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Asset Pricing with Heterogeneous Agents, Incomplete Markets and Trading Constraints

Tsvetanka Karagyozova
University of Connecticut and University of British Columbia

University of Connecticut

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341 Mansfield Road, Unit 1063
Storrs, CT 06269–1063
Phone: (860) 486–3022
Fax: (860) 486–4463
http://www.econ.uconn.edu/

Abstract

The consumption capital asset pricing model is the standard economic model used to capture stock market behavior. However, empirical tests have pointed out to its inability to account quantitatively for the high average rate of return and volatility of stocks over time for plausible parameter values. Recent research has suggested that the consumption of stockholders is more strongly correlated with the performance of the stock market than the consumption of non-stockholders. We model two types of agents, non-stockholders with standard preferences and stock holders with preferences that incorporate elements of the prospect theory developed by Kahneman and Tversky (1979). In addition to consumption, stockholders consider fluctuations in their financial wealth explicitly when making decisions. Data from the Panel Study of Income Dynamics are used to calibrate the labor income processes of the two types of agents. Each agent faces idiosyncratic shocks to his labor income as well as aggregate shocks to the per-share dividend but markets are incomplete and agents cannot hedge consumption risks completely. In addition, consumers face both borrowing and short-sale constraints. Our results show that in equilibrium, agents hold different portfolios. Our model is able to generate a time-varying risk premium of about 5.5explanation for the equity premium puzzle reported by Mehra and Prescott (1985).

Journal of Economic Literature Classification: G12, E44

Keywords: asset pricing, equity premium puzzle, prospect theory, heterogeneous agents

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I Introduction

The Consumption Capital Asset Pricing Model (CCAPM) developed by Lucas (1978) and Breeden (1979) is the standard economic framework for modeling security prices. Lucas (1978) provides a general equilibrium framework for asset pricing in an exchange economy. Assuming a one-good economy with rational identical agents, Lucas shows that in equilibrium trade does not occur as it is optimal for the representative agent to hold the asset he is endowed with and to consume the dividend. Thus, the model fails to answer the question of what drives trades in financial markets. However, this is not the only drawback of the model. The resulting Euler equations provide a tool for empirical tests of the model. Such tests have failed to validate the model (Hansen and Singleton, 1982; Hansen and Jagannathan, 1991; Ferson and Constantinides, 1991).

Mehra and Prescott (1985) demonstrate the inability of the model to generate the high risk premium of a representative portfolio of risky assets over relatively riskless assets observed in US historical data for plausible values of model parameters. While the historical real rate of return on a market portfolio of risky assets (such as Standard and Poor's 500 Composite Stock Index) has exceeded the real rate of return on relatively riskless assets (such as 3-month T-bills) by about 6% per year, Mehra and Prescott demonstrate that the CCAPM cannot generate a risk premium of more than 0.35% for the range of plausible parameter values that they assume. The result of their empirical test of the CCAPM was so striking, that they termed it the "equity premium puzzle". Subsequent empirical tests have shown that the equity premium puzzle is neither a sample-period phenomenon (Siegel, 1992; Mehra, 2003), nor a country-specific phenomenon (Dimson, Marsh, and Staunton, 2006; Campbell, 2003; Mehra and Prescott, 2003; however, Jorion and Goetzmann, 1999, find that the equity premium puzzle is largely a US phenomenon).

The equity premium is a puzzle only if we accept the restriction on the coefficient of relative risk aversion imposed by Mehra and Prescott. They suggested that the coefficient of relative risk aversion should not exceed 10 to be considered plausible. However, some empirical studies imply that people are more risk averse than economists believe and a coefficient of risk aversion as high as 30 is not implausible if small stakes are involved (see for example Kandel and Stambaugh, 1990). However, it is a general belief that an explanation of the equity premium puzzle should entail a low value of the coefficient of relative risk aversion. Lucas (1994) states that an explanation of the puzzle employing a coefficient of risk aversion greater than 2.5 is "likely to be widely viewed as a resolution that depends on a high degree of risk aversion" (p. 335).

In the core of the puzzle is the definition of risk. While the Capital Asset Pricing Model (CAPM) developed independently by Sharpe (1964), Lintner (1965), and Mossin (1966) defines risk as the covariance of a stock return with the return on a market portfolio, CCAPM defines risk as the covariance of consumption growth with the market return. Empirically, the puzzle is driven by the low correlation of stock market returns with the aggregate consumption or the low "quantity" of risk. Thus, stocks are not sufficiently risky to generate the high historical return and therefore, the price of risk or the coefficient of risk aversion must be high to reconcile the risk premium generated by the model with its historical counterpart.

A high coefficient of risk aversion resolves the equity premium puzzle but it gives rise to another puzzle as pointed out by Weil (1989). The standard preferences used in macroeconomics link the coefficient of risk aversion with the elasticity of intertemporal substitution. If an agent is highly risk averse he dislikes variability in consumption across states and requires a large premium to invest in stocks. As the elasticity of intertemporal substitution is the inverse of the coefficient of risk aversion, a risk averse agent dislikes variation in consumption across time as well. Yet, people do save enough at the low risk-free rate to generate an average growth rate of consumption of about 2% per year. This anomaly has been dubbed the "risk-free rate puzzle" by Weil.

The seminal work of Mehra and Prescott has stemmed a large volume of theoretical and empirical studies. This huge body of literature indicates the importance of the topic. Not only do we not have a model that is able to shed light on the return differentials across assets but the two puzzles point out to our inability to explain aggregate economic phenomena. As pointed out by Kocherlakota (1996, p. 33), "the risk free rate puzzle indicates that we do not know why people save even when returns are low: thus our models of aggregate savings behavior are omitting some crucial element"; the equity premium puzzle indicates that "we cannot hope to give a meaningful answer to R. Lucas' (1987) question about how costly individuals find business cycle fluctuations in consumption growth."

The literature on the equity premium puzzle can be divided into two broad categories: (1) Research that looks closely at the historical data used by Mehra and Prescott and claim that in fact, the equity premium is not as large as it is generally believed because of measurement errors in the data. For example, McGrattan and Prescott (2003) attribute the large equity premium in US data reported by Mehra and Prescott to taxes, regulatory constraints, and diversification costs. (2) Research that does not question the reliability of historical data but suggests that the equity premium puzzle can be attributed to the underlying assumptions of the model. As a result, a number of modifications and generalizations relaxing the assumptions of Mehra and Prescott have been offered: time-nonseparable preferences (Hansen and Constantinides, 1991; Heaton, 1993); recursive preferences (Weil, 1989; Epstein and Zin, 1991); state-nonseparable preferences (Nason, 1988; Abel, 1990); rare-event declines in aggregate consumption (Rietz, 1988); transaction costs (Luttmer, 1993); combined assumptions of consumer heterogeneity and incomplete consumption insurance (Mehra and Prescott, 1985). However, none of these alternatives can overcome the drawbacks of the original Lucas model without posing further complications (for surveys of the literature, see Kocherlakota, 1996; Cochrane, 1997; Mehra and Prescott, 2003). Our model contributes to this second body of research on the equity premium puzzle.

Mankiw and Zeldes (1991) raise an important objection to the empirical tests of the CCAPM. In the United States roughly only a third of the population holds stocks. Therefore, empirical tests based on the Euler equations of the model which employ aggregate consumption data are doomed to fail unless the consumption processes of stockholders and non-stockholders are highly correlated. They find that the two consumption processes differ substantially and "failures of the consumption CAPM might be rationalized by a model with two groups of consumers: stockholders and non-stockholders" (p. 99).

A related issue is that models which allow for agent heterogeneity typically use the same utility function to describe the individual preferences. However, there is no a priory reason that would lead us to believe that stockholders and non-stockholders have identical preferences. Barberis, Huang, and Santos (2001) suggest that stockholders have preferences which differ from the preferences of nonstockholders. They introduce elements of prospect theory into a standard asset pricing model and are successful in generating stock returns which are more volatile than the underlying dividends.

One of the well-regarded alternatives to expected utility theory is the prospect theory developed by Kahneman and Tversky (1979). In contrast to expected utility, which is a normative theory, prospect theory is a positive theory of choice under risk with objective probabilities. Prospect theory is based on the assumption that agents derive utility not from levels in wealth, but rather from *changes* in wealth. Further, agents are more sensitive to losses than to gains in wealth: a property known as "loss aversion".

Prospect theory is a static model of choice under risk and its incorporation into intertemporal decision-making is neither straightforward nor trivial. In their pioneering work, Barberis, Huang and Santos extend prospect theory to account for intertemporal decision-making. They model asset prices in a representative-agent economy with complete markets. Our work is related to theirs in the sense that the investor-type agent in our model is endowed with preferences similar to the ones specified by Barberis, Huang and Santos. Changes in financial wealth affect directly the utility of investors who have a large share of their wealth invested in securities. As a result, investors take anticipated fluctuations in their financial wealth explicitly into consideration when making decisions. However, in contrast to Barberis, Huang and Santos we model an economy with heterogeneous agents, incomplete markets and idiosyncratic income shocks.

In a complete market, representative-agent model, individuals completely insure the idiosyncratic shocks to their income and individual consumption is perfectly correlated with the aggregate per capita consumption. However, the impact of idiosyncratic shocks on individual consumption and asset prices is not straightforward once we allow for heterogeneous agents and market incompleteness. This impact has been shown to vary with the underlying assumptions of the model. As pointed out by Heaton and Lucas (1996), the impact of idiosyncratic shocks depends on (1) the size and correlation structure of the shocks; (2) whether the idiosyncratic shocks are transitory or permanent; and (3) the presence of trading frictions.

In the presence of aggregate uncertainty and transitory idiosyncratic shocks without trading costs, asset prices in an incomplete market setting do not differ significantly from those in complete market models (Telmer, 1993; Lucas, 1994) because agents are able to smooth consumption by buying assets after a good, high income, state and sell assets after a bad state. However, when there are short-sale and trading constraints the equity premium rises when the short-sale constraint is binding (Marcet and Singleton, 1999). Interestingly, Constantinides and Duffie (1996) show that when idiosyncratic shocks are permanent, trade does not take place, and the volatility of consumption increases in equilibrium. Their result indicates that allowing for agent heterogeneity is a necessary but not a sufficient condition for trade in financial markets.

While a number of modifications of the representative-agent model have been proposed, the literature on asset pricing with heterogenous agents has started to grow only recently (see, for example, Constantinides, Donaldson, and Mehra, 2002; Marcet and Singleton, 1999; Heaton and Lucas, 1996; Constantinides and Duffee, 1996). There are three avenues, which we explore in our model: (1) We contribute to the literature on heterogenous agent models by introducing a model which allows for preference heterogeneity. To our knowledge, this is a unique feature of this model. (2) We build on models in behavioral economics, which introduce alternatives to the expected utility theory in decision-making. However, typically these models do not allow for agent-heterogeneity and market incompleteness. In line with Mankiw and Zeldes, we allow for two types of agents: Type A exhibits standard preferences used in macroeconomics and Type B exhibits preferences that employ elements of the prospect theory developed by Kahneman and Tversky (1979). Type A agents are non-stockholders, i.e. individuals that either do not hold stocks or whose stock holdings represent a negligent proportion of their income. Type B agents are stockholders or investors: their stock holdings represent a significant proportion of their income and thus, their consumption pattern depends on their stock market performance. In addition to consumption, a Type B agent derives utility from changes in his financial wealth (the prospect theory element in preferences). (3) This research is also closely related to models which consider the impact of aggregate and individual-level uncertainty as well as trading frictions on asset prices in heterogeneous-agent, incomplete market dynamic stochastic general equilibrium models.

We allow for aggregate uncertainty in the form of shocks to the aggregate per share dividend and idiosyncratic labor income shocks which have both permanent and transitory component. We use data from the Panel Study of Income Dynamics (PSID) to calibrate the individual income processes. In addition, we explore the impact of short-sale and liquidity constraints on equilibrium consumption processes and asset prices in an incomplete market setting. However, in our model we do not allow for transaction costs. Heaton and Lucas (1996) show that sizable transaction costs or limited quantity of tradable securities generate about half of the observed risk premium. As a result, the equity premium generated by our model maybe biased downward in the sense that if we account for transaction costs as well, the equity premium should rise.

Our results suggest that heterogeneous preferences and idiosyncratic labor income shocks induce agents to hold different portfolios in equilibrium. Our model generates a substantial time-varying risk premium of stocks over bonds while maintaining a low risk-free rate and a low correlation between individual consumption and stock market returns. The paper is organized as follows: Section II presents the model; Section III discusses the model parametrization and Section IV discusses the solution algorithm; Section V presents the results; Section VI concludes.

II The Model

1. The Economy

Time is discrete and indexed by $t = 0, 1, 2, ...$ There are two assets in the economy: a risky asset (stock), which is a claim to a stream of stochastic dividends, and a risk-free asset (discount bond), which is a claim to one unit of the consumption good in period $t + 1$. There are two types of infinitely-lived agents in this pure exchange economy. The agents are price-takers in goods and securities markets.

1.1. Preferences

For clarity, the preferences of the two types of agents are discussed separately below.

A. Type A Agent

Agents of Type A maximize an additively-separable utility function which exhibits constant relative risk aversion:

$$E\left[\sum_{t=0}^{\infty}\rho^t\frac{(C_t^A)^{1-\gamma}}{1-\gamma}\mid\Omega(t)\right] \tag{1}$$

where C_t^A is the consumption of Type A agent at time t; $0<\rho<1$ is the subjective discount factor; and $\Omega(t)$ denotes the time t information set which is generated by the state variables in the model and is common to both agents. The coefficient of relative risk aversion, $0<\gamma<\infty$, controls for the curvature of the utility function. The utility function reduces to $\ln C_t^A$ when $\gamma=1$. It is continuous, concave and obeys the Inada conditions, i.e. $\lim_{C\to 0}U'(C_t)=\infty$ and $\lim_{C\to\infty}U'(C_t)=0$.

B. Type B Agent

A Type B agent derives utility from both consumption and anticipated fluctuations in financial wealth. His utility function is additive in these two sources of utility. The idea that individuals derive utility from changes in wealth rather than wealth levels was first postulated by Kahneman and Tversky (1979). The prospect theory that they developed is a positive theory of choice under uncertainty derived on the base of experimental evidence. The major building block of prospect theory is the assumption that individuals derive more dissatisfaction from a loss than satisfaction from a gain of an equal size, termed "loss aversion". It has been suggested that together with risk aversion and probability weights, loss aversion is a major component of risk attitudes (see for example Köbberling and Wakker, 2005). In the literature, several different ways of modelling loss aversion have been suggested. Based on experimental evidence, Kahneman and Tversky (1992) suggest the following form for the utility from gains and losses:

$$U(X)=\begin{cases}X^{\alpha} & \text{for}\quad X\geq 0\\ -\lambda(-X)^{\beta} & \text{for}\quad X<0\end{cases} \tag{2}$$

where X denotes changes in wealth with respect to a reference point and $\lambda > 1$ is a measure of loss aversion. Thus, the utility function can be represented by a piece-wise function which is steeper for losses ($X < 0$) than for gains ($X > 0$). Figure 1 plots the utility from gains and losses for $\alpha = \beta = 0.88$ and $\lambda = 2.5$, the parameter values Kahneman and Tversky obtained based on experimental data. The function is slightly concave in the positive domain (risk aversion) and slightly convex (risk seeking) in the negative domain with a kink at 0 (loss aversion). Notice that the function becomes nearly linear in its argument for large gains and losses.

In addition to consumption, a Type B agent explicitly takes into consideration expected fluctuations in financial wealth in his decision-making. In line with prospect theory, we assume that his preferences exhibit loss aversion with respect to changes in financial wealth. The prospect theory, however, is a static model of choice under uncertainty and to incorporate it into a dynamic model, additional assumptions on whether and how prior gains and losses affect decision-making have to be made. Barberis, Huang and Santos (2001) show that loss aversion by itself cannot explain the large equity premium of stocks over bonds observed in historical data within the frames of a representative agent, complete markets model. However, allowing for prior investment performance to influence current and future investment decisions improves the performance of their model.

With slight modifications to be discussed below, we adopt the preference specification suggested by Barberis, Huang and Santos for Type B agent. For simplicity, they assume that $\alpha = \beta = 1$, i.e. in line with prospect theory the utility from gains and losses exhibits the loss aversion property (the utility function from losses is steeper than that from gains and the function is kinked at 0) but the utility function is linear in gains and losses. In order to incorporate prospect theory in a dynamic model, the authors assume that the extent to which an investor is loss averse depends on his prior stock market performance. Thus, people are more willing to gamble after prior gains and more conservative after prior losses. This is the "house money" effect coined by Thaler and Johnson (1990). Thus, the Type B investor maximizes the following utility function suggested by Barberis, Huang and Santos:

$$E\left[\sum_{t=0}^{\infty}\left(\rho^t\frac{(C_t^B)^{1-\gamma}}{1-\gamma}+b_0\rho^{t+1}v(X_{t+1},z_t)\right)\mid\Omega(t)\right] \tag{3}$$

where C_t^B is the consumption of Type B agent in time t; S_t^B is the risky asset holdings of Type B agent in time t; z_t is a state variable that measures gains and losses prior to time t; $v(\cdot)$ is the utility the investor derives from financial gains or losses; X_{t+1} is the gain or loss from the risky asset holdings between t and $t+1$; b_0 is an exogenous scaling factor that controls for the relative importance of the "prospect theory" term in the utility function. If $b_0 = 0$, the model reduces to the standard preferences defined in Equation 1. Barberis, Huang and Santos scale the prospect theory term by $b_0\bar{C_t}^{-\gamma}$ where $\bar{C_t}^{-\gamma}$ is the aggregate per capita consumption at time t, exogenous to the investor in their model. As they allow for a constant growth rate in consumption and dividends, determined exogenously, this adjustment is necessary to ensure that the prospect utility term will not have an explosive impact on the utility function as the wealth in the economy grows. However, in our model this is not necessary as consumption is an endogenous process and we are looking for a stationary equilibrium where consumption is determined endogenously. In our model the individual and aggregate wealth are stationary over time.

The reference level with respect to which gains and losses are measured is usually assumed to be the status quo, in our case the value of the risky asset in period t. The gain or loss between t and $t+1$ is the difference between the value of the risky asset holdings in $t+1$ and t adjusted for the asset value in $t+1$ if instead, the value of the risky asset in t were invested in the risk-free asset:

$$X_{t+1} = S_t^B(P_{t+1} + d_{t+1}) - S_t^B P_t/P_{f,t} \tag{4}$$

where P_t and $P_{f,t}$ are respectively the (ex-dividend) price of the risky asset and the price of the risk-free asset measured in units of the consumption good at time t and d_t is the per-share dividend of the risky asset at time t.

Agents are identical in terms of their coefficient of risk aversion and discount factor. The first term in the utility function of Type B agent is identical with the instantaneous utility function of Type A agent. However, a Type B agent is also loss averse where the loss aversion property is captured by the second term in his utility function. Thus, the two types of agents essentially differ in their attitudes towards risk: a Type A agent dislikes only fluctuations in consumption while in addition, a Type B agent dislikes fluctuations in financial wealth as well.

Barberis, Huang and Santos further assume that a Type B agent keeps track of losses and gains over time. In line with the "house money" effect, losses are more painful when they occur after prior losses than after prior gains. We define the "historical benchmark level", Z_t, as the per unit price of the risky asset that the investor remembers. If $Z_t > P_t$, the investor has realized a loss in time t and future losses will be more painful. Conversely, if $Z_t < P_t$, the investor has realized a gain in the stock market and $S_t(P_t - Z_t)$ serves as a cushion for future losses. For simplicity, we define $z_t = Z_t/P_t$. The investor has had prior losses if $z_t > 1$ and prior gains if $z_t < 1$. The cushion of prior gains increases with the increase of the rate of return on the risky asset. The law of motion for z_t is given by:

$$z_t = \eta \left(z_{t-1} \frac{\bar{R}}{R_t} \right) + (1 - \eta) \tag{5}$$

where R_t is the real gross rate of return on the risky asset between $t-1$ and t and $\bar{R}$ is chosen in such a way that in equilibrium, the median value of z is 1; η can be thought of as a proxy for the investor's memory. If $\eta = 0$, the investor has "no memory" and the historical benchmark level adjusts immediately to changes in the price of risky assets. In contrast, if $\eta = 1$, the investor has a long memory and prior losses and gains affect his decisions for a long period of time.

Barberis, Huang, and Santos assume that the price-dividend ratio is only a function of z_t and thus, the (endogenous) rate of return on the risky asset in period t is only a function of z_t as well. As a result, z_t is an endogenous state variable. Note that the meaning of endogenous state variable in this case has a slightly different meaning from what is typically meant by the term. Conventionally, the term "endogenous state variables" is used to describe members of the state vector in period t which are endogenous variables in period $t-1$. One of the difficulties in solving the model of Barberis, Huang, and Santos is that we have to solve simultaneously for the endogenous state variable z_t and the price-dividend ratio in period t. In our model we do not impose any restrictions on the price-dividend ratio and z_t is an endogenous state variable in the conventional sense: the state variable z_{t-1} is used to solve for the values of the endogenous variables in time t. z_t is found from Equation 5 after solving for the stock price (and thus, the rate of return on stocks for a given realization of d_t) in period t.

The utility from gains and losses depends on the prior stock market performance of Type B agent. Let $Y_{t+1} = [P_t, P_{f,t}, P_{t+1}, d_{t+1}]$ denote the array of variables that affect the utility from gains and losses. Let

$$v(X_{t+1}(Y_{t+1}, S_t^B), z_t) = v(Y_{t+1}, S_t^B, z_t) \tag{6}$$

If $z_t = 1$ (neither prior gains nor losses), the utility from gains and losses is given by:

$$v(Y_{t+1}, S_t^B, z_t = 1) = \begin{cases} X_{t+1} & \text{for} \quad X_{t+1} \geq 0 \\ \\ \lambda X_{t+1} & \text{for} \quad X_{t+1} < 0 \end{cases} \tag{7}$$

where X_{t+1} is defined in Equation 4. Thus, the utility from a gain is given by the gain itself and the disutility from a loss is equal to the value of the loss penalized by a factor of $\lambda > 1$. λ is a measure of loss aversion; it indicates how much more painful a loss is than a corresponding gain. This utility function is a close approximation to the utility function suggested by Kahneman and Tversky for larger gains and losses.

If $z_t < 1$, the investor has accumulated prior gains that serve as a cushion if future losses occur. Losses which are completely cushioned by prior gains are not very painful but losses in excess of prior gains are penalized more severely. Thus, if the cushion created in period t is equal to or greater than the loss realized between t and $t+1$, i.e. if $S_t Z_t \leq S_t(P_{t+1} + d_{t+1})$, the disutility from a loss is equal to the loss itself. Losses in excess of prior gains are penalized more severely, by a factor of λ. More formally, if we update the cushion created in period t, $S_t(P_t - Z_t)$, by the risk free rate, for $z_t \leq 1$ we obtain:

$$v(Y_{t+1}, S_t^B, z_t) = \begin{cases} S_t^B(P_{t+1} + d_{t+1}) - \frac{S_t^B P_t}{P_{f,t}} & \text{for} \quad R_{t+1} \geq z_t R_{f,t} \\ \\ \frac{S_t^B P_t}{P_{f,t}}(z_t - 1) + \lambda S_t^B \left(P_{t+1} + d_{t+1} - z_t \frac{P_t}{P_{f,t}}\right) & \text{for} \quad R_{t+1} < z_t R_{f,t} \end{cases} \tag{8}$$

where $R_{f,t}$ is the gross real return on the risk-free asset between t and $t+1$. Notice that Equation 8 reduces to Equation 7 when $z_t = 1$.

If the investor has accumulated prior losses on the stock market ($z_t > 1$), subsequent losses are more painful and are penalized more severely than when the investor has had prior gains. The penalty factor in this case or alternatively, the measure of loss aversion, $\lambda(z_t) > \lambda$, is increasing in prior losses:

$$\lambda(z_t) = \lambda + k(z_t - 1) \tag{9}$$

where $k > 0$.

The utility from gains and losses in the case of prior losses ($z_t > 1$) is given by:

$$v(Y_{t+1}, S_t^B, z_t) = \begin{cases} S_t^B(P_{t+1} + d_{t+1}) - \frac{S_t^B P_t}{P_{f,t}} & \text{for} \quad R_{t+1} \geq R_{f,t} \\ \\ \lambda(z_t) S_t^B \left(P_{t+1} + d_{t+1} - \frac{P_t}{P_{f,t}} \right) & \text{for} \quad R_{t+1} < R_{f,t} \end{cases} \tag{10}$$

Figure 2 shows Type B agent's utility from gains and losses for different values of z_t. When there are no prior gains or losses, $z_t = 1$, the disutility from a loss is greater than the utility from a gain of an equal magnitude as the utility from losses is steeper than the utility from gains, i.e. the utility from gains and losses exhibits the loss aversion property. The utility from gains is the same regardless of Type B agent's prior stock market performance. However, the disutility from a loss differs depending on whether the investor has had prior gains, losses or neither gains or losses. When a loss comes on the heels of prior losses, it is more painful than when there are neither prior gains nor losses. The dashed green line on Figure 2 shows the utility from gains and losses when there are prior losses. It is drawn for $z_t = 1.25$. Compared to the case of $z_t = 1$, the slope of the dashed green line is steeper for losses implying that losses are more painful when there are prior losses. When there are prior gains, how painful a subsequent loss is depends on how large the created cushion and the incurred loss are. The red dash-dotted line is drawn for $z_t = 0.5$, e.g. the investor has had substantial prior gains. In this case losses which are completely cushioned by prior gains are not penalized, i.e. the disutility from the loss is equal to the loss itself. However, losses in excess of the cushion are penalized by a factor of λ.

1.2. Endowments

In addition to preferences, agents are heterogeneous with respect to their labor income as well. In each period t, a Type i agent receives an exogenous labor income y_t^i for $i = A, B$, which is subject to idiosyncratic shocks. In addition, agents receive income if they have invested in stocks and/or bonds. The stock is a claim to a stochastic stream of dividends and agents face aggregate uncertainty if they invest in stocks.

The one-period zero-coupon bond yields one unit of the consumption good in period $t+1$ with certainty. Agents of Type A and B face standard budget constraints:

$$C_t^A + P_t S_t^A + P_{f,t} B_t^A = y_t^A + (P_t + d_t) S_{t-1}^A + B_{t-1}^A \tag{11}$$

$$C_t^B + P_t S_t^B + P_{f,t} B_t^B = y_t^B + (P_t + d_t) S_{t-1}^B + B_{t-1}^B \tag{12}$$

where B_t^i is the riskless asset holdings of Type i agent for $i = A, B$ at time t and y_t^i is the stochastic labor income of agent i for $i = A, B$ at time t.

We assume that there is no population growth and normalize the size of the population to 1. Thus, the aggregate income in the economy y_t at any t is given by:

$$y_t = \theta y_t^A + (1-\theta) y_t^B + d_t \tag{13}$$

where θ is the share of Type A agents in the economy.

1.3. Borrowing and Short-Sale Constraints

Agents can trade securities to transfer wealth across states and time in order to smooth their consumption. There are only two assets in the economy: a riskless bond and a risky stock. However, agents cannot diversify away all risks as markets are incomplete and they cannot write contracts contingent on their expected labor income. In addition, individuals face state-dependent short sale and borrowing constraints in the asset markets. The short sale constraint, $K_{s,t}^i$ faced by agent i for $i = A, B$ in time t depends on the agent's income. In each period t the short-sale constraint is given by:

$$S_t^i \geq K_{s,t}^i \quad \text{where} \quad K_{s,t}^i = m y_t^i \tag{14}$$

where $m \leq 0$. In our basic model, we rule out short sales, i.e. $m = 0$ irrespective of the state of the economy. However, we test our results for sensitivity to this assumption.

Individuals may not be able to smooth their consumption over time because of credit rationing. In each period t agents can borrow only a fraction $h \leq 0$ of their income:

$$B_t^i \geq K_{b,t}^i \quad \text{where} \quad K_{b,t}^i = h y_t^i \tag{15}$$

where $K^i_{b,t}$ is the state-dependent borrowing constraint faced by agent i for $i = A, B$. The borrowing constraint is binding in some states but not in others. Besides being a realistic feature of financial markets, the borrowing constraint ensures that consumers will not rollover debt, or get involved in Ponzi schemes.

2. Market Equilibrium

The equilibrium consumption and asset holdings as well as asset prices are determined endogenously in our model. Each consumer maximizes his stochastic consumption stream subject to the budget and portfolio constraints for a given stream of prices $\{P_t\}_{t=0}^{\infty}$ and $\{P_{f,t}\}_{t=0}^{\infty}$. Employing the Kuhn-Tucker conditions, the relevant stochastic Euler equations for consumer i's maximization problem for $i = A, B$ are given by:

- Bonds

Either

$$(C^i_t)^{-\gamma} P_{f,t} = \rho E\left[\left(C^i_{t+1}\right)^{-\gamma} \mid \Omega(t)\right] \quad \text{and} \quad B^i_t > K^i_{b,t} \tag{16}$$

or

$$(C^i_t)^{-\gamma} P_{f,t} \geq \rho E\left[\left(C^i_{t+1}\right)^{-\gamma} \mid \Omega(t)\right] \quad \text{and} \quad B^i_t = K^i_{b,t} \tag{17}$$

for $i = A, B$

- Stocks

Either

$$(C^A_t)^{-\gamma} P_t = \rho E\left[\left(C^A_{t+1}\right)^{-\gamma} (P_{t+1} + d_{t+1}) \mid \Omega(t)\right] \quad \text{and} \quad S^A_t > K^A_{s,t} \tag{18}$$

or

$$(C^A_t)^{-\gamma} P_t \geq \rho E\left[\left(C^A_{t+1}\right)^{-\gamma} (P_{t+1} + d_{t+1}) \mid \Omega(t)\right] \quad \text{and} \quad S^A_t = K^A_{s,t} \tag{19}$$

Either

$$(C^B_t)^{-\gamma} P_t = \rho E\left[(C^B_{t+1})^{-\gamma}(P_{t+1} + d_{t+1}) \mid \Omega(t)\right] + b_0 \rho E\left[\hat{v}(Y_{t+1}, z_t) \mid \Omega(t)\right] \quad \text{and} \quad S^B_t > K^B_{s,t} \tag{20}$$

or

$$(C^B_t)^{-\gamma} P_t \geq \rho E\left[(C^B_{t+1})^{-\gamma}(P_{t+1} + d_{t+1}) \mid \Omega(t)\right] + b_0 \rho E\left[\hat{v}(Y_{t+1}, z_t) \mid \Omega(t)\right] \quad \text{and} \quad S^B_t = K^B_{s,t} \tag{21}$$

where for $z_t \leq 1$

$$\hat{v}(Y_{t+1}, z_t) = \begin{cases} P_{t+1} + d_{t+1} - \frac{P_t}{P_{f,t}} & \text{for} \quad R_{t+1} \geq z_t R_{f,t} \\ \\ \frac{P_t}{P_{f,t}}(z_t - 1) + \lambda \left(P_{t+1} + d_{t+1} - z_t \frac{P_t}{P_{f,t}} \right) & \text{for} \quad R_{t+1} < z_t R_{f,t} \end{cases} \tag{22}$$

and for $z_t > 1$

$$\hat{v}(Y_{t+1}, z_t) = \begin{cases} P_{t+1} + d_{t+1} - \frac{P_t}{P_{f,t}} & \text{for} \quad R_{t+1} \geq R_{f,t} \\ \\ \lambda(z_t) \left(P_{t+1} + d_{t+1} - \frac{P_t}{P_{f,t}} \right) & \text{for} \quad R_{t+1} < R_{f,t} \end{cases} \tag{23}$$

If the short sale and borrowing constraints are non-binding, the Euler equations are given by Equations 16, 18, and 20. The Euler equations for bonds are standard: if the consumer decreases incrementally his consumption in period t and invests his savings in the riskless asset, his utility cost in t should be equal to the discounted expected value of the utility benefit in $t+1$ adjusted for the rate of return on the riskless asset between t and $t+1$. This is a necessary condition for optimality for any t. Similarly, the Euler equation for stocks for Type A agent is standard and has a similar interpretation. However, the Euler equation for stockholders has a different interpretation. If the stockholder reduces his consumption by an infinitesimal amount in time t and invests the savings in the risky asset, his utility cost in t should be equal to the discounted value of the expected utility benefit in the next period adjusted for the expected rate of return on the risky asset plus the expected change in the value of the risky assets. When the investor realizes a loss, $\hat{v}(\cdot)$ is negative implying that he would require a higher expected rate of return to invest in stocks. How high the expected rate of return would be depends on whether the investor has had prior losses or gains. If he has had prior losses, he is more loss averse and he would require a higher rate of return on the risky asset to invest in it and conversely, if he has had prior gains, he would require a lower rate of return.

For simplicity, the outstanding shares of the risky asset are normalized to one. We only allow for private borrowing and lending and therefore, bonds are in zero net supply. Thus, the market clearing conditions for stocks and bonds in each period t are given by:

$$\theta S_t^A + (1-\theta) S_t^B = 1 \tag{24}$$

$$\theta B_t^A + (1-\theta) B_t^B = 0 \tag{25}$$

We assume that there is no population growth and the population size is normalized to one. Walras law guarantees that the goods market clear, i.e. $\theta C_t^A + (1-\theta) C_t^B = y_t$ is satisfied for each t. The aggregate income in the economy y_t is given by Equation 13.

Each agent faces idiosyncratic shocks to his labor income as well as aggregate shocks to the per share dividend. Markets are incomplete as while there are three sources of uncertainty, there are only two markets, the bond and the stock markets, to hedge consumption risks. In addition, borrowing and short sale constraints limit the agents' ability to smooth consumption across states and time.

Information is complete and symmetric, i.e. both agents know the past realizations of stock prices as well as shocks to their individual incomes and the per share dividend. There are eight endogenous variables in our model in each t: P_t, $P_{f,t}$ and C_t^i, B_t^i, S_t^i for $i = A$, B. We use the four (relevant) Euler equations, the equilibrium conditions (Equations 24 and 25), the income process (Equation 13) and the budget constraint for Type A agent to find the equilibrium distributions of the endogenous variables as a function of the state vector. Because of Walras Law, the budget constraint for Type B agent is redundant. For convenience, we discuss the state vector separately below.

3. State Variables

3.1. Exogenous State Variable

The exogenous state of the economy at every t is given by $[\ln(y_t^A)\ln(y_t^B)\ln(d_t)]'$ where ln denotes the natural logarithm. We use annual data on 632 households over the period 1968-1997 from the Panel Study of Income Dynamics (PSID) to calibrate the income processes of stockholders and non-stockholders and data from NIPA accounts to calibrate the aggregate dividend process. The individual incomes and the aggregate dividend are assumed to follow first-order autoregressive processes.

The income of Type i agent for $i = A, B$ is assumed to be a stationary first-order autoregressive process:

$$\ln(y_t^i) = \xi^i + \omega^i \ln(y_{t-1}^i) + \varepsilon_t^i \tag{26}$$

where $\varepsilon_t^i \sim Niid(0, (\sigma_\varepsilon^i)^2)$ for $i = A, B$. We use PSID data to classify individuals as stockholders and non-stockholders. Results from Abowd and Card (1989), Heaton and Lucas, and Marcet and Singleton suggest that aggregate shocks have little impact on the conditional mean and unconditional variance of individual incomes. As a result, we assume that lagged values of the aggregate dividend have no impact on the individual income processes.

The aggregate dividend is assumed to be independent of the individual income processes and follows a stationary first-order autoregressive process:

$$\ln(d_t) = a_1 + a_2 \ln(d_{t-1}) + e_t \tag{27}$$

where $e_t \sim Niid(0, \sigma_e^2)$. The section on calibration below provides details on data estimation and calibration.

3.2. Endogenous State Variable

The state vector contains endogenous variables as well. These are the elements of wealth defined in the previous period as well as prior investment outcomes, i.e. $B_{t-1}^A, S_{t-1}^A, z_{t-1}$.

III Calibration

1. Law of Motion of the Exogenous State Variables

We use data from the Panel Study of Income Dynamics (PSID) to calibrate the individual income processes of Type A and B agents. There is a huge body of literature on the dynamic process that governs the individual earnings recorded in longitudinal studies. While this process does not seem to be clearly understood as yet, it is clear that shocks to individual earnings are highly persistent and follow a complex dynamic structure.

Annual data on the individual labor processes from the PSID is used to calibrate the exogenous income processes of the two types of agents. The PSID is a longitudinal study of a sample of the US population conducted annually since 1968 and biannually since 1997. The original 1968 sample consists of two independent samples: a sample drawn by the Survey Research Center (SRC sample) that includes about 3,000 households representative of the US population and a sample of about 2,000 households drawn from the Survey of Economic Opportunity respondents (SEO sample) which represents low-income families. As we are interested in a representative sample of the US population, we only consider the SRC sample in line with Lillard and Willis (1978) who suggest dropping the SEO sample because of endogenous selection problem.

The PSID follows both the original families as well as their split-offs. We use both individual- and family-level data to find the total family money income as a sum of the reported taxable income of head and wife, as well as the taxable income of other earners in the family and transfer income received by family members from all sources. Transfer income and other sources of income are included to measure idiosyncratic shocks net of the implicit insurance offered by transfer payments and other sources of income. Taxable income includes labor income as well as income from other sources. Labor income includes the labor portion of income from all sources such as wages and salaries, bonuses, overtime, tips, commissions, professional practice or trade, and market gardening. Transfer income includes social security income, unemployment and workers compensation, child support, retirement income as well as other welfare transfers to the head and wife. The total family money income weighted by the number of family members and deflated by the CPI is used as a proxy for the individual labor income in our model.

The PSID survey is retrospective in the sense that it is administered at the beginning of the year and the income reported in a given year refers to the previous calendar year and is measured in previous year dollars. Thus, our sample refers to the period from 1967 to 1996. There are several restrictions that we impose on the data. We include in our sample only families that completed the survey in all years from 1968 to 1997. We exclude missing observations, i.e. families, which once in the survey, did not complete the survey in a given year. We also exclude families with zero reported income in a given year. We use data from the Wealth Supplements in 1984, 1989 and 1994 to categorize families as stockholders and non-stockholders. As our model does not allow individuals to move from one category to the other, we exclude from the sample all families that were stockholders in one year and non-stockholders in the others. We also exclude those families who declined to answer the question on whether they hold stocks and thus, cannot be categorized as stockholders or non-stockholders. In our sample we have a total of 652 families of which 431 (about 65% of the population) are non-stockholders and 221 (about 35%) are stockholders. Thus, we set the proportion of non-stockholders in the population $\theta = 0.65$ and the proportion of stockholders to 0.35.

To account for the observed agent heterogeneity in empirical data, we follow the approach suggested by Heaton and Lucas (1996). For each individual we use OLS to estimate his individual income process:

$$\ln(y_{jt}) = \xi_j + \omega_j \ln(y_{jt-1}) + \varepsilon_{jt} \tag{28}$$

where $\{\xi_j\}_{j=1}^N$ and $\{\omega_j\}_{j=1}^N$ are parameters and N is the number of individuals in our sample. Permanent differences in the individual labor incomes are captured by $\{\xi_j\}_{j=1}^N$ while $\{\omega_j\}_{j=1}^N$ captures the persistence of idiosyncratic income shocks to labor incomes. We assume that innovations to the income of individual j follow a white noise process with $E[\varepsilon_{jt}] = 0$, $E[\varepsilon_{jt}\varepsilon'_{it}] = \sigma_j^2$ if $j = i$ and 0 otherwise, and $E[\varepsilon_{jt}\varepsilon'_{jt-1}] = 0$. The parameters in Equation 26 for a Type i agent for $i = A, B$ are found as cross-sectional averages of the corresponding parameter estimates of all individuals who fall in the category of non-stockholders and stockholders, respectively. For example, if M denotes the number of non-stockholders in our sample, ξ^A in Equation 26 is given by:

$$\xi^A = \frac{1}{M}\sum_{j=1}^{M}\xi_j \tag{29}$$

The cross-sectional averages of the ordinary least squares estimates of the coefficients in Equation 28 and averages of their standard errors are reported in the first two rows of Table 1.

Income shocks to the individual labor incomes are highly persistent with the shocks to the labor income of non-stockholders being more persistent than the shocks to the labor income of stockholders. The estimated cross-sectional mean of the standard deviation of the idiosyncratic shocks to the labor income of non-stockholders is $\sigma_\varepsilon^A = 0.37$ with a standard deviation of 0.15 while the corresponding value for stockholders is $\sigma_\varepsilon^B = 0.32$ with a standard deviation of 0.14. It is somewhat counterintuitive that innovations to the labor income of non-stockholders are more volatile than for stockholders. The reason could be that stockholders, who on average have a higher income than non-stockholders, are more likely to have a more stable income as well.

Our results are consistent with empirical estimates based on microeconomic data. For example, based on PSID data MaCurdy (1982) finds that the standard deviation of the residual in a regression with real labor income per capita in logarithms as a dependent variable is 0.58. However, Deaton (1991) argues that MaCurdy's estimate overstates the true volatility of innovations to the individual labor income because of measurement errors. Deaton suggests that this volatility for shocks to the logarithm of income in first differences should be between 0.1 and 0.15. As a result, we scale down the variance of shocks to the individual income processes that we estimate by about 2/3, i.e. we assume that $\sigma_{\varepsilon}^{A} = 0.17$ and $\sigma_{\varepsilon}^{B} = 0.15$ thus placing Type B agent at the upper bound of the interval suggested by Deaton and Type A agent just above that bound.

The PSID does not provide data on the dividend income for the whole sample period. We use annual data on the net dividend from the National Income and Product Account (NIPA) tables published by the Bureau of Economic Analysis to calibrate the process of the aggregate dividend. To increase the precision of our estimates, we use all the available data, which spans the period from 1929 to 2006. We weigh the dividend by CPI and the U.S. population in a given year to obtain the real dividend per capita. Data on the U.S. population is obtained from the U.S. Census Bureau. The regression estimates of the parameters in Equation 27 for the detrended series of the real dividend in logarithms are presented in the third row of Table 1. The estimated standard error of the shock to the dividend process is 0.1. To be consistent with the assumed volatility of shocks to the individual income processes, we scale down this estimate by 2/3 as well. Thus, the standard deviation of innovations to the dividend process is $\sigma_e = 0.06$. The aggregate income in the economy in any given year is the weighted sum of the individual labor incomes of the two agents and the aggregate per capita dividend. The aggregate income is normalized, so that its average is 1.

2. Structural Parameters

Table 2 summarizes the chosen parameters for the model. The discount factor ρ is set equal to 0.96. There is still an ongoing debate on the average value of the coefficient of risk aversion (see, for example, Kocherlakota, 1997). As discussed above, the equity premium puzzle exists only if we assume that values of γ greater than 10 are implausible. We set $\gamma =$ 2, well into the plausible region suggested by Mehra and Prescott. Based on our data we set the share of non-stockholders in the population θ to 0.65. The importance of the prospect theory term in the overall utility of Agent B is controlled by b_0; k is a penalty factor for losses when they occur after prior losses and η is a proxy for investor's memory. For our base model we adopt the parameter values of k, η, and the lower bond of b_0 suggested by Barberis, Huang and Santos. However, we test the sensitivity of our model to these parameter values. λ penalizes losses when there are no prior gains or losses. We set it equal to 2.25, the value estimated by Tversky and Kahnemnan (1992) based on experimental data.

While it is intuitive that the borrowing constraint is a function of individual's income, it is not immediately clear what the lower bound of the constraint is. For our baseline model we set $h =$ -1/3 and therefore, the state-dependent borrowing constraint is given by $K^i_{s,t} = -1/3y^i_t$ for $i = A, B$. Even though our results presented below show that the borrowing constraint is rarely binding, we test the sensitivity of our results to this assumption. In our baseline model we rule out short sales, $m = 0$.

IV Solution Algorithm

We solve for the equilibrium numerically using a modification of the parameterized expectations algorithm (PEA) developed by Marcet (1988) and den Haan and Marcet (1990, 1994). Marcet and Singleton (1999) extend the algorithm to account for agent heterogeneity. The appendix to this chapter offers a concise discussion of the numerical algorithm. We simulate the equilibrium path of the economy for 2,000 periods and exclude the first 100 periods to eliminate any impact of the initial conditions on results. All computations are executed in Matlab.

V Results

1. Representative Agent Models

We first solve the representative agent model to see whether accounting for loss aversion improves the results. We essentially solve the model of Mehra and Prescott where the representative agent's labor income is set equal to the aggregate labor income in the economy which a weighted average of the individual incomes of the two agents. However, in contrast to Mehra and Prescott, who set the consumption of the representative agent equal to the aggregate per share dividend, we set consumption equal to the aggregate income in the economy, which is the sum of the per share dividend and the aggregate labor income. This is our Model A. We then perform the same exercise except for the fact that the representative agent's preferences account for loss aversion (Equation 3). This is our Model B which is similar to the model solved by Barberis, Huang, and Santos. Results are presented in Table 3.

Our results are consistent with results obtained by Heaton and Lucas. The equity premium generated by the models is higher than the premium generated by models based on aggregate data. The reason is that microeconomic data are more volatile than aggregate data. This can be corrected to some extent if we assume a higher value for the discount factor. Our results suggest that Model B outperforms Model A as it generates a higher equity premium for a lower correlation of consumption with stock returns. The reason is that Model B introduces a second source of risk aversion, namely loss aversion. Thus, allowing for heterogeneity in preferences may enable us to obtain a better match to the equity premium observed in historical data.

2. Heterogenous Agents: Loss Aversion

To evaluate the performance of our model, we have to compare our estimates to the corresponding values reported in empirical studies. While an average equity premium of 6% and a risk free rate of return of 1% in real terms are widely cited in the literature, these empirical values (and their volatility) are not robust to the sample period. Siegel (1999), for example, reports an equity premium of 4.1% based on U.S. data for the period 1802-1998. However, the equity premium has been more pronounced during the post World War II period predominantly due to a decrease in the risk-free rate. The moments of asset returns observed in historical data that we use as a base for comparison with our results are obtained from estimates reported by Mehra and Prescott (2003). As a benchmark we use the 30-year average of the U.S. equity premium over the period 1951-2000 reported by Mehra and Prescott (2003) because this period most closely matches the sample period of the data we use to estimate the law of motion of the individual income and aggregate dividend processes. Further, Mehra and Prescott report that the standard deviation of stock returns in real terms is about 20% per year while the standard deviation of returns to T-bills is about 4% per year. These are the volatility values of the variables of interest that we use to compare our results to. The empirical values of the mean and volatility of the price-dividend ratio are borrowed from Barberis, Huang and Santos.

The sample moments of the distributions of asset returns for our calibrated economy are reported in Table 4. Our model is able to generate a substantial average risk premium of 5.5% while maintaining a low risk-free rate. The risk-free rate generated by our model is a match to its historical counterpart. The risk premium of 5.5% generated by our model is very close to the historical risk premium of 6% which is widely cited in the literature. However, the rate of stock return generated by our model (while substantial at 6.9%) falls short from the chosen historical benchmark by about 2%.

In line with historical data, our model generates a time-varying risk premium (see Figure 3). Stocks are much more volatile than bonds and as a result, they offer higher returns. However, the average stock and bond volatility predicted by our model exceed the corresponding volatilities observed in historical data. Specifically, our model generates a risk-free rate of return which is nearly three times more volatile than its historical counterpart. We conclude that while our model matches the first moments of the empirical distributions of asset returns closely, it overstates the volatility of the risk free rate.

Our model also matches quite closely the mean and volatility of the price-dividend ratio. It is an important result as the representative agent model of Barberis, Huang, and Santos fails to match the volatility of the price-dividend ratio. For example, their model with $b_0 = 0.7$ and $k = 3$ (the same parameters that we choose) generates an equity premium of 1.3% with a standard deviation of 17.39%, and a mean price-dividend ratio of 29.8 with a standard deviation of 2.9. While our model matches the mean of the price-dividend ratio predicted by the model of Barberis, Huang, and Santos, we are also able to account for its volatility. Thus, our result suggests that allowing for agent heterogeneity and not imposing restrictions on the price-dividend ratio provide a better match to the data.

One of the implications of the complete market representative agent model is that in equilibrium, individual consumption is perfectly correlated with aggregate income. Our results suggest that the optimal consumption is less than perfectly correlated with aggregate income for both types of agents and therefore, the optimal consumption allocation in our model departs from the one predicted by a complete market model. The consumption of Type B agents is more strongly correlated with aggregate income than the consumption of Type A agents. This is due to the fact that the aggregate income is a weighted sum of the income processes of the two types of agents and the aggregate dividend. Even though Type B agents represent only 35% of the population, their income accounts for 56% of aggregate income. As suggested by microeconomic data, consumption of both types of agents is more strongly correlated with their own income than with aggregate income.

The consumption processes for the two agents for 1,000 periods are depicted on Figure 4. The consumption of Type B agent has higher mean and volatility than the consumption of Type A agent. The result is not surprising as Type B agent has higher average income and higher income volatility as well (see Figure 5). Furthermore, prior gains and losses on the stock market are an additional source of consumption volatility for Type B agent.

The consumption of Type B agent is negatively correlated with z (correlation coefficient of -0.41) implying that consumption tends to be high when the investor has had prior gains in the stock market (see Figure 6). In fact, prior gains and losses affect the consumption of Type A agent as well through general equilibrium effects. The equilibrium distribution of z is depicted on Figure 7. $\bar{R}$ is set equal to 1.03 to ensure that in equilibrium, the median value of z is 1, i.e. half of the time Agent B has losses and half of the time he has gains. z appears to be normally distributed. It is quite volatile with a standard deviation of 0.24. The distribution of z is consistent with the one obtained by Barberis, Huang and Santos.

Consistent with historical data, our model also predicts a low correlation between the individual consumption processes and the real rate of return on stocks. The consumption of non-stockholders is more highly correlated over time with stock returns than the consumption of stockholders. While this result may appear counterintuitive, it is not immediately clear what drives it. One possibility is that the result is driven by the different income processes. Another possibility is that it is driven by the preference heterogeneity. While for a Type A agent the source of risk is the correlation of his consumption process with the rate of return on risky assets, a Type B agent has a second source of risk as well, namely fluctuations in his financial wealth. Thus, the consumption process of a Type B agent is sensitive not only to fluctuations in the rate of return on risky assets but to changes in his financial wealth as well as prior losses and gains.

Agents can trade on the bond and/or stock market to smooth their consumption. The volume of trading in stocks and bonds is high as Figures 8, 9, 10, and 11 show. Through trading, both agents achieve smoother consumption than their individual income processes. The standard deviation of Agent A's consumption is 0.105 while the standard deviation of his individual income is 0.136. The standard deviation Type B's consumption is 0.28 while his income volatility is 0.32.

As expected, z_t and R_t in our data have a high negative correlation (correlation coefficient of -0.56). In the presence of prior losses, higher expected rates of return on the risky asset are necessary to induce a Type B agent to invest in the stock market.

The bond holdings and the bond constraints for Type A and B agents are depicted in Figures 8 and 9. The bond constraint is weak in the sense that it is rarely binding for both types of agents. On average, Type A agent is a lender and Type B agent is a borrower. The stock market is more volatile than the bond market as Figures 10 and 11 show. While Type B agents (stockholders) represent only 35% of the population, they hold about 50% of the stocks in the economy.

Figure 12 shows the income, bond and stock holdings of Type A agent for 200 periods. Type A agent uses predominantly the bond market to smooth his income. He mainly borrows when his income is low and lends when his income is high. In contrast, Type B agent uses predominantly the stock market to smooth his consumption as Figure 13 shows. Stock holdings are highly positively correlated with the individual income process of Type B agents (a correlation coefficient of 0.53) and weekly positively correlated with the individual income process of Type A agent (a correlation coefficient of 0.18). The converse is true for bond holdings: they are highly correlated with the income of non-stockholders (a correlation of 0.7) and weakly correlated with the income of stockholders (correlation of 0.21).

The stock price is quite volatile as Figure 14 shows. The income and consumption of Type A agent tend to be high when the stock price is high. However, the stock price is not as highly correlated with the income and consumption of Type B agent. The reason is that prior stock market performance affects the decision-making of stockholders. As a result, the stock price has a high negative correlation with z_t (correlation coefficient of -0.77) implying that Type B agents are less loss averse after prior gains ($z_t < 1$) and thus, require a lower rate of return to invest in stocks and are willing to pay a higher price.

We are particularly interested in quantifying the effect of the loss aversion term in the preferences of Type B agent on our results. To do so, we simulate a model (Model A) where we keep all the properties of the model presented above with one exception: we assume that both agents have the standard preferences of Type A agent. The results for this simulated economy are presented below.

3. Heterogenous Agents: Standard Preferences

Table 5 presents our results for a model with heterogeneous agents with identical standard preferences (Model C). The model generates a risk premium of 4.58% but as in the model with loss aversion, it fails to approximate the second moment of the empirical distribution of the risk free rate. Therefore, accounting for loss aversion increases the risk premium by about 17%. In addition to matching the first moments of the distributions of asset returns better, the introduction of loss aversion in the preferences of Type B agent improves the performance of the model in matching the second moments of the distributions as well. A comparison of the correlation coefficients obtained under the two models shows that the introduction of loss aversion has little impact on the co-movements of individual consumption processes with stock returns, individual incomes and the aggregate income. However, it appears that the loss aversion property in the utility function of a Type B agent leads to a decrease in the co-movements of consumption and the risky asset return.

VI Robustness

1. The Significance of the Prospect Utility Term in Type B Agent Preferences

On average, the share of the loss aversion term in the utility function of Type B agent in our model is 0.31. Therefore, it is of interest to explore the implications of changing the weight of the utility from gains and losses in the overall utility function. Table 6 presents the sensitivity of our results to changes in b_0, the term which controls for the importance of prior stock market outcomes in the utility function. Our results show that the performance of our model in terms of its ability to match the first moments of the empirical distributions of asset returns increases with the increase in b_0. When $b_0 = 1$, the model generates a risk premium of about 7% in real terms. This represents a 50-percent increase in the risk premium compared to a model without loss aversion. It is also notable that this increase in the equity premium is due to both a decrease in the risk-free rate and an increase in the return on the risky asset. When $b_0 = 0.6$, on average the loss aversion term weighs for 27% of the total utility of Type B agent while when $b_0 = 1$, the weight of the loss aversion term increases to 40.4%. This result indicates that accounting for loss aversion (and more precisely, for loss aversion and prior stock market performance) in preferences improves significantly the performance of the model and enables us to match closely the empirical distributions of asset returns.

2. Investor's Memory

Barberis, Huang, and Santos argue that loss aversion by itself is not able to account for the equity premium puzzle. Our results presented in Table 7 support their argument that tracking prior gains and losses is instrumental in accounting for the empirical value of the average risk premium. The risk premium is increasing in η implying that the longer the investor's memory, the higher the equity premium generated by our model.

3. Borrowing Constraints

We test our results for robustness to the specification of the borrowing constraints. Table 8 shows the averages for the variables of interest for different specifications of the budget constraint. Our results show that tightening the budget constraint increases the equity premium generated by our model as the risk-sharing opportunities decrease. This is true even though the borrowing constraint is rarely binding in our benchmark model (in less than 5% of the cases for both agents). The return on bonds decreases when the borrowing constraint is more restrictive.

Tightening the budget constraint decreases the ability of individuals to smooth idiosyncratic shocks through trading in financial markets. As a result, when the borrowing constraints are tighter, the correlation of agent's consumption with their individual incomes increases.

VII Conclusion

We have calibrated a general equilibrium model with heterogeneous agents, incomplete markets, and portfolio constraints. While Type A agents are non-stockholders and have the standard preferences used in macroeconomics, Type B agents are stockholders who explicitly take into consideration prior stock market performance when making consumption and savings decisions. In equilibrium, consumers hold different portfolios and use both the stock and bond markets for consumptions smoothing. Our results suggest that prior investment performance has a significant impact on the decision-making of Type B agents. Market clearing conditions imply that prior gains and losses in the stock market have spillover effects on the decision-making of Type A agents as well.

Our model generates a high average equity premium of about 5.5% while the risk-free rate is kept low at 1.5%. In line with historical data, the individual consumption has low correlation with stock returns. However, our model does not match as well the second moments of the asset return distributions and particularly, the volatility of the risk-free rate. This is a feature that our model shares with consumption-based capital asset pricing models.

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Appendix: The Solution Algorithm

As our model does not have an analytical solution, we use the parameterized expectations algorithm (PEA) developed by Marcet (1988) and den Haan and Marcet (1990, 1994) to solve numerically for the stationary distribution of the endogenous variables in our model. There are several advantages of using the algorithm for solving our model: (1) The algorithm performs well when the state space is large and there are a number of stochastic shocks in the conditional expectations as its wide use in a variety of economic environments has shown; (2) Theoretically, we are still lacking understanding on the existence, uniqueness, and properties of equilibrium when markets are incomplete. The PEA enables us to solve the model based on the Euler equations even though we may not know theoretically the properties of the solution. The flip side of the coin though, is that the PEA can provide an arbitrarily close approximation to models with a unique stationary and ergodic distribution but it does not guarantee that the obtained solution is a global maximum. This can be a problem in models with multiple equilibria.

In what follows we first briefly describe the algorithm and then we discuss its application to our model. For a detailed discussion of the algorithm the reader should refer to Marcet; den Haan and Marcet; and Marcet and Lorenzoni (1998). Marcet and Singleton discuss the application of the algorithm to a heterogenous agent, incomplete markets model. Our discussion below borrows from these sources.

The equilibrium in dynamic general equilibrium models with uncertainty is usually described by a set of Euler equations, budget constraints and equilibrium conditions. Let x_t be a vector of n endogenous non-state variables, y_t a vector of m endogenous state variables, and u_t a vector of s exogenous processes that follow a first-order Markov process. For each t the equilibrium relations can be summarized in the following system:

$$0 = g(E_t\{\phi(x_{t+1}, y_{t+1})\}, x_t, y_t, x_{t-1}, y_{t-1}, u_t) \tag{30}$$

where $g : R^p \times R^n \times R^m \times R^n \times R^m \times R^s \longrightarrow R^q$ and $\phi : R^{n+m} \longrightarrow R^p$ are known functions and E_t denotes the conditional expectations operator. The PEA considers solutions such that

$$E\{\phi(x_{t+1}, y_{t+1})|u_t, x_{t-1}, y_{t-1}, u_{t-1}, x_{t-2}, y_{t-2}, \ldots\} = E\{\phi(x_{t+1}, y_{t+1})|y_{t-1}, u_t\} \tag{31}$$

where y_t is a finite-dimensional vector. The PEA computes a recursive solution to (30) where the conditional expectation is given by a time-invariant function Υ such that

$$\Upsilon(y_{t-1}, u_t) = E_t\{\phi(x_{t+1}, y_{t+1})\} = E\{\phi(x_{t+1}, y_{t+1})|y_{t-1}, u_t\} \tag{32}$$

The PEA consists of finding an approximation to Υ by finding ξ and a flexible function $\psi_t(\xi; y_{t-1}, u_t) : R^{m+s} \longrightarrow R^p$ such that for all t

$$0 = g(\psi_t(\xi; y_{t-1}(\xi), u_t), x_t(\xi), y_t(\xi), x_{t-1}(\xi), y_{t-1}(\xi), u_t) \tag{33}$$

where $\xi \in R^{w\times p}$ denotes a vector of parameters. The function is such that as $w \longrightarrow \infty$ we can approximate $E_t\{\phi(\cdot)\}$ and therefore, $\Upsilon(y_{t-1}, u_t)$ arbitrarily well. For example, we can choose a polynomial function for $\psi_t(\xi; \cdot)$ as it can approximate any function when the order of the polynomial increases. The algorithm entails 4 different steps:

1. A major assumption is that the system g in (30) is invertible with respect to its second and third arguments. Thus, the first step is to ensure that the system in (30) is invertible with respect to x_t and y_t, so that the endogenous variables can be uniquely determined from (30). Choose starting values for the endogenous state variables y_0 and exogenous processes u_0. Draw a series $\{u_t\}_{t=1}^T$ from the specified distribution of u with T sufficiently large.

2. Specify the initial values of ξ. For these values and the realizations of u drawn in the previous step, substitute $\psi_t(\xi; \cdot)$ for the conditional expectations in (30). Use (33) to compute recursively the law of motion for the endogenous variables $[x_t(\xi), y_t(\xi)] = f(\xi; y_{t-1}(\xi), u_t)$ and the series $\{x_t(\xi), y_t(\xi)\}_{t=1}^T$. A necessary condition for the implementation of the algorithm is to choose ξ in such a way that $\{x_t(\xi), y_t(\xi)\}_{t=1}^T$ is an ergodic process.

3. Find the mapping $G(\xi) : R^{w\times p} \longrightarrow R^{w\times p}$ such that

$$G(\xi) = \arg \min_{\xi \in R^{w\times p}} \frac{1}{T} \sum_{t=0}^{T} \|\phi(x_{t+1}(\xi), y_{t+1}(\xi)) - \psi_t(\xi'; y_{t-1}(\xi), u_t)\|^2 \tag{34}$$

   where ξ' is the new set of parameters that minimize the difference between the estimated expectations and their realizations. Typically, given ξ^i a non-linear least squares regression (NLS) is used to compute ξ^{i+1} for $i = 1, 2, ...$ (for more information on NLS please see Pindyck and Rubinfeld (1981, sec. 9.4.1).

4. Iterate until

$$\xi_f = G(\xi_f) \tag{35}$$

by repeating steps 2 and 3. In the literature, the following iterative scheme is used to update ξ until a fixed point ξ_f is found:

$$\xi^{i+1} = (1-\tau)\xi^i + \tau G(\xi^i) \quad \text{for} \quad i = 1, 2, ... \tag{36}$$

The approximate solution at the fixed point is given by a series for the endogenous variables $\{x_t(\xi_f), y_t(\xi_f)\}_{t=1}^{T}$, a law of motion for the endogenous variables, $f(\xi_f; \cdot)$, and an approximation to $\Upsilon(\cdot)$ given by $\psi_t(\xi_f; \cdot)$. The algorithm ensures that $\psi_t(\xi_f; \cdot)$ is the best predictor of $E_t\{\phi(x_{t+1}, y_{t+1})\}$ and thus, consistent with the rationality assumption, if agents use $\psi_t(\xi_f; \cdot)$ to predict $E_t\{\phi(\cdot)\}$ on average, they do not make systematic errors.

Table 1: *Parameters of the first-order autoregressive models for the exogenous variables; standard errors are shown in parenthesis*

Dependent	c	$\ln y^A_{t-1}$	$\ln y^B_{t-1}$	$\ln d_{t-1}$
$\ln y^A_t$	4.276 (1.377)	0.521 (0.154)		
$\ln y^B_t$	3.492 (1.307)		0.644 (0.134)	
$\ln d_t$	1.028 (0.309)			0.789 (0.062)

Table 2: *Parameter values*

Parameter	**Value**
ρ	0.96
γ	2
θ	0.65
b_0	0.7
k	3
λ	2.25
η	0.9
h	-1/3
m	0

Table 3: *Moments of Asset Returns and Consumption Implied by a Representative Agent Model*

Sample Moments	Data	Model A	Model B
Bond return			
Mean	0.0141	0.0233	0.0208
Standard deviation	0.04	0.1198	0.1116
Stock return			
Mean	0.0898	0.0716	0.0784
Standard deviation	0.2	0.2594	0.2862
Equity premium	0.0758	0.0484	0.0577
Correlation of consumption with stock returns		0.4213	0.4153

Table 4: *Sample Moments of Asset Returns and Consumption Implied by the Heterogeneous Agent Model*

Sample Moments	**Data***	**Model**
Bond return		
Mean	0.0141	0.0142
Standard deviation	0.04	0.117
Stock return		
Mean	0.0898	0.0691
Standard deviation	0.2	0.272
Equity premium		
Mean	0.0758	0.0549
Standard deviation		0.2413
Price-dividend ratio		
Mean	25.5	29.54
Standard deviation	7.1	9.03
Average loss aversion		
Correlation of Agent A's consumption with stock returns		0.39
Correlation of Agent B's consumption with stock returns		0.32
Correlation of Agent A's consumption with aggregate income		0.80
Correlation of Agent B's consumption with aggregate income		0.91
Correlation of Agent A's consumption with own income		0.87
Correlation of Agent B's consumption with own income		0.92

* Source: Dividend-price ratio data are taken from Barberis, Huang and Santos (2001). All other statistics are from Mehra and Prescott (2003)

Table 5: *Sample Moments of Asset Returns and Consumption Implied by a Model Without Loss Aversion (Model C)*

Sample Moments	**Data**	**Model**
Bond return		
Mean	0.0141	0.0158
Standard deviation	0.04	0.118
Stock return		
Mean	0.0898	0.0615
Standard deviation	0.2	0.2582
Equity premium	0.0758	0.0458
Price-dividend ratio		
Mean	25.5	34.33
Standard deviation	7.1	9.95
Correlation of Agent A's consumption with stock returns		0.41
Correlation of Agent B's consumption with stock returns		0.34
Correlation of Agent A's consumption with aggregate income		0.81
Correlation of Agent B's consumption with aggregate income		0.91
Correlation of Agent A's consumption with own income		0.87
Correlation of Agent B's consumption with own income		0.92

Table 6: *Robustness:* b_0

Sample Moments	**Data**	$b_0 = 0.6$	$b_0 = 0.8$	$b_0 = 0.9$	$b_0 = 1$
Bond return					
Mean	0.0141	0.0145	0.0146	0.014	0.0139
Standard deviation	0.04	0.117	0.1168	0.1163	0.1162
Stock return					
Mean	0.0898	0.0684	0.0695	0.0699	0.0703
Standard deviation	0.2	0.27	0.2736	0.2752	0.2766
Equity premium					
Mean	0.0758	0.054	0.0553	0.0559	0.0564
Standard deviation	0.2	0.2396	0.2429	0.2443	0.2456
Price-dividend ratio					
Mean	25.5	30.23	30.09	30.03	29.98
Standard deviation	7.1	9.05	9.07	9.09	9.11
Average loss aversion		2.45	2.43	2.41	2.41
$\rho_{C^A,R}$ *		0.39	0.39	0.39	0.39
$\rho_{C^B,R}$ *		0.32	0.32	0.32	0.32
$\rho_{C^A,y}$ *		0.80	0.80	0.80	0.80
$\rho_{C^B,y}$ *		0.91	0.91	0.91	0.91
ρ_{C^A,y^A} *		0.88	0.88	0.88	0.88
ρ_{C^B,y^B} *		0.92	0.92	0.92	0.92

* ρ is the correlation coefficient of the respective variables

Table 7: *Robustness:* η

Sample Moments	**Data**	$\eta = 0.5$	$\eta = 0.7$	$\eta = 0.8$
Bond return				
Mean	0.0141	0.013	0.0141	0.0145
Standard deviation	0.04	0.1165	0.1172	0.1173
Stock return				
Mean	0.0898	0.0657	0.0671	0.0679
Standard deviation	0.2	0.2601	0.2638	0.2676
Equity premium				
Mean	0.0758	0.0527	0.0529	0.0534
Standard deviation	0.2	0.2313	0.2337	0.2371
Price-dividend ratio				
Mean	25.5	30.78	30.17	30.16
Standard deviation	7.1	9.09	8.82	8.89
Average loss aversion		2.26	2.31	2.36
$\rho_{C^A,R}$ *		0.4	0.39	0.39
$\rho_{C^B,R}$ *		0.32	0.32	0.32
$\rho_{C^A,y}$ *		0.80	0.81	0.80
$\rho_{C^B,y}$ *		0.91	0.91	0.91
ρ_{C^A,y^A} *		0.88	0.88	0.88
ρ_{C^B,y^B} *		0.93	0.92	0.92

* ρ is the correlation coefficient of the respective variables

Table 8: *Robustness: Borrowing constraint*

Sample Moments	**Data**	$h = 0.15$	$h = 0.2$	$h = 0.25$	$h = 0.35$	$h = 0.4$
Bond return						
Mean	0.0141	0.0113	0.0126	0.0133	0.015	0.0154
Standard deviation	0.04	0.1177	0.1172	0.1169	0.1167	0.1164
Stock return						
Mean	0.0898	0.0697	0.0692	0.0688	0.0695	0.0703
Standard deviation	0.2	0.282	0.278	0.2723	0.2718	0.2709
Equity premium						
Equity premium	0.0758	0.0584	0.0566	0.0554	0.0545	0.0549
Standard deviation		0.2517	0.2474	0.2417	0.2412	0.24
Price-dividend ratio						
Mean	25.5	31.75	31.28	30.39	29.68	28.91
Standard deviation	7.1	9.88	9.58	9.13	8.94	8.74
Average loss aversion		2.47	2.43	2.47	2.43	2.46
$\rho_{C^A,R}$ *		0.4	0.4	0.39	0.39	0.39
$\rho_{C^B,R}$ *		0.31	0.32	0.31	0.33	0.33
$\rho_{C^A,y}$ *		0.78	0.79	0.79	0.81	0.81
$\rho_{C^B,y}$ *		0.9	0.9	0.90	0.91	0.91
ρ_{C^A,y^A} *		0.89	0.89	0.88	0.87	0.87
ρ_{C^B,y^B} *		0.93	0.93	0.93	0.92	0.92

* ρ is the correlation coefficient of the respective variables

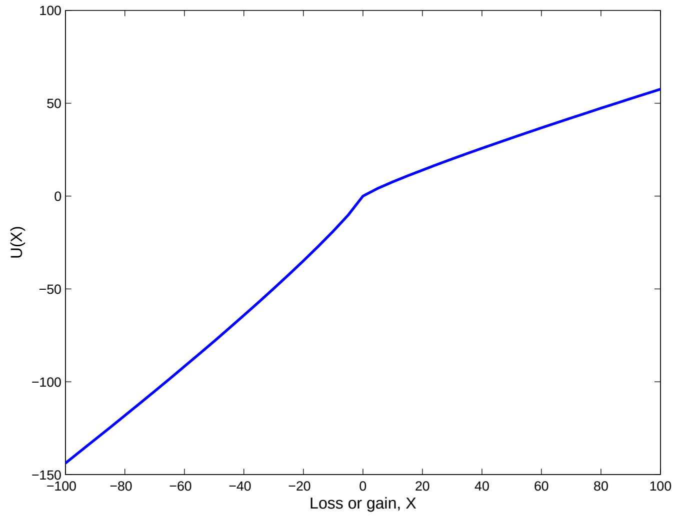

Figure 1: *Kahneman and Tversky's Value Function*

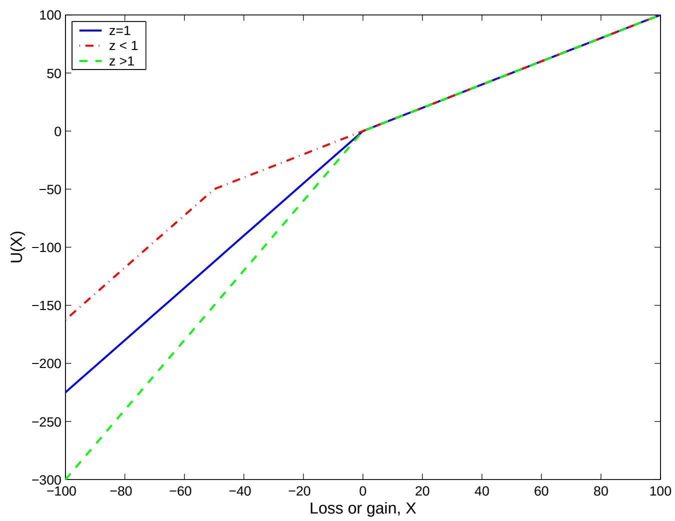

Figure 2: *Utility from gains and losses for different values of* z_t

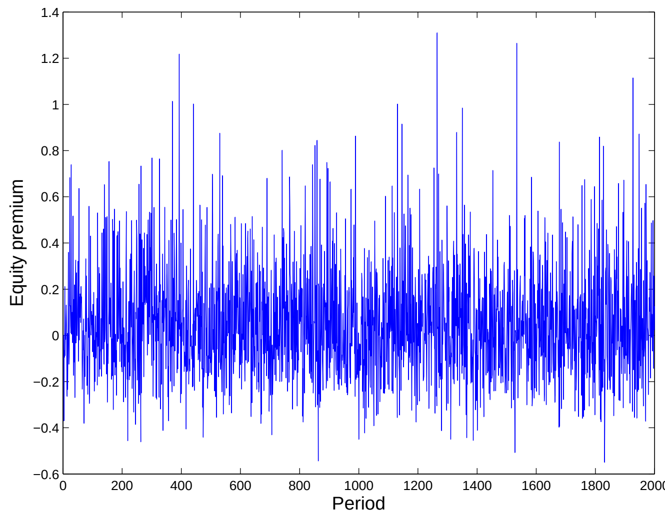

Figure 3: *The volatility of excess stock returns for t = 0: 2000*

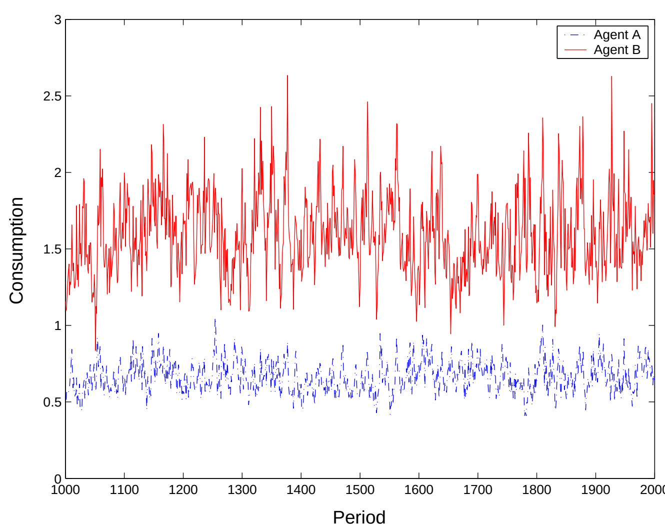

Figure 4: *Consumption for t = 1000:2000*

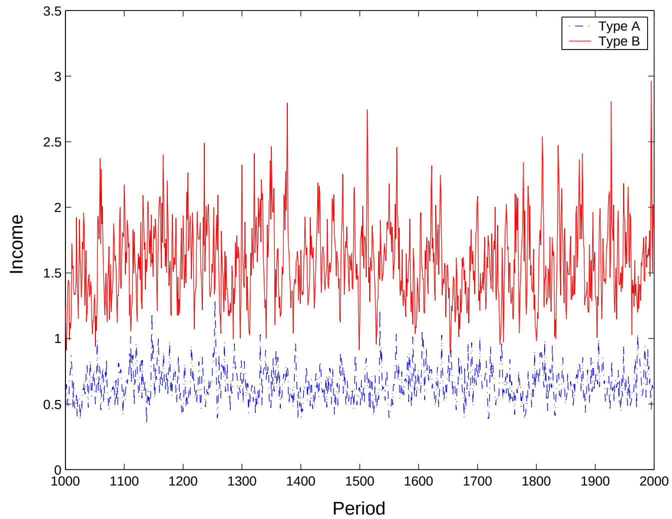

Figure 5: *Income for t = 1000:2000*

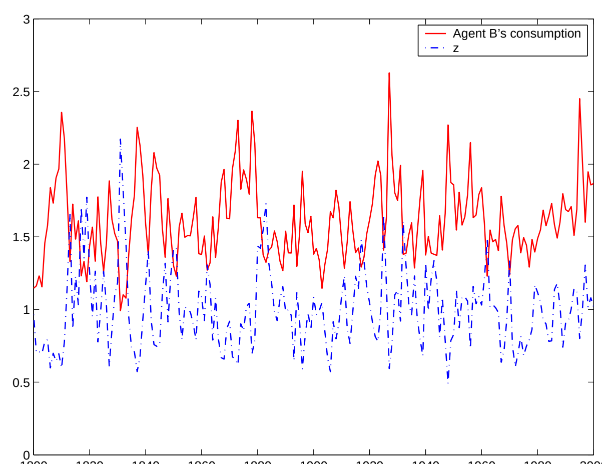

Figure 6: *Type B agent's consumption and z for t = 1800:2000*

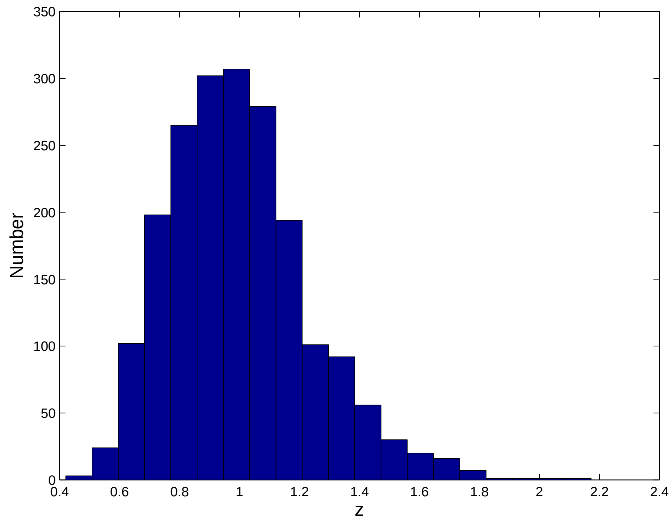

Figure 7: *Histogram of z*

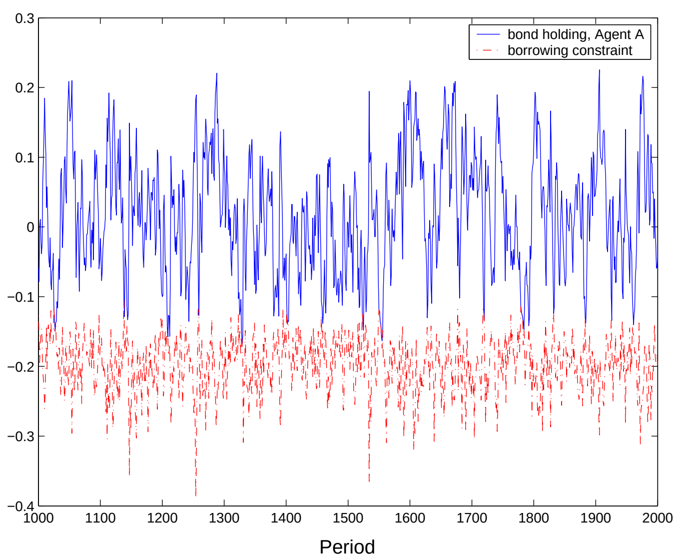

Figure 8: *Bond holdings and bond constraint for 1,000 periods, Type A agent*

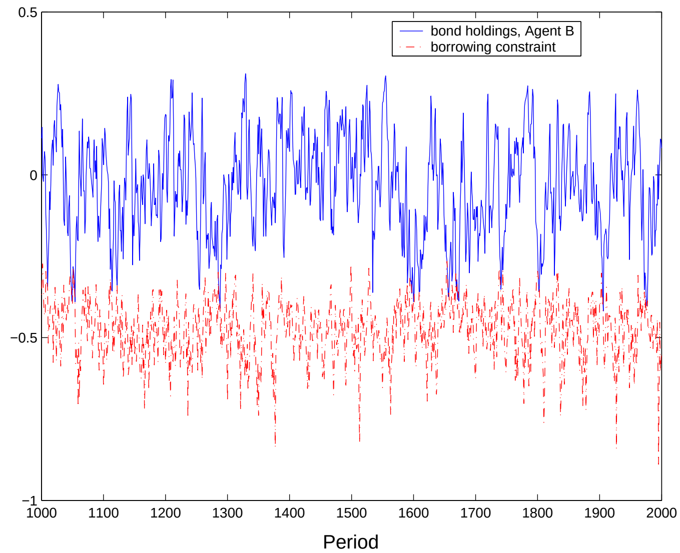

Figure 9: *Bond holdings and bond constraint for 1,000 periods, Type B agent*

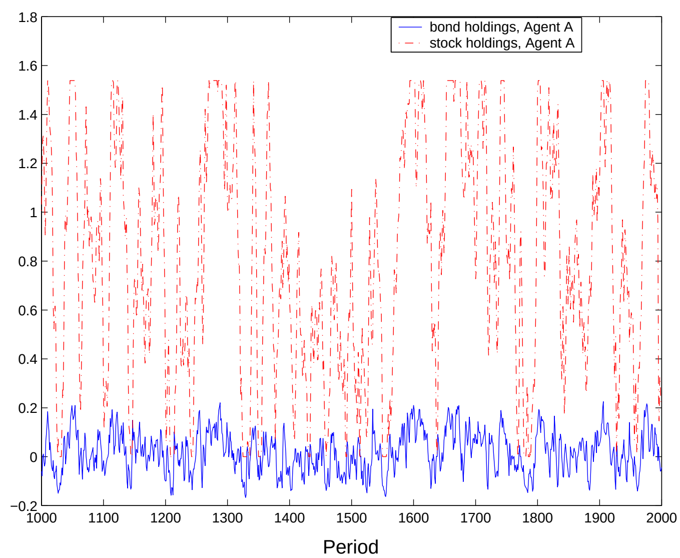

Figure 10: *Stock and bond holdings for 1,000 periods, Type A agent*

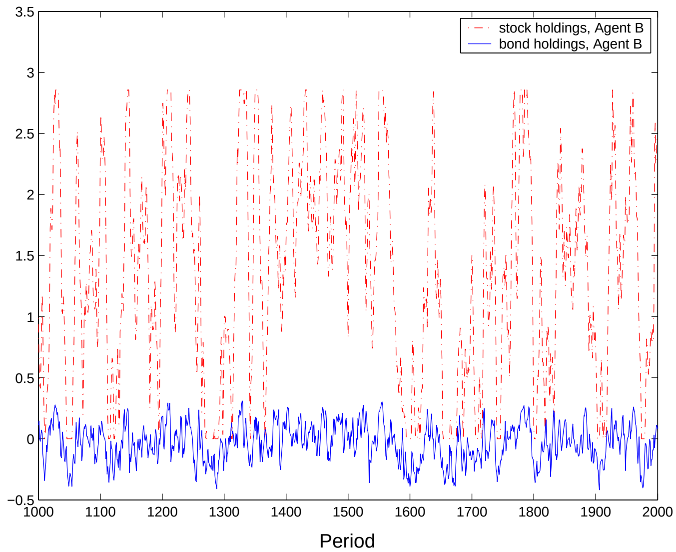

Figure 11: *Stock and bond holdings for 1,000 periods, Type B agent*

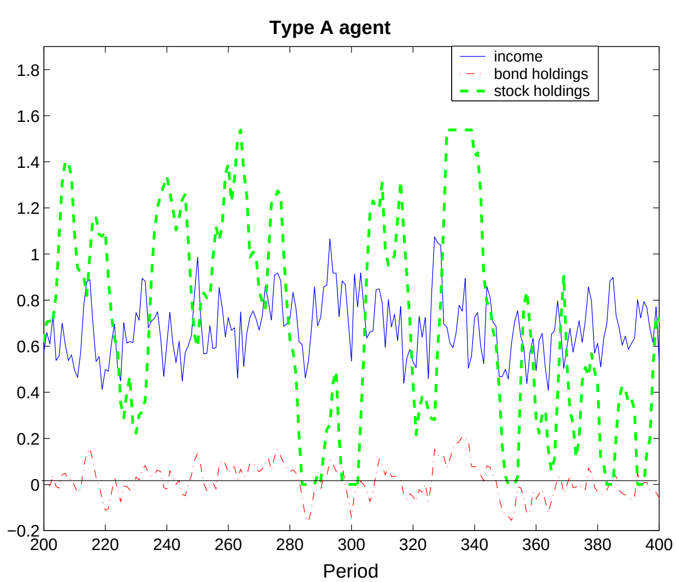

Figure 12: *Income, stock and bond holdings for 200 periods, Type A agent*

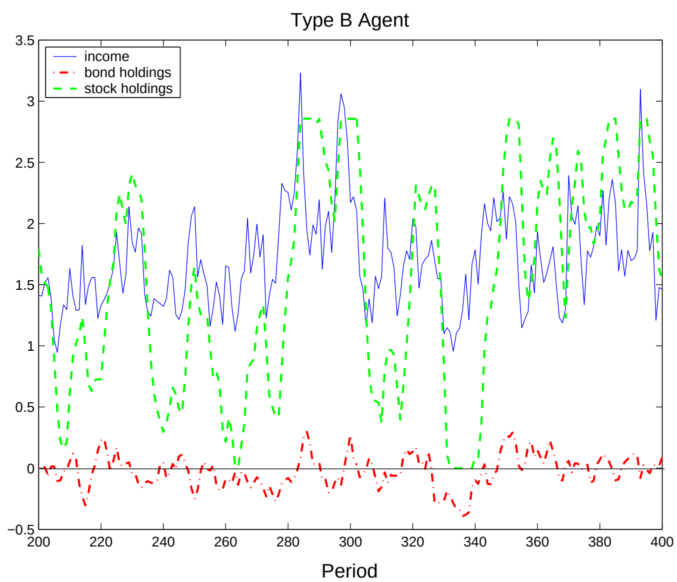

Figure 13: *Income, stock and bond holdings for 200 periods, Type B agent*

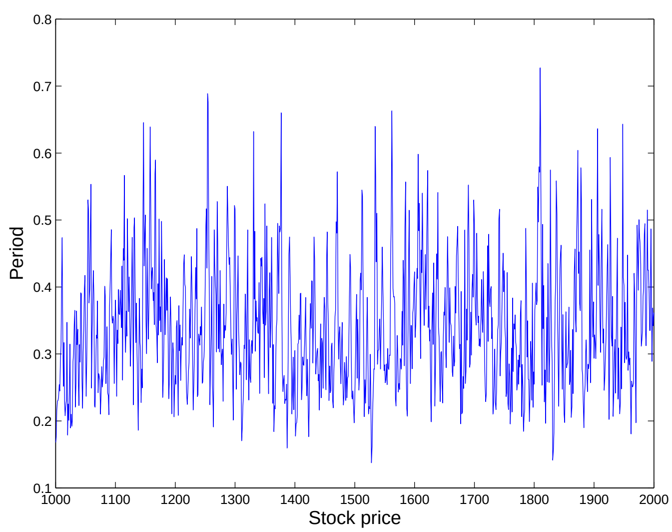

Figure 14: *Stock price for 1,000 periods*