1.292	1.109
Annual velocity money	0.452	0.500	0.541	0.592	0.675
Welfare	12.777	12.863	12.907	12.918	12.833

Notes:  $\alpha = 1$ ,  $\sigma = 0.25$ ,  $r = 0.01$  and  $M = 1$ .

wealth inequality.

In addition, as mentioned in Chapter 1 the seller's bargaining power has a negative effect on wealth inequality. Therefore in this environment, the net effect of money creation on wealth inequality is ambiguous. When the seller's bargaining power is low, money creation can reduce inequality only when the money growth rate is low. However, when the seller's bargaining power is high, poor buyers are in such a disadvantaged position that the redistributive effect of a money transfer policy can give them enough endogenous bargaining power and lead to a reduction in wealth inequality as they get more money transfer. The effects of money creation on the distribution of wealth are illustrated in Figure 2.1 and Figure 2.2.

The effect of money creation on price dispersion seems closely related to that on wealth inequality. In general, an increase in wealth inequality leads to an increase in price volatility. Since the higher the dispersion of money holdings, the higher the occurrence of large differences in money holdings between buyers and sellers and thus the more dispersed the distribution of prices. The distribution of prices is reported in Figure 2.3.

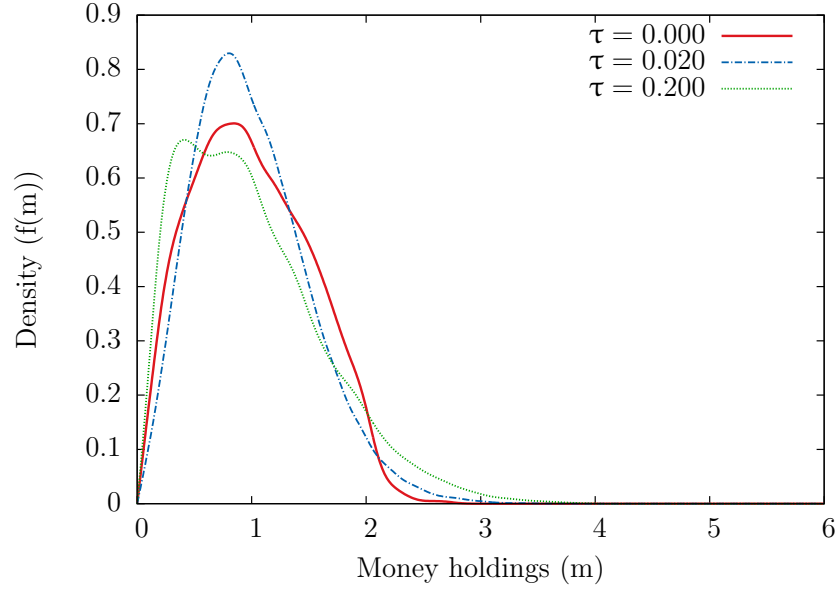
Since the real-balance effect reduces the incentive to hold money, there is a lower demand for money which will lead to smaller quantity of good exchanged. However, this negative

effect can be dominated by the redistributive effect by benefiting the poor and giving them the opportunity to consume and by disadvantaging the rich making them more willing to produce. The positive effect of wealth redistribution dominates the negative real-balance effect when both the money growth rate and the seller's bargaining power are low.

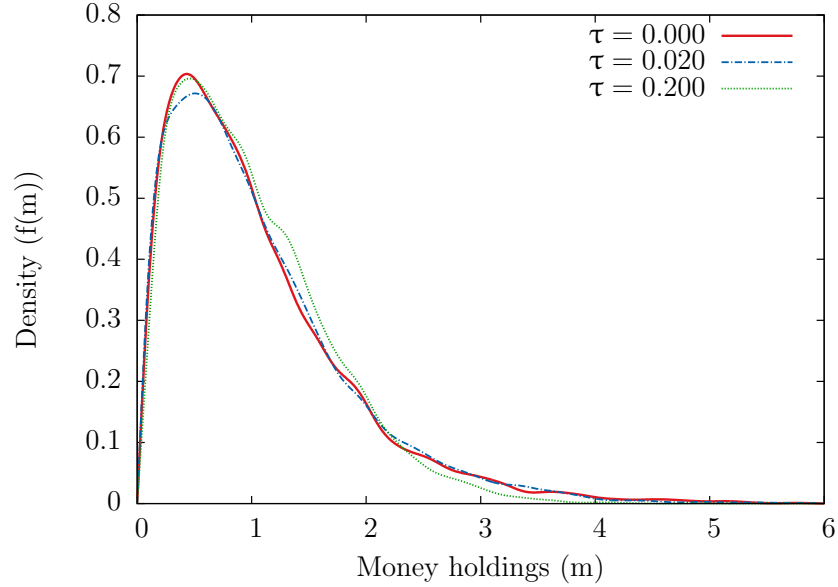
The intuition behind the results related to the average welfare is the following. The real-balance effect reduces agents' valuation of money by increasing the opportunity cost of holding money. However, the redistributive effect increases the valuation of money by poor agents and decreases that of rich agents. Therefore the net effect of money creation on welfare can either be positive or negative since in this environment, welfare is defined as the average valuation of money by the agents. In particular, money creation can be welfare improving when the money growth rate is low or even moderate in cases where the bargaining power of sellers is high. However, note that for any given level of the money growth rate, the higher the seller's bargaining power, the lower the average welfare.

Since money creation increases the opportunity cost of holding money thanks to the real-balance effect, agents try to limit their exposure to the negative effects of inflation by spending a higher proportion of their money holdings when they have the opportunity to trade. Therefore, the velocity of money will be increasing in the growth rate of money supply. Note also, that since the stock of money is increased through a lump-sum transfer to all agents, no agents will run out of money and thus they can afford to hold less money for self-insurance against the randomness of the trade matching process leading to a further increase in spending and thus a higher velocity of money.

Finally, the result related to the real balance can be explained as follows. Since the average normalized money supply is constant in equilibrium and the prices are increasing in the growth rate of the money the supply, the real balance will be decreasing in the growth rate of the money supply.

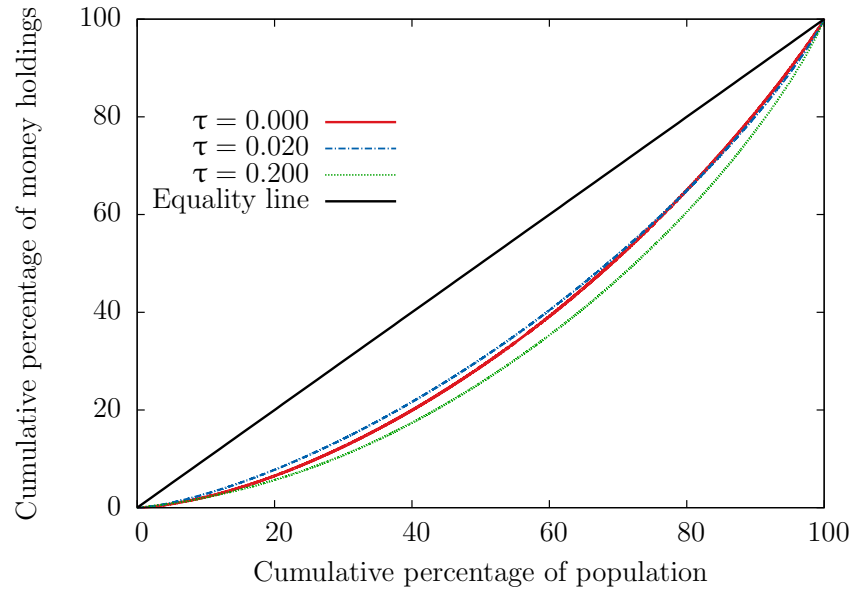


(a) Buyer's bargaining power equals to 0.80

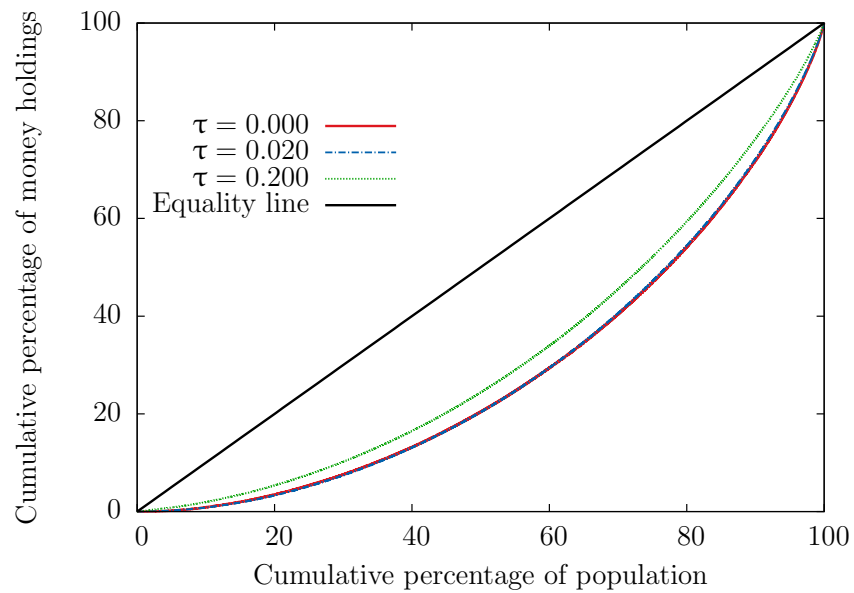


(b) Buyer's bargaining power equals to 0.40

Figure 2.1: Probability density of money holdings with varying money growth rates

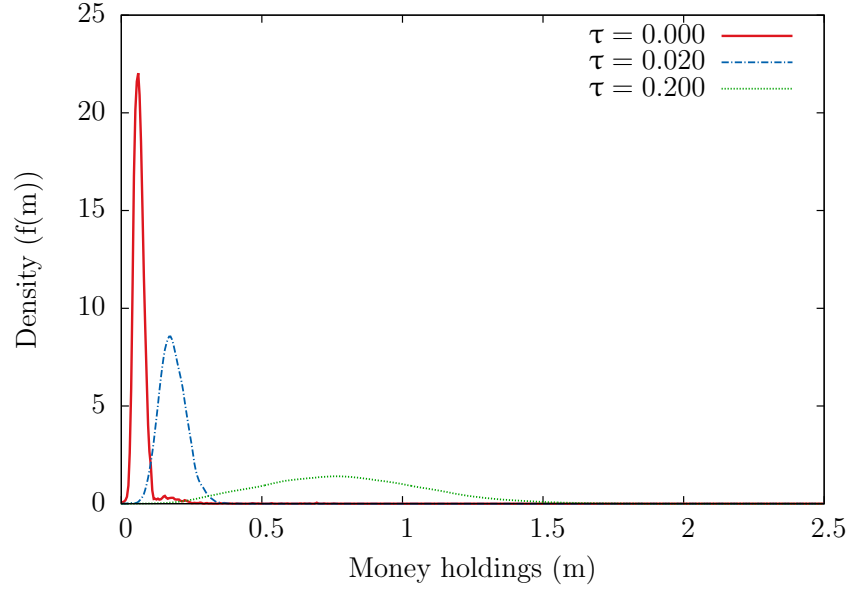


(a) Buyer's bargaining power equals to 0.80

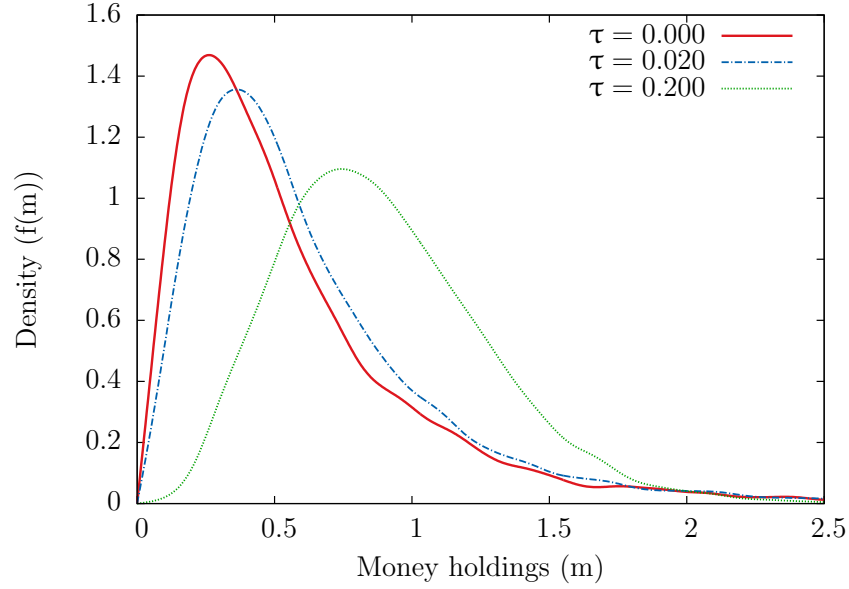


(b) Buyer's bargaining power equals to 0.40

Figure 2.2: Lorenz curves with varying money growth rates



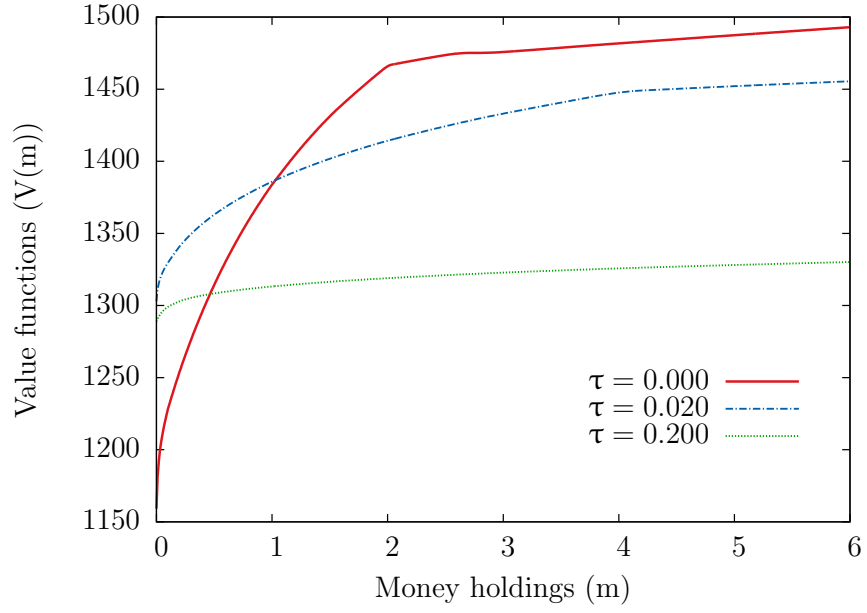
(a) Buyer's bargaining power equals to 0.80



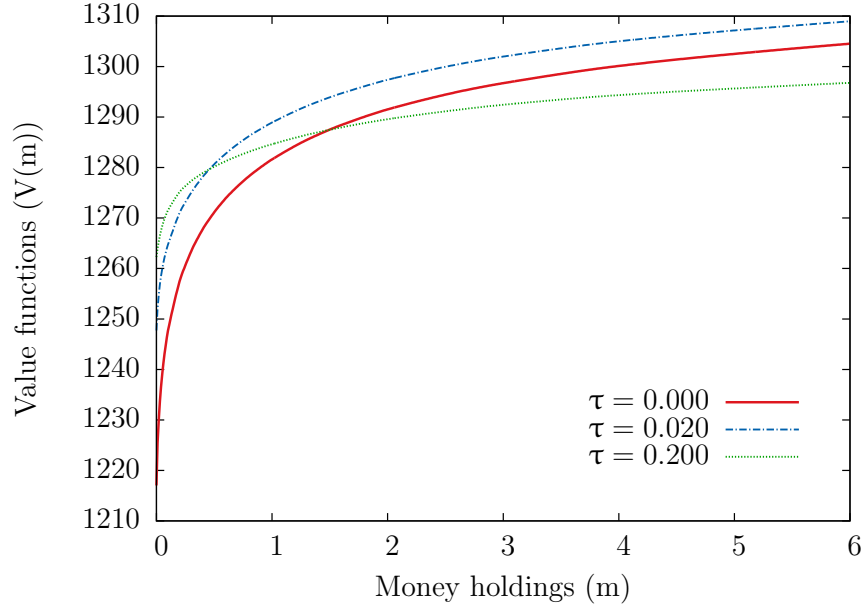
(b) Buyer's bargaining power equals to 0.40

Figure 2.3: Probability density of prices with varying money growth rates





(a) Buyer's bargaining power equals to 0.80



(b) Buyer's bargaining power equals to 0.40

Figure 2.4: Value functions with varying money growth rates

## 2.8 Conclusion

In this paper I investigated the economic effects of an expansionary monetary policy by extending the model used in Molico (2006) to situations where buyers do not have all the bargaining power. The results of the simulation show that the main result in Molico (2006), suggesting that an expansionary monetary policy can be welfare improving when the growth rate of money supply is low, is robust to changes in the buyers' bargaining power. The results also confirm that money creation can reduce inequality in particular when the bargaining power of the buyers is high. For sufficiently low buyers' bargaining power, an expansionary monetary policy can increase wealth inequality.

As pointed out by Molico (2006), the type of model used in this paper, where agents use money also for self-insurance purposes, cannot be calibrated to deliver levels of velocity of money observed in real economic data. It would be interesting to extend the model used in this paper so that it is amenable to calibration. This can be done by extending the present economic environment to include a centralized goods market as in Lagos and Wright (2005). The resulting environment will be similar to that in Chiu and Molico (2011) but without the assumption that buyers have full bargaining power.

For future research, it would also be interesting to add another asset such as a bond which could give agents the opportunity to diversify their asset portfolio and thus limit their exposure to the negative effects of inflation. To this end, one can develop a model that combines the present economic environment with that of Zhu and Wallace (2007) where fiat money coexists with government bonds.

## Chapter 3

# The Effects of Monetary Growth on Welfare and Wealth Inequality in a Money Search Model with Exogenous Breakdown

### 3.1 Introduction

The purpose of this essay is to examine the welfare and distributional effects of money creation in an economic environment with exogenous breakdown. This essay extends the model used in Chapter 2 in two directions. The first extension allows agents to meet other potential partners during the bargaining process. Following Trejos and Wright (1995), I call this bargaining procedure a bargaining with exogenous breakdown. The goal of this extension is to test the robustness of the results obtained in Chapter 2 to a change in the pricing mechanism. The second extension adds an alternative measure of welfare: the welfare cost of inflation. This extension offers the opportunity to check whether the welfare cost of inflation and the average welfare as a measure of aggregate welfare lead to the same conclusion with respect to the welfare effect of an increase in the money supply.

The traditional way of evaluating the welfare effect of inflation is by computing the welfare cost of inflation. One approach for finding that cost is the welfare triangle approach developed by Bailey (1956). In this approach, real money balances are treated like consumption goods

and inflation like a tax on real balances. Therefore, the welfare cost of inflation is computed by calculating loss of consumer surplus, the area under the demand curve for real money balances, due to a change in inflation. An alternative approach developed by Lucas (1981) is the compensating variation approach. In this approach, the welfare cost of inflation is found by measuring by how much consumption in an economy with zero percent inflation must be reduced to leave economic agents indifferent between this economy and the one with a higher inflation rate.

In their studies Bailey (1956) and Lucas (1981, 2000) use models where money was not essential. As pointed out by Craig and Rocheteau (2006), the estimations of the welfare cost of inflation in environments where money is not essential are in general lower than those in environments where money is essential.

Lagos and Wright (2005) estimate the welfare cost of inflation in the search model where money is essential by using Lucas' compensated variation approach. Chiu and Molico (2011) also evaluate the welfare cost of inflation in an environment similar to the one used by Lagos and Wright (2005). But unlike the model in Lagos and Wright (2005), the model in Chiu and Molico (2011) leads to a non-degenerate distribution of money holdings. However, as in Molico (2006), Chiu and Molico (2011) make the assumption that in a random matching between a seller and a buyer in the decentralized market, the buyer has all the bargaining power. In this paper I follow the Lagos and Wright (2005) and Chiu and Molico (2011) approach in estimating the welfare cost of inflation.

In the economic environment of this paper where the bargaining procedure is one with an exogenous breakdown, I find that expansionary monetary policies conducted through a lump-sum transfer to agents are not welfare improving when buyers do not have full bargaining power. The results of the simulations show that an increase in money supply leads to a decrease in average welfare and an increase in the welfare cost of inflation. However, the results show that money creation can reduce wealth inequality when the growth rate of money is high, although the opposite is true when the growth rate of money is low.

The remainder of the paper is structured as follows. Section 3.2 presents the economic framework. Section 3.3 presents the value function. In Section 3.4, I lay out the Nash bargaining problem. Section 3.5 defines the steady state equilibrium. In Section 3.6, I outline the computational strategy. Section 3.7 presents the simulation results, and Section 3.8 concludes the paper.

## 3.2 Economic environment

The economic environment is the same as that in Chapter 2 except that, in a random meeting between a buyer and a seller, agents can meet other agents during the bargaining process. In particular, in a random match, the buyer is selected with probability  $\theta$  to make a proposal and with probability  $(1 - \theta)$ , the seller is selected. When one of the traders is chosen to make the offer, the other trader can either accept or reject the proposal. If the offer is accepted, then the trade is executed and the game ends for those two agents for the current trading period. However, if the proposal is rejected, the parties take a break. During the break they can meet other potential partners. If at least one of the parties is successful in meeting a new trading partner, then the old trading match is dissolved, and the newly formed trading pair starts a bargaining process. However, if during the break nothing happens, then the agents continue the bargaining process and one of them is again chosen at random to make a proposal. The game continues in this manner until the end of the trading period.

As in Chapters 1 and 2, in equilibrium, no agent in a bargaining process ever leaves the bargaining table and no offer is ever rejected, meaning that agreement is reached in the first round of proposals. Here the fear of being abandoned motivates the trading partners to reach an agreement as soon as possible. The rest of the environment remains the same as in Chapter 2.

### 3.3 Value function

Using the same notations as in Chapter 2, the Bellman equation for the stationary economy is given by :

$$\begin{aligned}
V(\tilde{m}) = \frac{1}{1+r} \Big\{ & \alpha\sigma \int_0^\infty \left[ U(q(\tilde{m}, \tilde{m}_s)) + V\left(\frac{\tilde{m} - \tilde{d}(\tilde{m}, \tilde{m}_s) + \tau}{1+\tau}\right) - V\left(\frac{\tilde{m} + \tau}{1+\tau}\right) \right] dF(\tilde{m}_s) \\
& + \alpha\sigma \int_0^\infty \left[ -C(q(\tilde{m}_b, \tilde{m})) + V\left(\frac{\tilde{m} + \tilde{d}(\tilde{m}_b, \tilde{m}) + \tau}{1+\tau}\right) - V\left(\frac{\tilde{m} + \tau}{1+\tau}\right) \right] dF(\tilde{m}_b) \\
& + V\left(\frac{\tilde{m} + \tau}{1+\tau}\right) \Big\} \quad (3.1)
\end{aligned}$$

In words (3.1) means that in equilibrium, the value function of an agent with normalized money holding  $\tilde{m}$  is equal to the present discounted value of the sum of the agent's expected gain from trade and his continuation value.

### 3.4 Generalized Nash bargaining

To formulate the Nash bargaining problem, I need to specify the threat points of the agents. Since the bargaining process is one with exogenous breakdown, I follow Trejos and Wright (1995) by setting agents' threat points to their continuation values. Therefore in a single coincidence of wants meeting the quantities of consumption good and money exchanged are found by solving the following problem:

$$\max_{q, \tilde{d}} \left[ U(q) + V\left(\frac{\tilde{m}_b - \tilde{d} + \tau}{1+\tau}\right) - V\left(\frac{\tilde{m}_b + \tau}{1+\tau}\right) \right]^\theta \left[ -C(q) + V\left(\frac{\tilde{m}_s + \tilde{d} + \tau}{1+\tau}\right) - V\left(\frac{\tilde{m}_s + \tau}{1+\tau}\right) \right]^{(1-\theta)} \quad (3.2)$$

subject to

$$0 \leq q \leq 1 \quad (3.3)$$

$$0 \leq \tilde{d} \leq \tilde{m}_b \quad (3.4)$$

$$U(q) + V\left(\frac{\tilde{m}_b - \tilde{d} + \tau}{1+\tau}\right) \geq V\left(\frac{\tilde{m}_b + \tau}{1+\tau}\right) \quad (3.5)$$

$$-C(q) + V\left(\frac{\tilde{m}_s + \tilde{d} + \tau}{1+\tau}\right) \geq V\left(\frac{\tilde{m}_s + \tau}{1+\tau}\right) \quad (3.6)$$

$$\frac{\tilde{m}_s + \tilde{d} + \tau}{1+\tau} \leq \tilde{m} \quad (3.7)$$

Equations 3.3 to 3.6 have the same meaning as Equations 2.10 to 2.13.

### 3.5 Equilibrium definition

Given threat points and bargaining powers of agents, a stationary equilibrium can be defined as a time-invariant distribution of money holdings  $F(\tilde{m})$ , terms of trade  $q(\tilde{m}_b, \tilde{m}_s)$  and  $\tilde{d}(\tilde{m}_b, \tilde{m}_s)$ , and a value function  $V(\tilde{m})$  such that given the constant growth rate  $\tau$  the following requirements are met:

1.  $F$  is an invariant distribution given  $\tilde{d}$ ;
2.  $V$  satisfies the Bellman Equation (3.1) given  $(q, \tilde{d})$  and  $F$ ;
3.  $(q, \tilde{d})$  solves the generalized Nash bargaining problem Equations 3.2–3.7 given  $V$ .

### 3.6 Computational procedure

The computational procedure used in this chapter is based on the six-step procedure described in Chapter 1. However, we need to make various adjustments to the equations used. In particular, in step 4 of the numerical procedure described in Chapter 1 we need use the following updating equation for the value function from iteration  $k$  to iteration  $k + 1$ :

$$\begin{aligned}
 V^{k+1}(\tilde{m}) = & \frac{1}{1+r} \left\{ \alpha \sigma \int_0^\infty \left[ U(q(\tilde{m}, \tilde{m}_s)) + V^k \left( \frac{\tilde{m} - \tilde{d}(\tilde{m}, \tilde{m}_s) + \tau}{1 + \tau} \right) - V^k \left( \frac{\tilde{m} + \tau}{1 + \tau} \right) \right] dF(\tilde{m}_s) \right. \\
 & + \alpha \sigma \int_0^\infty \left[ -C(q(\tilde{m}_b, \tilde{m})) + V^k \left( \frac{\tilde{m} + \tilde{d}(\tilde{m}_b, \tilde{m}) + \tau}{1 + \tau} \right) - V^k \left( \frac{\tilde{m} + \tau}{1 + \tau} \right) \right] dF(\tilde{m}_b) \\
 & \left. + V^k \left( \frac{\tilde{m} + \tau}{1 + \tau} \right) \right\}.
 \end{aligned} \tag{3.8}$$

We also need to use a different set of equations for determining the terms of trade between a seller and a buyer. We need to use Equations 3.2 to 3.7. In addition, for updating the money holdings of the agents, the same law of motion as in Chapter 2 (Equation 2.6) is used.

### 3.7 Simulation results

In this section I present the results of the simulation of the economic implications of a change in the growth rate of money. Two sets of results are presented. One set is for the case where the buyer has high bargaining power and the other is for the case where the buyer has low bargaining power. The results are summarized in Table 3.1 and Table 3.2. The results are also illustrated in Figures 3.1, 3.2, 3.3, and 3.4, which present respectively, the distribution of money holdings, the Lorenz curves, the value functions and the distribution of prices for changing growth rates of money supply.

Before getting to the results, I need to give more detail about the computation of the welfare cost of inflation. To find the welfare cost of inflation, I ask the following question: what proportion of their consumption are agents willing to give up to have zero inflation rather than  $\tau$  inflation rate.

To answer the question let  $N(\tau)_{\Delta q_\tau}$  be the net average expected utility under an inflation rate  $\tau$  when the seller produces  $q_\tau$  and the buyer consumes a factor  $\Delta$  of  $q_\tau$ . The value of  $N(\tau)_{\Delta q_\tau}$  is given by the following equation:

$$N(\tau)_{\Delta q_\tau} = \frac{1}{1+\beta} \left\{ \alpha \sigma \int_0^\infty \int_0^\infty \left[ U(\Delta q_\tau(\tilde{m}_b, \tilde{m}_s)) - C(q_\tau(\tilde{m}_b, \tilde{m}_s)) \right] dF(\tilde{m}_b) dF(\tilde{m}_s) \right\} \quad (3.9)$$

where  $\beta$  is the discount factor.

To find the welfare cost of inflation we first find the factor  $\Delta$  that solves the following equation:

$$N(\tau)_{q_\tau} = N(0)_{\Delta q_0}. \quad (3.10)$$

Then the welfare cost of inflation, as a proportion of agents' consumption, of having a money growth rate of  $\tau$  instead of a zero inflation rate is given by  $1 - \Delta$ . Note that finding  $\Delta$  based on equation 3.10 amounts to solving a nonlinear equation. For this I use the Newton-Raphson method (see Yang et al. (2005)).

The results of the simulations show that, in general, wealth inequality is decreasing in



the growth rate of money supply. However, when the money growth rate is low, an increase in the money growth rate leads to an increase in wealth dispersion.

Also, the average price of goods exchanged in equilibrium is increasing in the growth rate of money supply while the opposite holds for the average quantity of good traded in equilibrium.

In addition, the higher the money growth rate, the lower the dispersion of prices. However, when the bargaining power of the buyer is low and the growth rate of money supply is also low, an increase in the growth rate of money has a limited effect on price dispersion.

Furthermore, an increase in the growth rate of money leads to a decrease in the real balances and an increase in the income velocity of money.

Finally, an expansionary monetary policy has a negative effect on welfare since an increase in the money growth rate decreases the average welfare and increases the welfare cost of inflation.

Table 3.1: Effect of monetary expansion when the buyer's bargaining power is 0.80

Money growth rate(%)	0	2	5	10	20
Standard deviation (money)	0.898	0.948	0.976	0.956	0.855
Gini coefficient	0.463	0.492	0.503	0.494	0.449
Average price	0.481	0.736	0.939	1.136	1.339
Coefficient variation (prices)	0.665	0.645	0.656	0.647	0.602
Average quantity (good)	0.778	0.756	0.721	0.678	0.623
Real balance(M/P)	2.081	1.359	1.064	0.880	0.747
Annual velocity money	0.389	0.580	0.715	0.827	0.908
Welfare (average)	12.837	12.528	12.190	11.849	11.403
Welfare cost of inflation	-	0.013	0.048	0.094	0.156

Notes:  $\alpha = 1$ ,  $\sigma = 0.25$ ,  $r = 0.01$  and  $M = 1$ .

The intuitions behind the results are similar to those exposed in Chapter 2 and rely on the one hand on the redistributive and real balance effects of a lump-sum money transfer and on the other hand on the self-insurance role of money and the hold-up effect of bargaining

Table 3.2: Effect of monetary expansion when the buyer's bargaining power is 0.40

Money growth rate(%)	0	2	5	10	20
Standard deviation (money)	1.440	1.448	1.369	1.241	1.018
Gini coefficient	0.659	0.661	0.634	0.591	0.509
Average price	1.561	1.762	1.848	1.924	2.078
Coefficient variation (prices)	1.088	1.040	0.972	0.870	0.701
Average quantity (good)	0.629	0.588	0.542	0.491	0.433
Real balance(M/P)	0.640	0.568	0.541	0.520	0.481
Annual velocity money	1.086	1.171	1.156	1.108	1.044
Welfare (average)	9.585	9.348	9.129	8.881	8.433
Welfare cost of inflation	-	0.061	0.129	0.206	0.293

Notes:  $\alpha = 1$ ,  $\sigma = 0.25$ ,  $r = 0.01$  and  $M = 1$ .

power. Therefore, in what follows I focus on discussing some intuition behind the differences in the welfare and wealth distributional effects of expansionary monetary policy between the current environment and that in Chapter 2.

Recall that in Chapter 2 the threat points of agents are zero while in this Chapter the threat points are equal to the agents' continuation values. Therefore, when the threat points are strictly positive, in a match the agent with the highest threat point will gain a larger share of the trading surplus as compared to the case where the threat points are zero. In particular, in a trading match between a rich buyer and poor seller, the buyer will be more willing to transfer a higher quantity of money in exchange of same quantity of good as shown by the higher velocity of money in Tables 3.1 and 3.2 as compared to Tables 2.3 and 2.4. Therefore, the average quantity of good exchanged will be lower and the prices will be higher.

So, higher money transfers between rich buyers and poor sellers will reduce the dispersion of money holdings. This reduction in inequality is made larger by higher money growth rates mainly thanks to the redistributive effect of policy of a lump-sum money transfer. However, when the money growth rate is very low, an increase in the growth rate of money can increase inequality. This is due to the fact that cash-constrained poor buyers can spend a higher

proportion of their current money holdings thus leading to an increase in the dispersion of money holdings which can dominate the reduction in the dispersion of money holdings due to differences in threat points between buyers and sellers.

To understand the differences in the welfare effects of monetary expansion between the environments in Chapters 2 and 3, let us compare the values presented in Figure 3.3 to those in Figure 2.4. The comparison reveals that values in Figure 3.3 are significantly lower than their corresponding values in Figure 2.4. This seems to suggest that the net result of strictly positive threat points is to exacerbate the hold-up problem. Note that a lower valuation of money leads to a lower average welfare since the average welfare is defined as the average of the value functions of the agents. Therefore, the additional hold-up can help to explain why even at low levels of money growth rates, an expansionary monetary policy is not welfare improving.

Note that if we compare the welfare costs of inflation when the seller's bargaining power is low (Table 3.2) to their corresponding values when the seller's bargaining power is high, we can conclude that the welfare cost of inflation is increasing in the bargaining power of the seller. This can be attributed to the fact that the higher the seller's bargaining power, the larger the hold-up effect. Lagos and Wright (2005) got a similar pattern of results when the authors calibrated their model to the United States economy.

Table 3.3: Average Welfare comparison for various buyer's bargaining power ( $\theta$ ) and money growth rate ( $\tau$ )

	$\tau = 0\%$	$\tau = 2\%$
$\theta = 1.00$	13.655	13.827
$\theta = 0.95$	13.740	13.584
$\theta = 0.80$	12.837	12.528
$\theta = 0.40$	9.585	9.348

Notes:  $\alpha = 1$ ,  $\sigma = 0.25$ ,  $r = 0.01$  and  $M = 1$ .

To conclude this section I present additional results and give more insights into my findings. First, I check whether in the current economic environments, the welfare effects of money creation are in line with those obtained in Chapter 2 when buyers have full bargaining power. The idea is that, when buyers have full bargaining power, the agents' threat point should have no impact on the outcomes of the generalized Nash bargaining problem since buyers keep all the surplus from the trade in a bilateral random matches. Thus, the effects of inflation should be similar in these cases.

To test this idea, I compute in the economic environment of this chapter the average welfare corresponding to the money growth rates  $\tau = 1$  and  $\tau = 0.02$  for  $\theta = 1$ . The results are reported in Table 3.3. Recall that from my replication of Molico's results (Table 2.1) that the average welfare for  $\tau = 1$  and  $\tau = 0.02$  are respectively 13.664 and 13.827. As expected, these values are indeed close to the those reported in Table 3.3 for the corresponding money growth rates. As we can notice, money creation is beneficial then buyers have all the bargaining power. As the results in Table 3.3 show however, money creation does not improve welfare when  $\theta < 1$  in the current environment.

Second, in order to get additional insight into these results, I use graphs to compare the terms of trade for a case where an expansionary monetary is beneficial ( $\theta = 1$ ) to another one where money creation is not welfare improving ( $\theta = 0.95$ ). The terms of trade presented in Figures 3.5 and 3.6 are for the case where the growth rate of money  $\tau = 0.02$ . A comparison of the graphs in Figure 3.5 reveals that in both cases the quantity of goods traded in a bilateral match is increasing in the buyer's money holding and is decreasing in seller's money holding. The quantity of goods traded is lower however, for the case where  $\theta = 0.95$  as compared to the case where  $\theta = 1$ .

A similar comparison exercise performed on the graphs in Figure 3.6 shows that the amount of money exchanged in a bilateral match is increasing in the buyer's and seller's money holdings. The quantity of money exchanged in a match is higher however, for the case where  $\theta = 0.95$  as compared to the case where  $\theta = 1$ .

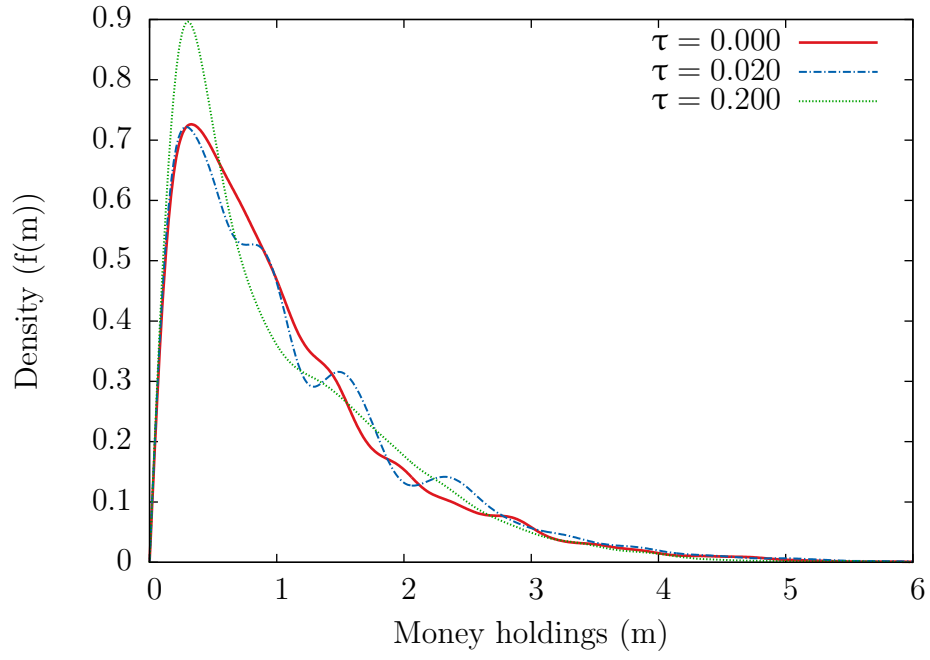
Those observations confirm a previous remark that when agents have positive threat point, giving some bargaining power to sellers makes the hold-up problem more severe which in turn decreases agents' marginal valuation of money. As a result of this, the average welfare is low.

## 3.8 Conclusion

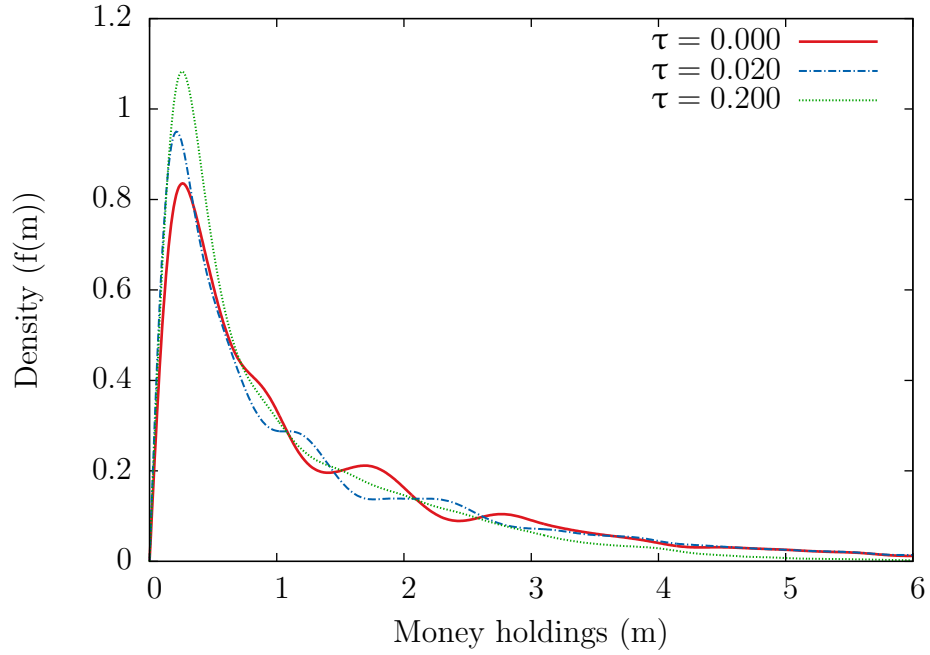
In this paper I examined the economic effects of money creation in an economic environment where agents are allowed to meet other potential partners during the bargaining process. The results of the computer simulations show that an expansionary monetary policy decreases the average welfare and increases the welfare cost of inflation when buyers do not have full bargaining power. However, the same policy can lead to a reduction in wealth inequality although, at lower money growth rates, an increase in the money growth rate increases wealth inequality.

The different results show that both the buyer's bargaining power and the detail of the bargaining protocol are important in assessing the effects of monetary policy. Therefore, those factors will be determinant in calibrating the model to a particular economy. Although in Lagos and Wright (2005) the authors made an attempt to estimate the bargaining power of buyers, they acknowledged that it is in fact hard to pin down the value of that parameter. So, further research is needed in that direction.

Additional research is also required to determine conditions under which a bargaining procedure without exogenous breakdown or one with exogenous breakdown is more compatible with a specific country's economic environment.

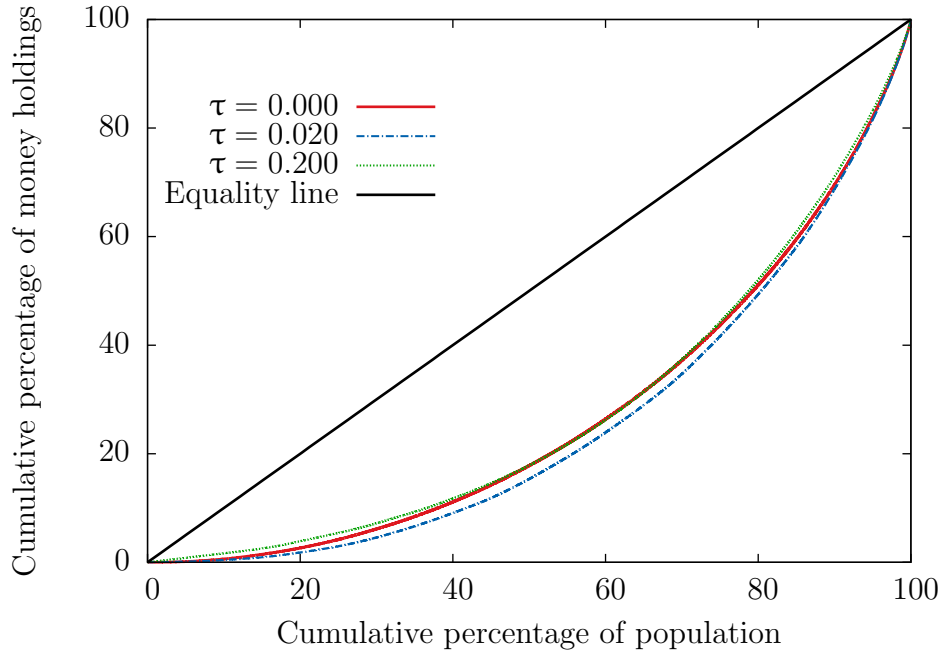


(a) Buyer's bargaining power equals to 0.80

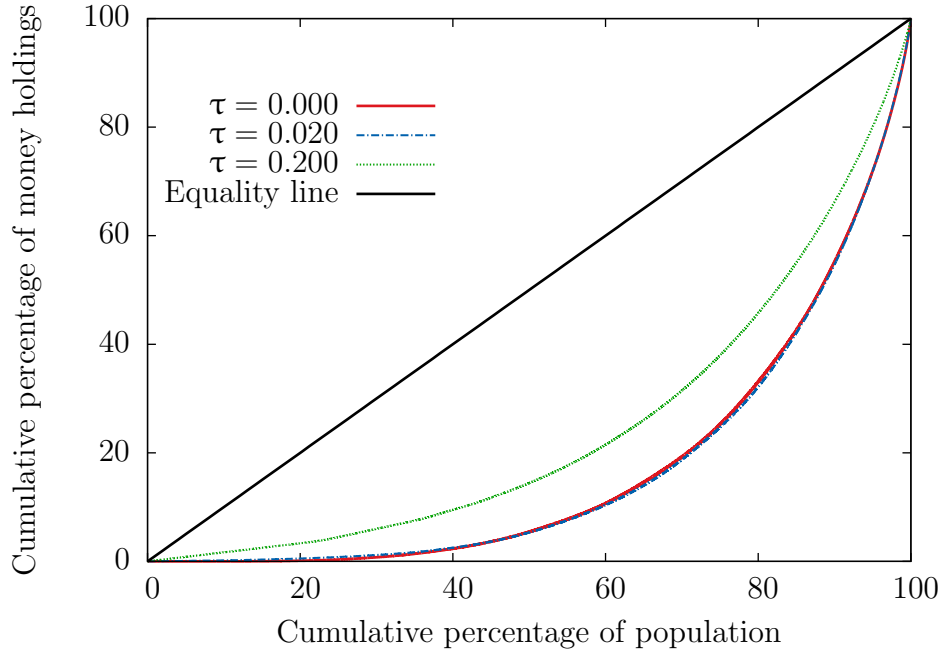


(b) Buyer's bargaining power equals to 0.40

Figure 3.1: Probability density of money holdings with varying money growth rates

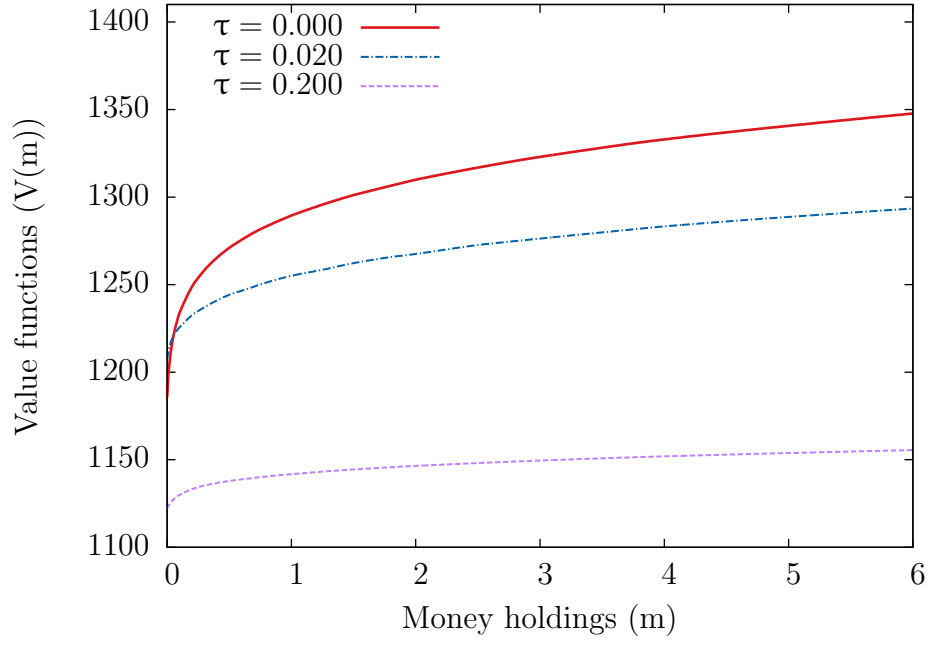


(a) Buyer's bargaining power equals to 0.80

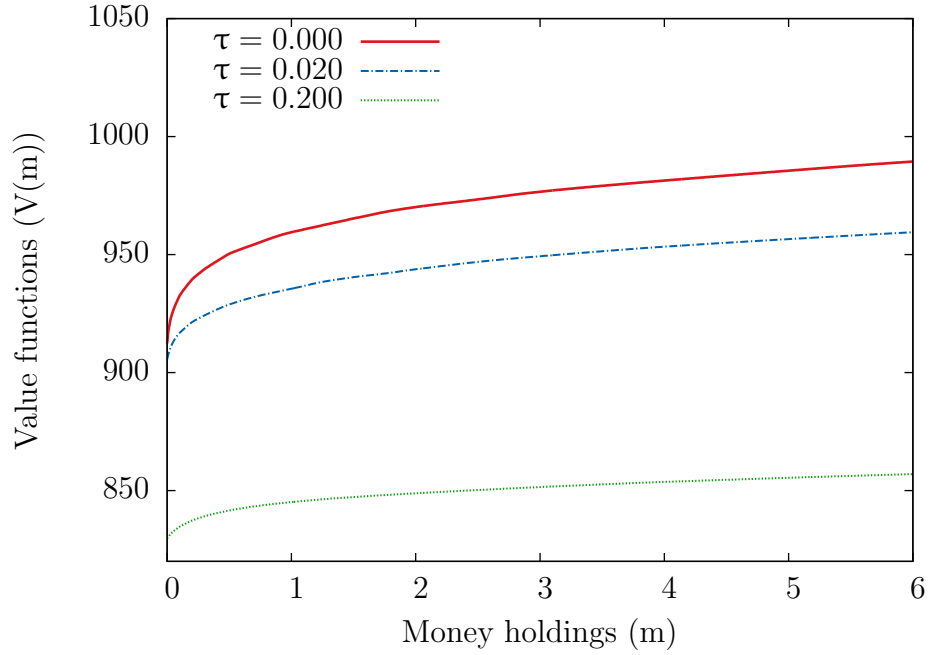


(b) Buyer's bargaining power equals to 0.40

Figure 3.2: Lorenz curves with varying money growth rates



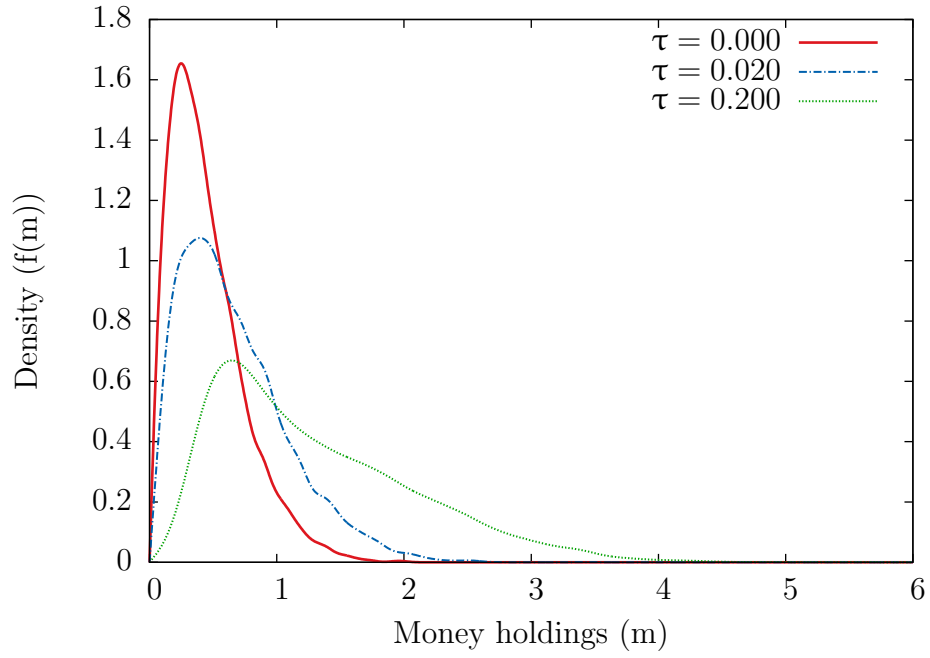
(a) Buyer's bargaining power equals to 0.80



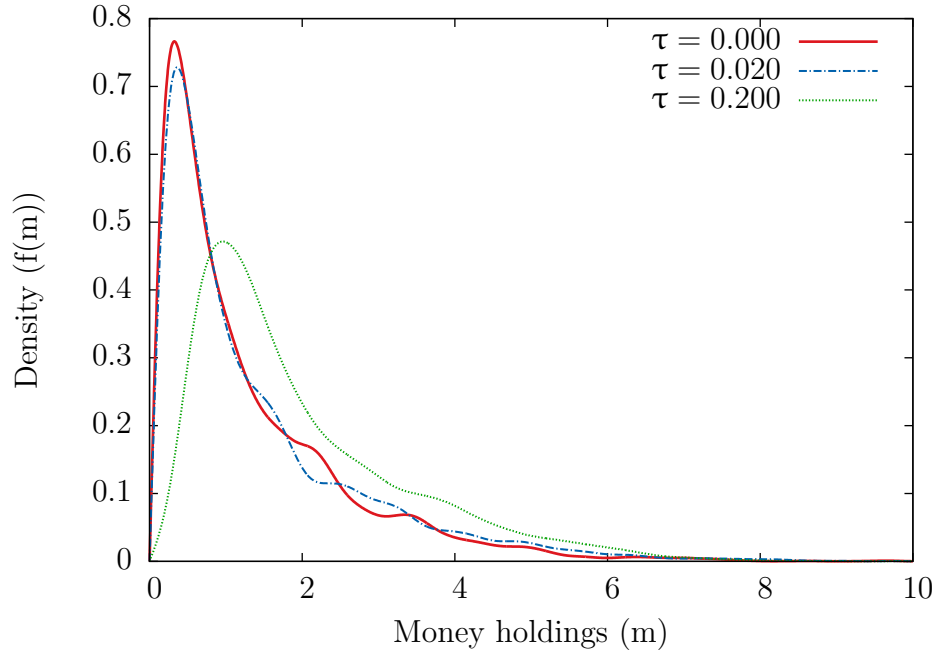
(b) Buyer's bargaining power equals to 0.40

Figure 3.3: Value functions with varying money growth rates



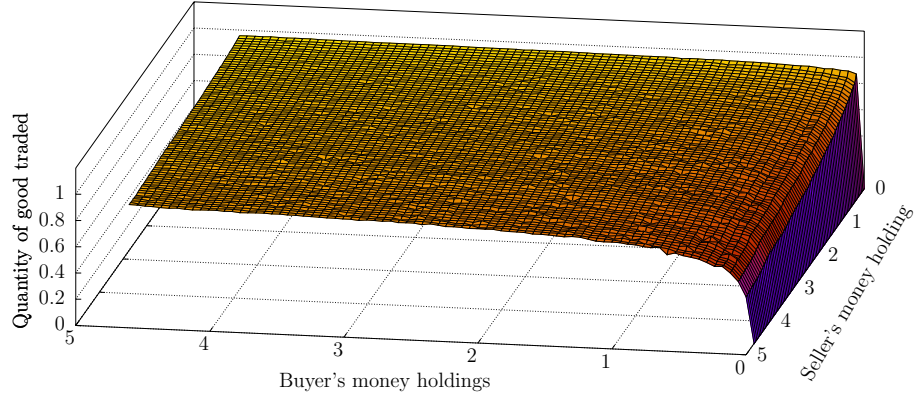


(a) Buyer's bargaining power equals to 0.80

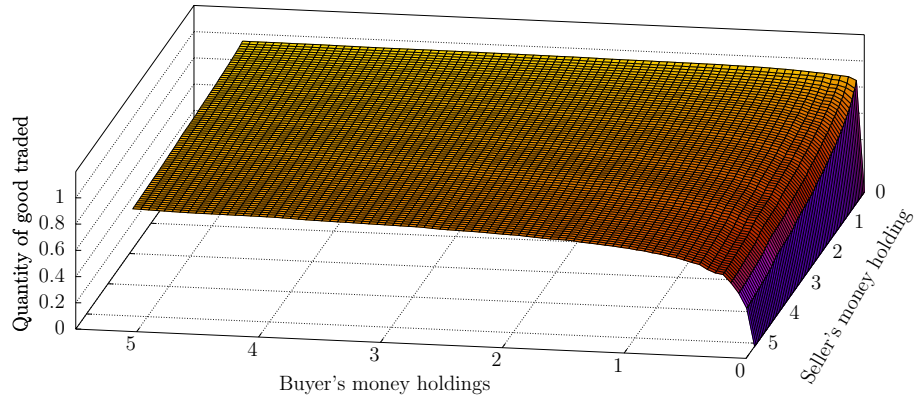


(b) Buyer's bargaining power equals to 0.40

Figure 3.4: Probability density of prices with varying money growth rates

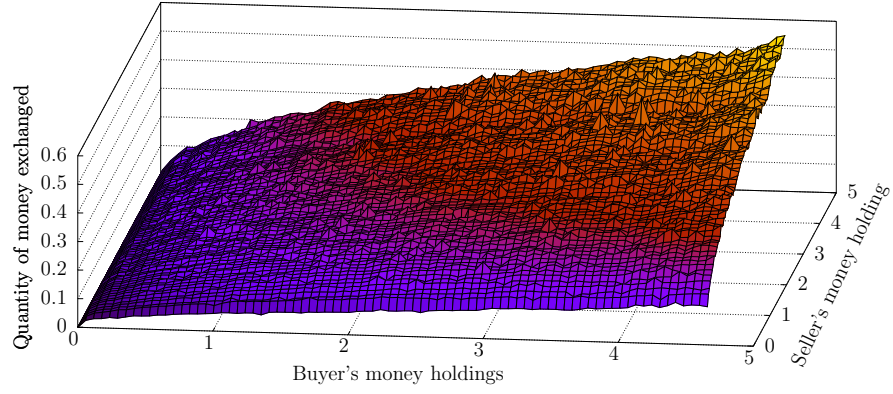


(a) Buyer's bargaining power equals to 1.00

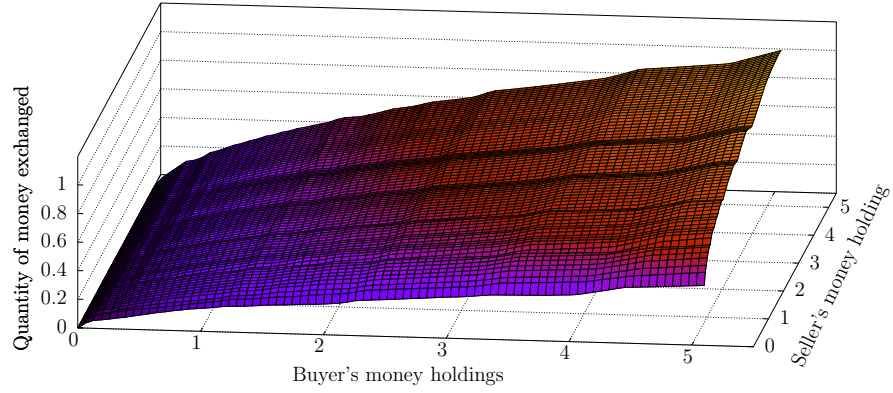


(b) Buyer's bargaining power equals to 0.95

Figure 3.5: Quantity of good traded as a function of buyer's and seller's money holdings (case  $\tau = 0.02$ )



(a) Buyer's bargaining power equals to 1.00



(b) Buyer's bargaining power equals to 0.95

Figure 3.6: Quantity of money exchanged as a function of buyer's and seller's money holdings (case  $\tau = 0.02$ )

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