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Game Theoretic Bidding Strategies for Auctions in Green Electricity Markets

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I. Abstract

Electricity markets in the United States presently employ an auction mechanism to determine the dispatch of power generation units. In this market design, generators submit bid prices to a regulation agency for review, and the regulator conducts an auction selection in such a way that satisfies electricity demand. Most regulators currently use an auction selection method that minimizes total offer costs ["bid cost minimization" (BCM)] to determine electric dispatch. However, recent literature has shown that this method may not minimize consumer payments, and it has been shown that an alternative selection method that directly minimizes total consumer payments ["payment cost minimization" (PCM)] may benefit social welfare in the long term. The objective of this project is to further investigate the long term benefit of PCM implementation and determine whether it can provide lower costs to consumers. The two auction selection methods are expressed as linear constraint programs and are implemented in an optimization software package. Methodology for game theoretic bidding simulation is developed using EMCAS, a real-time market simulator. Results of a 30-day simulation showed that PCM reduced energy costs for consumers by 12%. However, this result will be cross-checked in the future with two other methods of bid simulation as proposed in this paper.

II. Introduction

The world is presently facing a bitter energy crisis of massive scale. While the total consumption and demand for fossil fuels is rising with the growth in world population, our supply of fossil fuels is quickly dwindling and the price of energy is accordingly escalating at an alarming pace. The United States Department of Energy projects that by 2012, there will be a shortage of fossil fuels, and that the shortage will have to be made up by still "unidentified projects." Furthermore, by 2030, the DOE expects that approximately half of the energy needs that would normally have been supported by fossil fuels will have to be supplied by these unidentified projects [14].

This entails that an increase in the efficiency of energy usage will be required if we are to meet the consumer demands for electricity. Power engineers and scientists have discussed various options for closing this shortage; such initiatives as investment in renewable sources of energy and development of the power transmission grid have been developed in the United States and are in effect [5, 12]. However, these are still very new technologies and, while they are expected to have a significant effect in the long term, they may not be enough to satisfy aggregate consumer demands and prevent the imminent energy crisis.

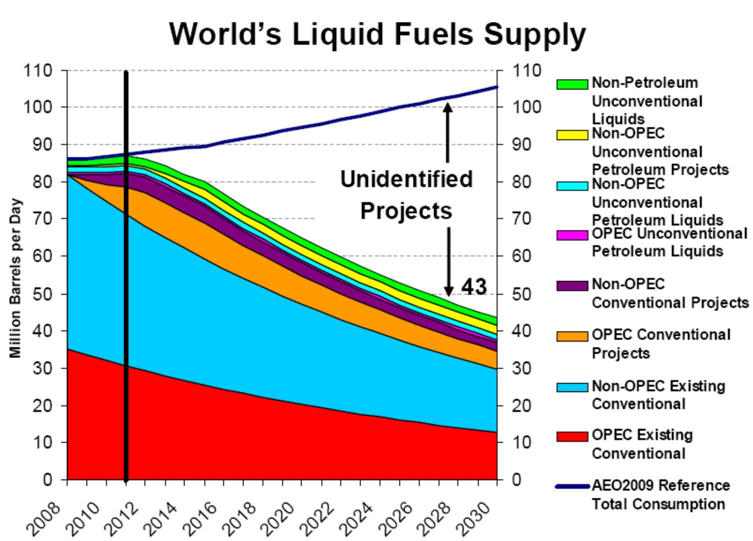

Figure 1: The supply of fossil fuels will likely decrease steadily into the near future and a shortage is expected to occur. *Source*: EIA, US Department of Energy [14].

Meanwhile, humankind’s past two centuries of industrial development have caused steady and severe harm to the environment. Unless air pollution is heavily restricted throughout the world, constant emissions of carbon dioxide in developed and developing nations are expected to raise the global temperature until gradually global warming permanently damages coastal civilizations and other regions of the world.

The power industry is responsible for the vast consumption of fossil fuels and release of harmful pollutants to the air [11]. Thus, the development of our electricity infrastructure – from power generation and transmission to distribution and consumption – becomes ever more critical. Indeed, United States President Barack Obama highlighted the importance of rebuffing the American power grid when in 2009 he called on the country’s engineers and bright minds to “build a new smart grid that will save us money, protect our power sources from blackout or attack, and deliver clean, alternative forms of energy to every corner of our nation” [16].

This project seeks to address these issues. The overall objective of this project is the maximization of social welfare. However, this will be accomplished via two sub-objectives: (1) the minimization of consumer payments for energy, and (2) the reduction of carbon dioxide emissions by power generators. These two objectives will be tackled through investigation and analysis of the auction selection mechanism used by American electricity regulators.

III. Standard Electricity Market Structure

In any region, there is a certain demand for electricity. In Connecticut, this demand is approximately 4000 MW in the day-ahead market on a peak load day. To meet this demand, power generators interconnected through high-voltage transmission lines produce electricity. However, the installed capacity in a region generally exceeds the demand. In other words, power generators can produce more energy than is demanded. At the aggregate level, Connecticut's generators may be able to produce up to 6000 MW or more. Thus, regulators face the problem of having to decide which generators should be on and which should be off in order to meet real-time energy demand [1].

To handle this issue, deregulated electricity markets in the United States use an auction mechanism to determine the daily dispatch of generators. In this scheme, generators that produce publicly available energy submit a bid to the regional Independent System Operator (ISO) [7]. Connecticut generators, for instance, submit their bids to ISO New England for review. Usually, a generator's bid consists of its power capacity constraints (minimum and maximum generation levels), bid price [$/MW(h)], and startup cost [$]. Once bids from each of the regional generators are collected, ISO New England conducts an auction selection to determine the dispatch of generators so as to meet the system demand. Simultaneously, ISOs must consider the physical constraints of the generators, the transmission constraints of the regional power system, and any other constraints. Further, a primary objective for ISOs is to minimize the price of energy to consumers.

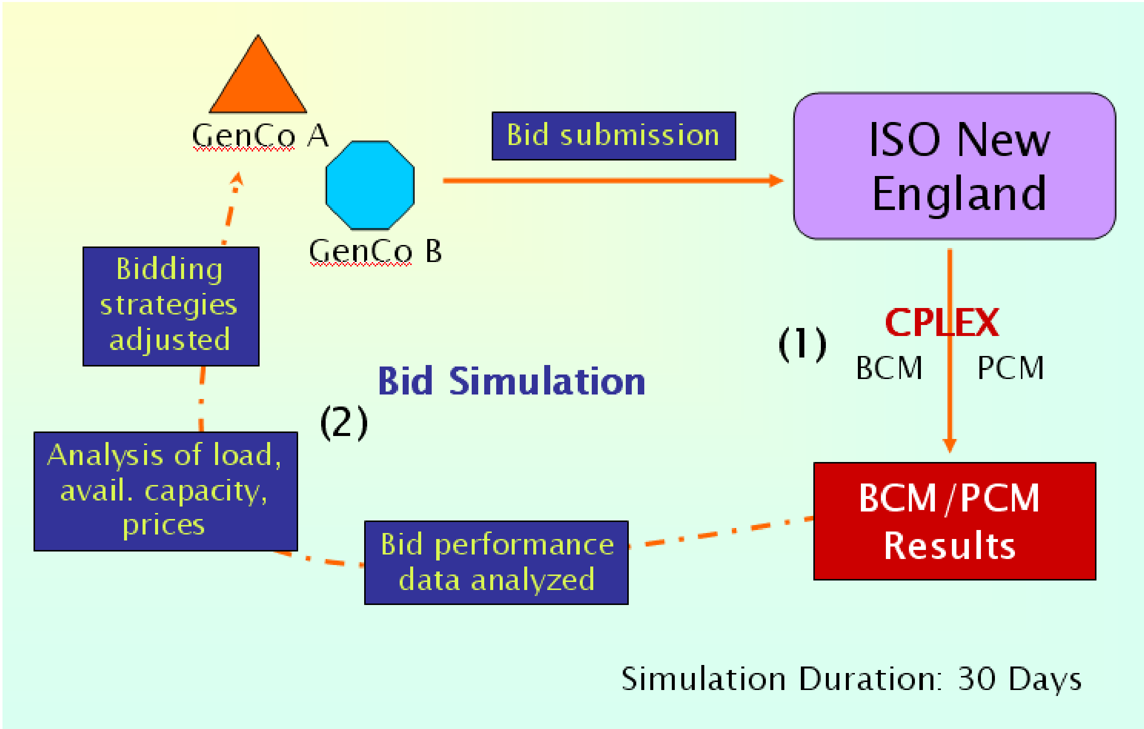

Figure 2: Overview of electricity market operation. Generators submit their bids to the regional ISO in the day-ahead and real-time energy markets, and the subsequent auction is conducted by the ISO. The auction results are analyzed by suppliers for bids for the next day. Simulation of this market can be used to compare the BCM and PCM auction selection methods.

These economic issues make the auction selection process for ISOs critical to the welfare of the society. The outcome of an auction selection directly determines the price of electricity [1]. Also, these policy decisions have significant effects on the environment in the long run. Therefore, it is crucial to fully investigate the auction mechanism in electricity markets so as to determine which method of selection is the most beneficial for society from the perspective of the consumer in the long term.

Previous studies have compared BCM and PCM [13, 21, 22]. However, this project is novel because it compares the two auction selection methods using a continuous, holistic bidding method with industry-standard applications to simulate the bidding process and auction selection. Furthermore, in previous studies, generators were restricted to discretized bidding so that they could submit bid prices only at certain levels [22]. This discretization is a simplification, as in real markets, generators may set their bids at any level on a continuous range within regulated bounds. In this study, generators will be able to bid in this continuous range for profit maximization. This will be accomplished by using a real-time market simulator to be described in V.

The auction selection methods will also be implemented in a linear constraint programming application built for mathematical optimization known as CPLEX. This integration of industry-standard computer applications allows for thorough comparison of the two auction selection methods.

IV. BCM and PCM as Constraint Programs

The advantage of payment cost minimization over bid cost minimization can be seen from the mathematical formulation of the two methods. The objective function for the bid cost minimization selection method can be given as

$$\min_{\{p_i(t)\}} J = \sum_{t=1}^{T} \sum_{i=1}^{I} \{C_i(p_i(t), t) + S_i(t)\}$$

Equation 1: Objective function for the bid cost minimization selection method.

while that of payment cost minimization is

$$\min_{\{MCP(t)\},\{p_i(t)\}} J = \sum_{t=1}^{T} \sum_{i=1}^{I} \{MCP(t)\, p_i(t) + S_i(t)\}$$

Equation 2: Objective function for the payment cost minimization selection method.

where t is the index number of the auction, i is the generator index, C_i is the total cost of unit i, S_i is the startup cost of unit i, MCP is the market clearing price of the auction, and p_i is the power generated by unit i. Constraints of these selection methods generally include system demand and any physical constraints of the generating units, including minimum up/down times, maximum and minimum capacity, and ramp rates [7, 13].

In the objective function for bid cost minimization, total offer costs are minimized. However, this does not minimize final consumer payments to the market; instead, it only minimizes the total cost of all bid prices submitted to the market by generators. Conversely, payment cost minimization directly minimizes final consumer payments by considering the system-wide market clearing price (MCP) or transmission-constrained locational market price (LMP) in the formulation of its objective function [1, 10, 18]. The fact that BCM fails to provide an auction selection that results in minimal consumer payments in all cases is illustrated below. System demand and bid data is given in Table 1 for a four-generator system. Comparison of Tables 2 and 3 shows that implementation of PCM results in lower payment costs than BCM implementation does.

Generator	**Min. MW**	**Max. MW**	**$/MW**	**Startup Cost**
Unit 1 Bid	0	50	10	0
Unit 2 Bid	0	40	15	0
Unit 3 Bid	0	10	80	0
Unit 4 Bid	0	50	20	2000
System Demand: 100 MW				
Single Hour Auction				

Table 1: Problem definition with bid data for four-generator auction.

The four bids shown in Table 1 contain all four critical elements of a standard bid. Each gives the unit capacity constraints that the physical generator associated with the bid must adhere to. In this example, Units 1 and 2 have very low bid prices ($10/MW and $15/MW respectively) and no bid startup cost. Thus, it is natural that the regulator will select them in an auction so as to minimize consumer payments. Together, these two units will be dispatched to produce 90 MW, consisting of the maximum 50MW capacity of Unit 1 and the maximum 40MW capacity of Unit 2. This amount of generation covers all of the demand but for the remaining 10MW. The regulator will have to select either Unit 3 or 4 to satisfy this remaining portion of the demand. The difference between BCM and PCM is illustrated in this final portion of the dispatch.

***Bid Cost Minimization Results*, MCP = $80**				
			Pay as Bid	MCP
Generator	**MW**	**$/MW**	**Bid Costs**	**Payment Costs**
Unit 1	50	10	500	4000
Unit 2	40	15	600	3200
Unit 3	10	80	800	800
Unit 4	0	20	0	0
Total Costs			**1900**	**8000**

Table 2: Bid cost minimization results for the bid data of Table 1.

In Table 2, we see that, following the objective of minimizing total bid (offer) costs, the BCM method yields $1900 worth of bid costs and $8000 worth of payment costs. BCM selects Unit 3 to provide the remaining 10MW of demanded electricity, setting the market clearing price (MCP) to $80. In the pay-as-bid settlement scheme, this would minimize consumer payments since Unit 4 is not selected so its startup cost is not paid. Units 1 and 2 are paid at the rate of their bid price for their generation. But as the settlement scheme used by most American ISOs is to pay each unit at the rate of the MCP, the high bid price of $80 is paid to all generators and covers all 100MW of the demand, so that the total payment cost amounts to $80/MW x 100MW = $8000.

***Payment Cost Minimization Results*, MCP = $20**				
			Pay as Bid	MCP
Generator	**MW**	**$/MW**	**Bid Costs**	**Payment Costs**
Unit 1	50	10	500	1000
Unit 2	40	15	600	800
Unit 3	0	80	0	0
Unit 4	10	20	2200	2200
Total Costs			**3300**	**4000**

Table 3: Payment cost minimization results for the bid data of Table 1.

With the PCM selection method as shown in Table 3, however, payment costs are minimized. Here, the total bid cost is higher than it was in the case of the BCM selection method because minimization of total bid cost is not the objective. Instead, the objective is minimization of total consumer payment cost, so under the MCP settlement scheme, PCM yields lower total payment costs. Here, Unit 3 is not selected because it has a very high bid price, and the MCP would have been set to $80/MW as it was in the case of the BCM auction. However, here, Unit 4 is selected because it has a low bid price of $20/MW. Although this unit still sets the MCP, the total energy cost is now only $20/MW x 100MW = $2000. Adding the startup cost for Unit 4, we obtain the total payment cost amount of $4000. In this hypothetical example, PCM implementation would save half of the consumer's original payment for energy.

This example assumes that bids are given. That is, Table 1 contains the bid data, and BCM and PCM are then applied to determine which method provides lower payment costs. PCM provides lower payment costs than BCM in all examples where bid data is given. But now, we consider the case where generators know that PCM is the selection method used by regulators. Would generators then bid differently, and would PCM really provide lower costs in the long term? This question is addressed in V.

V. Project Methodology

To determine whether BCM or PCM is more beneficial to consumers in the long term, a proper method of simulating the behavior of the generators is required. The primary goal of a generating company is profit maximization. Generators know their cost structure and the regulatory procedures for determination of daily dispatch. They are also aware that there are many other generators present in the market and that they are competing for dispatch of electricity [22].

Thus, a situation of an economic game arises, where each generator seeks to maximize profit by finding the appropriate Nash equilibrium strategy of bidding. Simulation of this behavior is very complex, as each generator must analyze its competitors' strategies and determine its own best course of action. A software package released by Argonne National Laboratory, the Electricity Market Complex Adaptive System (EMCAS), was used to simulate this stage of the market [6]. The advantage of using EMCAS is that all external factors concerning the market are considered in the simulation, and also that generating companies are able to bid at any level. In previous studies, this was not possible; in the study by Zhao et. al., bids were discretized with matrix games to find the discrete Nash equilibrium [22]. This can result in the loss of the continuous Nash equilibrium. With EMCAS, however, this equilibrium strategy is not lost as bidders are able to bid at any level. At the end of this study, though, some questions were raised about the bid simulation process used by EMCAS and other methods were used for bid simulation to cross-check the results derived with EMCAS.

A four-generator, four-generating company (GenCo) model was developed in EMCAS as shown in Figure 3, and a 30-day simulation was conducted as shown in Figure 2. For this method, one day was simulated in EMCAS, day-ahead bids for the four generators were extracted, and auctions for PCM and BCM were executed using the CPLEX implementations described later in this section. Auction results were then used as historical data input for the second day in EMCAS, and the second day's bidding events were subsequently determined and extracted for the auction to run in CPLEX. This cycle was repeated for 30 days, and total consumer payments were calculated for the BCM and PCM selection methods. Units Base 1A, 2A, and 4A had no startup cost, while Base 3A had a startup cost of $12,000.

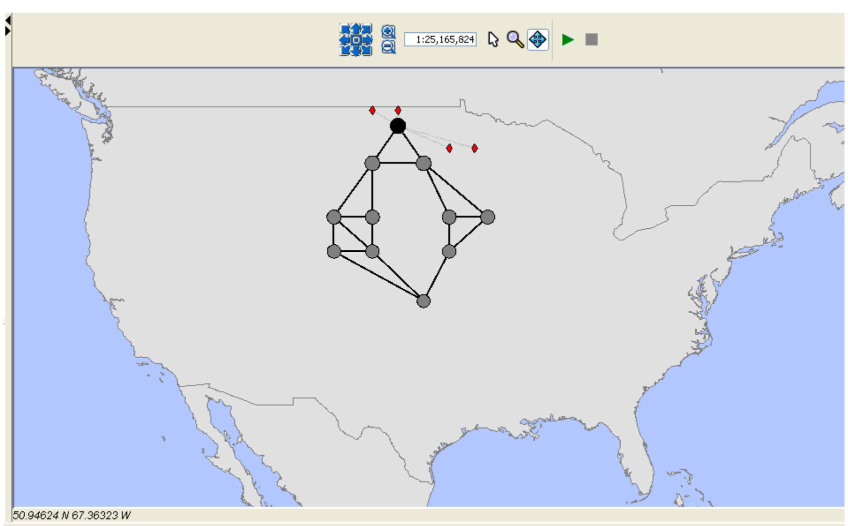

Figure 3: EMCAS case study power system topography. Note that there are four generators at one node and no transmission constraints present in the model.

A simplification made in this model is the assumption of single-block bidding. While in the New England market, GenCos are able to bid in incremental bid blocks, single bid blocks were used in this study to avoid an added layer of complexity. Furthermore, transmission constraints are not considered in this model and uniform MCPs are used as opposed to transmission congestion-based LMPs. Generator startup costs are assumed to be fully compensated. These computational simplifications are consistent with those of previous studies of electricity markets [7, 13, 21, 22].

To simulate the auction selection conducted by the regulating ISO marked as (1) in Figure 2, BCM and PCM were implemented as constraint programs with CPLEX, an industry-standard linear optimization package released by IBM. The key to this step was to first formulate the BCM and PCM constraint programs linearly. Subsequently, these linear formulations were implemented with data used from the EMCAS bid simulations. For illustration and reference, the specifications for the constraint programs for both auction selection methods appear in the appendix.

VI. Simulation Results with EMCAS and CPLEX

Results from the simulation using EMCAS for bid generation and CPLEX for auction selection show that the PCM auction method for bid selection provides lower consumer payments than that of BCM. Results are provided in Tables 4 and 5.

Unit ID	Capacity (MW)	Startup Cost ($)	Average Bid Price [30-day sim] ($/MW)
Base 1A	50	0	18.34
Base 2A	40	0	35.06
Base 3A	50	0	46.59
Base 4A	60	12,000	23.53

Table 4: Example case study, generator characteristics and bid results

Objective Cost	BCM ($)	PCM ($)	Difference ($)
Total (First Seven Days)	752,352	536,948	215,404
Total (All 30 Days)	2,864,344	2,522,565	341,779

Table 5: Example case study, objective payment results

While these results seemingly show that the PCM auction mechanism is preferable from the perspective of consumers as it yields lower final payments, this is not entirely clear. Further inspection of the full data results suggest that, though the BCM-PCM payment gap is high early in the early days of the simulation, in the last week, the gap is very small or zero. This result raises some issue with regard to the reality of the bid generation method employed by suppliers in EMCAS.

VII. Results Discussion and Future Testing

Discrete game theoretic approach

In light of the results from the 30-day simulation with EMCAS for the simulation of bids, it is desirable to cross-check the effectiveness of the PCM auction mechanism by using other methods of bid generation. Zhao et. al. have used a game theoretic approach to solve this problem, with the Nash Equilibrium for bidders determined through a matrix game [22]. Their simplified model uses two bidders that can bid at three discrete levels. Auction selections are executed using the BCM and PCM algorithms for each strategy tuple, and Nash Equilibriums are determined using the matrix game concept. In the case that no equilibrium is found from the discrete matrix game, approximate equilibriums are sought in a methodical manner. If any equilibriums are found using this strategy, they constitute the optimal bidding strategy for the two suppliers in the game.

This method will be implemented in MATLAB and CPLEX to cross-check the results found from 30-day simulation example case with bid generation provided by EMCAS. It is expected that the findings of Zhao et. al. will be confirmed for the example case of this study to support the results of the 30-day simulation case.

Probabilistic bidding model

An alternative bidding model with four generators that maximizes each generator's expected profit based on historical data will also be implemented for cross-checking the results of the 30-day simulation example case. Inputs for each generator include historical demand levels and historical bid prices. Each generator has access to the other generators' bids from six months before the auction, as well as the historical load levels.

The expected profit for one generator is calculated using the probability distributions of historical bids of the other three generators, the probability distribution of the load level, the expected level of power generation dispatched given the other generators' bids (which are cycled-through with a step equal to the bid interval specification), and the expected payout per megawatt (which would either be equivalent to the MCP if selected or 0 if not selected). The expected profit given the first generator's bid can be expressed as

$$EP=\sum_{i_{G2}}^{n_b}\sum_{i_{G3}}^{n_b}\sum_{i_{G4}}^{n_b}\sum_{i_D}^{n_d}[\Pr(q^1_{2,i_{G2}}=q_2)*\Pr(q^1_{3,i_{G3}}=q_3)*\Pr(q^1_{4,i_{G4}}=q_4)*\Pr(d_{i_D}=D)]*[P]*[payout]$$

Equation 3: Expected profit function for the first of four generators engaged in a game based on the proposed probabilistic model of bidding.

where the $q^1_{ind,i_{Gind}}$ are the predicted generator bids, $\Pr(q^1_{ind,i_{Gind}}=q_{ind})$, are the probabilities that the predicted generator bid is equal to the actual generator bid, P is the expected generation level given other generators' bids $q^1_{ind,i_{Gind}}$ and the first generator's bid, and *payout* is the expected level of payout per megawatt given the same information (the payout per megawatt being the MCP if $P > 0$ or 0 if $P = 0$). This expected profit method is a function of the bid of the first generator; all other inputs to this method are static user inputs (including capacities, the historical bid and demand data, and number of intervals for the demand data, n_d, and number of intervals for the historical bid data, n_b). The program implementation cycles through the range of possible bids for the first generator and determines the bid that will maximize the expected profit depending on the behavior of the competing generators.

VIII. Numerical Testing Apparatus

The selection methods for BCM and PCM auctions were implemented in CPLEX and executed on an Intel Xeon E3510 PC at 1.60 GHz with two processors and 8.00 GB of

RAM. The EMCAS market simulation model was constructed and run on an Intel Core 2 Duo PC at 2.20 GHz with 2.00 GB of RAM. The same PC was used to implement the game theoretic bidding model and probabilistic bidding model in MATLAB.

IX. Conclusions

The decreasing supply of fossil fuels is expected to cause an energy crisis of immense magnitude in the near future. Increased efficiency in electricity markets can relieve some of this pressure. Implementation of PCM may reduce the price of energy for consumers and have some effect on the quantity of harmful emissions. This project has attempted to determine what effect use of the PCM auction selection method has on energy price.

In review, this study was fruitful in showing that implementation of PCM as an auction selection method may reduce consumer payments in the long term. However, we cannot draw any strong conclusions at this time and will seek to cross-check simulation results with alternative methods of bid simulation. Successful implementation of these alternative bid simulation methods may support the results found in this study. Subsequently, more concrete conclusions may be stated with respect to the efficaciousness of employing the PCM auction selection method in electricity markets. The discrete game theoretic model presented in section VII can show through determination of the discrete Nash equilibrium that PCM may minimize total consumer payments in the long term. The probabilistic model can be used to arrive at the same result using historical data associated with the generators.

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XII. Appendix

BCM, objective function specification

```
dvar int Take[providers][time] in 0..1;
dvar float Power[providers][time] in 0..maxDemand;
dvar float MCP[time] in 0..90;
dvar int u[providers][time] in 0..1;

minimize
  sum (t in 1..nbHours) (sum (p in providers) suppliers[p][2] * u[p][t]) + // Startup Costs
  sum (t in 1..nbHours) (sum (p in providers) suppliers[p][1] * Power[p][t]); // Prices every hour
```

PCM, objective function specification

```
dvar int Take[providers][time] in 0..1;
dvar float Power[providers][time] in 0..maxDemand;
dvar float MCP[time] in 0..100;
dvar int u[providers][time] in 0..1;

minimize
  sum (t in 1..nbHours) (sum (p in providers) suppliers[p][2] * u[p][t]) + // Startup Costs
  sum (t in 1..nbHours) (MCP[t] * demand[t]); // Prices every hour
```

Constraint Specification for BCM and PCM

```
subject to {
  sum (p in providers) Take[p][0] == 0;
  sum (p in providers) Power[p][0] == 0;
  forall (t in 1..nbHours)
  {
    sum (p in providers) Power[p][t] == demand[t];
    forall (p in providers) {
      MCP[t] >= (suppliers[p][1] * Take[p][t]);
      u[p][t] >= 0;
      u[p][t] >= (Take[p][t] - Take[p][t-1]);
      ((Power[p][t] >= suppliers[p][4] && Power[p][t] <= suppliers[p][3]) || Power[p][t] == 0);
      ((Take[p][t] == 1 && Power[p][t] >= 0.0001) || (Take[p][t] == 0 && Power[p][t] == 0));
    }
  }
}
```