Spring 5-12-2013

Collusion in Peer Evaluation

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A research paper submitted to the University Honors Program in partial fulfillment

of the requirements as Honors Scholar in Economics

May 2013

Contents

Abstract

The exact consensual and impartial division function by DeClippel et al (2008) offers a procedure for dividing a fixed award among partners. The peer evaluation mechanism proposed by DeClippel et al (2008) offers participating partners incentives to tell the truth. This paper examines and demonstrates that, when agents form at most one coalition and report wrong values of the relative shares, they capture the total amount of money being divided and their share improves by exactly the same percentage. When multiple coalitions emerge, the division function fails to assign payoffs to each partner. This paper also includes a discussion of possible ways including monitor hiring and changing aggregator to mitigate such collusion among agents.

1. Introduction.

A team of three or more partners receives a reward after successfully accomplishing a project. They would like to divide the reward up and distribute it fairly amount the partners. By “fairly” we mean the partners are rewarded according to their relative contributions in completing the project. The problem, however, is more complicated than it seems. Since partners with distinct talents collaborate to finish the project, it is difficult to measure each single partner’s contribution. There are no simple indicator of each agent’s workload and contribution either. A typical example would be partners in a game development team who need to divide the bonus they earned after selling the game. In the game development process, storywriters develop the ideas and storylines; programmers code and test the algorithms; graphic designers sketch and color the layout and marketers promote the products and negotiate with potential investors. It is rather challenging to directly assign shares of the partners based on their positions due to the non-linearity and verifiability in their nature of work and individual talent.

The partners need a proper division rule that solves the problem of division of a fixed reward. The division rule will be a function that takes partners’ reports about each other’s share and yields their individual payments. Tideman and Plassmann (2008) suggest that such division rule should be incentive compatible, objective and consensual. An incentive compatible division rule prevents any partners from affecting his own share by altering his reports. The partners would not accept any rule that is not incentive compatible since it induces strategic behaviors and is unfair to other partners. Objectivity of the rule ensures that a partner could not affect other partners’ shares by making subjective claim of his own share. Lastly, the division rule should be consensual, which means the resulting shares assigned by the division rule are consistent with the reports by the partners. If the shares do not reflect their opinions then some partners must be underpaid or overpaid.

There are many similar discussions on dividing a fixed divisible good among a fixed number of agents. Brams and Taylor (1995) introduced Brams–Taylor theorem to produce an envy-free division of an infinitely divisible good among a certain number of players. Brams et al. (2006) provided a more recent analysis of the cake-cutting problem where each party has private value of its share. Thomson (2003) formulated properties of rules used in division process and also presented resolution of conflicting claims. De Clippel et al (2008) introduced a division mechanism with an impartial, exact and consensual division function for a group of at least three partners. The inputs of the function come from the reports on the relative share of pairs of partners other than the reporter. The division mechanism is incentive compatible since the share of one partner is determined exclusively by other partners. Individuals could not make subjective claims and have absolutely no direct influence on their share. When there are more than three partners in the group, one particular pair of partners is evaluated more than once. The division mechanism adopts an aggregator to aggregate reports.

There are many different methods that can be used to aggregate reports. For certain aggregators, always reporting the truth is not a dominant strategy for agents. A subgroup of agents can indirectly improve its share under such division mechanism. Therefore the mechanism is not coalition strategy proof. The coalition has incentive to increase its profit and distribute it among its members. The agents' profit maximizing strategy would be to take fully advantage of the property of the division mechanism by reporting false conflicting reports. To achieve this goal, the agents have to form a coalition to indirectly raise their ratings since they would be submitting ratings of each other.

This paper analyzes the coalition formation among the peer evaluating partners and the collusion results with two commonly used aggregators. The coalition is able to capture all the money being divided and distribute it among its members. The co-conspirators can improve their individual share equally percentage wise. The coalition as a whole maintains a constant value and is not affected by its size and individual shares of its members. The existence of multiple coalitions will lead the division rule into trouble.

We first start with examples of the five agents and then generalize to n agents ($n > 3$). The following section provides an alternative way to evaluate collusion results in the context of cooperative game theory. The last part of this paper discusses some possible ways including monitor hiring and changing aggregator to mitigate such collusion.

2. The Peer Evaluation Mechanism.

A simple division rule would be available if all the partners have objective claims of their own share. Then, we could divide the dollar proportionally. Unfortunately, such rule is not feasible since people are poor judges of themselves, and all the claims incorporate a certain level of subjectivity. De Clippel et al (2008) proposed a way to take away the subjectivity in the reports by asking them to report relative shares of other partners. The report on relative shares of an individual is determined exclusively by other agents. The division function will take the proposed reports as inputs and yield the individual shares for the partners.

The model is set up as follows: Let $N = \{1, \dots, n\}$ be a set of at least 3 agents ($|N| \geq 3$). For each subset M of N, define $\mathcal{R}[M]$ the set of consistent share ratios for subset M, to be the following subset of $\mathbb{R}^{M \times M}$:

$$\mathcal{R}[M] = \left\{ r \in \mathbb{R}^{M \times M} : \forall\, i\,, j\,, k \in M, r_{ij} > 0 \textit{ and } r_{ij} r_{jk} r_{ki} = 1 \right\}$$ (De Clippel et al, 2008)

Each agent submits a vector of the size $\binom{n-1}{2}$ containing relative shares of pairs of partners other than pairs contain himself. For all $i, j \in N \backslash \{k\}$, define $\frac{w_i}{w_j} = r_{ij}^k$ the ratio of w_i and w_j are the relative contributions of i comparing to j according to agent k. Notice that w_i and w_j does not necessary reflect agent *i* and *j*'s payoff. Then define conversions $r_{ij}^{k} = \frac{1}{r_{ji}^{k}}$ including conventions $\frac{1}{+\infty} = 0$, $\frac{1}{0} = +\infty$ and $i < j$. There are *n-2* reports about a particular pair *i* and *j*. Define R_{ij} to be the aggregated report for *i* and *j*. The arithmetic mean aggregator: $R_{ij} = \frac{\sum_{k\neq i,j} r_{ij}^{k}}{n-2}$ and geometric mean aggregator: $R_{ij} = \sqrt[n-2]{\prod_{k\neq i,j} r_{ij}^{k}}$ are adopted to aggregate the individual reporting. Define the aggregated reports are consistent if and only if $\underbrace{R_{ij} \cdot R_{jk} \cdots R_{mn} \cdot R_{ni}}_{n\ aggregated\ R} = 1$ for all $i, j, k, m, n \ldots \in N$. The shares of partner *i* is given by the exact, impartial and consensual division function $S_i = \frac{1}{1+\sum_{t\neq i} R_{ti}}\ for\ t \in N\backslash\{i\}$. De Clippel et al (2008).

The division function has the desired properties to be a proper division rule for the partners. It avoided the situation where agents judge their own contributions by requiring them reporting relative shares of other partners. The impartiality is reflected by the two properties it possesses: each partner reports relative share of others and no one makes any claim about their own share and the share of a particular agent is determined exclusively by the reports of others.

This division rule is exact, which means it always divides the total amount of money or reward when the aggregated reports are consistent. The consistency of the reports indicates that the information partners provided agrees with each other. Tideman and Plassmann (2008) demonstrated graphically (see Figure 1) that the division rule distributes strictly less than the total amount when the reports are not consistent. The example shows the surplus as a function of r_{32}^{1} and r_{13}^{2} over the range of 0.2 to 1, for $r_{21}^{3} = 3$. The surplus is zero when $r_{13}^{2} \cdot r_{32}^{1} = \frac{1}{3}$. The graph plots surplus as a function of two proposals at the three-partner level. The surplus increases as the product of the proposals deviate from the consistent value $\frac{1}{3}$. The surplus is required to be wasted to prevent strategic behaviors. Finally, The division is consensual such that the resulting shares by the division rule reflect the information the partners provided.

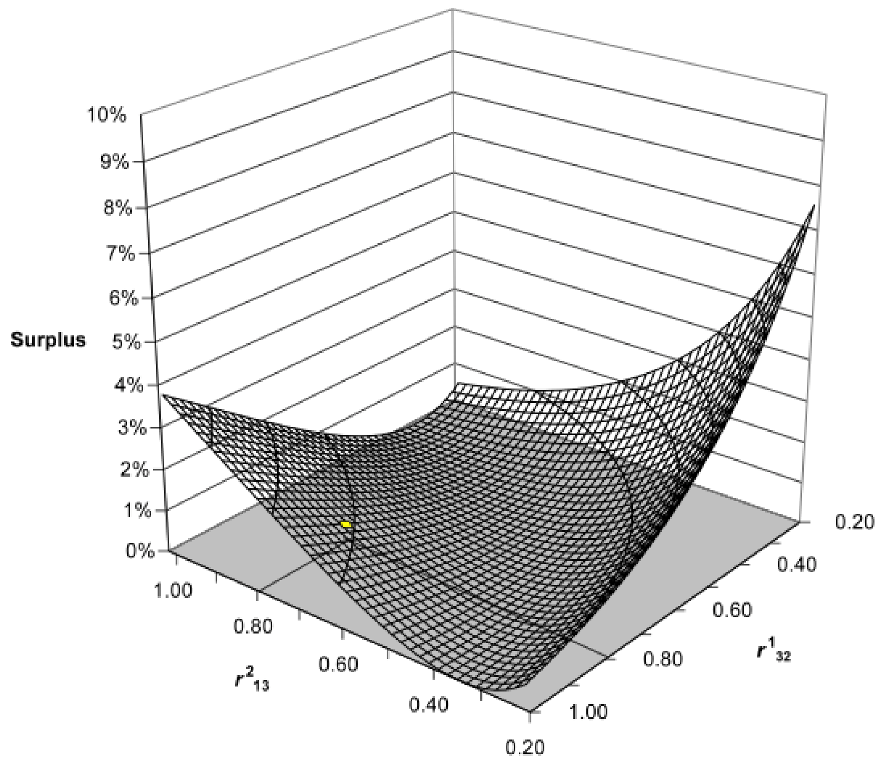

Figure 1

Surplus as a function of proposals (Tideman and Plassmann, 2008)

3. Examples of Five-Agent Peer Evaluation.

3.1 Preliminaries.

We first analyze the peer evaluation mechanism and collusion on the five-agent level. Let $P = \{1, 2, 3, 4, 5\}$ ($|P| = 5$) be the set of participating agents. Agents submit peer evaluations

r=(r^1, r^2, r^3, r^4, r^5) where Agent 1's evaluation is r^1=($r^1_{23}, r^1_{24}, r^1_{25}, r^1_{34}, r^1_{35}, r^1_{45}$) $\in [0, +\infty] \times [0, +\infty] \times [0, +\infty] \times [0, +\infty] \times [0, +\infty] \times [0, +\infty]$. For simplicity, the total amount of money being divided is assumed to be one dollar. Payments are assigned by exact and impartial division function $S_i = \frac{1}{1+\sum_{t\neq i} R_{ti}}\ for\ t \in N\backslash\{i\}$.

The reports that correctly reflect the ratio of actual relative shares for the agents are considered "true" reports. Define true report for agents i and j as q_{ij}. Assume agents know the value of ratios q of the partners they are evaluating but do not know if other agents possess the same information. That means each agent can choose whether to report the value and only he knows if he is telling the truth or lying. If all the partners choose to report truthfully, their reports on a certain pair will all agree, so will the aggregated result for this pair. Therefore, true reports are always consistent provided that q_{ij} are always consistent. If partners do not report honestly, there will be disagreements among the partners. For a true ratio $q_{ij} = m$, if partner k reports $r^k_{ij} = n$ ($n \neq m$), it means k is reporting a false report. The false reports are often inconsistent with other partners' reports before being aggregated. However, the aggregated reports may or may not be consistent. It is the consistency of aggregated reports that determines whether there will be a surplus. One example of peer evaluation profile for five partners is:

$r^3_{12} = r^4_{12} = r^5_{12} = 3 \quad r^2_{13} = r^4_{13} = r^5_{13} = 12 \quad r^2_{14} = r^3_{14} = r^5_{14} = 2 \quad r^2_{15} = r^3_{15} = r^4_{15} = 6$

$r^1_{23} = r^4_{23} = r^5_{23} = 4 \quad r^1_{24} = r^3_{24} = r^5_{24} = \frac{2}{3} \quad r^1_{25} = r^3_{25} = r^4_{25} = 2 \quad r^1_{34} = r^2_{34} = r^5_{34} = \frac{1}{6}$

$r^1_{35} = r^2_{35} = r^4_{35} = \frac{1}{2} \quad r^1_{45} = r^2_{45} = r^3_{45} = 3$

When all the agents are being honest, their reports agree with each other, thus we can conclude the true reports as follows:

$$q_{12} = 3 \quad q_{13} = 12 \quad q_{14} = 2 \quad q_{15} = 6 \quad q_{23} = 4$$

$$q_{24} = \frac{2}{3} \quad q_{25} = 2 \quad q_{34} = \frac{1}{6} \quad q_{35} = \frac{1}{2} \quad q_{45} = 3$$

The evaluations profiles satisfy $q_{12}q_{23}q_{34}q_{45}q_{51} = 1$, are consistent. Then we use the exact, impartial and consensual division function $S_i = \frac{1}{1+\sum_{t\neq i} R_{ti}}$ to calculate each individuals' share. The shares we calculated using true reports are the actual shares each individual partner deserves.

$$S_1 = \frac{1}{1+q_{21}+q_{31}+q_{41}+q_{51}} = \frac{12}{25} \quad S_2 = \frac{1}{1+q_{12}+q_{32}+q_{42}+q_{52}} = \frac{4}{25} \quad S_3 = \frac{1}{1+q_{13}+q_{23}+q_{43}+q_{53}} = \frac{1}{25}$$

$$S_4 = \frac{1}{1+q_{14}+q_{24}+q_{34}+q_{54}} = \frac{6}{25} \quad S_5 = \frac{1}{1+q_{15}+q_{25}+q_{35}+q_{45}} = \frac{2}{25}$$

3.2 Collusion and Coalition Formation.

The incentive property of this mechanism depends on the aggregator. The reporting process prevents individual agents from directly affecting their own ratios. However, with arithmetic mean or geometric mean as aggregator, it is possible for them to improve their evaluations indirectly. Both arithmetic mean and geometric mean are subject to highly inflated ratings. Each offers agents incentive to form a coalition to increase their shares as a group. When a coalition is formed, it gains the ability to affect its own share through highly inflated ratings.

On the coalition level, it is optimal for all the members to behave uniformly since higher evaluations result in a higher payoff given the division function. The coalition can capture a greater share of the money and distribute it among its members. On individual level, the distributive rule is already determined since the relative shares of co-conspirators are reported truthfully by partners outside the coalition, the shares of co-conspirators are determined by division function and distributed accordingly. It is optimal for the colluding partners to increase the gain of coalition as much as possible. The profit maximizing strategy for colluding partners is to rate their co-conspirators as high as possible against agents not in the coalition and report the true values for other pairs. The overall share of the coalition will keep rising until its members increase the false reports up to $+\infty$. Colluding partners could raise their share without deducting the total amount of reward. There would be no surplus of the money due to the conflicting reports since aggregated reports remain consistent.

We first assume that only one coalition forms. The multiple coalitions cases are much more complicated and results are unpredictable. Co-conspirators have to acquire significantly more information to behave strategically in order to benefit from forming a certain kind of coalition of a particular size. The situations where more coalitions emerge will be further discussed later.

Case 1. Suppose 4 out of the 5 partners discreetly form a coalition to improve their share of the dollar by reporting wrong evaluation of the 5[th] partner. For example, partner 1, 2, 3, 4 collude against partner 5. The coalition will try to increase its share by letting co-conspirators act uniformly to rate their group members $+\infty$ compared to partner 5, therefore, the new evaluation profiles are:

$r_{12}^3 = r_{12}^4 = r_{12}^5 = 3$ $r_{13}^2 = r_{13}^4 = r_{13}^5 = 12$ $r_{14}^2 = r_{14}^3 = r_{14}^5 = 2$ $r_{15}^2 = r_{15}^3 = r_{15}^4 = +\infty$

$r_{23}^1 = r_{23}^4 = r_{23}^5 = 4$ $r_{24}^1 = r_{24}^3 = r_{24}^5 = \frac{2}{3}$ $r_{25}^1 = r_{25}^3 = r_{25}^4 = +\infty$ $r_{34}^1 = r_{34}^2 = r_{34}^5 = \frac{1}{6}$

$r_{35}^1 = r_{35}^2 = r_{35}^4 = +\infty$ $r_{45}^1 = r_{45}^2 = r_{45}^3 = +\infty$

The share of each partner also changed. The percentage gain of the co-conspirators is calculated to reflect the relative gain for each partner. Percentage gain for agent *i* is: $P_g^i = \frac{S_i' - S_i}{S_i} \times 100\%$

$S_1' = \frac{1}{1+r_{21}+r_{31}+r_{41}+r_{51}} = \frac{12}{23}$ Percentage gain for agent 1: $P_g^1 = \frac{S_1' - S_1}{S_1} = 8.7\%$

$S_2' = \frac{1}{1+r_{12}+r_{32}+r_{42}+r_{52}} = \frac{4}{23}$ Percentage gain for agent 2: $P_g^2 = \frac{S_2'-S_2}{S_2} = 8.7\%$

$S_3' = \frac{1}{1+r_{13}+r_{23}+r_{43}+r_{53}} = \frac{1}{23}$ Percentage gain for agent 3: $P_g^3 = \frac{S_3'-S_3}{S_3} = 8.7\%$

$S_4' = \frac{1}{1+r_{14}+r_{24}+r_{34}+r_{54}} = \frac{6}{23}$ Percentage gain for agent 4: $P_g^4 = \frac{S_4'-S_4}{S_4} = 8.7\%$

$S_5' = \frac{1}{1+r_{15}+r_{25}+r_{35}+r_{45}} = 0$

The result demonstrates that all the co-conspirators benefit from lying about the evaluation of the 5th partner. The coalition captures the total amount of money and improves its members' outcomes by the same percentage. The evaluations are still consistent. The individual shares of the dollar add up to 1; no money is wasted.

Case 2. Suppose 3 out of the 5 partners discreetly form a coalition to improve their share of the dollar by reporting wrong evaluation of the 4th and 5th partner. For example, partner 1, 2, 3 collude against partner 4, 5. The new evaluation profiles are:

$r_{12}^3 = r_{12}^4 = r_{12}^5 = 3$ $r_{13}^2 = r_{13}^4 = r_{13}^5 = 12$ $r_{14}^2 = r_{14}^3 = +\infty, r_{14}^5 = 2$

$r_{15}^2 = r_{15}^3 = +\infty, r_{15}^4 = 6$ $r_{23}^1 = r_{23}^4 = r_{23}^5 = 4$ $r_{24}^1 = r_{24}^3 = +\infty, r_{24}^5 = \frac{2}{3}$

$r_{25}^1 = r_{25}^3 = +\infty, r_{25}^4 = 2$ $r_{34}^1 = r_{34}^2 = +\infty, r_{34}^5 = \frac{1}{6}$ $r_{35}^1 = r_{35}^2 = +\infty, r_{35}^4 = \frac{1}{2}$

$r_{45}^1 = r_{45}^2 = r_{45}^3 = 3$

There are disagreements in some pair of evaluations. We could either apply arithmetic or geometric mean to aggregate the evaluations. In this case, arithmetic and geometric mean yields the same result. We choose the arithmetic mean as the aggregator here.

$r_{14} = \frac{r_{14}^2+r_{14}^3+r_{14}^5}{3} = +\infty$ $r_{15} = \frac{r_{15}^2+r_{15}^3+r_{15}^4}{3} = +\infty$ $r_{24} = \frac{r_{24}^1+r_{24}^2+r_{24}^5}{3} = +\infty$

$r_{25} = \frac{r_{25}^1+r_{25}^3+r_{25}^4}{3} = +\infty$ $r_{34} = \frac{r_{34}^1+r_{34}^2+r_{34}^5}{3} = +\infty$ $r_{35} = \frac{r_{35}^1+r_{35}^2+r_{35}^4}{3} = +\infty$

Calculate the share of each partner and percentage gain using the new evaluation profile:

$S_1' = \frac{1}{1+r_{21}+r_{31}+r_{41}+r_{51}} = \frac{12}{17}$ Percentage gain for agent 1: $P_g^1 = \frac{S_1'-S_1}{S_1} = 47\%$

$S_2' = \frac{1}{1+r_{12}+r_{32}+r_{42}+r_{52}} = \frac{4}{17}$ Percentage gain for agent 2: $P_g^2 = \frac{S_2'-S_2}{S_2} = 47\%$

$S_3' = \frac{1}{1+r_{13}+r_{23}+r_{43}+r_{53}} = \frac{1}{17}$ Percentage gain for agent 3: $P_g^3 = \frac{S_3'-S_3}{S_3} = 47\%$

$S_4' = \frac{1}{1+r_{14}+r_{24}+r_{34}+r_{54}} = 0$ $S_5' = \frac{1}{1+r_{15}+r_{25}+r_{35}+r_{45}} = 0$

The result of three-partner collusion is similar to the four-partner collusion. The evaluation turns out to be consistent and no money was wasted. Total amount of money is captured and all the co-conspirators gain from colluding by the same percentage. The percentage gain of each co-conspirator increases as the number of co-conspirators decreases.

Case 3. Suppose only 2 out of the 5 partners discreetly form a coalition to improve their share of the dollar by reporting wrong evaluation of the 3rd, 4th and 5th partner. For example, partner 1, 2 collude against partner 3, 4, 5. The new evaluation profiles are:

$r_{12}^3 = r_{12}^4 = r_{12}^5 = 3$ $r_{13}^2 = +\infty, r_{13}^4 = r_{13}^5 = 12$ $r_{14}^2 = +\infty, r_{14}^3 = r_{14}^5 = 2$

$r_{15}^2 = +\infty, r_{15}^3 = r_{15}^4 = 6$ $r_{23}^1 = +\infty, r_{23}^4 = r_{23}^5 = 4$ $r_{24}^1 = +\infty, r_{24}^3 = r_{24}^5 = \frac{2}{3}$

$r_{25}^1 = +\infty, r_{25}^3 = r_{25}^4 = 2$ $r_{34}^1 = r_{34}^2 = r_{34}^5 = \frac{1}{6}$ $r_{35}^1 = r_{35}^2 = r_{35}^4 = \frac{1}{2}$

$r_{45}^1 = r_{45}^2 = r_{45}^3 = 3$

Use arithmetic mean as aggregator to settle the difference:

$r_{13} = \frac{r_{13}^2+r_{13}^4+r_{13}^5}{3} = +\infty$ $r_{15} = \frac{r_{15}^2+r_{15}^3+r_{15}^4}{3} = +\infty$ $r_{24} = \frac{r_{24}^1+r_{24}^2+r_{24}^5}{3} = +\infty$

$r_{25} = \frac{r_{25}^1+r_{25}^3+r_{25}^4}{3} = +\infty$ $r_{23} = \frac{r_{23}^1+r_{23}^4+r_{23}^5}{3} = +\infty$ $r_{14} = \frac{r_{14}^2+r_{14}^3+r_{14}^5}{3} = +\infty$

Calculate the share of each partner and percentage gain using the new evaluation profile:

$S_1' = \frac{1}{1+r_{21}+r_{31}+r_{41}+r_{51}} = \frac{12}{16}$ Percentage gain for agent 1: $P_g^1 = \frac{S_1' - S_1}{S_1} = 56.25\%$

$S_2' = \frac{1}{1+r_{12}+r_{32}+r_{42}+r_{52}} = \frac{4}{16}$ Percentage gain for agent 2: $P_g^2 = \frac{S_2' - S_2}{S_2} = 56.25\%$

$S_3' = \frac{1}{1+r_{13}+r_{23}+r_{43}+r_{53}} = 0$ $S_4' = \frac{1}{1+r_{14}+r_{24}+r_{34}+r_{54}} = 0$ $S_5' = \frac{1}{1+r_{15}+r_{25}+r_{35}+r_{45}} = 0$

The two-partner collusion game yields similar results as previous games. The percentage gain is increasing with the decrease of number of co-conspirators. The two partners successfully captured the money and improved their outcomes by reporting wrong evaluations of other partners. Their percentage gains are also the highest among all forms of collusions.

3.3 Collusion Result for Co-conspirators.

By comparing the result of the three different forms of collusion under five-agent evaluation, we can see that the percentage gain of the co-conspirators is 8.7% for case 1. The number increases to 47% in case 2 and to 56.25% in case 3. The leap in percentage gain from case 1 to case 2 is because the colluding group succeeded in claiming partner 4’s share. Partner 4’s share is $\frac{6}{25}$ which is the second highest share among the five. It is also the highest among the victims. The coalition only moved from 47% to 56.25% by claiming partner 3’s share. We could conclude that the percentage gain is highly associated with the number of colluders and the victim’s share of the dollar. This might lead the co-conspirators to consider who to collude with and what it the optimal size of their coalition. It is obvious that with fewer people colluding, the co-conspirators gain more percentage-wise. The best size for the colluding group is two people given the division rules and aggregator. Partners with the least amount share of the dollar can benefit the most from colluding. Therefore, in this particular five-agent peer evaluation, partner 3 and partner 5 can form a coalition and benefit the most from colluding than any other possible teams.

4. Collusion in N-Agent Peer Evaluation.

***Proposition 1**:* Colluding partners can capture the total amount of money being divided.

Proof. Define: $N = \{1, \dots, n\}$, V= Partners do not belong to the colluding group.

$V = \{a, b, c, d \dots\}, (V \subset N)$. Payment for agent *a* : $S_a = \frac{1}{1+\sum_{t\neq a} R_{ta}}$ for $t \in N\backslash\{a\}$.

For agent *a,* the evaluation comes from both agents in and outside the coalition:

$$\sum_{t\neq a} R_{ta} = \sum_{t\notin V} R_{ta} + \sum_{t\in V} R_{ta}$$

When collusion happens, the aggregated reports for agents in set *V* are $+\infty$:

$$R'_{ba} = R'_{ca} = R'_{da} = R'_{fa} = \cdots = +\infty$$

Therefore: $\sum_{t\in V} R'_{ta} = +\infty$ *and* $\sum_{t\in V} R'_{ta} = \sum_{t\in V} R_{ta} = +\infty$

$$S'_a = \frac{1}{1+\sum_{t\notin V} R'_{ta}+\sum_{t\in V} R'_{ta}} = \frac{1}{1+\sum_{t\notin V} R_{ti}+\infty} = 0$$

For all agents in set *V* : $S'_a = S'_b = S'_c = S'_d = \cdots = 0$

The evaluations are consistent; the total amount of money is 1: $\sum_{t\notin V} S_t + \sum_{t\in V} S_t = 1.$

From the collusion we know that: $\sum_{t\in V} S_t = \sum_{t\in V} S'_t = S'_a + S'_b + S'_c + S'_d + \cdots = 0.$

Therefore: $\sum_{t\notin V} S_t = 1$

Colluding partners capture the total amount of money.

***Proposition 2**:* Colluding partners improve their share equally percentage-wise.

Proof. $N = \{1, \dots, n\}$, V= Partners do not belong to the colluding group.

$V = \{a, b, c, d \dots\}, (V \subset N)$ *and* $i, j \notin (N\backslash V)$.

Payment for agent *i* : $S_i = \frac{1}{1+\sum_{t\neq i} R_{ti}}$ for $t \in N\backslash\{i\}$ Percentage gain for agent *i* : $P_g^i = \frac{S'_i - S_i}{S_i}$

For agent *i,* the evaluation comes from both agents in and outside the coalition:

$$\sum_{t \neq i} R_{ti} = \sum_{t \notin V} R_{ti} + \sum_{t \in V} R_{ti}$$

When collusion happens, the aggregated reports for agents outside the coalition are zeros:

$$R'_{ai} = R'_{bi} = R'_{ci} = R'_{di} = \cdots = 0,$$

Therefore: $\sum_{t \in V} R'_{ti} = 0 \;\; and \;\; \sum_{t \notin V} R'_{ti} = \sum_{t \notin V} R_{ti}$

The share for agent *i* is: $S'_i = \frac{1}{1+\sum_{t \notin V} R'_{ti} + \sum_{t \in V} R'_{ti}} = \frac{1}{1+\sum_{t \notin V} R_{ti} + 0}$

The percentage gain for agent *i* is:

$$P_g^i = \frac{S'_i}{S_i} - 1 = \frac{(1+\sum_{t \neq i} R_{ti}) - (1+\sum_{t \notin V} R_{ti})}{1+\sum_{t \notin V} R_{ti}} = \frac{\sum_{t \in V} R_{ti}}{1+\sum_{t \notin V} R_{ti}}$$

Same procedure holds for agent *j* : $P_g^j = \frac{\sum_{t \in V} R_{tj}}{1+\sum_{t \notin V} R_{tj}}$

Comparing the percentage gain between *i* and *j*

$$P_g^i - P_g^j = \frac{\sum_{t \in V} R_{ti}}{1 + \sum_{t \notin V} R_{ti}} - \frac{\sum_{t \in V} R_{tj}}{1 + \sum_{t \notin V} R_{tj}} = \frac{\sum_{t \in V} R_{ti} \cdot \sum_{t \notin V} R_{tj} - \sum_{t \notin V} R_{ti} \cdot \sum_{t \in V} R_{tj}}{(1 + \sum_{t \notin V} R_{ti}) \cdot (1 + \sum_{t \notin V} R_{tj})}$$

Given the property of the evaluations: $R_{tj} = R_{ti} \cdot R_{ij}$

$$\sum_{t \in V} R_{ti} \cdot \sum_{t \notin V} R_{tj} = \sum_{t \notin V} R_{ti} \cdot \sum_{t \in V} R_{tj}$$

Therefore: $$P_g^i - P_g^j = 0$$

Colluding partners improve their share equally percentage-wise.

Proposition 3: The percentage gain of the colluding partners increases as the number of colluding agents decreases.

Proof. Define: $V = \{a, b, c, d \dots\}$ = Partners do not belong to the colluding group. Let *n* = total number of partners $(n \geq 3)$. There are $n - 2$ different sizes of collusion for one coalition case. Let *x* and *y* be the number of co-conspirators in any two different forms of collusion and satisfies:

$$2 \leq x < y \leq n - 2$$

$V_x = \{a, b, c, d \dots\}$ = Set of partners do not collude with *x* co-conspirators

$V_y = \{a, b, c, d \dots\}$ = Set of partners do not collude with *y* co-conspirators

$$P_g^i(x) = \frac{\sum_{t \in V_x} R_{ti}}{1+\sum_{t \notin V_x} R_{ti}} \qquad P_g^i(y) = \frac{\sum_{t \in V_y} R_{ti}}{1+\sum_{t \notin V_y} R_{ti}}$$

Compare Percentage gain of player *i* under the two forms of collusions.

$$P_g^i(x) - P_g^i(y) = \frac{\sum_{t \in V_x} R_{ti}}{1 + \sum_{t \notin V_x} R_{ti}} - \frac{\sum_{t \in V_y} R_{ti}}{1 + \sum_{t \notin V_y} R_{ti}} = \frac{\Phi}{(1 + \sum_{t \notin V_x} R_{ti}) \cdot (1 + \sum_{t \notin V_y} R_{ti})}$$

$$\Phi = \sum_{t \in V_x} R_{ti} - \sum_{t \in V_y} R_{ti} + \sum_{t \in V_x} R_{ti} \cdot \sum_{t \notin V_y} R_{ti} - \sum_{t \notin V_x} R_{ti} \sum_{t \in V_y} R_{ti}$$

Given that: $2 \leq x < y \leq n - 2$

$$\sum_{t \in V_x} R_{ti} > \sum_{t \in V_y} R_{ti} \ and \sum_{t \in V_x} R_{ti} \cdot \sum_{t \notin V_y} R_{ti} > \sum_{t \notin V_x} R_{ti} \cdot \sum_{t \in V_y} R_{ti}$$

$$P_g^i(x) > P_g^i(y)$$

Therefore $P_g = \frac{\sum_{t \in V} R_{ti}}{1+\sum_{t \notin V} R_{ti}}$ is a decreasing mapping for the number of co-conspirators. The percentage gain of the colluding partners increases as the number of colluding agents decreases.

The conclusion for five-agent collusion results holds in the *n* player game as well. All co-conspirators have the incentive to maximize $P_g = \frac{\sum_{t \in V} R_{ti}}{1+\sum_{t \notin V} R_{ti}}$ by either maximize $\sum_{t \in V} R_{ti}$ or minimize $1 + \sum_{t \notin V} R_{ti}$ Therefore, partners that faces the lowest share can benefit the most from forming a coalition to collude against the rest partners. The optimal number of colluding partners will still be two given the division rule and aggregator in the n partner game.

5. Collusion in Peer Evaluation as a Constant-sum Game.

We now examine the collusion in peer evaluation in the context of cooperative game theory to gain deeper insight of how partners behave strategically as individuals as well as a coalition. There are also other useful tools to evaluate partners' performance. The collusion in peer evaluation could be viewed as a game in coalitional form. All the partners are players and their payoffs are their share of the dollar assigned by the division function. Their strategy is either to report truthfully or to submit wrong reports. We will first set up the model and demonstrate some characteristics of the game. We will then introduce coalition value as another tool to evaluate co-conspirators' performance through collusion. We will again analyze the example of the five-agent game and then generalize the results to n-agents ($n > 3$).

5.1 One Coalition.

Let $N = \{1, 2, 3, 4, 5\}$. Define S as a subset of N: $S \subset N$ and a coalition C to be a subset of N that has at least 2 elements. The set of all subsets is denoted by 2^N. There are five players; the total number of subsets is 32. The characteristic function *v* of the game is defined on the set 2^N. For $i = 1, 2, 3, 4, 5$ X_i is the strategy set for player *i* and $u_i(x_1 \dots x_5)$ is the payoff function for player *i*. x_i is the evaluation vector reported by player *i*. In order to maximize his share through collusion, there is no reason for the colluding partner to report a number that is less than $+\infty$ if $+\infty$ is allowed. Therefore, the evaluations can either equal to the true value or take the value $+\infty$ assuming that only partners in the coalition are aware of the existence of it. Then define *v(S)* for each $S \in 2^N$ as the coalition value of the subset *S*. Let coalition value be $v(S) = \text{Val}(\sum_{i \in S} u_i\,(x_1 \ldots x_5))$. For partner *i,* the payoff function is:

$$u_i(x_1 \ldots x_5) = \begin{cases} \dfrac{1}{1 + \sum_{t \in C} R_{ti}} & \textit{when i is in the coalition} \\ v(\{i\}) = 0 & \textit{when i is not in the coalition} \end{cases}$$

$x_1 \ldots x_5$ is as following:

$x_1 = (r_{23}^1, r_{34}^1, r_{45}^1, r_{25}^1, r_{24}^1, r_{35}^1)$ $x_2 = (r_{13}^2, r_{14}^2, r_{15}^2, r_{34}^2, r_{45}^2, r_{35}^2)$ $x_3 = (r_{12}^3, r_{14}^3, r_{15}^3, r_{24}^3, r_{25}^3, r_{45}^3)$

$x_4 = (r_{12}^4, r_{13}^4, r_{15}^4, r_{23}^4, r_{25}^4, r_{35}^4)$ $x_5 = (r_{12}^5, r_{13}^5, r_{14}^5, r_{23}^5, r_{24}^5, r_{34}^5)$

R_{ij} is defined to be the aggregated result from all the reported values of r_{ij}. The aggregator is arithmetic or geometric mean.

In peer evaluation games, $\sum_{i=1}^{5} v(\{i\}) = v(N) = 1$. Therefore it is an inessential game. This means there is an imputation $\boldsymbol{x} = (v(\{1\}), v(\{2\}), v(\{3\}), v(\{4\}), v(\{5\}))$ so that $\boldsymbol{x}$ is the solution to the game. Each player can expect what he deserves. In this case, $\boldsymbol{x} = \left(\frac{12}{25}, \frac{4}{25}, \frac{1}{25}, \frac{6}{25}, \frac{2}{25}\right)$. The evaluations are consistent given that they can only take true value or $+\infty$. No money is wasted in five-agents peer evaluation; the 1 dollar is always completely divided up among the partners with no surplus. We have $\sum_{i \in S} u_i\,(x_1 \ldots x_5) = 1$. It is also true that $u_i(x_1 \ldots x_5) = 0$ for any *i* not in C. Coalitional form peer evaluation game is therefore a constant-sum game.

When all the agents are being honest and report the true values of the evaluations, it is the same as all the partners together form the grand coalition N. The true values of the evaluations are:

$r_{12}^3 = r_{12}^4 = r_{12}^5 = 3$ $r_{13}^2 = r_{13}^4 = r_{13}^5 = 12$ $r_{14}^2 = r_{14}^3 = r_{14}^5 = 2$ $r_{15}^2 = r_{15}^3 = r_{15}^4 = 6$

$r_{23}^1 = r_{23}^4 = r_{23}^5 = 4$ $r_{24}^1 = r_{24}^3 = r_{24}^5 = \frac{2}{3}$ $r_{25}^1 = r_{25}^3 = r_{25}^4 = 2$ $r_{34}^1 = r_{34}^2 = r_{34}^5 = \frac{1}{6}$

$r_{35}^{1} = r_{35}^{2} = r_{35}^{4} = \frac{1}{2} \quad r_{45}^{1} = r_{45}^{2} = r_{45}^{3} = 3$

The grand coalition is $\{1, 2, 3, 4, 5\}$. Then we use the exact and impartial division function $S_i = \frac{1}{1+\sum_{t \neq i} R_{ti}}$ to assign the share to each partner. The payoff for each partners are:

$u_1(x_1, x_2, x_3, x_4, x_5) = \frac{12}{25} \quad u_2(x_1, x_2, x_3, x_4, x_5) = \frac{4}{25} \quad u_3(x_1, x_2, x_3, x_4, x_5) = \frac{1}{25}$

$u_4(x_1, x_2, x_3, x_4, x_5) = \frac{6}{25} \quad u_5(x_1, x_2, x_3, x_4, x_5) = \frac{2}{25}$.

The value of the grand coalition is $v(N) = u_1 + u_2 + u_3 + u_4 + u_5 = 1$

Case 1. Partner 1, 2, 3, 4 forms a coalition and leave out partner 5. The coalition in this situation is $C = \{1, 2, 3, 4\}, \{5\}$. The payoff for each partners are: $u_1'(x_1', x_2', x_3', x_4', x_5') = \frac{12}{23}$

$u_2'(x_1', x_2', x_3', x_4', x_5') = \frac{4}{23} \quad u_3'(x_1', x_2', x_3', x_4', x_5') = \frac{1}{23} \quad u_4'(x_1', x_2', x_3', x_4', x_5') = \frac{6}{23}$

$u_5'(x_1', x_2', x_3', x_4', x_5') = 0$. The value of the coalition C is $v(C) = u_1' + u_2' + u_3' + u_4' = 1$

Case 2. Partner 1, 2, 3 forms a coalition and leave out partner 4, 5. The coalition in this situation is $C = \{1, 2, 3\}, \{4\}, \{5\}$. The payoff for each partners are: $u_1'(x_1', x_2', x_3', x_4', x_5') = \frac{12}{17}$

$u_2'(x_1', x_2', x_3', x_4', x_5') = \frac{4}{17} \quad u_3'(x_1', x_2', x_3', x_4', x_5') = \frac{1}{17} \quad u_4'(x_1', x_2', x_3', x_4', x_5') = 0$

$u_5'(x_1', x_2', x_3', x_4', x_5') = 0$. The value of the coalition C is $v(C) = u_1' + u_2' + u_3' = 1$

Case 3. Partner 1, 2 forms a coalition and leave out partner 3, 4, 5. The coalition in this situation is $C = \{1, 2\}, \{3\}, \{4\}, \{5\}$. The payoff for each partners are: $u_1'(x_1', x_2', x_3', x_4', x_5') = \frac{12}{16}$

$u_2'(x_1', x_2', x_3', x_4', x_5') = \frac{4}{16} \quad u_3'(x_1', x_2', x_3', x_4', x_5') = 0 \quad u_4'(x_1', x_2', x_3', x_4', x_5') = 0$

$u_5'(x_1', x_2', x_3', x_4', x_5') = 0$. The value of the coalition C is $v(C) = u_1' + u_2' = 1$

By comparing the value of the coalitions across different forms of collusion in the five-agent peer evaluation. We can see that the value of coalition remains the same and equals the total amount of money that is being divided.

Proposition 4: The coalition value, regardless of the size of the coalition, remains the same if only one coalition is allowed to form.

Proof. Define S as a subset of N: $S \subset N$ and a coalition C to be a subset of N that has at least 2 elements. The set of all subsets is denoted by 2^N. Define *v(S)* for each $S \in 2^N$ as the coalition value of *S*. Define $v(S) = \text{Val}(\sum_{i \in S} u_i\,(x_1 \dots x_n))$ to be a non-negative extended real value set function. The characteristic function *v* of the game is defined on the set 2^N and satisfies the following:

(1) $v(\phi) = 0$ (2) If S and T are disjoint coalitions, then $v(S) + v(T) \leq v(S \cup T)$

For $i = 1, 2 \dots n$. X_i is the strategy set for player *i*. The evaluations can either equal to the true value or take the value $+\infty$. $x_i = (r_{jk}^i \dots r_{mn}^i \dots)$. The division functioned used to assign individual shares is $S_i = \frac{1}{1+\sum_{t \neq i} R_{ti}}\ for\ t \in N\backslash\{i\}$.

$u_i(x_1 \dots x_n)$ is the payoff function for player *i* defined on 2^N :

$$u_i(x_1 \dots x_n) = \begin{cases} \dfrac{1}{1 + \sum_{t \in C} R_{ti}} & when\ i\ is\ in\ the\ coalition \\ v(\{i\}) = 0 & when\ i\ is\ not\ in\ the\ coalition \end{cases}$$

The n-agents peer evaluation is a constant-sum game ($n \geq 3$). When all the partners are being honest. $\sum_{i \in N} u_i\,(x_1 \dots x_n) = v(N) = 1$. Let *C* be the coalition that has no less than 2 players. Let *p* be the number of elements in *C* ($n \geq p \geq 2$).

The value of coalition is $v(C) = \text{Val}(\sum_{i=1}^{p} u_i'\,(x_1' \dots x_p'))$

$$\sum_{i=1}^{p} u_i'\,(x_1' \dots x_p') = \sum_{i=1}^{p} \frac{1}{1+\sum_{t \in C} R_{ti}} \quad \sum_{i=1}^{n} u_i\,(x_1 \dots x_n) = \sum_{i=1}^{n} \frac{1}{1+\sum_{t \neq i} R_{ti}}$$

The exactness property of the division function: $\sum_{i=1}^{n} \frac{1}{1+\sum_{t\neq i} R_{ti}} = \sum_{i=1}^{p} \frac{1}{1+\sum_{t\in C} R_{ti}} = 1$

$$\sum_{i=1}^{p} u_i' \left(x_1' \dots x_p'\right) = \sum_{i=1}^{n} u_i \left(x_1 \dots x_n\right) = 1$$

Therefore, for any form of collusion in the n-player peer evaluation, the value of the coalition

$$v(C) \equiv 1$$

5.2 Multiple Coalitions.

The existence of multiple coalitions makes it more challenging for the colluding agents to decide how to behave. If agents maintain their strategy from the one coalition case, the reports would often be inconsistent, which often results in a great loss of the total amount of reward. There are cases where colluding agents could still gain a significant amount, however, it is much harder to achieve under the presence of multiple coalitions. When inconsistency happens, it may cause agents to receive less money than they actually deserve by reporting honestly. There may be cases where they end up with self-conflicting conclusions, which make it impossible to divide the reward. A short story would illustrate the idea. Suppose Irene and two of her friends Jack and Kate are among the peer evaluating partners. Jack and Kate are not friends and joined two distinct coalitions. They both invited their mutual friend Irene to join their coalitions. Irene accepted both offers and joined both teams. When agents submit their evaluation results. Irene faces a problem. Since she is in Jacks coalition, she is supposed to assign Jack $+\infty$ against everyone not in Jack's side. She is also in Kate's coalition, she again should assign Kate $+\infty$ against everyone not in Kate's side. Irene ends up in a self-conflicting situation, she should assign Jack $+\infty$ against Kate and also Kate $+\infty$ against Jack. Since the rating of an ordered pair

r_{jk} could not be both $+\infty$ and zero, the evaluations could not be aggregated at this stage and no money is divided.

Besides situations in which the reports could not be aggregated, such as ones caused by Irene's dilemma. Another possible outcome is that $R_{ij} = \infty$ for all players for some i and j. Every partner is assigned a share of zero, thus all the money is wasted. In the more general situations, results are more complex and unstable when multiple coalitions emerge. In the case of two coalitions, denoted as A and B. They could come from a partition of N ($A \cup B = N$). They could also be mutually exclusive sets and both are proper subset of N ($A \subsetneq N, B \subsetneq N \; and \; A \cap B = \phi$). The situation that some players are in both A and B ($A \cap B \neq \phi$) is possible as well. Another possibility would be A or B is a subset of the other ($A \subset B \; or \; B \subset A$). The situation becomes increasingly complicated as the number of coalitions increases. Colluding partners under such situations do not have a dominant strategy as they did at the one coalition case. The evaluations reported often come out inconsistent thus generating a huge surplus of the money. Colluding players would need to have enormous amount of information to increase their share through colluding. Therefore, it requires much more effort and information for a group of colluding partners to improve their shares when more than one coalition emerges under the mechanism by De Clippel et al (2008).

6. Mitigating Collusion in Peer Evaluation.

The previous sections demonstrated that the peer evaluation mechanism by De Clippel et al (2008) was incentive compatible but not coalition strategy proof with ordinary arithmetic and geometric mean aggregators. Participating agents have the incentive to form a coalition to indirectly improve their individual shares. The colluders' strategy is trivial; they would rate their partner infinity against agents not in their coalition. The coalition could easily extract the victim's money and improve its own shares. The adopted aggregator failed to detect and prohibit such behavior. In this section, we will discuss some possible methods such as hiring monitor and changing aggregator to solve the collusion problem that arises in peer evaluation.

6.1 Monitor.

One way to discourage partners from submitting highly inflated reports is to hire an unbiased monitor to audit the aggregating process. The monitor has to come from a party that does not have any involvements with any of the participating agents. The monitor could examine the reports submitted by the agents and assess if they are consistent to detect dishonest reports. If several reports about a particular pair i and j are obviously highly inflated or deflated, then the reports are likely to be dishonest and should not be further aggregated. The monitor could detect some misbehavior such as the profit-maximizing strategy by the colluding partners.

Although the extra evaluation step by the monitor would cause co-conspirators to adjust their reports towards the true values, there could still be noises in the reports and the size of deviations are not clear. The existence of a monitor does not necessarily guarantee truth telling. The judgment of the monitor is subjective and may be false occasionally. Another fact that makes hiring the monitor not so appealing to the partners is that hiring the monitor is not free; it results in a deduction of the total amount of reward. Although hiring a monitor eliminates certain kinds of misbehaviors including extreme reports like $+\infty$ and 0 when they are clearly outliers. The effectiveness of this method is limited. It is unclear how close this method would lead the partners to tell the truth. They could still report slightly inflated reports to increase their share even though that decreases the total amount of money. The cost of using this method would also make it less favorable to the partners.

6.2 Changing Aggregator.

There are in principle infinitely many ways to aggregate individual proposals. However, only a subset of aggregators is realistic and practical. De Clippel et al. (2008) proved that the sum of the individual shares does not exceed 1 for any set of proposed share ratios if and only if the any pair of reported values satisfies the condition $R_{ij}R_{ji} \geq 1$. This result serves as an important criterion for selecting appropriate aggregators. $R_{ij}R_{ji} \geq 1$ holds for both arithmetic and geometric mean. However, from the previous sections we observe that arithmetic and geometric aggregator are subject to extreme values. Colluding partners can easily manipulate aggregated results to favor themselves. Another possible way to mitigate such behavior is to adjust the aggregator to aggregate proposals. In this section, we will analyze two different aggregators that might ameliorate the collusion problem that emerges with ordinary arithmetic/geometric aggregator. The two candidates are: harmonic mean and geometric median.

6.2.1 Harmonic Mean.

The harmonic mean is the reciprocal of the arithmetic mean of the reciprocals of individual reports. The formula for aggregated report R_{ij} using harmonic mean for partner *i* and *j* is $R_{ij} = \left(\frac{1}{n-2} \cdot \sum_{k \neq i,j} \frac{1}{r_{ij}^k}\right)^{-1}$. If colluding partners maintain their strategy and report infinity against partners outside their coalition, the infinity reports vanish during the aggregating process since harmonic mean sums up reciprocals of individual reports.

Using harmonic mean as an aggregator certainly discourages highly inflated reports. In fact, colluding partners do not necessarily benefit from reporting a proposal that deviates greatly from its true value. It is unclear how much percentage or absolute gain they would have by reporting false values. The highly inflated reports no longer dominate and would instead induce inconsistency.

Although aggregated results would not be dominated by a single or several extreme reports. Such reports would still inflate the aggregated result, which may favor the colluding agents. Because highly inflated reports reduce the size of denominator and raise the value of the whole ratio. The profit maximizing strategy for colluding partners is to increase their aggregated result as much as possible; they have to therefore reduce the size of the denominator. In order to introduce multiple zeros into the denominator and raise the aggregated reports, the co-conspirators are required to form a large coalition. The previous analysis showed that the co-conspirators could raise their share dramatically if the size of the coalition is small. With a large coalition, each of the colluding partners benefit far less. Furthermore, if colluding partners maintain a relatively “small” coalition as well as the reporting strategy, the property of harmonic mean guarantees that aggregated result converges to true report when the number of participating agents increases.

The harmonic mean partially solves the collusion problem that arises with arithmetic and geometric mean. It is resistant to highly inflated reports and discourages coalition formation when n is large. The colluding partners face difficulty to manipulate harmonic mean to favor their coalition. Regardless of the size of the coalition, they are not able to capture the total amount of money. They will eventually either not to increase their share at all or capture a really small percentage gain depending on the coalition size and relative sizes of their actual true reports. It would be much less appealing to collude. However, for small partnership, the collusion problem remains. They could increase their share with a reasonable percentage gain since harmonic mean could not eliminate the impact of highly inflated reports at this level.

Tideman and Plassmann (2008) also suggested that harmonic mean is not a good aggregator since it does not satisfy the $R_{ij}R_{ji} \geq 1$ condition which might lead the sum of shares to exceed 1.

6.2.2 Geometric Median.

The geometric median of a set of sample points is the point that minimizes the sum of distance to the sample points. It is defined as $R_{ij} = argmin \sum_{k \neq i,j} \left\| r_{ij}^{k} - y \right\|_2$ where y is the argument that minimizes the sum of the Euclidean distances of the sample points. Geometric median is highly resistant to inflated reports and satisfies the $R_{ij}R_{ji} \geq 1$ condition.

Tideman and Plassmann (2008) showed that geometric median is a preferred aggregator when $n \geq 5$ since it prevents the aggregated result from being manipulated by extreme reports. Since geometric median has a high degree of resistance to strategizing, individuals have less opportunity to manipulate the aggregated results through their infinity reporting strategy.

With the presence of a coalition, the only way for co-conspirators to increase one's aggregated result dramatically is to form a very large coalition such that its size is close to the grand coalition. However, the gains for the members are so small that is undesirable for them to do so. Geometric median also avoided the problem that harmonic mean had. It is highly resistant to misbehaviors like highly inflated reports even at small partnership level. A disadvantage of implementing geometric median aggregator is that it is more computational complicated than other aggregators. It requires more steps and computing powers to calculate especially with large partnership. Despite the computational complexity, geometric median is the most effective aggregator out of the four aggregators we have mentioned.

7. Conclusion.

We have explored and analyzed the effect of collusion and coalition formation under the peer evaluation mechanism by De Clippel et al (2008) with particular aggregators. We demonstrated that arithmetic and geometric mean aggregators are vulnerable to collusion. The co-conspirators could take fully advantage of the arithmetic and geometric mean aggregators under a simple contract. The aggregated results are dominated by extreme reports from the colluding partners. They could extract the shares from the victims and increase their own share equally percentage wise. For colluding partners as a whole, the value of the coalition is constantly one, which means the colluding partners always extract the total amount of reward and spilt it among themselves. The collusion results are no longer valid when multiple coalitions emerge. The complexity of several coalitions case could lead the mechanism into trouble and is unfavorable to the agents.

In the discussion of possible methods to mitigate such misbehaviors, we examined the effectiveness of monitor hiring and changing aggregators. It is worth mentioning that out of the four aggregators we have discussed, none of them is perfect in collusion mitigation. Geometric median is the most attractive one that serves our purpose of discouraging coalition formation.

However, adopting the geometric median aggregator is not the only way to offer incentive for truth telling; there are infinitely many other options for us to explore. Aside from the methods we have discussed, another direction to construct coalition strategy proof mechanism that follows from here could be a modification of the division function. Such modification may cause the loss of exactness, impartiality, or consensuality properties. There is a tradeoff between coalition strategy-proofness and other properties of the division function. We have modeled the peer evaluation as a one-shot game. A possible extension from there could be turning the peer evaluation process into a repeated game and analyzing the equilibrium. A mechanism that evolves over time could probably help reach our goal as a coalition strategy proof mechanism.

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