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Understanding Volatility: An Analysis of the Stock Market Return-Variance Correlation

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Table of Contents

Abstract

This paper attempts to explain the negative correlation between stock market returns in the United States (measured by the risk premium of the S&P 500 Index) and the respective volatility of these returns. The academic research regarding two primary schools of thought on this issue, the volatility feedback effect and the leverage effect, is furthered as potential explanations for this phenomenon. A tertiary explanation relating to investor behavior is also explored as a viable cause. In order to empirically study this relationship, I examine the risk premium quintiles and the corresponding CBOE Volatility Index levels for the time-series dating from January 2, 1990 to June 29, 2018. This approach differentiates from the stochastic and autoregressive volatility models that attempt to explain this relationship, by examining the distribution of the negative return-volatility correlation. Results from this analysis serve to better understand the properties of volatility and their impact on investments.

INTRODUCTION

Defining Volatility

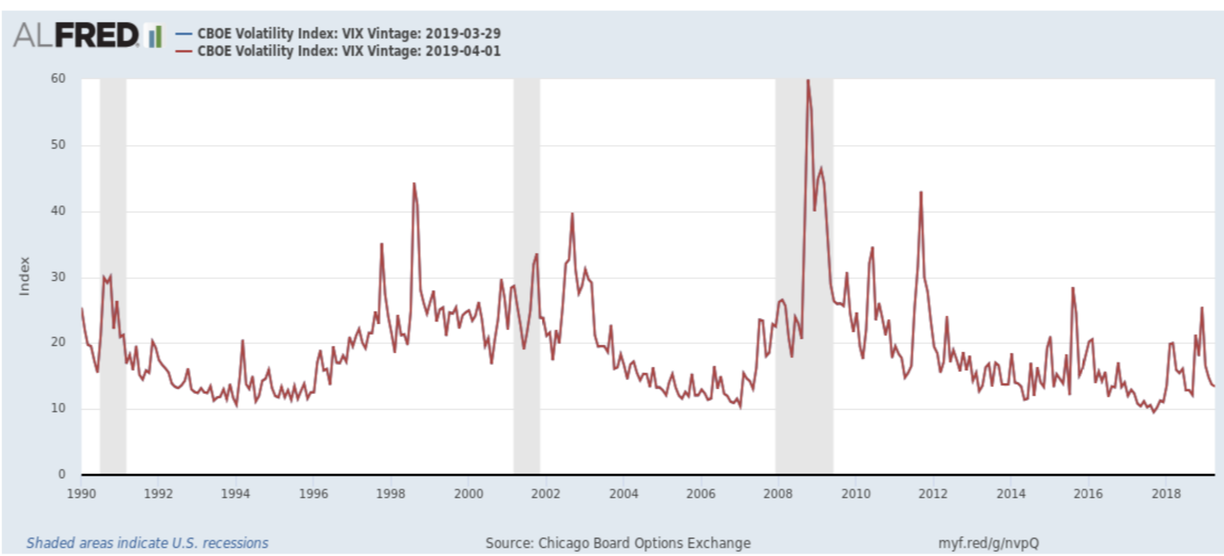
Volatility has long had its place in the financial markets. Defined in a general sense as the standard deviation of asset returns (usually annualized), it provides an important metric of dispersion that investors can use to guide their decisions. An essential distinction, however, is that volatility is not equivalent to risk. Premier investor Warren Buffet spoke of this from a more macro standpoint, stating that equities will always be a more volatile asset class than cash-equivalents, but that does not mean they are *riskier* (Buffet, 2014). He was referring to the common practice of aligning risk with volatility, despite risk reflecting downside fluctuations and volatility representing fluctuations in both directions. Volatility, hence, has no information regarding the shape of equity returns (Poon, 2005).

While volatility as a concept appears rather simple in nature, it in itself has several variations that should be distinguished. It can vary across different asset classes as Buffet noted, different individual securities, as well as across different lengths of time. It also can be referred to as realized volatility, having occurred across a historical time horizon, or as implied volatility, an estimate for the volatility at a future point in time. For this paper, the volatility in question will be measured by the Chicago Board Options Exchange (CBOE) Volatility Index, referred to as the VIX.

VIX

As the primary metric in study, an overview of the VIX is warranted. The CBOE Volatility Index measures the implied volatility of the S&P 500 Index with an average expiration of 30 days. It uses near-term (weekly) and next-term (monthly) call and put options with 23 to 37 days to expiration, and calculates a weighted 30-day variance. Upon taking the square root of this variance and multiplying by 100, the result is the VIX level noted on the index (CBOE, n.d.) Quoted in percentage points, it represents the expected standard deviation of the S&P 500 Index over the next year -- at a 68% confidence level or one standard deviation. It is of note that the VIX has been observed to overestimate the volatility of the stock market when compared to the realized volatility for the respective time horizon, a phenomenon referred to as the volatility or variance premium. This premium, as measured by implied minus realized volatility, averages approximately 3% (Eraker, 2007). The VIX will be used as a proxy for conditional or implied volatility throughout this paper. A graph of the VIX is found below, spanning from 1990 until present day.

Figure 1 (FRED, 2019).



Introduction

This forward-looking, option-derived metric possesses an incredibly interesting characteristic that has long been a subject of interest in academic research -- it is highly negatively correlated to the stock market's returns. As illustrated by the figure above, the VIX tends to spike significantly during recessions and fall during strong bull markets. Two schools of thought dominate the aforementioned relationship. The first, the volatility feedback effect, primarily stemmed from French, Schwert, and Stambaugh in 1987. Their premise was that as volatility increases, the required rate of return for the security increases, thus further discounting the cash flows and lowering the stock price. (French, Schwert, Stambaugh 1987). Additional analysis between the weighted average cost of capital for a large-cap equity sample and accompanying VIX levels indicate support for this claim, as well as academic research on price shocks and their volatility impact. The second school of thought regarding this relationship is referred to as the leverage effect. First introduced by Fischer Black in 1976, the leverage effect was theorized to occur when there was a decline in stock price, which would then increase the relative level of leverage at the firm level -- leading to further volatility (Black, 1976**).** This effect faced both support and opposition, however, as it was found to vary upon the time horizon in study, and failed to incorporate all-equity financed companies. Lastly, a third explanation for the return-volatility association appeals to the behavioral aspects of investing, citing the investor's tendency for loss aversion as a cause for the asymmetry.

While these theories have been extended to international equities, applied to high-frequency data, and modeled in several environments, the distribution of the return-volatility association lacks significant exploration. My analysis of using a quintile and decile approach sheds light on the asymmetry of this already asymmetric relationship, while providing validation to its extreme nature. The concentration of correlations in extreme quintiles in this time series provide support for primarily the volatility feedback effect, which favors large short-term changes in asset prices and volatility. These findings can then be applied to the ever-increasing securitized volatility market, with implications in hedging, arbitrage, and predictive strategies.

VOLATILITY FEEDBACK EFFECT

The premise of volatility having a negative relationship with stock market returns due to some kind of "feedback" has been the focal point of several prominent financial scholars. French, Schwert and Stambaugh first explore this relationship in the context of expected returns and volatility, finding a positive relationship. They claim that this then induces a negative relationship between realized returns and "unpredicted" (i.e. implied) volatility, in which an increase in volatility feeds back into each firm by requiring a higher discount rate. Cash flows remaining unchanged, the increased discount rate would lower the net present value of the firm -- and thus the current stock price in accordance with a discounted cash flow methodology (French, Schwert, Stambaugh 1987).

Additionally pivotal to their findings was their methodology for returns. Rather than using the stock market returns for their indices in study, the market risk premium was used in its place. Concordant with Fama and French's Three Factor Model, the market risk premium accounts for different interest rate regimes, as well as the expected equity return in excess of the risk-free security rate (Fama, French 1992). The market risk premium (equity premium) is often used to explore the return-volatility relationship, and will be used in this paper's analysis later on.

Unaddressed by French, Schwert, and Stambaugh, however, was the question as to what would cause the initial changes in volatility. Volatility can be impacted on the macro level by political events or global economic data, but can also be effected at the firm level by the release of company-specific information. The latter was the focus of Campbell and Hentschel, who provided the example of a dividend announcement inducing a volatility feedback effect. Logically, a "negative" announcement about dividends would increase future volatility because of the concept that volatility is persistent. The required rate of return would thus be higher, decreasing the stock price and further amplifying the volatility. Likewise, a "positive" announcement increases volatility, but the volatility of the stock's returns is dampened due to a higher discount rate. Caught in the midpoint between these two extremes is no news (lack of volatility), which was found to increase the stock market return (Campbell, Hentschel 1991).

In order to provide evidence to the existence of this volatility feedback effect, a methodology of modeling the stock returns and volatility must be used. A primary technique first used in the context of feedback by Campbell and Hentschel was a generalized autoregressive conditional heteroscedasticity (GARCH) process. The crux of this type of model is that it assumes error variance is not uniform but rather heteroskedastic. Additionally, the error variance is autoregressive; thus it depends on the previous time period's variance. The GARCH process is frequently used to model the volatility of stock market returns, for it exhibits several heteroskedastic and autoregressive properties (Ruppert, 2011).

International Evidence

The use of these models has allowed significant evidence of this effect to compile**.** On the international front, corroboration of this feedback was found in several studies. GARCH models for the volatility of indices in Hong Kong and Taiwan markets (Hong Kong Hang Seng Index and Taiwan Stock Exchange Weighted Index) provide support for an asymmetric volatility-return relationship (Yeh, Lee 2000). Using a sample period from May 22, 1992 to August 27, 1996, it was found that the impact of a negative unexpected shock (i.e. bad news) on future volatility was significantly larger than the impact of a positive unexpected shock (good news) -- indicating an asymmetric relationship attributable to volatility feedback. Additional international evidence can be found in emerging markets, specifically in India. Equities in emerging markets are essential to consider because of their different properties from typical developed countries and economies. These differences include a low correlation with developed markets, more predictable returns, higher average returns, and higher volatility (Bekaert and Wu, 2000). These distinguishable properties thus pose a potential resistance to the existence of this particular effect. Using closing stock price data for the primary index in India (the BSE 500 Index) from July 26, 2000 to January 20, 2009, asymmetric volatility was found to indeed still be prevalent in India, confident at the 1% level when modeled through a nonlinear asymmetric model (GARCH) (Goudarzi, Ramanarayanan 2011). The same occurrence of negative shocks inducing more volatility than a positive shock attributed to the existence of a feedback effect, despite different equity market characteristics.

Extreme Cases

In order to further examine the validity of the volatility feedback effect, it is important to consider the extremes; i.e. the extreme downside movements in stock market prices and volatility levels. Wagner and Aboura (2010) use sample data from January 2, 2000 to September 30, 2008, comprised of S&P 500 Index and the aforementioned CBOE Volatility Index closing levels --

which includes extreme observations from the dotcom bubble, 9/11 terrorist attacks, as well as the 2007-2008 financial crisis. Using a GARCH model for these volatility-shock periods, they found significantly stronger evidence of volatility asymmetry in which VIX spikes corresponded with large market price drawdowns; more so than normal volatility levels (Wagner, Aboura 2010). The exaggerated asymmetry during periods of financial or economic shock provides evidence that a specific effect, namely "extreme volatility feedback", is causing the conditional volatility to spike in response to a large drop in security prices.

Many of the aforementioned studies examined price data of the S&P 500 Index using daily prices across several years, occurring decades in the past. The environment and nature of the United States equity market and equity markets worldwide, however, have changed significantly. As of 2017, quantitative and passive investing accounted for about 60% of trades in the United States. Of the segment comprising quantitative investing, high frequency trading accounts for 52% of the 60% -- with technology and speeds nonexistent a decade earlier (Cheng 2017). Understanding the volatility-return relationship requires an analysis of this high-frequency data, in order to see its implications for the validity of effects like the volatility feedback effect. Such analysis could also assist in explaining the causality and timing of the relationship: Under the feedback effect does higher volatility lead to lower returns, or do lower returns lead to higher volatility?

In their study of the leverage and volatility effects, Bollerslev, Litvinova, and Tauchen address these essential issues. In order to obtain high frequency data, they used tick-by-tick S&P 500 futures data from January 4, 1998 to March 9, 1999 because of the trading frequency and low trading costs in this market. They discovered that asymmetric volatility did in fact appear, in which large declines in five-minute S&P 500 futures data were accompanied by a spike in market volatility (Bollerslev, Litvinova, Tauchen (2006). Their findings came to question, however, the logistics and practicality of the volatility feedback effect. How could companies be revising their required rate of return (which would thus lower the stock price) on a tick-by-tick basis? Or does the market instantly price in volatility changes as immediate revisions to costs of capital? Perhaps the explanation is another effect, or combination of effects, entirely.

Empirical Test

A potential way to determine the validity of the volatility feedback effect is to examine the relationship between the aggregate weighted average cost of capital of a group of firms (or an index), and the VIX. According to the volatility feedback effect, these two should be positively correlated -- as volatility increases, the cost of capital rises as a result. Using a random sample of 10 companies from the Dow Jones Industrial Index and standardized values for the VIX and WACC on a three month basis from March 31, 2000 to December 31, 2018, the resulting correlation was 0.0789. The regression results are listed below.

Table 1.

Source	SS	df	MS		
				Number of obs =	76
				F(1, 74) =	0.46
Model	.222622548	1	.222622548	Prob > F =	0.4982
Residual	35.5566701	74	.480495542	R-squared =	0.0062
				Adj R-squared =	-0.0072
Total	35.7792926	75	.477057235	Root MSE =	.69318

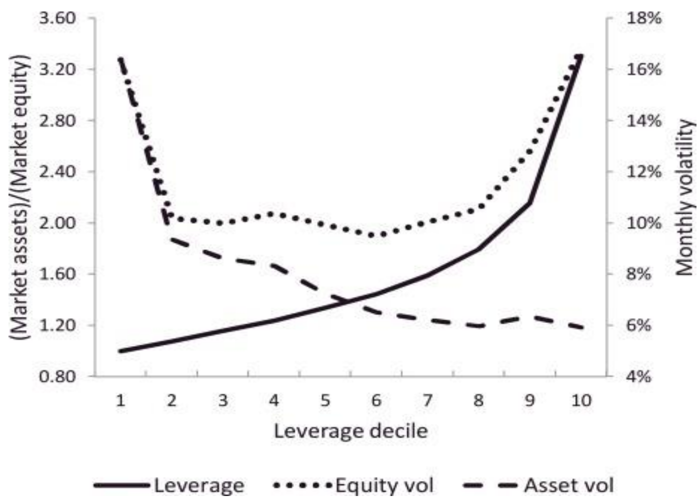
averagedow10	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
vix	.0541225	.079513	0.68	0.498	-.1043105	.2125555
_cons	2.10e-09	.079513	0.00	1.000	-.158433	.158433

While not significant at the 95% level, the positive association between the standardized values of the VIX and the average of 10 Dow Jones Industrial Index companies' weighted average cost of capital provides some insight as to the relevance of the volatility feedback effect. A larger sample of firms, as well as more frequent data for WACC values could yield results in further support of this effect, and would be a valuable area for future research.

LEVERAGE EFFECT

A competing school of thought for the explanation behind the volatility-return relationship in equities is referred to as the leverage effect. First defined by Fischer Black in his paper *Studies of Stock Prices Volatility Changes*, it cites leverage as the primary reason for stock return volatility. The reasoning is as follows: As a firm experiences a decline in price (and thus in its market value of equity), its relative amount of debt compared to equity increases. With more leverage, the firm is likely to be more volatile, consistent with the well-documented research between debt/equity ratios and volatility (Black, 1976). The following figure (Figure 2) illustrates this relationship, sorting firms by leverage decile and plotting against monthly volatility of equity (Chorro et al., 2018). It is important to note that while the extreme deciles of leverage do experience higher equity volatility, it is difficult to ascertain a relationship between debt/equity and volatility for the middle deciles.

Figure 2.



While Black's theory appeals logically, the explanatory power of the leverage effect was found to be rather limited -- if present at all. The study by Chorro et al. examined the returns of the S&P 500 Index over a 25-year time horizon, using a GARCH model to disentangle the leverage effect's contribution to the asymmetrical volatility-return relationship. In line with the plot illustrated above, leverage effects were found to be a statistically relevant characteristic in only 30% of firms in the S&P 500. When viewed in aggregate as compared to at a firm level, the explanatory contribution of the leverage effect decreased even more so.

The implications of financial leverage impacting returns and volatility in an asymmetric manner also rely on the time horizon in question. Pan and Liu utilize several extensions of GARCH models to examine the short-term and long-term impacts of leverage on volatility, using historical daily closing levels of the S&P 500 Index from January 2, 1991 to December 31, 2015. They concluded that the leverage effect existed in some manner for the short-term, but lacked statistical significance in the long-term. Additionally, negative shocks to the S&P 500 index had a much larger effect on volatility via the leverage effect than shocks in the positive direction

(Pan, Liu 2017). Much like the analysis for the volatility feedback effect, however, the impact of the effect could be different when examined over an even shorter horizon: high frequency. Due to the noise associated with high frequency data including bid-ask variations, liquidity constraints, and trading mechanics, disentangling these biases has proved difficult for researchers (Ait-Sahali, Fan, Li 2011). Using a new class of models called multivariate high-frequency-based volatility models (abbreviated HEAVY), however, Noureldin, Shephard, and Sheppard were able to find evidence for the leverage effect in high frequency data while accounting for this noise (Noureldin et al., 2011). The authors do not separate this effect from the volatility feedback effect, however, and thus one must approach their estimations with caution.

The general conclusion amongst scholars regarding the leverage effect is that it has some explanatory merit for the asymmetric volatility puzzle, but not as much as the volatility feedback effect or other underlying characteristics. Some, however, make the claim that this relationship is not due to leverage at all. If the leverage effect explains the negative relationship between volatility and stock market returns, then companies with no financial leverage should exhibit different return-volatility relationships than their levered counterparts. This was the premise of Hasanhodzic and Lo in their study of all-equity financed firms from 1972 to December 2008. They found an equally strong (if not stronger) negative relationship between stock returns and their respective volatility for these all-equity financed firms when compared to the study's universe of levered companies (Hasanhodzic and Lo, 2011). They thus directly dispute Black's claim that leverage plays an explanatory role in the return-volatility relationship. The leverage effect therefore appeals logically but appears to contribute little to nothing towards the phenomenon of asymmetric volatility.

INVESTOR BEHAVIOR

While mathematical models for volatility have adduced explanatory power to the volatility feedback effect, and to some extent the leverage effect, much about the relationship remains unexplained. This has led some to claim that the underlying cause of asymmetric volatility may be simply due to the nature of investor behavior.

The CBOE Volatility Index (VIX), despite being a measure of aggregate implied volatility, has become known as the “Fear Gauge”. It has in some cases become synonymous with risk, and yet does not fit risk’s own definition. The reasoning behind this is likely due to investor behavior. In an examination of past volatility’s effect on investor’s future judgements, Du and Budescu displayed historical data for 80 stocks to participants in the study, and asked them to forecast the prices of these stocks in the future. They found a negative relationship between volatility and forecasts, meaning that for stocks with higher volatility, individuals underestimated the actual price of the stock in the future (Du and Budescu, 2007). These implications could feasibly be carried out in the stock market and pose an explanation to the volatility asymmetry. As volatility increases, an investor’s expected return for a stock decreases or the stock is avoided altogether (sold or lack of demand), thus lowering the price of the stock.

This concept of loss aversion is highly related to volatility. Loss aversion refers to the tendency to have a higher sensitivity to losses as opposed to gains (Barberis and Huang, 2001). While volatility merely represents the standard deviation of stock returns, investors in accordance to loss aversion would be more wary of larger variations because they represent the possibility of a larger loss. Since the relationship in study is between realized returns and *conditional* (implied) volatility, the negative association becomes even stronger. As stock prices decrease, investors are highly sensitive to the loss and thus forecast future volatility to be even higher. Reversing the causality, as volatility increases, the potential for a larger loss rises and stock prices drop to reflect the loss aversion. While this explanation is linked to the previous two effects tied to asymmetric volatility, it differs in the sense that the aversion is present at the investor level, as compared to at the firm level.

Asymmetric volatility at the market level is the crux behind behavioral explanations for its causation. Hibbert et al. cite "affect heuristics" (rules of thumb or practices based on instinct/intuition) as a factor in the negative return-volatility relationship. The aforementioned loss aversion, for instance, is the primary affect heuristic in the focus of their analysis. Investors trading on behalf of this heuristic influences the demand for equities, as they are more likely to purchase stocks during non-volatile markets and sell them during volatile ones (Hibbert et al, 2008). This further carries over to the derivatives market (and subsequently the VIX), as traders bid up prices for put options and ultimately implied volatility through extrapolation bias. They argue that due to the fact that this relationship was occurring contemporaneously and with such strong correlations in the tails, that this must be behavior driven and not the result a lagged effect such as leverage or volatility feedback.

ANALYSIS

In order to truly understand the underlying cause of the negative relationship between volatility and stock market returns, one must examine the association itself. Concordant with French, Schwert, and Stambaugh in their analysis of volatility asymmetry, this paper uses the market risk premium as a proxy for returns, because it takes into account several interest rate regimes throughout the sample range of January 2, 1990 to June 29, 2018. This paper also uses the CBOE Volatility Index (VIX) in reference to volatility, examining it in the context of implied or conditional volatility. In agreement with Hibbert et al, the VIX provides a suitable measurement for conditional volatility because of its inclusion of options extending across a variety of strike prices and its constantly updating implied volatilities.

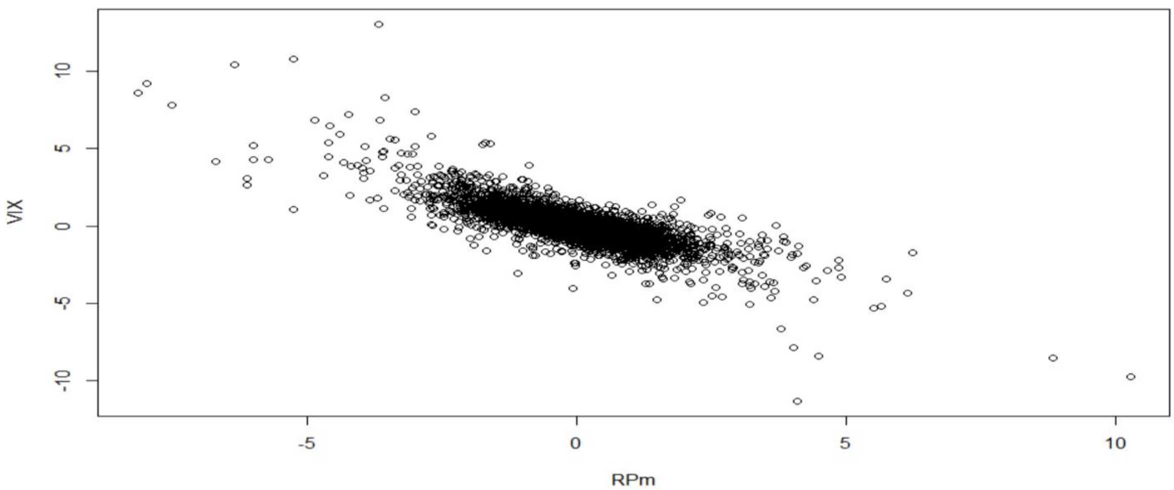
When the values of the change in the VIX and the risk premium are standardized and plotted, the negative relationship becomes quite clear (Figure 3). Spanning over the aforementioned time horizon, the correlation between the two variables is a staggering -0.789. Examining the direction of the S&P 500 index returns on a daily basis compared to the VIX yields similar results. From January 2, 1990 to March 3, 2019, the S&P 500 Index and the VIX moved in opposite directions 77.79% of the time. The following table describes these movements.

Table 2.

S&P 500 Up	VIX Index Down	Percent Opposite
3922	3851	78.5%
S&P 500 Down	VIX Index Up	Percent Opposite
3423	3458	77.0%

Total Movements Opposite (VIX and SPX):	5717
Total Movements in Tandem (VIX and SPX):	1632
Percentage Opposite:	77.79%

Figure 3.

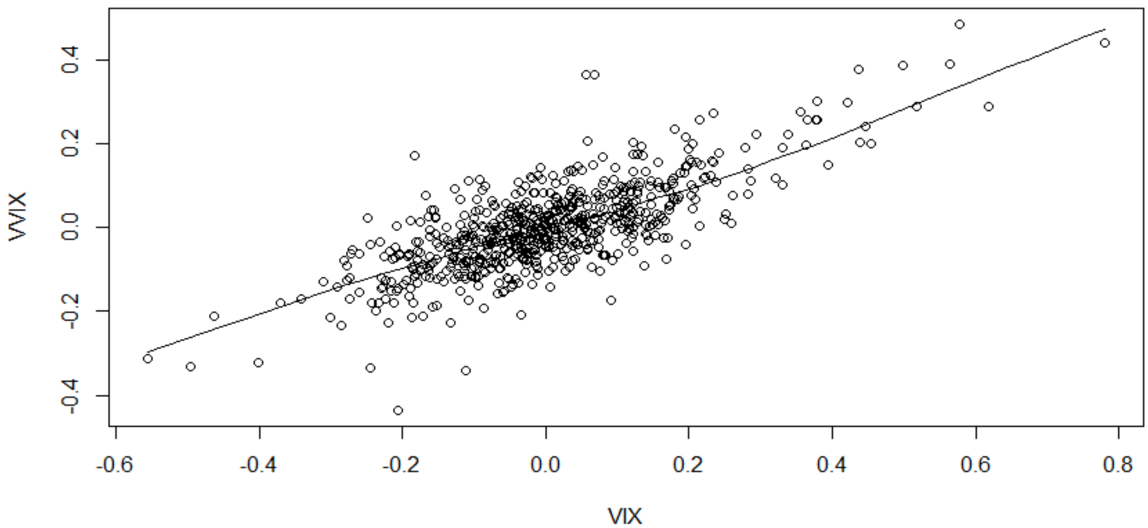


With such consistent movement in the opposite direction, one would expect a significantly negative beta to the market. This is indeed the case, with a beta over the same span of -4.288, using daily closing SPX and VIX data. This beta also changes depending on the type of market. For example, for the sample period from January 1990 to February 2010, the beta of the VIX versus the S&P 500 Index was -2.2 in bull markets, - 3.9 in bear markets, and -3.5 overall (Stanton, 2011).

The distribution of this relationship provides further insights. While much is documented about the negative relationship between returns and conditional volatility, little is discussed about the asymmetry of this relationship itself, rather than the relationship of solely volatility. There are a few scholarly works that do address the fact that the relationship between returns and volatility is nonlinear -- and thus becomes more significant during extreme shocks or periods (Hibbert et al, 2008). Their focus is primarily on the downside case, in which large decreases in the stock market price cause spikes in volatility through effects such as feedback or leverage. This paper's research expands beyond this view to incorporate the whole relationship -- why does volatility decrease (increase) by an almost equal amount during a positive return shock versus a negative one in absolute terms?

As mentioned previously, an essential part of the volatility-return relationship is that it is nonlinear. If it was linear, then the impact of a negative return of one percent on volatility would have a proportionate impact as a five percent negative return, and this is not the case. Following the methodology of the Federal Reserve, a way to test for this relationship is through the volatility of volatility; i.e. the derivative (Park, 2015). The CBOE VVIX (volatility of volatility) Index derives its value using the same methodology as the VIX, except for the fact that it uses VIX options instead of options from the S&P 500 Index (CBOE, n.d.). Plotted against the VIX, a positive relationship between the VIX and the VVIX would indicate that stock market volatility is a nonlinear construct. The figure below illustrates the relationship from January 2, 1990 to March 8, 2019 of their standardized daily returns, with an R-squared value of .56 and a test statistic statistically significant beyond the 99th percentile.

Figure 4.



With nonlinearity established in the relationship between stock returns and conditional volatility, examining the risk premium quintiles provides insight as to which periods or days are contributing to this overall negative correlation. For this analysis, each risk premium and volatility value on a daily basis were treated as a pair in the time series. Each quintile represents the group of risk premium-volatility pairs that fell into the particular percentile ranking. As illustrated by the plots in Appendix 1-5, the second, third, and fourth quintile display little to no relationship between the change in volatility and the change in the risk premium. The first quintile (lowest returns) and fifth quintile (highest returns), however, have a very strong negative association. The table below summarizes the results for the following regression per quintile, controlling for the Fama French Factors of SMB (market capitalization) and HML (book to market ratios).

Where:

$$y_t = \alpha + \beta vix_t + \beta smb_t + \beta hml_t + \varepsilon_t$$

y_t = The standardized market risk premium at time t

β = The parameter of interest to estimate the relationship between different factors and the market risk premium

vix_t = The standardized value of the CBOE Volatility Index (VIX) at time t.

smb_t = The standardized value of the aggregate SMB factor returns for the S&P 500 Index at time t

hml_t = The standardized value of the aggregate HML factor returns for the S&P 500 Index at time t

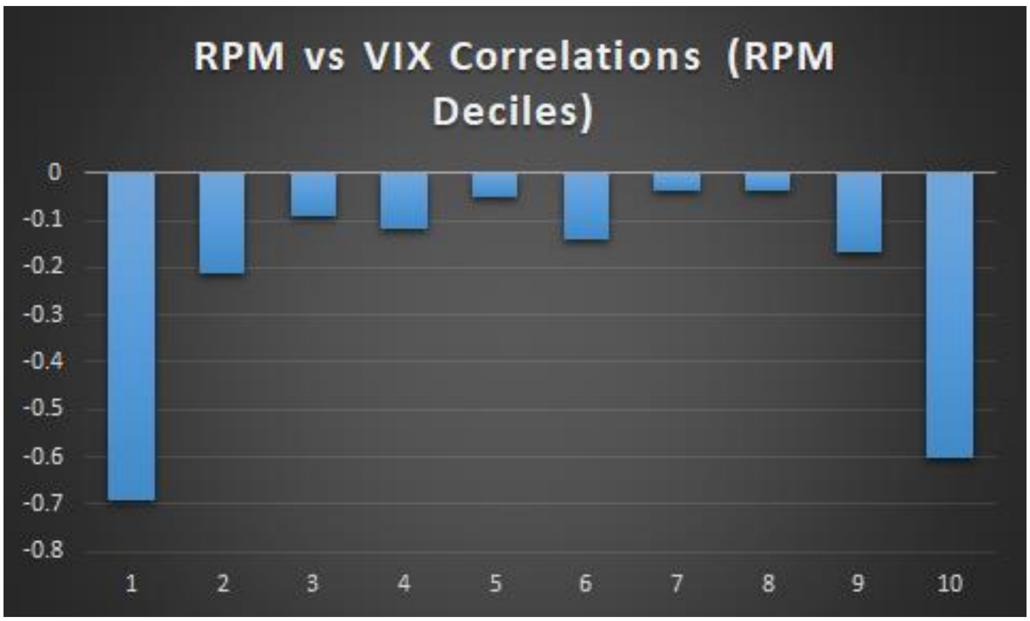
Table 3:

Quintile	Variable	Coefficient	Standard Error	T Statistic	P Value	R-squared
Quintile 1	vix	-0.495	0.0129	-38.26***	0.000	0.514
	smb	0.00689	0.0133	0.52	0.605	
	hml	0.0164	0.01119	1.38	0.167	
Quintile 2	vix	-0.0629	0.00795	-7.92***	0.000	0.044
	smb	0.00124	0.00398	0.31	0.755	
	hml	-0.00919	0.00477	-1.93	0.054	
Quintile 3	vix	-0.0304	0.00546	-5.57***	0.000	0.0216
	smb	0.00315	0.00309	1.02	0.309	
	hml	0.0301	0.00369	0.1	0.923	
Quintile 4	vix	-0.0424	0.00751	-5.65***	0.000	0.0243
	smb	0.00153	0.00373	0.41	0.681	
	hml	-0.00692	0.0047	-1.47	-1.470	
Quintile 5	vix	-0.495	0.0167	-29.62***	0.000	0.396
	smb	-0.0545	0.0133	-4.09***	0.000	
	hml	0.0047	0.0124	0.38	0.705	

*** indicates statistically significant at the 99% confidence level

Similar results appear when analyzing the correlations of the market risk premium of the S&P 500 Index versus the VIX from a decile perspective (Figure 5). The first and tenth deciles both exhibit the strong negative correlation described in academic literature, but the second through ninth decile exhibit very weak negative correlations with hardly any relationship at all. It is the tenth decile that is of particular interest. Why does the VIX react so strongly negative to large increases in the market risk premium or return?

Figure 5.



This finding contrasts several of the assumptions made earlier that helped support the effects explaining asymmetric volatility. For instance, Campbell and Hentschel in their work illustrating the volatility feedback effect relied upon the concept that volatility is persistent. Thus, forecasted volatility is conditional on today's volatility levels, and can often exhibit momentum-like properties. This appears to be the case for the first quintile or decile, in which large spikes in present-day volatility (via large negative stock returns) can reverberate into larger implied volatilities in the future. However, a large spike in stock return volatility resulting from a large increase in the market risk premium actually *decreases* future volatility, because investors now believe there to be less volatility in future. Hwang et al in their study of volatility found a lack of support for the persistency of true volatility -- a key input for the GARCH models used to demonstrate the volatility feedback and leverage effects (Hwang et. al, 2007).

In regards to the leverage effect, the distribution of the relationship is better explained. As the stock market decreases significantly (1st quintile), the relative level of equity compared to debt decreases, causing the volatility to increase. Conversely, as the stock market experiences a positive return shock (5th quintile), the leverage ratio decreases due to a larger proportion of equity, resulting in lower volatility.

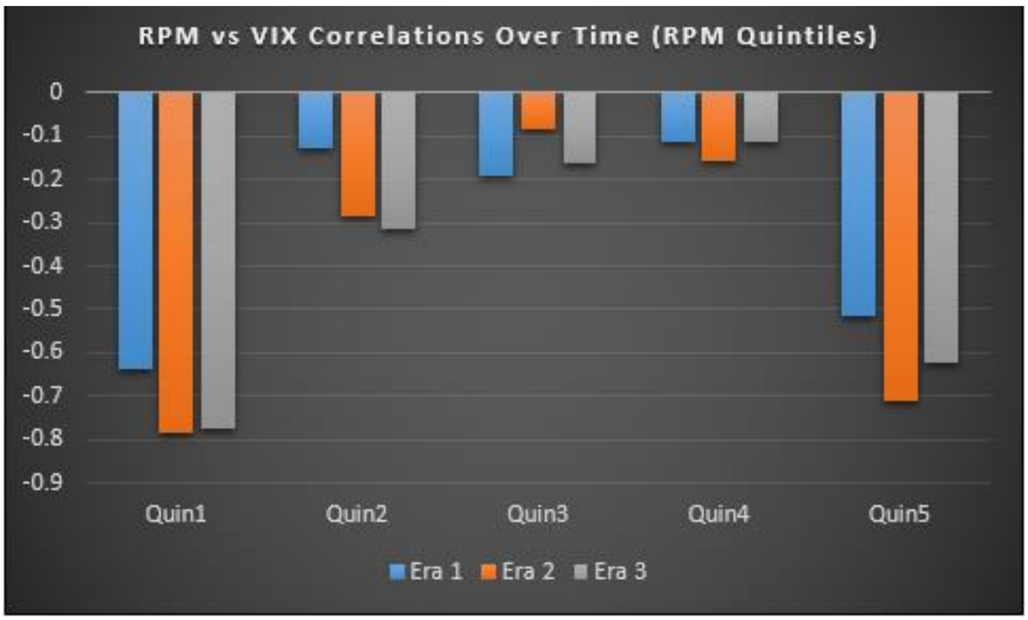
As the most open-to-interpretation explanation for the return-volatility relationship, the investor behavior reasoning may have best chance of capturing the entirety of the relationship distribution. Loss aversion does not only affect decisions relating to losses, but also spills over to gains. On the negative return side, the investor sensitivity to losses is indicated by the extreme volatility that accompanies the extreme negative return shocks in the first quintile and decile. The fifth quintile (positive return shocks), however, can potentially be thought of as the sensitivity to the lack of losses -- a higher sensitivity than an investor's sensitivity to gains. A large positive return in the stock market signals that the risk of losses has declined, leading to a significant decrease in the implied/conditional volatility.

Historical Analysis

Because the sample period spans nearly 30 years of data, it is also important to examine the risk premium – volatility relationship in a historical context, throughout different eras of the stock market. The first era in study was from the beginning of the sample (January 2, 1990) to the end of the dot-com bubble, which ended at its lowest point at the end of September 2002. During this time horizon, the S&P 500 Index returned a compounded annual return of 6.4%, with an average VIX level of 19.86. The second era spanned from the endpoint of the dot-com bubble to the end of the financial crisis, defined as ending in June 2009 by the National Bureau of Economic Research (NBER, 2012). While this period did not contain the same amount of trading days, it encompassed both the bull market before the Financial Crisis as well as the ensuing recession, with an average VIX level of 20.82 and the S&P returning approximately 1.78% annually. The last era marked the beginning of a historically long bull market, starting on June 30, 2009 and spanning to the end of the sample on June 29, 2018. During this period, volatility was rather low at an average VIX level of 17.44, while returns were much higher at 12.05% annually.

Using the same approach as the overall relationship analysis, each era was divided into a time series of risk-premium and VIX z-scores, which were then separated into quintiles. The resulting correlations between the z-scores in each quintile are illustrated below (Figure 6). All quintiles largely follow the same relationship as the entire time series, but the second era notably contains more extreme correlations in the first and fifth quintile. This was largely the result of the Great Recession falling within its sample, in which many extreme values of risk premiums and volatility levels occurred. In contrast, the first era lacked as strong of a negative correlation overall and in the quintiles of focus, alluding to the fact that the relationship has become stronger over time – perhaps due to increased volume and prevalence of high frequency trading. Further, despite the low levels of volatility found in the bull market following the Great Recession, the asymmetric relationship between returns and volatility remained very strong, emphasizing the prevalence of volatility declines during positive return shocks.

Figure 6.



IMPLICATIONS

The defining characteristic of volatility-based instruments is the fact that they are negatively correlated with the stock market and consequently the market risk premium. When compared to other major asset classes or factors as listed by the St. Louis Fed hedge fund factors, volatility stands alone in this regard. The following table illustrates the correlations of prominent factors with the market risk premium over the time period from January 2, 1990 to June 30, 2018.

Table 4.

SMB	HML	LIBOR	USD	Oil	Gold	Credit	Term	VIX
-0.003	-0.067	-0.002	-0.059	0.119	-0.010	0.001	0.006	-0.789

The market of volatility based-instruments has grown substantially over the past decades as investors sought to take advantage of the characteristics of stock market volatility. The Chicago Board Options Exchange offers Volatility Index Futures in order to utilize as an investment or hedging tool. Options on the VIX remain prevalent as well, with millions of contracts bought and sold daily. There are also several exchange-traded notes (ETNs) and exchange-traded funds (ETF) that attempt to track the VIX as a benchmark through volatility futures, or even take leveraged or inverse positions on volatility. For example, the VelocityShares Daily 2x Leveraged VIX Short-Term ETN attempts to track twice the return of the VIX, and has over $800 million in assets under management (Yahoo Finance, 2019). There are many ways to trade volatility -- but it is important to first understand the implications of the return-volatility relationship.

This was not the case for the once-popular inverse volatility exchange traded note XIV, which was created by Credit Suisse and had over $2 billion in assets under management. The large spike in volatility on February 5, 2018 resulted in a decline in XIV of 85% in one day, before being liquidated a few days later after losing most of its value (Franck, 2019). Its downfall sheds light on some of the important aspects discussed in this paper. Volatility has a strong negative correlation to the market return in aggregate, but the strongest relationship occurs within the first and fifth quintiles. Extreme negative return shocks produce even more extreme increases in volatility, while large positive return shocks result in extreme decreases in volatility. Individuals or institutions investing in inverse volatility ETNs or shorting volatility instruments themselves should be wary of the leverage, feedback, and investor behavior effects that drive the asymmetric relationship between stock market returns and conditional volatility. These can lead to margin calls or significant fluctuations in value that one must be prepared to endure.

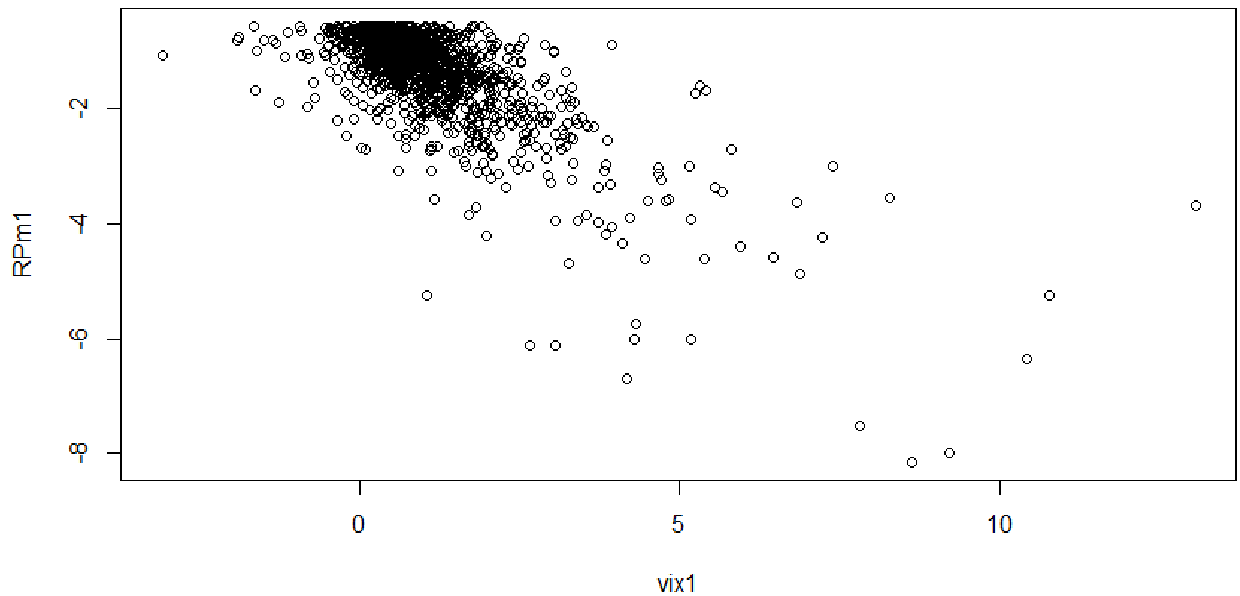
CONCLUSION

The stock market and volatility, as measured by the VIX, share a very strong negative relationship. Two competing schools of research attempt to explain this phenomenon, primarily through GARCH models of volatility. The volatility feedback effect finds reason in the process of a negative return shock increasing volatility and raising the required return -- thus decreasing the price and amplifying the level of volatility. Support has been found in emerging markets in India, as well as in high-frequency tick-by-tick data, but lacks full explanation for the instantaneousness of cost of capital revisions. A second school of thought, the leverage effect, states that as asset prices decline, the debt to equity ratio increases, resulting in increased volatility. This effect has significantly less explanatory power, and is called into question by firms that are all-equity financed and still display the same characteristics of return-volatility relationships. A final explanation encompasses several ideas through general investor behavior. Because investors are more sensitive to losses, the potential of larger decreases in stock prices due to higher volatility serves as reason to cause the negative relationship between returns and volatility.

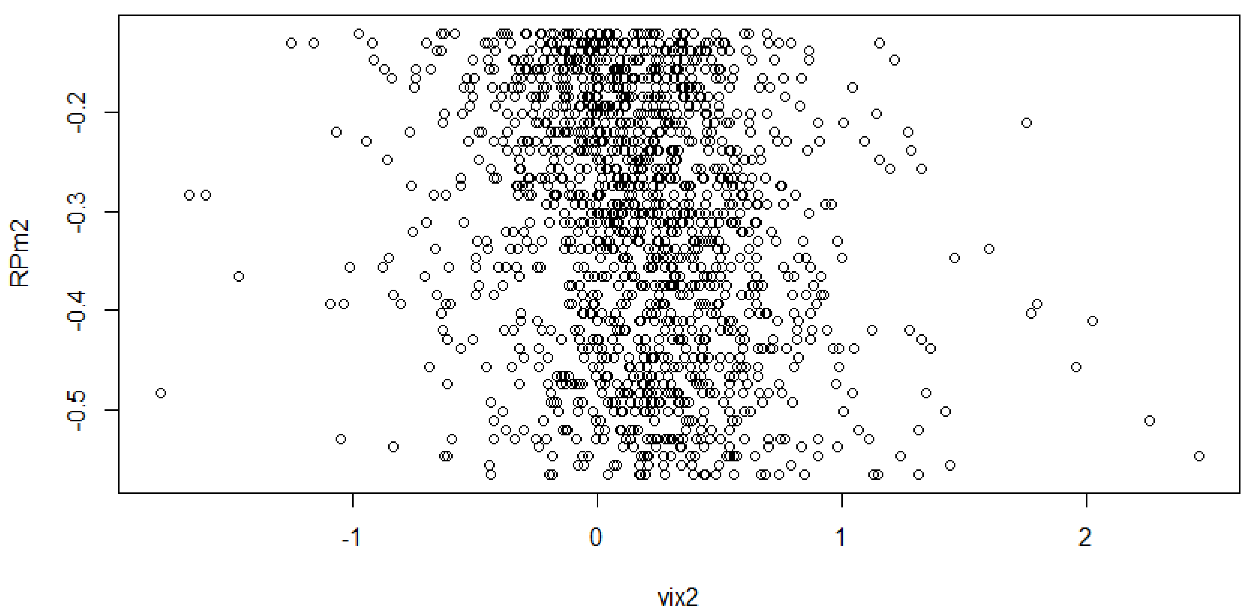
This paper furthers the academic research on these effects by taking a quintile and decile approach, in order to examine the distribution of the volatility asymmetry. The relationship was found to be nonlinear, with the first and fifth quintiles (i.e. the extremes) of the risk premium correlating the most negatively with the corresponding volatility time-series. These findings do not appeal completely to the volatility feedback effect, particularly in the extreme positive shocks to returns. They are better explained by the leverage and investor behavior effects, in which the strong negative correlations in extreme quintiles could be a result of leverage or loss aversion. They also have significant implications on the large volatility instrument market, in which they help explain the extreme losses of certain exchange-traded notes, and provide guidance to future investments in volatility.

Appendix

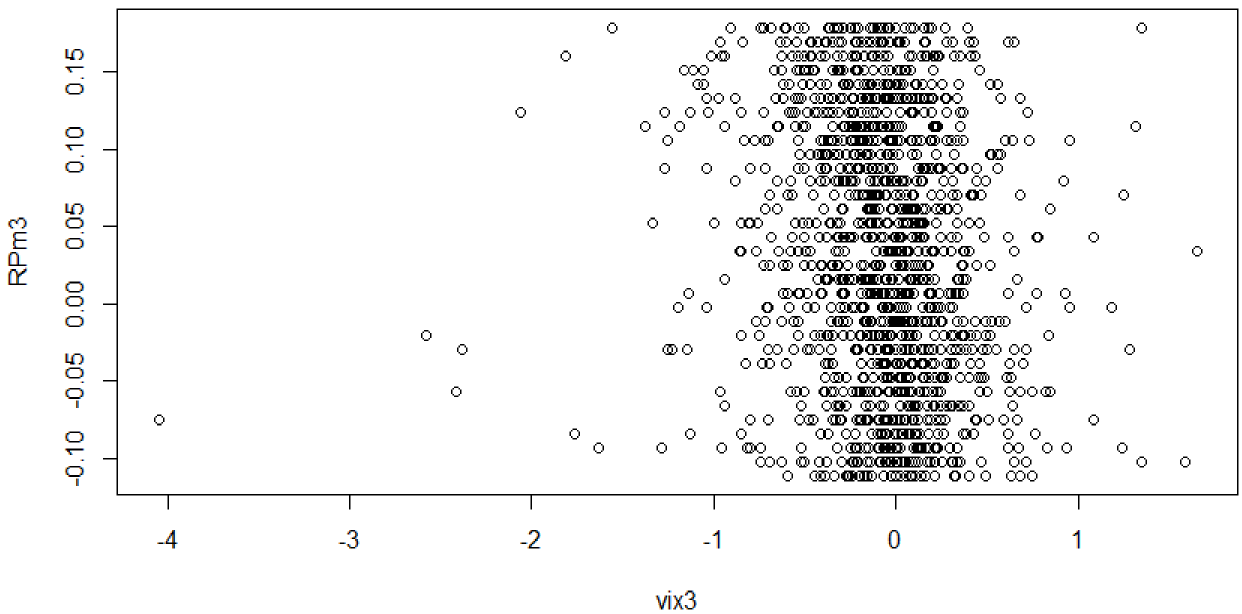
Appendix 1: RPm Quintile 1



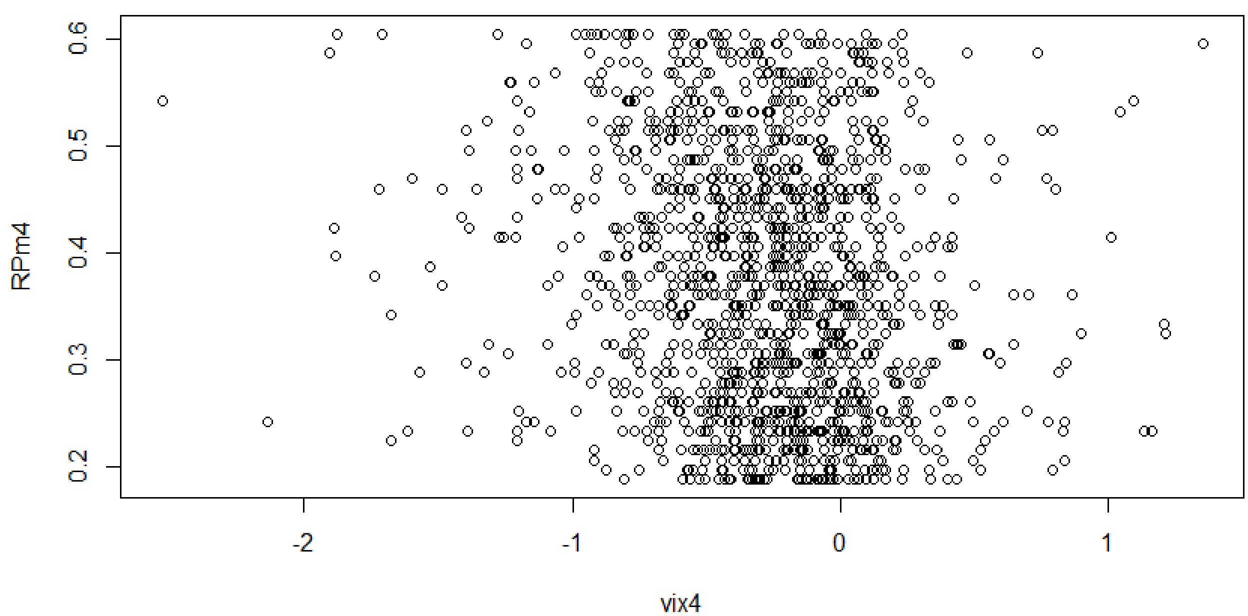
Appendix 2: RPm Quintile 2



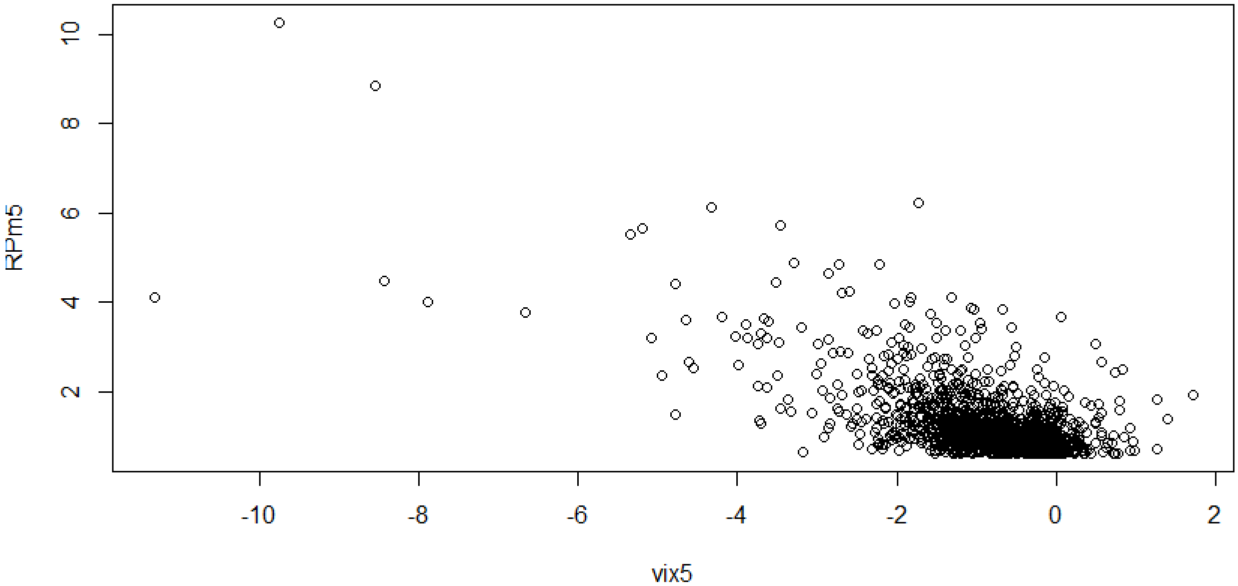
Appendix 3: RPm Quintile 3



Appendix 4: RPm Quintile 4



Appendix 5: RPm Quintile 5



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