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Abstract

We propose a nonparametric model for global cost minimization as a framework for optimal allocation of a firm's output target across multiple locations, taking account of differences in input prices and technologies across locations. This should be useful for firms planning production sites within a country and for foreign direct investment decisions by multi-national firms. Two illustrative examples are included. The first example considers the production location decision of a manufacturing firm across a number of adjacent states of the US. In the other example, we consider the optimal allocation of US and Canadian automobile manufacturers across the two countries.

Keywords: Cost minimization; Data Envelopment Analysis; Heterogeneous technology; Location efficiency

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INPUT PRICE VARIATION ACROSS LOCATIONS AND A GENERALIZED MEASURE OF COST EFFICIENCY

1. Introduction

In the typical neoclassical optimization model of a cost-minimizing firm, input prices are treated as parameters, not subject to choice by the firm. Competitive price taking behavior in the input market is at the core of neoclassical duality theory. For example, the classic survey paper by Diewert (1982) deals primarily with competitive markets and considerations of non-competitive approaches to microeconomic theory and duality feature more like an afterthought. The decision making problem consists of selecting the input bundle that would produce the target output (bundle) at the least cost, *given the applicable prices of inputs.* The firm has the ability to *set* prices of inputs only if the input market is non-competitive. However, in such markets, the degree of monopsonistic power is limited by the elasticity of the input supply.

Even when input markets are competitive, input prices may vary across locations (like countries or regions within a country) although they are given *at any particular location*. Such variations in prices may occur due to lack of mobility of inputs. A firm effectively *chooses* between the input price vectors by producing its output at one location or another.[1] However, if the firm produces all or part of its output in any particular location, it must use inputs purchased locally at prices applicable in that location. There are many real life situations that fit this description. A multi-national company takes the local input prices (especially for labor and real estate) as an important factor while deciding to make foreign direct investment in a given country. Output produced in some other country must be from inputs purchased there at that country's prices.[2] Likewise, within a given country, a firm decides to locate its production facilities in one state or another based on the cost-competitiveness of the different states.

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[1] In a related context, Tone (2002) considers the case where input prices vary across firms. He constructs a production possibility set using the actual input costs of the different firms at these prices as a measure of an aggregate input. As argued by Banker et al. (2007) this would lead to a valid measure of efficiency only if all firms were allocatively efficient. It should be noted that this indirectly constructed production possibility set will obviously be price dependent and would differ from the technology obtained from the free disposal convex hull of actual inputs and outputs.

[2] Lower input prices in developing countries largely account for the extent of business process outsourcing by the US firms to countries like India and China.

In the context of multi-national choice of locations another pertinent consideration is the difference in technology across countries. Such technological diversity arises due to differences in the physical and legal environment across countries.

In this paper we utilize the nonparametric method of Data Envelopment Analysis (DEA) and introduce a model that allows the firm to *choose* the prices it pays for the inputs while at the same time treats the input prices as parameters. In the proposed setup a firm is allowed to locate its production facilities at multiple locations in light of input price differences. By contrast, the standard model is one where a specific location (and input price vector) is pre-specified[3]. We develop a global measure of cost efficiency, one component of which is the firm's *location* or *input price efficiency*. We then generalize the model further to accommodate technological heterogeneity across countries. This permits us to decompose the location efficiency of the firm into two distinct components representing *technology choice* and *input price efficiency*. The proposed methodology should be useful for firms planning production sites within a country and for foreign direct investment decisions by multi-national firms. We then provide two examples to illustrate the application of our proposed methodology. The first example considers location choice when technology is homogeneous but input prices vary across locations. For this we consider the choice of production location(s) by a US manufacturing firm across a number of adjacent states of the US. In the second example, both the technology and input prices vary across the potential locations. Here we assess the potential for cost saving by US or Canadian auto makers by distributing the output target across both countries.

The paper unfolds as follows. Section 2 provides the methodological background and develops the proposed models for measuring global cost efficiency. Section 3 further generalizes the model to allow for technological variation across countries. Section 4 contains the two illustrative examples. Section 5 concludes.

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[3] See for example, Ray (2004).

2. The Nonparametric Methodology

Consider an industry producing m outputs from n inputs. An input-output bundle $(x \in R_n^+, y \in R_m^+)$ is considered feasible when the output bundle y can be produced from the input bundle x. The technology faced by the firms in the industry can be described by the production possibility set

$$T = \{(x, y): y \text{ can be produced from } x\}. \quad (1)$$

The method of Data Envelopment Analysis introduced by Charnes, Cooper, and Rhodes (1978) and further extended to non-constant returns to scale technologies by Banker, Charnes, and Cooper (1984) provides a way to construct the production possibility set from an observed data set of input-output bundles.

Suppose that (x^j, y^j) is the input-output bundle observed for firm j $(j = 1,2,\ldots, N)$. Then the smallest production possibility set satisfying the assumptions of convexity and free disposability that includes these observed bundles is

$$S = \{(x, y) : x \geq \sum_{j=1}^{N} \lambda_j x^j ; y \leq \sum_{j=1}^{N} \lambda_j y^j ; \sum_{j=1}^{N} \lambda_j = 1; \lambda_j \geq 0; (j = 1,2,..., N)\}. \quad (2)$$

One can obtain various measures of efficiency of a firm using the set S as the reference technology. If the output-bundle of the firm is treated as an assigned task, efficiency lies in producing the target output bundle y^0 at the minimum cost. For the specific output bundle y^0, we define the input (requirement) set as

$$V(y^0)=\{x : y^0 \text{ can be produced from } x\} \quad (3)$$

An input requirement set that corresponds to S defined in (2) would be

$$V(y) = \{x : x \geq \sum_{j=1}^{N} \lambda_j x^j ; y \leq \sum_{j=1}^{N} \lambda_j y^j ; \sum_{j=1}^{N} \lambda_j = 1; \lambda_j \geq 0; (j = 1,2,..., N)\}. \quad (4)$$

Suppose that the firm faced the input price vector w^0. Then its actual cost is $C^0 = w^{0\prime} x^0$. However, the minimum cost of producing the target output is

$$C (w^0, y^0) = \min \ w'x : x \in V(y^0) \quad (5)$$

Here, the choice variable is the input vector x. The standard DEA model for cost minimization is

$C(w^0, y^0) = \min w'x$

$$\text{s.t.} \quad \sum_{j=1}^{N} \lambda_j y^j \geq y^0;$$

$$\sum_{j=1}^{N} \lambda_j x^j \leq x; \tag{6}$$

$$\sum_{j=1}^{N} \lambda_j = 1;$$

$$\lambda_j \geq 0; (j = 1,2,..., N).$$

A measure of the cost efficiency of the firm is

$$\gamma^0 = \frac{C(y^0; w|V(y^0))}{C^0}. \tag{7}$$

Input Price Variation and Location Efficiency

In the foregoing analysis it was assumed that the firm faces a specific vector of input prices and any input it uses to produce its output must be purchased at these prices. We now consider a situation where input prices vary across locations, even though they are fixed at any given location. The firm can take advantage of the input prices in a given location by producing all or part of its output at that location. In this model the firm can choose to produce its total output at multiple locations if it wants but cannot create its own input price vector by "cherry picking" individual inputs from different locations. Any output produced at a given location must be produced only from inputs procured locally at the applicable prices.

Suppose that the firm can produce some or all of its output in one or more of R different locations. The vector of input prices in location r is $w^r (r= 1,2,\ldots,R)$. The firm decides to produce output $y^r (\geq 0)$ at location r. The input requirement set for output bundle y in that location is

$V^r(y^r) = \{ x : y^r$ can be produced from x at location r $\}$.

The firm's problem is to allocate its output across the alternative locations and select appropriate input bundles to minimize the total cost. This can be stated as

$$\min \sum_{r=1}^{R} w^{r\,\prime} x^r$$

$$\text{s.t. } y^r \in V^r(y^r); \sum_{r=1}^{R} y^r \geq y^0. \tag{8}$$

Homogeneous Technology

If we assume that the firm can access the same production technology in all locations,

$V^r(y) = V(y)$; $(r = 1, 2, \ldots, R)$.

Define the matrix of input prices from the different locations

$$W = \{w^1 \mid w^2 \mid \ldots \mid w^r \mid \ldots \mid w^R\}. \tag{9}$$

Then the multi-location minimum cost function can be evaluated as

$$C^{ML}(y^0; W \mid V(y^0)) = \min \sum_{r=1}^{R} w^{r\prime} x^r$$

$$\begin{aligned}
\text{s.t.} \quad & \sum_{j=1}^{N} \lambda_{rj} x^j = x^r ; \ (r = 1, 2, \ldots, R); \\
& \sum_{j=1}^{N} \lambda_{rj} y^j = y^r ; \ (r = 1, 2, \ldots, R); \\
& \sum_{r=1}^{R} y^r \geq y^0 ; \\
& \sum_{j=1}^{N} \lambda_{rj} = B_r ; \ (r = 1, 2, \ldots, R); \\
& B_r \in \{0, 1\}; \ (r = 1, 2, \ldots, R); \\
& \lambda_{rj} \geq 0; (j = 1, 2, \ldots, N; r = 1, 2, \ldots, R).
\end{aligned} \tag{10}$$

The vector y^r is the output bundle produced at location r and x^r is the corresponding input bundle used there. The binary variable (B_r) indicates whether the firm produces any output at all in location r. Note that the firm does not have to produce at every location. When location r is not selected, both y^r and x^r would be null vectors. But because every (x^r, y^r) is some convex combination of observed input-output bundles (x^j, y^j), x^r and y^r will be null vectors only when every λ_{rj} (and hence , $B_r = \sum_{j=1}^{N} \lambda_{rj}$) equals zero for that particular location, r. On the other hand, for (x^r, y^r) to be strictly positive, at least one λ_{rj} must be positive and convexity requires that B_r should equal 1. Note that if w^r is constrained to be the same across all locations then a comparison of the optimal solutions of (6) and (10) would reveal whether $C(w^0, y^0)$ is greater than $\sum_r C(w^0, y^r)$, where $\sum_r y^r = y^0$. This is a test of sub-additivity of the cost function (Baumol, Panzar, and Willig, 1982).

A Generalized Measure of Cost Efficiency and its Decomposition

We now consider a firm actually producing output y^0 from input x^0 at location 0 where the input price vector is w^0. For any observed matrix of input prices W, a generalized or *global* measure of its cost efficiency will be

$$\gamma_G^0 = \frac{C^{ML}(y^0; W \mid V(y^0))}{C^0}. \tag{11}$$

The expression in (11) can be further decomposed as

$$\gamma_G^0 = \left(\frac{C(y^0; w^0 \mid V(y^0))}{C^0}\right) \cdot \left(\frac{C^{ML}(y^0; W \mid V(y^0))}{C(y^0; w^0 \mid V(y^0))}\right). \tag{12}$$

The first factor on the right hand side of (12) is the conventional cost efficiency measure (γ^0) which we may describe as the *local cost efficiency*. The second factor is its *location efficiency*

$$\gamma_L^0 = \left(\frac{C^{ML}(y^0; W \mid V(y^0))}{C(y^0; w^0 \mid V^0(y^0))}\right). \tag{13}$$

Note that location efficiency in (13) depends on the firm's ability to take advantage of the input price variation across locations (shown by the matrix W in the numerator). In that sense, it can also be viewed as its *input price efficiency.* A firm's location efficiency equals unity only when producing its entire output at its current location leads to the minimum cost. In that case the *global* and *local* measures of cost efficiency would be identical.

3. The Case of Heterogeneous Technologies

We have so far assumed that the firm has access to the same technology irrespective of where it locates its production facilities. This is not an unreasonable assumption in the context of a firm choosing between locations across different states within the same country. For example, the production possibility sets will differ little between two General Motors auto assembly plants – one in Arlington, TX and the other in Pontiac, MI. But this is unlikely to be the case when comparing two plants located in different countries. An input-output combination that is feasible at the assembly plant in Arlington may not be feasible at the GM assembly plant in Hokkaido, Japan or at the Silao assembly plant in Mexico. Such differences arise out of a variety of factors that include differences in regulation, labor laws and work culture, levels of human capital, and the physical environment. Hence, in location choice decisions a firm must often take account of differences in both the input prices and technologies.

In order to construct different production possibility sets and the corresponding input requirement sets for different locations, we first group the observed input-output bundles by the locations of the corresponding firms. Define the index set of observations $J = \{1, 2, ..., N\}$ and partition it into non-overlapping subsets

$J_r = \{j$: firm j is from location r ; $(r = 1,2,...,R)\}$.

In this case, the input requirement set at location r will be

$$V^r(y) = \{x : x \geq \sum_{j \in J_r} \lambda_{rj} x^j ; y \leq \sum_{j \in J_r} \lambda_{rj} y^j ; \sum_{j \in J_r} \lambda_{rj} = 1; \lambda_{rj} \geq 0\}; \quad (r = 1,2,..., R). \tag{14}$$

Define the collection of input requirement sets

$$\Omega(y^0) = \{V^1(y^0), V^2(y^0), ..., V^r(y^0), ..., V^R(y^0)\}. \tag{15}$$

The revised DEA mixed integer programming problem is

$$C_H^{ML}(y^0; W \mid \Omega(y^0)) = \min \sum_{r=1}^{R} w^{r\prime} x^r$$

$$\text{s.t.} \quad \sum_{j \in J_r} \lambda_{rj} x^j = x^r ; \; (r = 1,2,..., R);$$

$$\sum_{j \in J_r} \lambda_{rj} y^j = y^r ; \; (r = 1,2,..., R);$$

$$\sum_{r=1}^{R} y^r \geq y^0 ; \tag{16}$$

$$\sum_{j\in J_r}\lambda_{rj} = B_r\;;\;(r=1,2,...,R);$$

$$B_r \in \{0,1\};\;(r=1,2,...,R);$$

$$\lambda_{rj} \geq 0;\left(j\in J_r; r=1,2,...,R\right).$$

In the case of heterogeneous technologies, the firm's global efficiency becomes

$$\gamma^0_{G|H} = \frac{C_H^{ML}(y^0;W\mid\Omega(y^0))}{C^0}. \tag{17}$$

As in the case of homogeneous technology, $\gamma^0_{G|H}$ in (17) can also be decomposed as

$$\gamma^0_{G|H} = \left(\frac{C_H^{ML}(y^0;w^0\mid V^0(y^0))}{C^0}\right)\left(\frac{C_H^{ML}(y^0;W\mid\Omega(y^0))}{C_H^{ML}(y^0;w^0\mid V^0(y^0))}\right). \tag{18}$$

The first factor on the right hand side of (18) is the firm's local cost efficiency under technological heterogeneity (γ^0_H). It depends on both the local input prices, w^0, and the local technology, $V^0(y)$. The second factor is a measure of the location efficiency of the firm. However, in the present case, location efficiency will equal unity only if the firm has distributed its output target optimally across locations in light of differences in technologies as well as in prices. To separate these two sources of inefficiency, we may further decompose the location efficiency term as

$$\gamma^0_{L|H} = \left(\frac{C_H^{ML}(y^0;W\mid\Omega(y^0))}{C_H^{ML}(y^0;w^0\mid V^0(y^0))}\right) = \left(\frac{C_H^{ML}(y^0;w^0\mid\Omega(y^0))}{C_H^{ML}(y^0;w^0\mid V^0(y^0))}\right)\left(\frac{C_H^{ML}(y^0;W\mid\Omega(y^0))}{C_H^{ML}(y^0;w^0\mid\Omega(y^0))}\right). \tag{19}$$

In this decomposition, the first factor is the technology choice component

$$\gamma^0_V = \left(\frac{C_H^{ML}(y^0;w^0\mid\Omega(y^0))}{C_H^{ML}(y^0;w^0\mid V^0(y^0))}\right). \tag{20}$$

The other factor is the input price efficiency

$$\gamma^0_w = \left(\frac{C_H^{ML}(y^0;W\mid\Omega(y^0))}{C_H^{ML}(y^0;w^0\mid\Omega(y^0))}\right). \tag{21}$$

Thus,

$$\gamma^0_{G|H} = \left(\gamma^0_H\right).\left(\gamma^0_V\right).\left(\gamma^0_w\right). \tag{22}$$

When technology is homogeneous across locations, γ_V^0 equals unity and any location inefficiency comes from input price inefficiency. Only when the input price efficiency (γ_w^0) also equals unity, the local and the global measures of cost efficiency will coincide.

In the following section we provide two examples in order to illustrate how the proposed model can be used in empirical applications.

4. Empirical Applications

Example 1. Homogeneous Technology Case and Location Choice in US Manufacturing

In this example we consider the cost-minimizing choice of production locations by a manufacturing firm in the US. For this, we conceptualize a 1-output, 5-input production technology and use data constructed from the US 2002 Economic Census - Manufacturing. We assume that there is no difference in the technology across the states within the continental US. Given the fact that the market for manufactured goods is nationally integrated, we assume that the output price does not vary across states so that the value of output is a reasonable measure of the quantity produced. Input prices, however, do vary across states. We treat the average (i.e. per firm) input-output bundle *as a data point* from each state and the production possibility set is constructed as the free disposal convex hull of these points.

Output is measured by the gross value of production. The inputs included are (a) production labor (L_1), (b) non-production labor (L_2), (c) capital (K), (d) energy (E), and (e) materials (M). Production labor is measured by the number of hours worked. The corresponding input price is wage paid per hour to production workers (w_1). The other labor input is the number of non-production employees. The corresponding wage rate (w_2) is total emolument per employee. The capital input is the average of beginning-of-the year and end-of the year (nominal) values of gross fixed assets. The capital input price (i.e. user cost), p_k, is measured by the sum of depreciation, rent, and (imputed) interest expenses per dollar of gross value of capital. The quantity of the energy input (E) is constructed by deflating the expenditure on purchased fuels and electricity by state specific energy price (p_E)[4]. Total expenditure on materials parts and containers is used as measure of the materials input quantity (*M*). By implication, materials price (p_M) was set equal to unity for every state.

A firm with a target output level to produce is looking for the cost-minimizing location of its production plant(s) in one or several states within a particular region of the US. In order to minimize differences in transportation costs across locations, we restrict the firm's choice to four South Atlantic states (namely, Georgia (GA), North Carolina (NC), South Carolina (SC) and Virginia (VA)) and Tennessee (TN) from the Old South West. For the output target, we select the gross manufacturing output per firm in GA at current prices in 2002 ($14.249m). For this example, we solve both the standard DEA problem (6) for each individual input price vector as well as the DEA problem (10) above with N equal to 48 (the continental US states) and R equal to 5 (GA, NC, SC, VA, and TN).

[Table 1 about here]

Table 1 reports the input prices for the five states under consideration for location. The last column (C_j^*) shows how much it would cost to produce the target output if the firm had to locate all of its production in any one of the five states under consideration. At these different input prices, the lowest cost of production for the target output of $14.249m would be $8.320m if the firm located in Georgia. Decision to locate in any of the other four states would result in higher cost. Interestingly, the typical firm in GA actually incurs a cost of $9.503m (as shown in Table 2). Thus, it has a local cost efficiency of 0.8754.

[Table 2 about here]

Table 2 shows that when allowed to produce its target output in multiple locations over the five states considered, the firm would produce $7.8 m worth of the output in GA and the remaining $6.450m in SC. The corresponding costs would be $4.294m in GA and $3.628m in SC adding up to a total cost of $7.922m. Thus, if the firm decided to produce

[4] We use the industrial sector total energy price in 2002 (measured in nominal dollars per million Btu). Source: US Energy Information Administration.

the entire output in GA alone, even if it was fully cost efficient at the local prices, its multi-location cost efficiency would be 95.2%. Because, technology is assumed to be homogeneous across locations, the 4.8% inefficiency is entirely price inefficiency. It is important to note here that the optimal input-output bundles of the firm at its plants in GA and SC are not the actual (or scaled) average bundles of the firms observed in theses two states. Even though it is producing in GA, availing itself of the input prices of GA, the optimal input mix at the GA plant would be different from what is actually observed for an existing plant in GA. At the optimal solution to this problem, $\lambda_{GA|AZ} = 0.678$ and $\lambda_{GA|NM} = 0.322$. That is, the optimal bundle of the plant in GA is a weighted average of the input-output bundles of a typical firm in Arizona (AZ) (67.8%) and New Mexico (NM) (32.2%). Similarly, at this optimal solution, $\lambda_{SC|NM} = 1$ and all other $\lambda_{SC|j} = 0$ for any other state j. That is, the optimal bundle in its SC plant would be exactly the same as the actual average input-output bundle observed in NM. Overall, in an act of domestic outsourcing, the firm in GA would shift over 45% of its output from GA to SC.

Example 2. Heterogeneous Technology Case and International Choice of Location for Automobile Production in North America

In this example we consider a location decision across geographical regions where both input prices and the technologies differ. For this, we use a well known annual data set on the Auto Industries from the US and Canada for the years 1961-1984 constructed by Fuss and Waverman (1992). A limitation of this data set is that neither the US nor the Canadian data is a cross section data set. Instead, for each year we have only one observation on the output and input quantities per firm. This precludes construction of annual production possibility sets based on cross section data. Our analysis of necessity has to be based on the available time series data. At the same time, pooling data points from different years to construct the production possibility frontier would not be legitimate if there is technical change over time. We circumvent this problem by assuming that any technical change that has taken place over years is *non-regressive* so that input-output bundles observed in the past are deemed feasible in later periods, although input-output bundles observed in subsequent years may not be feasible.[5] In order to maximize the number of data points for construction of the non-parametric frontier, we analyze the data from the year 1984, which is the latest year for which data were available.[6]

For the automobile production technology, we consider a single output (total automobile production) and three inputs (labor, capital, and materials). Our data is obtained from Table 4.C.3 (for US) and Table 4.C.4 (for Canada) of Fuss and Waverman (1992), pages 112-113. In the above tables, output in both countries is reported as a multi-lateral quantity index, with Canada 1971 treated as the base. As for the inputs, the tables report the cost per plant in thousands of Canadian dollars (CST), the cost shares of the three individual inputs (i.e. SL, SK, and SM) as well as the price of the three individual inputs (PL, PK, and PM) in Canadian dollars. The input prices are also normalized with Canada 1971 as the base. For our analysis the input quantities (*L*, *K*, and *M*) were obtained from the reported total cost, the cost shares and the input prices.

The individual elements of the input vector $x' = (L, K, M)$ represent the quantities of the labor, capital, and material inputs. Similarly, y is the associated output. For this application, the input-output bundle (x_t^{US}, y_t^{US}) represents the actual output and inputs (per firm) in the US auto industry in year t. Similarly, the input-output bundles from Canada are (x_t^{CA}, y_t^{CA}). The corresponding input price vectors are w_t^{US} and w_t^{CA}. For the

[5] For a cross-sectional data it is obviously true that all input-output bundles actually observed simultaneously are feasible at the point in time when they were observed. But note that the assumptions of free disposability and convexity imply the feasibility of all other points in the free disposal convex hull of the observed points. Clearly, these other points in the production possibilities set were not observed. In a similar manner, when using time series data, if we assume that technical change is *non-regressive*, then input-output bundles observed in the past are deemed feasible in later periods. Even though they were not actually observed, they could have been (see Tulkens and Vanden Eeckaut, 1995).

[6] Another, limitation of the data set used in this example, is that by now it is over 20 years old and its relevance for the present state of the North American automobile industry is marginal. It needs to be recognized, however, that the objective of this example is to provide an empirical illustration of the proposed methodology rather than to carry out an in depth investigation of substantive issues related to the current state of the auto industry. One only needs to browse through the relevant chapters of the Fuss and Waverman book to appreciate the extent of the detailed information that one must have access to in order to update the data to the present times. Given the illustrative nature of this example, we considered that the data in hand, even though outdated, would be quite adequate for this example.

multi-location cost efficiency of US firms in 1984, we solve the following mixed integer programming problem:

$$
\begin{aligned}
&\min\ w_{1984}^{US\prime} x^{US} + w_{1984}^{CA\prime} x^{CA} \\
\text{s.t.}\quad &\sum_{t=1961}^{1984} \lambda_t^{US} x_t^{US} = x^{US}; \\
&\sum_{t=1961}^{1984} \lambda_t^{US} y_t^{US} = y^{US}; \\
&\sum_{t=1961}^{1984} \lambda_t^{US} = B^{US}; \\
&\sum_{t=1961}^{1984} \lambda_t^{CA} x_t^{CA} = x^{CA}; \\
&\sum_{t=1961}^{1984} \lambda_t^{CA} y_t^{CA} = y^{CA}; \\
&\sum_{t=1961}^{1984} \lambda_t^{CA} = B^{CA}; \\
&y^{US} + y^{CA} \geq y_{1984}^{US}; \\
&\lambda_t^{US}, \lambda_t^{CA} \geq 0\,(t = 1961, 1962, .., 1984); \\
&B^{US}, B^{CA} \in \{0,1\}.
\end{aligned}
\tag{23}
$$

At the optimal solution of this problem $\left(\lambda_{1980}^{US} = 0.09056, \lambda_{1982}^{US} = 0.90943 : \lambda_{1961}^{CA} = 1\right)$ while all other λ-weights take the value 0.

[Table 3 about here]

In Table 3, the row labeled as 'US actual' shows the actual inputs used and output produced in the year 1984. On the other hand, the row marked as 'US-optimal' shows the combined input bundle from both locations along with the total output produced. The other two rows 'in US' and 'in Canada' show optimal input bundles and output quantities at the US and Canadian locations of the firm. At the optimal solution of the problem, of the 2.14249 units of the output produced in the US in 1984, 20.5% would be produced in Canada and the remaining 79.5% would be produced in the US. A comparison of the actual and the optimal quantities of labor use in the US implies that 42.28% of labor would be eliminated. At the same time, production by the US firms in Canada would lead to increase in labor input by 54.76% of the actual labor employed in Canada. Thus, although the employment in the US auto industry would go down, total input of labor at the two locations would actually increase. It is also found that if US firms are restricted to the US technology and prices only, there is no evidence of cost inefficiency. Thus, local cost efficiency in the US in 1984 was 100%. But multi-location or global cost efficiency was only 85.28%.

Next, we solve the multi-location problem where in both US and Canada, the US firms would be paying the US input prices.[7] This neutralizes the effect of input price variations and allows us to extract the potential for any cost reduction by exploiting technology heterogeneity alone. The analysis shows that even in the absence of differences in input prices US firms could lower their actual cost by 8.96% by locating part of the production in Canada. Hence, the technology choice efficiency is 0.9108. By implication, input price efficiency is 0.9367.

[Table 4 about here]

We also looked at the multi-location cost minimization problem for Canadian automobile manufacturers for the same year. The results reported in Table 4 show that unlike in the case of US, for which the local cost efficiency was 100%, in case of Canada it is as low as 77.22%. That is, even in its present location, a Canadian firm could lower its production cost by 22.78% at the input prices it is actually paying. Attaining full cost efficiency would require that Canadian firms cut down labor by 25.89% and materials by 29.17%. It is interesting to note, however, that it would not be cost efficient for a Canadian firm to produce any part of its actual output in the US. Its global and local cost efficiency levels are identical.

5. Summary

The DEA model for global cost minimization provides a framework for optimal allocation of output targets across multiple locations that takes account of observed variation in input prices as well as in technologies across locations. This should be useful

[7] This corresponds to $C_H^{ML}(y^0; w^0 \mid \Omega(y^0))$, which is the numerator of equation (20).

for firms planning production sites within a country and for foreign direct investment decisions by multi-national firms. A note of caution is warranted here. Our approach considers only production costs. Depending on the spatial distribution of product markets, added transportation costs and taxes might lead to a different optimal allocation from what our model would suggest.

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Summary.

Table 1. Minimum Cost of Production in Single Location

State	w_1	w_2	p_E	p_M	p_K	C_j^*
GA	14.780	51.099	5.21	1	0.126	8.320
NC	14.068	50.299	6.84	1	0.128	8.330
SC	15.565	53.528	6.11	1	0.121	8.419
TN	15.595	51.748	5.97	1	0.126	8.419
VA	15.784	52.787	5.68	1	0.121	8.409

Note: The cost (in $ million) reported in the last column relates to the cost of producing the entire output target of GA (i.e., $14.249m) in each of the five states.

Table 2. Minimum Cost of Production in Multiple Locations

		Actual in GA	In GA		In SC	
Output	Y	14.249	Y^{GA}	7.800	Y^{SC}	6.450
Production Labor	L_1	78.82	L_1^{GA}	37.66	L_1^{SC}	29.03
Non-production Labor	L_2	12.14	L_2^{GA}	10.56	L_2^{SC}	5.85
Energy	E	44.72	E^{GA}	12.07	E^{SC}	13.81
Materials	M	6.743	M^{GA}	2.695	M^{SC}	2.386
Capital	K	5.872	K^{GA}	3.477	K^{SC}	3.236
	Total Cost	9.503	Cost_GA	4.294	Cost_SC	3.628
Minimum Multi-Location Cost 7.922						

Note: Output (Y), Materials (M), and Capital (K) and Cost are in $ million; Production labor (L_1) is in hrs (1,000); Non-production labor (L_2) is in number of persons; Energy (E) is in billion Btu.

Table 3. Optimal Output Allocation for US Firms

US-CA					
		L	K	M	Y
US actual	X_0^{US}	59852	90511	282013	2.14249
US optimal	X_*^{US}	67326	112646	216231	2.14249
in US	X_{US}^{US}	34549	76872	145889	1.70306
in CA	X_{CA}^{US}	32778	35774	70342	0.43943
Price in US	P_{US}	4.820	2.557	3.435	…
Price in CA	P_{CA}	3.272	2.811	2.807	…
Actual cost	C_0^{US}	1488488			
Actual average cost	AC_0^{US}	694747			
Optimal cost	C_*^{US}	1269534			
Optimal average cost	AC_*^{US}	592551			
Reduction in labor input	$L_0^{US} - L_{US}^{US}$	25303	(42.28% of actual labor employed)		
Increase in labor input	X_{CA}^{US}	32778	(54.76% of actual labor employed)		
Tech. choice comp.	γ_V^0	0.9104			
Input price comp.	γ_w^0	0.9367			
Location efficiency	$\gamma_{L\|H}^0$	0.8528			
Local cost efficiency	γ_H^0	1			
Global efficiency	$\gamma_{G\|H}^0$	0.8528			

Note: Details of data construction are available in Chapter 2 and 4 of Fuss and Waverman (1992).

Table 4. Optimal Output Allocation for Canadian Firms

CA-US					
		L	K	M	Y
CA actual	X_0^{CA}	52162	84594	303186	1.90402
CA optimal	X_*^{CA}	38656	86561	214743	1.90402
in CA	X_{CA}^{CA}	38656	86561	214743	1.90402
in US	X_{US}^{CA}	0	0	0	0
Price in US	P_{US}	4.820	2.557	3.435	…
Price in CA	P_{CA}	3.272	2.811	2.807	…
Actual cost	C_0^{CA}	1259683			
Actual average cost	AC_0^{CA}	661591			
Optimal cost	C_*^{CA}	972723			
Optimal average cost	AC_*^{CA}	510879			
Reduction in labor input	$L_0^{CA} - L_{CA}^{CA}$	13507	(25.89% of actual labor employed)		
Increase in labor input	X_{US}^{CA}	0			
Tech. choice comp.	γ_V^0	…			
Input price comp.	γ_w^0	…			
Location efficiency	$\gamma_{L\|H}^0$	1			
Local cost efficiency	γ_H^0	0.7722			
Global efficiency	$\gamma_{G\|H}^0$	0.7722			

Note: Details of data construction are available in Chapter 2 and 4 of Fuss and Waverman (1992).