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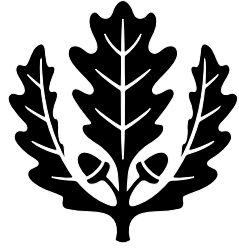
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**The Perception of Government Bonds and Money as Net Wealth:
An Integrated Approach**

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Abstract

Although much work examines whether government bonds constitute net wealth, little attention focuses on whether government money does. Most analysts merely assert that government money is net wealth. In an inflationary environment, however, money experiences "expected-inflation discounting" just as bonds experience "tax discounting." Indeed, Chiang and Miller (1988) find empirical evidence suggesting that the private sector discounts money more heavily than bonds. This paper provides the theoretical underpinnings for the two types of discounting in an integrated approach, where both new money and new bonds can finance the interest on outstanding bonds. We first analyze the objective aspect of bond- and money-discounting assuming that the private sector fully recognizes the economic consequences of bond and money issue. We then offer some conjectures on the subjective aspect of discounting by focusing on reasonable assumptions about the awareness of individual agents. Both aspects lend theoretical support for the view that more discounting of money exists than discounting of bonds. Finally, stability analysis of balanced growth equilibria further buttresses our theoretical findings.

Keywords: tax discounting, expected inflation discounting, net wealth

"What men perceive as real is real in its consequences." - Anonymous

1. Introduction

Much theoretical and empirical work exists on the question of whether interest-bearing government debt -- bonds -- constitute net wealth (e.g., Barro 1974, 1976; Feldstein 1976; Buchanan 1976; Kochin 1974; Tanner 1970, 1979; Kormendi 1983; and Seater and Mariano 1985). The implied future tax obligations associated with debt service provides the offset to the asset value of bonds. Little attention, however, focuses on the question of whether non-interest-bearing government debt -- money (more specifically, base money) -- constitutes net wealth. Government money, unlike bonds, does not bear interest. Thus, no debt-service obligation arises that entail future tax liabilities as offsets to the asset value of money. Nevertheless, government money does indeed have a value-offsetting factor of its own. In a secular inflationary environment, the expected erosion of the purchasing power of money from inflation could well detract from the asset value of money holdings. Thus, just as government bonds may be subject to "tax-discounting," government money may be subject to "expected-inflation discounting." This discounting due to expected inflation is unique to money and does not apply to bonds, since the nominal interest rate on bonds already includes a premium for expected inflation, so that the future tax liabilities associated with debt-servicing automatically allow for the expected-inflation tax on bonds.

The expected-inflation discounting of money, alluded to in Seater (1982), is analyzed and empirically tested in Chiang and Miller (1988). Using annual U.S. data, they find that when both types of discounting are accommodated in the model, the private sector discounts much more heavily its money holdings for expected inflation than its holdings of government bonds for the

expected tax liability. This distinction has important policy implications regarding the relative potency of monetary and fiscal policy as discussed by Chiang and Miller (1988, Section V).

At the outset, we must emphasize that the expected-inflation discounting does not concern itself with the ex post erosion of purchasing power that occurs as the price level responds to an expansionary monetary policy, or with even the ex post adjustments to changes in the expected inflation rate -- for example, a higher expected inflation rate leads to lower real money demand as well as other subsequent adjustments.¹ Rather, we consider the ex ante effect in a balanced growth equilibrium of expected future money growth and inflation on the net wealth content of government bonds and money in a Barro (1974, 1976) and Feldstein (1976) world. In this world, the focus is not on the effects of changing the path of future money growth and inflation, but on the effects of the existing balanced growth path.

The bond-financing literature customarily considers only two alternative methods of financing bond interest -- new tax revenue and new bond issue. The debt-service-induced tax liability is squarely faced only when it is no longer possible to raise unlimited funds through new borrowing. In other words, the tax liability is obviated so long as the avenue of new debt remains open. It is for this reason that in a growing economy, the relative magnitude of the rate of interest and the rate of growth of GDP becomes a matter of significance (see, for example, Barro 1976). In contrast, however, when discussing government budgets, it is customary to consider three alternative sources of finance: new tax revenue, new bond issue, and new money issue.² Since new money as well as new bonds can indeed finance bond interest, the neglect of money can

¹ McCafferty (1990, 212-14) provides a discussion of this Mundell-Tobin effect.

² As a matter of practice, the United States Treasury finances all government deficits with bond sales. Nevertheless, monetization of the deficit by the Federal Reserve essentially produces a money-financed deficit in result.

cause inaccurate conclusions about the tax-discounting of bonds.³ More specifically, the use of money to pay bond interest allows the government to shift some of the burden of financing bond interest from taxpayers to money holders. That is, the explicit tax is replaced by an implicit inflation tax on money holdings. Consequently, the need for tax-discounting of bonds is reduced somewhat. Chiang and Miller (1988) do consider the discounting of money in addition to bonds, but they do not allow paper money as a source of finance of bond interest. The two types of discounting thus appear as two parallel but separate processes. The present paper reconsiders these two types of discounting in an integrated framework where all three sources of finance occur. We first explore in Section 2 the objective aspect to the discounting of bonds and money. By "objective," we mean the matter-of-fact calculations of the prospective additional taxes to pay the interest on bonds, and of the inflation tax on money holdings, where the private sector fully recognizes such tax implications. We then take up in Section 3 the subjective aspect of discounting by explicitly considering the degree to which the private sector consciously expects the tax consequences associated with its bond and money holdings. Section 4 considers the conditions necessary for the existence of a permanent primary budget deficit in stable balanced growth equilibria. We recapitulate our conclusions in Section 5.

³ Sweeny (1988, 237-39) considers this issue where new money finances a portion of the interest expense, and concludes that government bonds are not net wealth. He gets this result because he assumes that the terminal stock of government bonds is zero (all bonds are paid off). Our balanced growth model has a constant ratio of government bonds to GDP. Thus, government bonds are never liquidated (the bond stock grows continuously). Such a model does not prohibit the standard Barro bond-discounting result (see below). Barro (1987, 394) also argues that the Ricardian equivalence theorem continues to hold when the government uses money to pay some bond interest. This conclusion relies, in our view, on the imposition of an intertemporal government budget constraint that rules out perpetual debt finance (see Barro 1979, 942). We do not rule out such an outcome. In fact, we explicitly consider in Section 4 the conditions necessary for the existence of a permanent primary budget deficit in a stable balanced growth equilibrium. Our constraint on government behavior takes the form of an "acceptable ratio" of the primary budget deficit to GDP (see footnote 8).

2. The Objective Aspect of the Discounting of Government Bonds and Money

Consider a steadily growing economy with a real growth rate of GDP equal to γ and an expected inflation rate equal to π , which we also assume is equal to the actual inflation rate. The nominal growth rate in this economy is therefore given by

$$(1) \quad g = \gamma + \pi.$$

The outstanding nominal stock of privately held government bonds at time t is $B(t)$, and $B(t)$ grows at the rate b :

$$(2) \quad B(t) = B(0) e^{bt}.$$

Thus, the rate of new debt issue is $dB(t)/dt = bB(t)$. Similarly, nominal money -- more specifically, base money, denoted by H -- grows at the rate h :

$$(3) \quad H(t) = H(0) e^{ht}.$$

And the rate of new money issue is $dH(t)/dt = hH(t)$.

Given the initial stock of the private sector's bond and money holdings, $B(0) + H(0)$, future issues of bonds and money, duly discounted to their present value (PV), will also add to the value of that stock. As offsets against this, however, the private sector must -- at the objective level of analysis -- subtract the PV of the projected interest expenses on bonds that will require government financing, as well as the PV of the inflation tax on money.⁴ Let ρ denote the real interest rate and r the nominal interest rate with

$$(4) \quad r = \rho + \pi.$$

Then the net wealth content of government debt, $NW[B + H]$, is represented as follows:

$$(5a) \quad NW[B + H] = B(0) + H(0) \quad [\text{value of initial stock}]$$

⁴ We concentrate on the bond interest and ignore the bond principal since the repayment of the principal is not an issue in a growth model. Since the bond stock grows at a constant rate, the government must issue new bonds, first,

$$(5b) \quad + \int_0^{\infty} bB(t)e^{-rt} dt \quad [\text{PV of future bonds}]$$

$$(5c) \quad + \int_0^{\infty} hH(t)e^{-rt} dt \quad [\text{PV of future money}]$$

$$(5d) \quad - \int_0^{\infty} rB(t)e^{-rt} dt \quad [\text{PV of future bond interest}]$$

$$(5e) \quad - \int_0^{\infty} \pi H(t)e^{-rt} dt \quad [\text{PV of future inflation tax}]$$

Note that, as pointed out above, since the nominal interest rate r includes π as a component, equation component (5d) already includes an inflation tax on bonds.⁵ Also note that while the future bond interest in (5d) necessitates explicit new tax revenue to the extent that the interest is not covered by new bonds and new money, the inflation tax on money holdings in (5e) is always implicit.

A more revealing decomposition of the net wealth content of government debt focuses on the two types of offsets to the value of the initial stock, namely, (a) the discounted value of the future explicit tax required to finance the interest expense not covered by new bonds and new money, and (b) the future implicit inflation tax associated with money balances. Since new bonds and new money can finance a portion of the bond interest expense, the explicit tax at each point in time is given by $[rB(t) - bB(t) - hH(t)]$. On the other hand, the implicit tax at each point of time is $\pi H(t)$. Thus, the net wealth content of government debt is alternatively expressed as follows:

$$(6a) \quad NW[B + H] = B(0) + H(0) \quad [\text{value of initial stock}]$$

$$(6b) \quad - \int_0^{\infty} [rB(t) - bB(t) - hH(t)]e^{-rt} dt \quad [\text{PV of explicit taxes}]$$

to cover the repayment of an principal due and, second, to keep the outstanding bond indebtedness growing at a constant rate.

⁵ Equation (5) evaluates the net wealth content of government debt with debt expressed in nominal terms. Alternatively, we could reformulate this equation to state all debt variables in real terms. In this case, however, the appropriate discount rate is the real interest rate. When so adjusted, our findings are unaltered.

$$(6c) \quad - \int_0^{\infty} \pi H(t) e^{-rt} dt \quad [\text{PV of implicit taxes}]$$

Note that equation (6) is equivalent to equation (5), for (6b) is the sum of (5b), (5c), and (5d), whereas (6c) is identical with (5e).

Further insight emerges by relating (6b) to the government budget constraint

$$(7) \quad \dot{B}(t) + \dot{H}(t) = G(t) - T(t) + rB(t),$$

where G denotes nominal government expenditure net of bond interest, T , nominal tax revenue, and a "." over a variable means the time derivative. Substituting for the time derivatives of government bonds and government money gives the following:

$$(7') \quad rB(t) - b\dot{B}(t) - h\dot{H}(t) = T(t) - G(t).$$

In this light, the explicit taxes in the integrand of (6b) represent the primary budget surplus -- the amount by which the tax revenue exceeds what is needed to finance the government expenditure net of bond interest. For later reference, note that to have a permanent primary budget deficit in a steadily growing economy, it is necessary to have both sides of (7') negative (i.e., it is necessary for the new debt of the government, $b\dot{B}(t) + h\dot{H}(t)$, to exceed the ongoing bond interest expense).

Long-Run Growth Equilibrium

Following the method of analysis contained in Barro (1976) and Feldstein (1976), we now consider a long-run growth equilibrium. A sustainable long-run growth equilibrium requires that the stocks of bonds and money must grow at the same rate, for otherwise portfolio adjustments would occur. Besides, the common rate of growth of bonds and money must match

the nominal rate of growth of GDP.⁶ Thus, we shall assume for the present discussion that $b = h = g$.

The Traditional Treatment of Bond Discounting

The traditional treatment of bond discounting emerges as a special case of (6) in which H is absent. Thus, we can, by dropping all terms involving H from (6), and setting $b = g$, specialize the net wealth expression to:

$$(8) \quad \begin{aligned} NW[B] &= B(0) + \int_0^{\infty} (g - r)B(0)e^{(g-r)t} dt \\ &= B(0) + B(0) \left[e^{(g-r)t} \right]_0^{\infty}, \end{aligned}$$

remembering that $B(t) = B(0)e^{bt} = B(0)e^{gt}$. Thus, in the traditional treatment of tax discounting of bonds, we have that

$$(9) \quad NW[B] = \begin{cases} 0 \\ B(0) \\ \infty \end{cases}, \text{ as } r \begin{cases} > \\ = \\ < \end{cases} g.$$

If the nominal interest rate exceeds the rate of growth of bonds, then $NW[B] = 0$, and bonds are not net wealth. This is essentially the case discussed by Barro (1976), who argues that r is greater than g is the more probable scenario in the economy. Even though he considers it unlikely to have r less than g (or r equal to g) in a steady state, however, he is unable to rule it out on a priori grounds and, with it, the possibility that the objective valuation of government bonds qualifies them as net wealth.

If, contrary to Barro, we do find $r = g$ or $r < g$, then government bonds indeed are wealth. In particular, if $r < g$, then the net wealth content of B is infinite. The "infinity" of $NW[B]$ in (9) links to what Barro (1976, 344, n.4) describes as the infinity of "the government's

⁶ Suppose that the growth rate of government debt exceeds the nominal growth rate of GDP. Then the debt/GDP ratio will explode, which is an unsustainable situation in the long run. We shall take up this question of the stability of government finance in Section 4.

collateral" (the government's future tax capacity). When $r < g$, the growth in government debt (bonds in his model) makes it possible to finance future interest expense with funds to spare. Thus, the government does not have to draw on its future tax capacity, which implies that the government's collateral for future bond issues is infinite. Similar infinity results are encountered again in the following discussion.

The Integrated Approach of Bond Discounting

When money is considered along with bonds, the government can finance bond interest with new money as well as new bonds. Thus, the explicit tax required for debt servicing in the integrated approach is lower than what the traditional approach implies. The present value of the future explicit taxes needed to finance that portion of the interest expense on bonds not covered by new money and new bond issues is given by (6b).

Using (6b) to offset the initial bond stock, and setting $b = h = g$, we then have the following:

$$(10) \quad \begin{aligned} NW[B] &= B(0) + \int_0^{\infty} \{(g - r)B(0)e^{(g-r)t} + gH(0)e^{(g-r)t}\} dt \\ &= B(0) + \{B(0) + gH(0)/(g - r)\} [e^{(g-r)t}]_0^{\infty}. \end{aligned}$$

Thus, in the integrated approach, we find that

$$(11) \quad NW[B] = \begin{cases} gH(0)/(r - g) \\ \infty \\ \infty \end{cases}, \text{ as } r \begin{cases} > \\ = \\ < \end{cases} g.$$

Comparing this to the traditional result in (9), we see that when money is integrated with bonds, the net wealth content of government bonds is enhanced. It is now always positive. In fact, it is infinite except in the case of $r > g$. Even in the latter case, where Barro stripped government bonds of all net wealth content, we now see a positive value for $NW[B]$ that is tied to the magnitudes of the initial money stock $H(0)$, the nominal rate of interest r , and the nominal rate of

growth of GDP g . The value of $NW[B]$ can fall below $B(0)$, in which case bonds are discounted, but only partially, and never completely. On the other hand, $NW[B]$ can also be equal to or exceed $B(0)$, in which case bonds are not discounted.

More specifically, in the Barro world of $r > g$, we have that

$$(11') \quad NW[B] \begin{cases} > \\ = \\ < \end{cases} B(0), \text{ as } g[H(0) + B(0)] \begin{cases} > \\ = \\ < \end{cases} rB(0).$$

The net wealth content of bonds exceeds (falls short of) $B(0)$ when the increase in total government debt, $g[H(0) + B(0)]$, exceeds (falls short of) the bond interest. Referring to (7') and the related discussion, the third result in (11'), where $NW[B]$ is less than $B(0)$, indicates that (partial) discounting of bonds can occur only if the government runs a primary budget surplus. With a primary budget deficit, on the other hand, the net wealth content of government bonds must exceed their face value.

Money Discounting

What about the discounting of money? Using the expression for the present value of the implicit taxes on money in (6c) as the offset, we can write the net wealth content of money as follows:⁷

$$(12) \quad \begin{aligned} NW[H] &= H(0) - \int_0^{\infty} \pi H(0) e^{(g-r)t} dt \\ &= H(0) - \{ \pi H(0) / (g-r) \} [e^{(g-r)t}]_0^{\infty} \\ &= \begin{cases} H(0)(r-g-\pi)/(r-g) \\ -\infty \\ -\infty \end{cases}, \text{ as } r \begin{cases} > \\ = \\ < \end{cases} g. \end{aligned}$$

⁷ Remember that the new money issue has already been allocated to help finance bond interest [see equation (10)]. Thus, new money issue is not available to enter equation (12).

The striking feature of this result is that the net wealth content of $H(0)$ can be negative infinity, which occurs when r is less than or equal to g . What does minus infinity mean in this case? The clue lies in the integral in (12) that represents the government's collateral from the inflation tax (its future inflation tax capacity). When $r < g$ (or $r = g$), the capacity for future inflation taxes becomes infinite, and this makes the net wealth content of money negative infinity. When this happens, discounting of money is more than complete. But more-than-complete discounting of money can occur even when $r > g$. In the latter case, the algebraic sign of the expression $(r - g - \pi)/(r - g)$ depends on the relative magnitude of $(r - g)$ and π , and we have

$$(13) \quad NW[H] \begin{cases} > \\ = \\ < \end{cases} 0, \text{ as } (r - g) \begin{cases} > \\ = \\ < \end{cases} \pi \text{ (when } r > g \text{)}.$$

Thus, complete discounting is implied by $(r - g) = \pi$, and more-than-complete discounting, by $(r - g) < \pi$. Moreover, so long as π is positive, the expression $(r - g - \pi)/(r - g)$ is a positive fraction, and $NW[H] < H(0)$, implying at least partial discounting of money. In short, if there is inflation in the growth equilibrium, then discounting of money is inevitable.

3. The Subjective Aspect of Discounting of Government Bonds and Money

The preceding discussion dealt with the objective aspect -- the matter-of-fact calculations -- of the discounting of government bonds and government money. But whether, and to what extent, the private sector collectively takes these objective calculations into account in forming its subjective evaluation of the net worth of bonds and money, is a different matter. If the private sector is keenly forward-looking, then it is likely to base its subjective asset evaluation on the objective calculations outlined above. If, on the other hand, the private sector is only vaguely cognizant of the effects of the existing offsets to asset values, then those objective calculations will produce little effect.

When we say that the private sector is forward-looking, what is really meant is that a large proportion of the individual agents in the private sector is sufficiently perceptive to be aware of the asset-value implications of the bond-induced tax liability and/or the inflation-induced erosion of the purchasing power of money. It is our conjecture that the private sector's (average) perceptiveness of the discounting of money dominates that of the discounting of bonds.

The key to this conjecture lies in the differential incidence of the two types of discounting. For money, the incidence of the expected-inflation tax is strictly proportional to the amount of money held by any individual. Since all holders of money assets face the identical proportional inflation tax, they are all likely to be equally aware of the tax burden. This results in a high average perceptiveness of the discounting of money. In sharp contrast, the incidence of the bond-induced tax liability is not directly linked to the amount of bonds held, but falls on the taxpaying public, in general. Hence, bondholders, who are not called upon to bear the full tax burden, need not be overly concerned about it. At the same time, non-bondholders, with no bonds in their portfolios, may be quite ignorant about any future tax burden that government bonds may entail. Thus, the average perceptiveness of and concerns about bond-related tax burdens is likely to be low in the private sector. At the subjective level, therefore, we also expect more money-discounting than bond-discounting.

4. The Stability of Government Finance

The above discussion deals with the wealth-perception implications of government debt - - bonds and money -- in a growth equilibrium. Since bond and money growth play the leading roles in the government budget constraint, and since the existence of the government budget constraint raises the issue of the stability of government finance, it is only natural to ask whether

there are linkages between the subject of discounting of bonds and money on the one hand, and the subject of the stability of government finance on the other. In the present section, we explore this issue by asking: Would the imposition of some dynamic stability condition on government finance enable us to make a more definitive pronouncement on the net wealth content of bonds and money?

As a prerequisite for stable government finance, the primary budget deficit ($G - T$) must be kept within bounds. We shall assume that the primary budget deficit is restricted to an acceptable constant fraction of GDP. In so doing, we implicitly accept the notion that government debt (bonds and money) serves a useful productive function in the macroeconomy. Accordingly, economic growth necessitates growth in the supplies of government bonds and money, which, in turn, requires a permanent primary deficit.⁸ Any bonds that are issued to finance the on-going primary deficit will incur interest obligations, to be financed with new bonds or new money. The stability question is "Under what conditions can we be sure that the debt-service requirement will not generate an explosive time path for the total-debt/GDP ratio?"

To study this question, we adopt the following symbols: $x = (G - T)/Y$ denotes the primary-deficit/GDP ratio; $D = B + H$ denotes the total government debt; and $d = D/Y$ denotes the total-debt/GDP ratio. Then the government budget constraint, $\dot{B} + \dot{H} = G - T + rB$, can be written as $\dot{D} = xY + r(D - H)$, or, after dividing through by D :

$$(14) \quad \dot{D}/D = xY/D + r - rH/D,$$

where the "." over the variable denotes the time derivative. This enables us to express the rate of growth of the total-debt/GDP ratio as follows:

$$\dot{d}/d = \dot{D}/D - Y/Y = x/d + r - rH/D - g,$$

or, after multiplying through by d :

$$(15) \quad \dot{d} = x + (r - g)d - rH/Y.$$

If we know the dynamic behavior of the last term ($-rH/Y$), we can then solve (15) as a differential equation in the single variable d , and derive the stability condition for the total-debt/GDP ratio.

Money Growing at a Constant Rate

Let money H grow at a constant rate h . Then

$$(16) \quad H/Y = [H(0)e^{ht}]/[Y(0)e^{gt}] = Ae^{(h-g)t}, \text{ where } A = H(0)/Y(0).$$

And (15) becomes

$$(17) \quad \dot{d} - (r - g)d = x - rAe^{(h-g)t},$$

which is a differential equation with a constant coefficient and a variable term. In the solution of this equation [see Chiang (1984, 542-43 or 487-88)], the particular integral is non-constant over time, giving us a moving equilibrium $\bar{d}(t)$. The solution path is

$$(18) \quad d(t) = [d(0) - \bar{d}(0)]e^{(r-g)t} + \bar{d}(t), \text{ where}$$

$$\bar{d}(t) = x/(g - r) + [rAe^{(h-g)t}]/(r - h).$$

In order to attain stability of finance -- with a convergent total-debt/GDP ratio -- it is necessary not only for the time path $d(t)$ to converge to the moving equilibrium $\bar{d}(t)$, but also for the moving equilibrium not to explode. Thus, the stability condition consists of two inequalities:

$$(19a) \quad h \leq g \quad [\text{for } \bar{d}(t) \text{ to be non-explosive}],$$

⁸ By imposing such an "acceptable ratio" of the primary deficit to GDP, we constrain the government from irresponsible fiscal behavior. Thus, it becomes unnecessary to impose an intertemporal government budget

$$(19b) \quad r < g \quad [\text{for } d(t) \text{ to converge to } \bar{d}(t)].$$

The former inequality is aimed at the money side of government finance; it forbids monetary growth in excess of the growth of GDP. The latter inequality is aimed at the interest payment on bonds; it forbids the nominal interest rate to exceed, or even equal, the growth rate of GDP.

Would the imposition of these stability conditions narrow down our conclusions regarding the net wealth content of bonds and money? In (11) and (12), which satisfy (19a) by assumption of a steady state, the effect of condition (19b) is to specialize our results to:

$$(20) \quad NW[B] = \infty, \text{ and } NW[H] = -\infty,$$

which represents the strongest possible case for our argument that bonds are net wealth, and money is not.

The Barth-Iden-Russek Analysis

Barth, Iden, and Russek (1986) [hereafter BIR] analyze the stability of government finance on the specific assumption that money grows at the same rate as GDP, and derive the stability condition $r < g$.⁹ That is, they derive (19b) without (19a). Although their analysis differs from ours in that they (a) use the traditional bond-discounting-only rather than our integrated framework, (b) consider the bonds/GDP ratio rather than the total-debt/GDP ratio, and (c) employ discrete time rather than continuous time, we can nevertheless adapt their analysis as a special case of ours.

constraint.

⁹ The specification that money grows at the same rate as GDP mirrors that used in the debate between Sargent and Wallace (1981), Darby (1984), and Miller and Sargent (1984). In this debate, the fiscal authorities choose to run a permanent primary budget deficit. In such a situation, the monetary authorities cannot implement an independent policy when $r > g$, but can when $r < g$.

If the dynamic behavior of money follows the pattern that money and GDP grow at the same rate ($h = g$), then (19a) is automatically satisfied. The $\bar{d}(t)$ term in the solution (8) now becomes

$$(21) \quad \bar{d}(t) = (x - rA)/(g - r) = \text{a constant},$$

giving us a stationary intertemporal equilibrium. But the complementary function remains unchanged and, consequently, the condition for $d(t)$ to converge to the stationary equilibrium is still $r < g$. This is why BIR, by assuming away (19a), obtain only (19b). In that sense, their analysis is a special case of ours.

The Tobin Analysis

Tobin (1982), unlike BIR, analyzes the problem of stability of government finance by assuming that bonds and money are each a constant fraction of the total debt. His stability condition is $r_D < \gamma$, where r_D denotes the (weighted) average real interest rate on the debt, and γ , the real growth rate of GDP.¹⁰ We now show that the Tobin result can also be derived from our differential equation (17).

Let $H/D = \theta$ (a constant). Then we can write

$$(22) \quad -rH/Y = -r(H/D)(D/Y) = -r\theta d.$$

Substituting this into equation (15), and simplifying, gives us the following differential equation:

$$(23) \quad \dot{d} - [r(1 - \theta) - g]d = x,$$

which has the solution

¹⁰ More specifically, in terms of our notation,
 $r_D = \rho(B/D) + (-\pi)(H/D),$

where (B/D) and (H/D) are both taken as constants. Note that while Tobin employs the symbol γ to represent the ratio of bonds to the total debt, we are using γ to mean the real growth rate of GDP.

$$(24) \quad d(t) = [d(0) - \bar{d}]e^{[r(1-\theta) - g]t} + \bar{d}, \text{ where}$$

$$\bar{d} = x/[g - r(1 - \theta)] = \text{a constant.}$$

This time the stability condition is $r(1 - \theta) < g$, which is equivalent to the Tobin stability condition that $r_D < \gamma$.¹¹ Since Tobin's result can be derived from Equation (17) on a particular condition, his analysis can also be considered a special case of ours.

In this particular case, condition (19a) vanished from the scene. The mathematical explanation for this is that, on Tobin's assumption that (H/D) is a constant, the exponential term $[Ae^{(h-g)t}]$ on the right-hand side of the differential equation (17) is absorbed into another term on the left-hand side, thereby producing a stationary intertemporal equilibrium in (24), and thus obviating the need for any stability condition on the original moving equilibrium. The economic explanation is that, by restricting money H to be a constant fraction θ of total debt D , Tobin eliminates the possibility of the H variable becoming explosive on its own, independently of bonds. Thus, it is no longer necessary to state a separate stability condition (19a) aimed at money per se. At the same time, by restricting bonds B to be a constant fraction $(1 - \theta)$ of the total debt D , Tobin also eliminates the possibility of the B variable becoming explosive on its own. This serves to "tame" the problem of the interest cost on bonds, so that the stability condition (19b) can be relaxed from the form $r < g$ to the form $r(1 - \theta) < g$,

The new condition $r(1 - \theta) < g$ can allow a higher interest rate while still maintaining stability of finance. Thus, a situation of $r > g$ is now permissible. On the other hand, the new condition is equally consistent with the opposite situation of $r < g$. Thus, the Tobin condition $r(1$

¹¹ By using equations (4) and (1), we can transform the inequality $r(1 - \theta) < g$ as follows:

$$\begin{aligned} (\rho + \pi)(1 - \theta) &< \gamma + \pi, \\ \rho(1 - \theta) + (-\pi)\theta &< \gamma, \\ r_D &< \gamma. \end{aligned}$$

- $\theta < g$ offers no definite clue for narrowing down our conclusions regarding the net wealth content of bonds and money. That is, his condition is neutral on our results in (11) and (12).

5. Conclusions

Our analysis shows that there are reasons, at both the objective and the subjective levels, to believe that the tax-discounting of government bonds is not as substantial as the expected-inflation discounting of money.

Objectively, bond discounting occurs only when the rate of interest exceeds the rate of growth of GDP, $r > g$. And even in this case, our integrated approach shows that bonds are at most only partially discounted (i.e., they are never discounted to zero as suggested by the traditional approach). Discounting is only partial because the government can shift the burden of financing the interest expense on bonds from taxpayers to money holders. And the larger the share of bond interest financed by new money, the lower the level of bond discounting.

The expected-inflation discounting of money, on the other hand, is an inescapable occurrence so long as there is positive inflation. Even in the least serious case, when $r > g$, money is at least partially discounted. Moreover, it is possible to have complete or more-than-complete discounting of money, even in that least serious case.

At the subjective level, by comparing the different patterns of incidence of the two types of discounting, we see yet another reason to find more expected-inflation discounting of money than tax-discounting of bonds. This is because the expected-inflation tax liability is linked directly to the amount of money held by individual agents, whereas the bond-induced tax liability is independent of the amount of bonds held. Thus, the results of the subjective consideration buttress the conclusions from the objective consideration.

Finally, by linking the subject of bond- and money-discounting to the subject of the stability of government finance, we try to ascertain whether the imposition of a stability condition on the total-debt/GDP ratio would enable us to narrow down our conclusions regarding the net wealth content of bonds and money. Our finding is that either such a condition offers no definite clue and is thus neutral, or -- by ruling out the $r > g$ possibility -- it strengthens our argument that bonds are, and money is not, net wealth.

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