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Math and Voting: Voting Methods, Fair Representation, and the Electoral College

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ABSTRACT

Voting is in integral part of any functioning democracy, but there exist more than just one way to count votes. Some voting methods use only a voter's top-choice candidate, while others require a ranking of all candidates, most-preferred to least-preferred, from each voter. We examine some of these ranked-choice voting methods, including the anti-plurality method, Hare's method, and Coomb's method.

Because of the variety of voting methods, we introduce criteria, which allow for an evaluation of the advantages and disadvantages of each method. The criteria give various definitions for what "good" or "bad" voting methods look like, depending on context. Some important criteria include the monotonicity, independence, and decisiveness criteria. Arrow's theorem then gives us restrictions on which criteria can coexist and which are incompatible.

Next, we explore how the state of Maine uses ranked-choice voting, specifically Hare's method, for both state and federal primaries and general federal elections. Hare's violation of the monotonicity and independence criteria is explored and proved.

Finally the results of the 2020 United States presidential election are dissected. The apportionment of congressional seats and thus Electoral College votes is cause for complaint from some citizens, and reapportionment may allow fairer representation for citizens through the Electoral College. This reapportionment of electoral votes, as well as a reallocation of electoral votes within each state is investigated.

Many thanks are due to my thesis advisor, Dr. Myron Minn-Thu-Aye, for his continued support and never-ending knowledge over the past year. If it were not for his guidance and grace, my thesis would have been little more than a collection of half-finished definitions and blank spaces.

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Chapter 1

Introduction

The opportunity for people to vote in elections is fundamental to the democratic process, as it gives individuals a structured way to have their opinions heard and for a group to make fair decisions. Most people in the 21st century understand the importance of voting. Similarly, most Americans also have their own complaints and gripes with the way elections are decided in the United States. The plurality method, also known colloquially as first-past-the-post voting, has boiled down to the presidential election being decided between the two major political parties' nominees. A common phenomena and criticism of the plurality method, especially in regards to the presidential election, is that a vote for any candidate besides the Democrat or Republican nominee is a vote wasted.

If so many people are unhappy with the system in place, it begs us to consider what other ways exist to count votes. One U.S. state that has taken matters into its own hands is Maine, which now uses ranked-choice voting for its state and federal primary elections, as well as federal general elections. Ranked-choice voting is a term used for any voting method that allows, and in some cases requires, voters to indicate their ranking of the election's candidates. There are a plethora of ranked-choice voting methods, each with its own benefits and drawbacks. Exploring and understanding these advantages and disadvantages allows us to decide which method is best for specific scenarios. Determining how to measure what is considered *best* is where politics meets math. In order to compare voting methods, each method needs to be defined uniquely and distinctively. Clear inputs, outputs, and procedures are defined for each method, which we can then use to ascertain how each method acts according to different criteria. No one method meets every indicator of what “best” can mean for a voting method, so we are forced to decide which characteristics we are willing to compromise on for each election.

Another area of electoral politics that is interlaced with mathematics is apportionment and representation. Instead of having every citizen have a direct say in every decision, we have a system of representation that only sometimes succeeds at its attempt of fairness. In the context of this paper, we explore how equally the Electoral College represents the American people and why, mathematically, criticisms of the Electoral College exist. The 2020 presidential election is analyzed under different types of apportionment between states and allocation of votes within states.

Chapter 2

Voting Methods

Given ballots with two or more candidates and voter preferences for some or all of the candidates, there are a variety of ways to determine a winner of the election. A voting method, also known as a social choice function, is the function by which winners of an election are decided. We will cover a subset of possible voting methods in this chapter.

The social choice function that most people are familiar with is the plurality method. It is the method used to decide the winner of U.S. presidential, congressional, and gubernatorial elections, among many others.

Definition 2.1. The plurality method is simple. First, each candidate's first-place votes are tallied up. Then, whichever candidate or candidates have the most first-place votes win the election.

The plurality method works for ranked-choice ballots, as well as in elections that do not involve ranked-choice voting, since only the first-place votes are considered. Ranked-choice ballots are ballots in which voters rank each candidate in order of preference. This type of ballot is used for many different social choice functions, several which we will discuss in this chapter.

In contrast to the plurality method, which elects the candidate(s) with the most *first*-place votes, the anti-plurality method elects the candidate(s) with the fewest *last*-place votes.

Definition 2.2. The anti-plurality method begins by tallying up each candidate's last-place votes. Then, the winner or winners of the election are the candidate or candidates with the fewest last-place votes.

The anti-plurality method chooses to elect the least-disliked candidate(s), rather than the most-liked candidate, who the plurality method elects. Since the anti-plurality method requires voters to rank the candidates in order of who they think most highly of down to who they like the least, it requires ranked-choice ballots. The next voting method also utilizes ranked-choice ballots, but rather than only using the number of first or last place votes, it uses all the information a voter supplies on their ballot.

Definition 2.3. The Borda count method works in the following way: given n candidates in the election, a voter's first-ranked candidate gets $n-1$ points, the second-ranked candidate gets $n-2$ points, and so on and so forth until the last-place candidate gets 0 points. Once each candidate's points are tallied across all voters, the candidate with the most points is the winner. If multiple candidates are tied for first place, they are all winners of the election.

One benefit of the Borda count method is that it always uses all the information provided by voters on their ranked-choice ballots. In other words, the Borda count method never overlooks or ignores any of a voter's rankings. However, some methods do not always use all of this information. The following is an example of a method that does not necessarily use all of a voter's provided ranked-choice ballot information to decide a winner.

Definition 2.4. Hare's method decides an election's winner by first tallying up each candidate's first-place votes across all ballots. If a candidate has the majority of first-place votes, they are the winner. Otherwise the following process occurs: provided that not all candidates have the same number of first-place votes, eliminate the candidate with the fewest first-place votes. On each voter's ballot, eliminate this candidate and move each lower-ranked candidate up one ranking. Continue this process of tallying first-place votes and eliminating the candidate with the fewest until there is one candidate remaining or until all of the remaining candidates have the same number of first-place votes. The candidate or candidates who were not eliminated win the election.

Hare's method eliminates the least-loved candidate each round until it decides on its winner(s). In practice, this means that voters can indicate their support for smaller candidates while still having a voice in the inevitable decision between larger candidates. One criticism of the plurality method, in regards to presidential elections, is that voting for any candidate besides the two major parties' nominees could be considered a throw-away vote. Hare's method helps eliminate this phenomenon by allowing people to support small candidates until their elimination and then support the remaining, larger candidates.

A voting method that uses logic similar to Hare's method is Coomb's method. In contrast to Hare's method, which eliminates the least-loved candidate each round, Coomb's method eliminates the most-hated.

Definition 2.5. In Coomb's method, the last-place votes of each candidate are tallied up. Provided that all candidates do not have the same number of last-place votes, eliminate the candidate(s) with the most last-place votes. Then, move up any lower-ranked candidates on each ballot and repeat the process of counting last-place votes and eliminating the most-hated candidate(s). This method results in a winner or winners when there is only one candidate left or when all remaining candidates have an equal number of last-place votes.

Coomb's method ensures that if a large enough portion of the electorate does not like a candidate, that candidate will not win the election, regardless of how everyone else ranks that candidate. This may allow more neutral, centrist candidates to survive longer in the election, as they may not draw strong negative opinions from most voters. This is in contrast to how centrist candidates might perform under Hare's method, where, if they are not favored enough, they will be eliminated early on in the election.

The voting methods defined up until now are not the only voting methods that exist or even the only ones that have been used in practice. The following three voting methods are not likely to be preferred by voters looking for a democratic election process.

Definition 2.6. The monarchy method has one candidate who is identified as the monarch. The monarch always wins the election.

Definition 2.7. The dictatorship method has one voter who is identified as the dictator. Whichever candidate the dictator votes for is the winner of the election.

Definition 2.8. The all-ties method will always result in a tie between all candidates in the election.

It is obvious that there is no lack of voting methods. Clearly defining each one is the first step to understanding their advantages and disadvantages. Next, we need to define clear measurements for how we might determine "good" or "bad" methods.

Chapter 3

Criteria

Due to the wide variety of voting methods, we need to identify various properties of these methods in order to differentiate and compare them. These properties are called criteria, and they allow us to evaluate the advantages and disadvantages of each voting method. They give different definitions for how a voting method can be deemed ”good” or ”bad”, depending on the context and ideal outcome. The following section will explain some of these specific criteria.

The first criterion is one that most people will recognize, as it is a characteristic of presidential, congressional, and many other elections in the United States.

Definition 3.1. The neutrality criterion is satisfied when a voting method treats all candidates as equal in the election, rather than giving preference to one candidate over another. If candidate A beats candidate B in an election, but then all voters switch their opinions (all of A's voters vote for B and all of B's voters vote for A), then candidate B should be the new winner of the election.

The main objective of the neutrality criterion is the idea that the rules of an election should apply to all candidates equally, and that no candidate should be given special treatment. If enough of the electorate feels positively about a candidate that they elect them, but then all those voters decide to vote for a different candidate, that different candidate should then win the election. This criterion means that the election treats all candidates fairly and applies the election's rules equally. Similar to the neutrality criterion, which treats candidates equally, the anonymity criterion treats *voters* equally.

Definition 3.2. The anonymity criterion is satisfied if the outcome of an election does not change when any two voters exchange completed ballots.

The plurality method, used in most U.S. presidential and congressional elections, satisfies both the neutrality and anonymity criteria. If, using the plurality method, candidate A beats candidate B in an election, that means A has n more votes than B, for some integer n. Now suppose the voters of candidates A and B switch their votes to the opposite candidate. Then candidate B has n more votes than candidate A, meaning B is the new winner under the plurality method. This method treats all candidates fairly and applies its rules equally, so it satisfies the neutrality criterion. If any two voters exchange their ballots, the overall number of votes for each candidate remains constant, so the winner before and after the ballot switch would be the same. The plurality method also satisfies the next criterion.

Definition 3.3. The monotonicity criterion tells us that a candidate cannot be negatively affected by being more favored by a voter or voters. Suppose candidate A wins an election, beating candidate B. Then, one or more voters switch their preference from supporting candidate B (loser) to supporting candidate A (winner). Candidate A thus gained voters by this switch. Then the election is run again. The winner of the first round, candidate A, must also be the winner of the second round for the voting method to satisfy the monotonicity criterion.

When thinking about elections, it makes little sense for a candidate to lose an election for the sole reason that *more* voters support them. However, this criterion is in fact violated by multiple voting methods including Hare's method, which will be explained in more detail in the following chapter. Hare's method also violates the following criterion, which deals with elections of three or more candidates.

Definition 3.4. The independence criterion holds for a voting method if, when candidate A beats candidate B in an election, no change in voters' opinions of a third candidate C can cause B to beat A. Stated another way, suppose votes are cast, ballots are counted, and candidate A is declared winner of the election, using some social choice function. Then, suppose one or more voters change their opinions on candidate C, but keep their ordering of candidates A and B constant compared to their first ballot. This change in opinion about the third candidate C cannot then allow candidate B to beat candidate A. If the voting method satisfies the independence criterion, candidate A will still win this election.

The main idea of this criteria is that if candidate A beats candidate B, no reordering of any other candidates besides A and B should then allow B to beat A. The only voting methods that satisfy the independence criterion are the monarchy method, the dictatorship method, and the all-ties method.

The following are some other criteria that are used to better understand voting methods.

Definition 3.5. The majority criterion states that if a candidate receives a majority, or more than half, of the votes in an election, that candidate is the sole winner of the election.

Definition 3.6. The decisiveness criterion is satisfied if a voting method always results in exactly one sole winner of the election.

Definition 3.7. The Pareto criterion is satisfied by a voting method if, for two candidates A and B, every voter prefers A over B, the method can never choose B to win the election.

This last definition follows the logic that if the entire electorate unanimously likes one candidate over another, it does not make sense to select the less-preferred candidate to win.

All of these criteria, along with more that are not discussed here, give us grounds to dissect and compare voting methods to decide which is best given a specific scenario. However, there is an important theorem that dictates the limits to which criteria a voting method can possibly satisfy [4]:

Theorem 3.8 (Arrow's theorem)**.** *If a voting method with at least three candidates satisfies both the Pareto and independence criteria, then the voting method must be a dictatorship.*

The implications of this theorem are that if you require the voting method in use to satisfy Pareto *and* independence, the only method you can possibly use is dictatorship. Obviously dictatorship is not the most desirable method when it comes to making fair democratic decisions, but if independence and Pareto are of upmost importance, dictatorship is the only choice. To rephrase Arrow's theorem, we get the following corollary:

Corollary 3.9. *It is impossible to find a voting method for three or more candidates that is nondictatorial and satisfies both the Pareto and independence criteria.*

Building off of this corollary, we know that a dictatorship does not satisfy the anonymity criterion, as the dictator's opinion, and therefore ballot, is the deciding factor in any election decided using the dictatorship method. So, the corollary can be extended to say the following:

Corollary 3.10. *Any voting method for at least three candidates that satisfies the anonymity criterion cannot also satisfy both the independence and Pareto criteria.*

For more information about the incompatibility of criteria and the proof of Arrow's theorem, the reader can consult *The Mathematics of Politics* by E. Arthur Robinson, Jr. and Daniel H. Ullman [4].

Chapter 4

Maine

Given all of the voting methods listed in Chapter 2 and others not mentioned, one method that is in practice in various locations and that we will focus on in this chapter is Hare's method. In the state of Maine, Hare's method, also known as instant run-off voting, is used to decide the winners of both state and federal primaries and general federal elections [3].

As a reminder, Hare's method works in the following way:

1. Tally up each candidate's first-place votes.
2. Eliminate the candidate(s) with the fewest first-place votes.
3. Move any candidates up that were ranked behind the eliminated candidate(s).
4. Repeat until there is one candidate left or until all remaining candidates have an equal number of first-place votes.

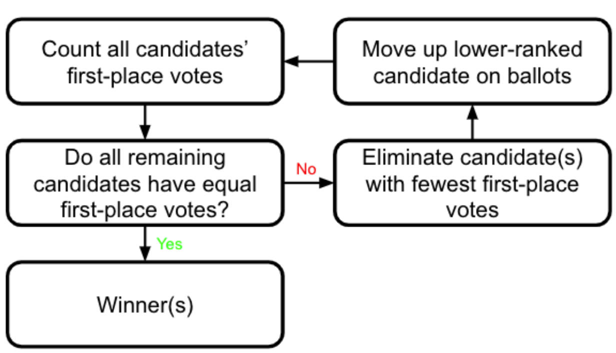

Figure 4.1: Diagram of Hare's method

Before going into greater detail of the benefits and drawbacks of Hare's method, we will first explore how Maine started using ranked-choice voting for their presidential and congressional elections. In 2001, the first bill to establish a ranked-choice system was introduced in the Maine Legislature, but died in committee. Between the years of 2005 and 2013, multiple ranked-choice voting bills were proposed and rejected by the Legislature. Leading up to the 2016 election, advocates for ranked-choice voting began soliciting signatures for a citizens' initiative petition, hoping to get the petition on the 2016 ballot. They received enough signatures, meaning that unless the Legislature enacted the proposed law, it qualified for the 2016 election. Maine voters ended up approving the ranked-choice voting question, so it became a law [6].

On January 7^{th}, 2017, the law took effect, but did not apply to elections until January 1^{st}, 2018. However, in February of 2017 the Maine Senate consulted with the Maine Supreme Judicial Court to check the constitutionality of the Ranked-Choice Voting Act. The issue was that the wording used in the law stated that ranked-choice voting would apply to elections for U.S. Senate, Congress, Governor, State Senate, and State Representative. In May of 2017, the Main Supreme Judicial Court found that, according to the Maine Constitution, ranked-choice voting was unconstitutional for the elections for State Representative, State Senator, and Governor, as those elections had to be decided by plurality. After that ruling, the Maine legislature could not agree whether to amend the Maine Constitution or amend the Ranked-Choice Voting Act. In October 2017, the legislature passed a bill which stated that ranked-choice voting would not be used in the 2018 or 2020 elections and unless the Maine Constitution was amended, the Ranked-Choice Voting Act would be repealed on December 1^{st}, 2021 [6].

However, similar to the initial petition that eventually got ranked-choice voting on the ballot, a petition was started for a people's veto of the legislature's latest bill. They gathered enough votes to repeal the legislature's bill that would delay ranked-choice voting, resulting in the June 2018 election using ranked-choice voting to decide its winners. It is important to note that the people's veto did *not* block the legislature's repeal of ranked-choice voting being used for governor and state legislature. This preserved the constitutionality of the Ranked-Choice Voting Act [6]. Ranked-choice voting was again used in the November 2018 midterm elections, and has been and will continue to be used in state and federal primary elections, as well as federal general elections, moving forward.

The voting method laid out in Maine's Ranked-Choice Voting Act is Hare's method, also called instant run-off voting. Although Hare's method is widely-used, is not a perfect voting method. It violates some of the criteria listed in Chapter 3, which we will investigate further in the rest of this chapter.

Theorem 4.1. *Hare's method violates the monotonicity criterion.*

As a reminder, the monotonicity criterion states that a candidate cannot be negatively affected by being more favored by a voter or voters. Figure 4.2 showcases this violation. To begin with, there are 7 voters whose candidate rankings are $A > B > C$, 8 voters whose rankings are $B > C > A$, 10 voters whose rankings are $C > A > B$, and 4 voters whose rankings are $A > C > B$. In the first round of this election, the first-place votes are tallied up for each candidate. Candidate B has the fewest first-place votes with 8, so B is eliminated from the election. When B is eliminated, the new ballot counts are 11 in favor of A over C and 18 in favor C over A, which means A is eliminated and C is the winner of this election.

Now, in order to show the monotonicity violation, suppose the 4 voters who originally ranked the candidates $A > C > B$ (indicated by the asterisk and light gray background in Figure 4.2) change their rankings to $C > A > B$, thereby *increasing* their support of candidate C. Then the total ranking counts are 7 voters who rank $A > B > C$, 8 voters who rank $B > C > A$, and 14 voters who rank $C > A > B$. Since A only has 7 first-place votes, A is eliminated. Then, the final ranking counts are 15 for $B > C$ and 14 for $C > B$, meaning B wins. This violates the monotonicity criteria because those 4 voters who changed their opinions ranked C higher on their ballots than the first time around when C won the election, so there should be no way that C is negatively impacted by the positive change in these voters' ballots. Since B wins the second time around, Hare's method clearly violates the monotonicity criterion.

Election 1

Number of this type of ballot	Rankings 1	2	3
7	A	B	C
8	B	C	A
10	C	A	B
4*	A	C	B

First place votes:

A	B	C
11	8	10

B has fewest first-place votes, so B is eliminated.

↓

	1	2
11	A	C
18	C	A

First place votes:

A	C
11	18

C wins.

The 4 voters marked with the * now change their opinions, ranking C above A.

Election 2

Number of this type of ballot	Rankings 1	2	3
7	A	B	C
8	B	C	A
14	C	A	B

First place votes:

A	B	C
7	8	14

A has fewest first-place votes, so A is eliminated.

↓

	1	2
15	B	C
14	C	B

First place votes:

B	C
11	18

B wins.

Figure 4.2: Example ballots showcasing how Hare's method violates the monotonicity criterion.

Theorem 4.2. *Hare's method violates the independence criterion.*

The independence criterion says that when candidate A beats candidate B in an election, no change in voters' opinions of candidate C can cause B to beat A. In other words, if the ordering of A and B remains constant, even if opinions about C change, A should remain the winner of the election. Figure 4.3 shows an example where Hare's method violates this criterion. To begin, 3 voters rank the candidates $A > B > C$, 6 rank them $A > C > B$, and 7 rank them $B > A > C$. Since C has 0 first-place votes, C is eliminated. After this elimination, the ballot counts are 9 voters who favor A over B and 7 who favor B over A. Therefore, A is the winner.

Now, suppose that the 6 voters who ranked the candidates $A > C > B$ (indicated by the asterisk and light gray background in Figure 4.3), change their opinion about candidate C and thus change their ballots to be $C > A > B$. Note that the ordering of candidates A and B never changes; A is always favored over B in both the first ballots and these new modified ballots. This change in rankings means that A has 3 first-place votes, B has 7, and C has 6. Therefore, A will now be eliminated first. After A's elimination, B has 10 first-place votes while C only has 6, so B is the winner. Since A won the first election and B won the second but all voters' orderings of A and B remained constant, Hare's method clearly violates the independence criterion.

Theorem 4.3. *Hare's method satisfies the Pareto criterion.*

The Pareto criterion tells us that if every voter prefers one candidate over another, then the less-preferred candidate can never win the election. If every voter prefers candidate A over candidate B, for example, then candidate B would be eliminated in the Hare's method process, meaning that B would not win the election. There is no time when directly comparing the votes for A and B when B could beat A with fewer votes when using Hare's method,

Election 1

Number of this type of ballot	Rankings 1	2	3
3	A	B	C
6*	A	C	B
7	B	A	C

First place votes:

A	B	C
6	7	0

C has fewest first-place votes, so C is eliminated.

↓

	1	2
9	A	B
7	B	A

First place votes:

A	B
9	7

A wins.

The 6 voters marked with the * now change their opinion on candidate C, but their ordering of A vs. B remains in tact

Election 2

Number of this type of ballot	Rankings 1	2	3
3	A	B	C
6	C	A	B
7	B	A	C

First place votes:

A	B	C
3	7	6

A has fewest first-place votes, so A is eliminated.

↓

	1	2
10	B	C
6	C	B

First place votes:

B	C
10	6

B wins.

Figure 4.3: Example ballots showcasing how Hare's method violates the independence criterion.

so Hare's method satisfies the Pareto criterion.

It is clear that the criteria defined in Chapter 3 allow us to evaluate and dissect voting methods. Although Hare's method is used in the state of Maine and at many other local and city levels, as well as national levels in other countries, it still has its drawbacks and negative aspects.

Chapter 5

Electoral College

Up until this point, we have referenced the U.S. presidential election as an election decided using the plurality method. However, it is more complex than that. The Founding Fathers did not trust 18^{th} century voters, as they thought they lacked education and resources to make an informed vote. So, a system of compromise was established, where states appointed special people called "electors" to act as an intermediary between citizens and the presidential election. These electors would technically cast the only true votes for president. This system is still in place today and is called the Electoral College. Many people hold strong opinions about the Electoral College, some stating that it is outdated, considering the accessibility to education about candidates that was not around in the 1700s. Others still support the Electoral College, stating that it gives smaller states the voice they deserve without getting completely drowned out by larger states.

While the Electoral College is not a completely unknown system in this day and age, the process of apportioning electoral votes is less well known. There are a total of 538 electoral votes, accounting for all the seats in congress (535) plus three votes for the District of Columbia. Article II, Section 1 of the Constitution lays out the rules for how many electoral votes each state gets: the number of House representatives for that state plus two, for the number of Senators [2]. Thus, each state has at least three electoral votes, for the minimum of one House representative and two senators. The District of Columbia was granted three electoral votes in the twenty-third amendment, as Washington D.C does not have any voting members of congress and thus did not have any electoral votes assigned [2]. For simplicity's sake, we may refer to all 50 states and the District of Columbia as "states" moving forward, although Washington D.C. is technically not a state at the time of writing. The apportionment of the 435 House seats occurs every ten years, after the Census is completed, and thus electoral votes are reapportioned every ten years. We will focus on the 2020 election, which used the apportionment established after the 2010 Census.

After citizens cast their votes for president, each state is responsible for establishing guidelines that their electors must follow when they cast their electoral votes. In 48 out of the 50 states, the electors are mandated to vote for their state's winner by the plurality method, meaning all of that state's electoral votes go to the state's plurality winner. In both Nebraska and Maine, two electoral votes must go to the state's winner by the plurality method, while their remaining electoral votes go to each district's winner by the plurality method. The District of Columbia's three electoral votes go to the district's winner by plurality. In order for a presidential candidate to win the country-wide election, they must receive a majority of the electoral votes, at least 270 total.

After understanding the basics of how the Electoral College functions, we will now explore various criticisms of the system and possible changes that could be made to make voting more equal and fair across the country. Census data from the Census Bureau and election data from the Harvard Dataverse will be used for the following computations and plots [5] [1]. The first area of complaint about the Electoral College is that because the minimum number of electors per state is three, states with very low populations are over-represented within the Electoral College. To understand this criticism more mathematically, we first calculate the citizen per elector in each state, by dividing the state's total number of citizens (taken from the 2010 census) by the number of electors that state is assigned. For big states like California, New York, and Texas, the number of citizens per elector is over 650,000, whereas for small states like Vermont, Washington D.C., and Wyoming, the number is closer to 200,000. Clearly there is an incredibly large range between states with a high level of representation (low citizen per elector number) compared to states with a much lower level of representation (high citizen per elector number), seen in Figure 5.1, sorted by highest ratio to lowest. The problem with having such variability in this range is that it means some U.S. citizens are more represented in the presidential election, disobeying the democratic idea of “one person, one vote” that this country holds near and dear.

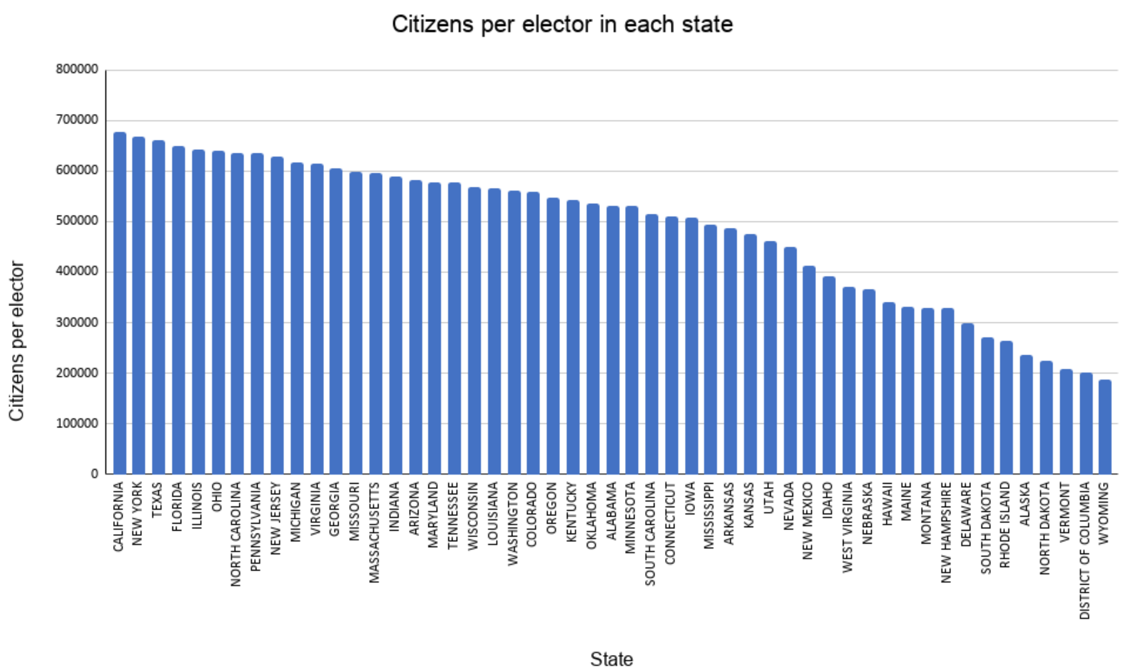

Figure 5.1: Graph representing the number of citizens per elector in each state.

One possible way to remedy this problem is to change how electoral votes are apportioned. Rather than assigning a number of electoral votes equal to each state’s number of seats in congress, another option is to assign a number of electoral votes directly proportional to the percentage of the national population each state has. So, for example, California, Texas, and New York contain 12.07%, 8.14%, and 6.28% of the overall national population, respectively. Logically it makes sense that each of these states should get 12.07%, 8.14%, and 6.28% of the 538 possible electoral votes. As with any process, this type of apportionment is not perfect. In the theoretical new apportionment, South Dakota has a very low number of citizens per elector, as seen in Figure 5.2. This is due to the fact that after rounding and assigning electoral votes to each state, there were not 538 total electoral votes. Rounding up and down meant that there were only 537 electors assigned and since South Dakota had the highest ratio of citizens to electors, it was assigned the extra elector. Even so, it is easy to visually see the difference between the original apportionment of electoral votes in Figure 5.1 to the theoretical reallocation in Figure 5.2. In the reapportionment, Alaska has the highest ratio of citizens to electors at 710,231. There are only three states that have a citizen to elector ratio lower than 500,000: Montana (494,708), Delaware (448,967), and South Dakota (407,090). All other states range between 510,000 and 710,000, a much narrower range than originally calculated for the 2010 census apportionment.

One piece of information that may be important to consider when deciding whether or not this reapportionment is valuable overall is the average citizen per elector across all states. For the original apportionment, the average citizen per elector is equal to 486,228. For the new, more proportional apportionment, the average citizen per elector is 576,823. So, while the reapportionment creates a more equal system for citizens across the board, it actually leads to more citizens being represented by each elector when the whole country is considered. For context, dividing the entire U.S. population by the total number of electors results in the following calculation: $308,745,538/538 = 573,876$, a sort of national average of citizens per electoral vote. So, since the new apportionment is nearly equal to this number and more fairly represents citizens across the board, it can be concluded that an allocation of electoral votes based purely off of a state's percentage of the overall national population is more fair than the system in place now.

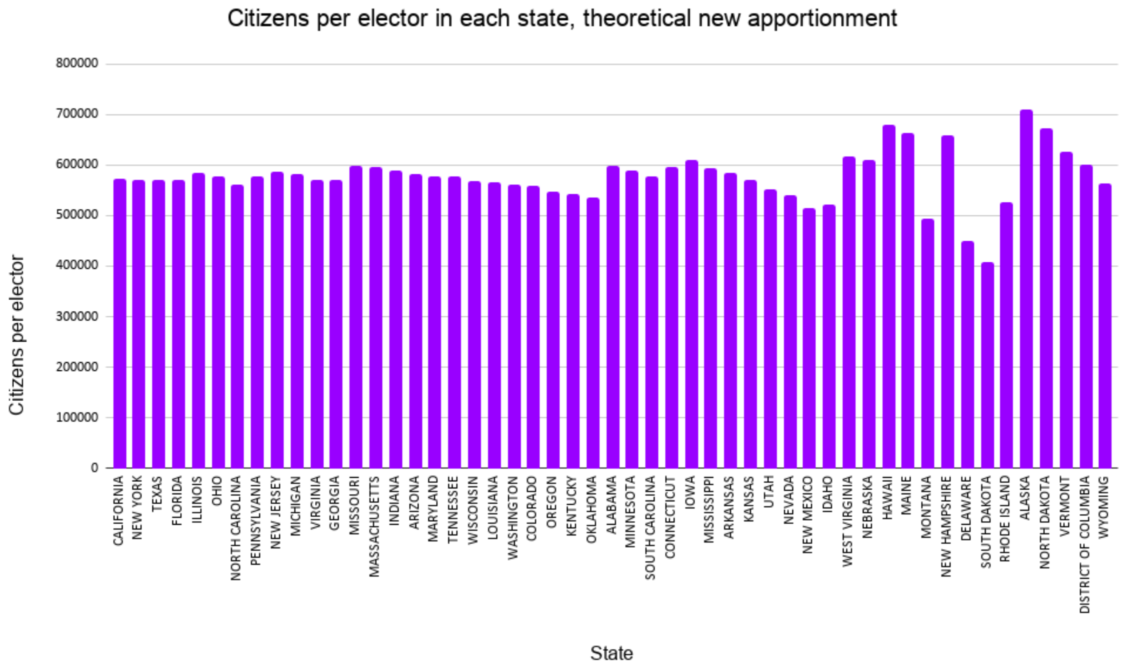

Figure 5.2: Graph representing the number of citizens per elector in each state after theoretically reapportioning electoral votes, using the same state order as Figure 5.1.

Aside from reapportionment, another way to possibly alleviate some complaints about the Electoral College is to allocate each state's electors in a more fractional manner to the candidates. Namely, assign a portion of each state's electoral votes proportional to the number of people who voted for each candidate in that state. More clearly, suppose a state S has 8 electoral votes. Assume that under the current system, this state would assign all 8 votes to whichever candidate wins the state's plurality vote. Call the winning candidate W and the losing candidate L. In this example, suppose W won 62.5% of the overall votes and L won 37.5% of the votes. If the electoral votes were allocated to the candidates fairly, then logically W should get 62.5% of the electoral votes and L should get 37.5% of the electoral votes. So, instead of W getting all 8 electoral votes, W would get 5 and L would get 3. This system helps solve a problem that many people see with the electoral college, that a winner-takes-all approach creates battleground states while discouraging voters in states

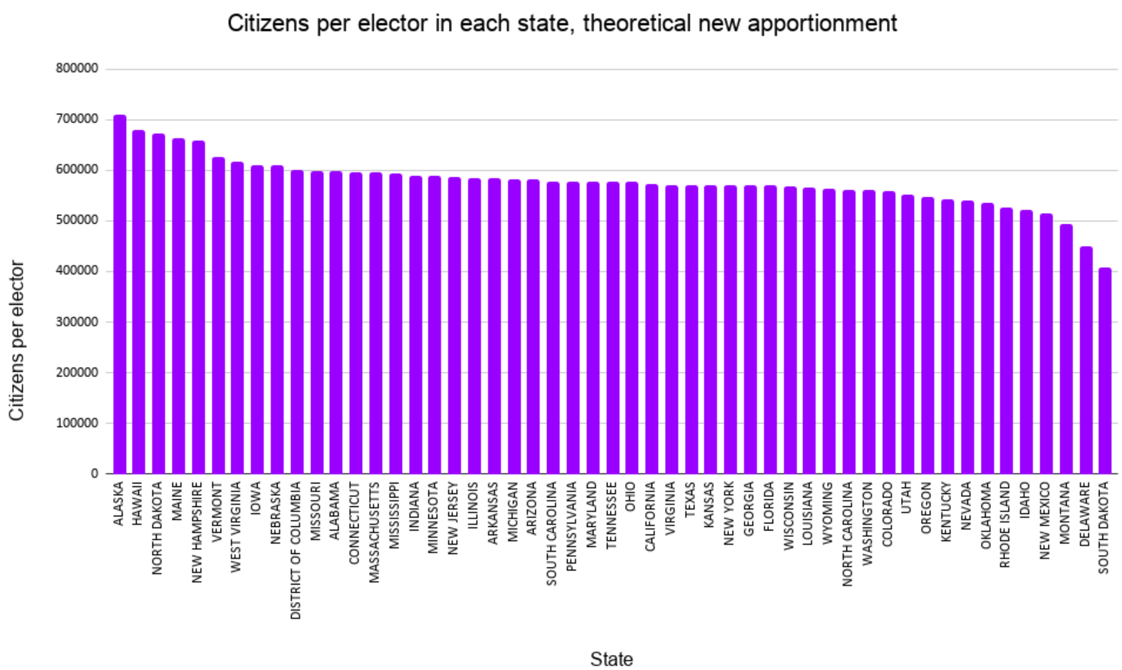

Figure 5.3: Graph representing the number of citizens per elector in each state after theoretically reapportioning electoral votes, re-sorted highest to lowest.

that consistently vote for one party over the other.

In terms of the 2020 election, the votes were split 306 for the Democratic nominee, Joe Biden, and 232 for the Republican nominee, Donald Trump. Both Maine and Nebraska split their votes according to their allocation rules, but every other state and the District of Columbia followed the winner-takes-all system. The Electoral College results can be seen in Figure 5.4, with the states in alphabetical order.

Now, imagine that instead of each state giving all of their Electoral College votes to one candidate (and instead of Maine and Nebraska following their rules), each state split these votes according to the percentage each candidate won their state. To do this calculation, we multiply the percentage of votes each candidate won in a state by the number of that state's electoral votes, and round to the closest integer. Again, because of rounding, some

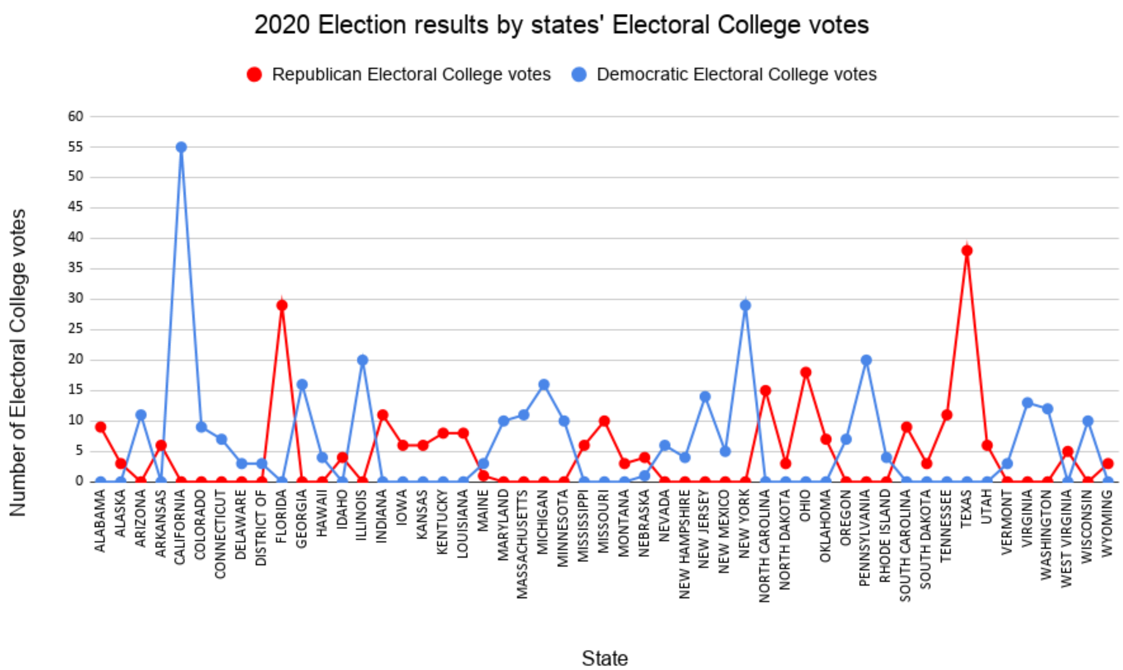

Figure 5.4: 2020 Election results by states' Electoral College votes.

states end up with fewer Electoral College votes given out to candidates than the number of electoral votes that state was assigned by the census. In this case, the candidate with a higher decimal in their win percentage receives the extra vote. Using this method, we can calculate the theoretical number of votes both Joe Biden and Donald Trump would have received in 2020: 277 and 261, respectively. The theoretical vote breakdown by state can be seen in Figure 5.5.

While Joe Biden would have still won in this theoretical election, there is no way to predict for sure what the outcome would have been if this allocation system had been established prior to the election. The winner-takes-all system means that states who vote close to 50% Democrat and 50% Republican become battleground states, as candidates try to ensure they receive all of those votes. Changing the vote-allocating system maybe change where and how candidates campaign, as it relies slightly more on popularity in bigger states, in order for a candidate to win a lot of votes from those states. This attention to big states is also a reason people do not want to switch to a nation-wide plurality system for president; people are afraid that candidates will campaign in dense locations, in order to use their time effectively and win as many votes as possible, thus ignoring smaller states.

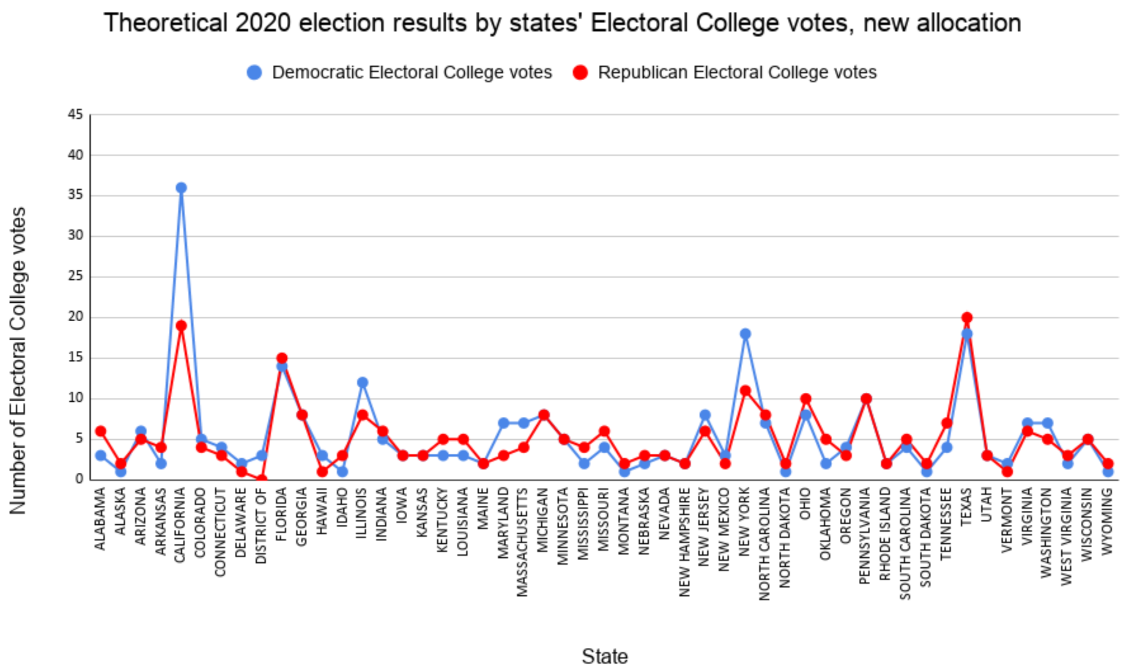

Figure 5.5: Theoretical 2020 election results by states' Electoral College votes using the representative allocation system.

We can also combine these two possible changes to the Electoral College, reapportioning Electoral College votes and proportionally dividing electoral votes in each state. We will use the same apportionment as is represented in Figure 5.2, but will more fairly divvy up the electoral votes in each state, represented in Figure 5.5. We follow the same type of calculations for each step:

- Reapportionment: For each state, the number of electoral votes is equal to the following, rounded to the nearest integer:

$$(State\ Population/National\ Population) * (538)$$

- Reallocation: For each state, the number of electoral votes each candidate receives is equal to the following, rounded to the nearest integer:

$$(Candidate\ Votes/Total\ State\ Votes) * (State's\ Electoral\ College\ Votes)$$

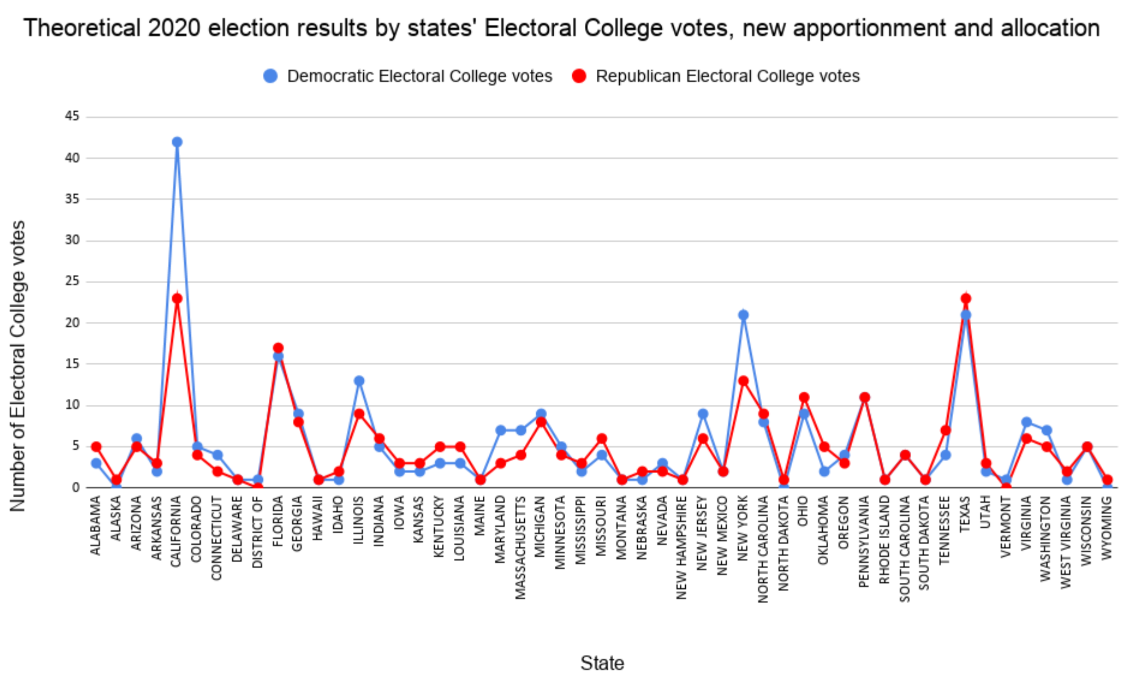

Figure 5.6: Theoretical 2020 election results by states' Electoral College votes using the established reapportionment and representative allocation systems.

Doing these calculations for each state results in the data seen in Figure 5.6 above. If this method of both reapportioning Electoral College votes across the country and more fairly allocating electoral votes within each state had been put in place for the 2020 election, the results would have been 281 votes for Joe Biden and 257 votes for Donald Trump. Again, Joe Biden would have still theoretically won under these conditions, but there is no way to say this for certain, as different characteristics of voting encourage or discourage citizens from voting, across the country.

This combination of changes to the Electoral College helps alleviate some of the concerns and criticisms many people have with the current system. They help equal out citizen representation through the Electoral College system and reduce the impact of swing or battleground states. While implementing ideas like a federal popular vote or ranked-choice voting across the country may take an incredible amount of campaigning and years of fighting, slightly more manageable changes like the aforementioned Electoral College changes could be helpful stepping stones. That is not to say that putting these Electoral College changes into policy would be simple or easy, but they could help citizens better understand how they affect elections in this country and to what extent their voice is heard compared to people in other states.

It is clear that the current systems do not represent citizens as equally across states as they could or accurately portray the opinions of citizens within states, so any positive change would be beneficial to the country as a whole. The state of Maine chose to take steps towards this representation equity through their implementation of Hare's method as well as their division of Electoral College votes in presidential elections, and the rest of the country should consider following Maine's lead. Voting and fair representation were incredibly important to the early beginnings of our country's formal government, but over time the systems established in the 1700s have proven to lose their equity. So, it is time to rethink how we maintain the democracy that many Americans cherish. Furthermore, this mathematical analysis of existing voting structures does not begin to comprehend the countless other systems that prevent people of this country from fairly voicing their opinions in our democratic processes. Obstacles including voter suppression and strict voter eligibility laws undermine any sense of fairness that is assumed to exist in existing structures. In order to truly have democratic processes, these issues, along with the representation problems analyzed in this paper, can only be solved by large changes to all existing systems.

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