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The Optimal Response to Default: Renegotiation or Extended Maturity?

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Abstract

This paper reinforces the argument of Harding and Sirmans (2002) that the observed preference of lenders for extended maturity rather than renegotiation of the principle in the case of loan default is due to the superior incentive properties of the former. Specifically, borrowers have a greater incentive to avoid default under extended maturity because it reduces the likelihood that they will be able to escape paying off the full loan balance. Thus, although extended maturity leaves open the possibility of foreclosure, it will be preferred to renegotiation as long as the dead weight loss from foreclosure is not too large.

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In a recent article, Harding and Sirmans (2002) examined the question of why, in

the presence of default, borrowers and lenders tend to prefer extending the maturity of the

loan rather than renegotiating the loan balance.[1] This observation is puzzling, they argue,

because, whereas renegotiation eliminates the deadweight costs of default by avoiding a

foreclosure sale, maturity extension can at best postpone that outcome. Harding and

Sirmans (2002) attempt to resolve this puzzle by arguing that extended maturity offers an

offsetting benefit by reducing two forms of agency costs in loan contracts. First, it gives

borrowers a greater incentive to invest in the asset during the term of the loan, and

second, it (possibly) reduces the tendency for borrowers to undertake practices that

increase the riskiness, or volatility, of the asset.

The purpose of this paper is to elaborate on this argument in favor of maturity

extension by examining more carefully the role of agency costs in the choice of default

options in loan contracts. To that end, we consider a loan contract with a fixed maturity

date. During the term of the loan, we suppose that the borrower makes a decision that

affects the probability that he will be able to pay off the loan at maturity. In a

commercial real estate context, this could involve a choice of the level of maintenance of

the building, the choice of tenant mix, monitoring effort, or any other decision that

influences the borrower's expected cash flow. This situation involves an agency (or

moral hazard) problem because the lender generally cannot observe and/or contract on

the borrower's effort choice. Thus, we examine how the structure of the loan contract—

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[1] For evidence on this, see Asquith, Gertner, and Scharfstein (1994) and Mann (1997).

specifically, the lender's response to default—affects the borrower's incentives to invest in effort during the term of the loan.

Our main conclusion is that borrowers will invest in greater effort under extended maturity as compared to renegotiation. Intuitively, because extended maturity reduces the likelihood that borrowers will be able to escape paying off the full balance of the loan, they have a greater incentive to invest in effort that reduces the probability of default. As a result, the moral hazard problem is less severe under the extended maturity option because it better aligns the interests of borrowers and lenders. And, if this effect is strong enough, it will outweigh the savings in foreclosure costs promised by renegotiation. In this sense, our results reinforce the claims of Harding and Sirmans (2002).

The Model

We derive the above conclusions in a model of a loan contract between a risk neutral lender and borrower. The borrower takes out a loan of L dollars at time $t=0$ in order to purchase an asset that produces an uncertain income for an infinite number of periods into the future, beginning at $t=1$. Specifically, suppose that in each period beginning in $t=1$, the asset produces positive income of y with probability $p(e)$ and zero income with probability $1-p(e)$, where e is the borrower's expenditure on effort to produce the high income state ($p'>0$, $p''<0$, and $p(0)\equiv p_0>0$).[2] Thus, as of period $t=0$, the expected present value of the asset is given by

$$V(e) = p(e)y/r, \tag{1}$$

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[2] The analysis is not limited to income-generating assets. For example, we could apply it to residential real estate by interpreting e as the borrower's effort to earn income in the labor market.

where r is the discount rate. In order for the initial investment to be profitable, it must be the case that $V(e)–e>L$, or

$$p(e)y/r – e > L, \qquad\qquad (2)$$

which we assume is true for all e.

Suppose that the loan matures at $t=1$, at which time the borrower owes principal plus interest equal to $L(1+i)$, where $i \geq 0$ is the interest rate on the loan. We will assume throughout that the lender operates in a perfectly competitive market. Thus i will adjust so that the lender expects to earn zero profit on the loan. Let the opportunity cost of funds to the lender be equal to the discount rate r.

Suppose that the only income the borrower has to pay off the loan in $t=1$ is that which is generated by the asset. We assume that

$$y > L(1+i), \qquad\qquad (3)$$

which implies that the borrower is able to repay the loan at maturity if the high income state occurs. However, if zero income is realized in $t=1$, the borrower is in default.[3] In this case, we suppose that the lender has three possible courses of action: (i) immediate foreclosure and forced sale of the asset, (ii) renegotiation of the principal, (iii) and refinancing of the debt. We assume that the borrower and lender choose the option at the time the loan is originated to maximize their joint returns, and that they cannot deviate from that choice in the event of default. [4]

Foreclosure

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[3] Even if the borrower has other assets with which to pay off the loan, the law of many states shields those assets from the lender. For example, in the case of real estate, laws against deficiency judgments prevent lenders from going after borrowers' non-housing assets to pay off a mortgage (Harding, Miceli, and Sirmans, 2000).

[4] The parties must commit to the default option up front because, once default occurs, any previously made choices by the borrower are sunk. Thus, the parties would have an incentive to choose that default option which promises the lowest cost going forward. In the current context, this would always be renegotiation. (See note 8 below.)

Consider immediate foreclosure first. Since the asset generates expected per-period income of $p(e)y$ in perpetuity, its market value as of $t=1$ is also $V(e)$ as defined in (1). However, because foreclosure requires a forced sale, we assume that it entails a one-time transaction cost of $T \geq 0$. Thus, the net proceeds from a sale are $V(e)-T$.[5] We assume that this amount is sufficient to cover the loan balance,[6] or that

$$V(e) - T \geq L(1+i), \tag{4}$$

again for all e. As a result, the lender is fully repaid at maturity, regardless of whether or not the borrower defaults. Competition among lenders therefore ensures that $i=r$. (We assume without loss of generality that the borrower pays the transaction costs.)

We can therefore write the borrower's expected return under foreclosure, as of $t=0$:

$$R_1 = \frac{1}{1+r} [p(e)(y-L(1+r)+V(e)) + (1-p(e))(V(e)-T-L(1+r))] - e.$$

The first term in square brackets is the borrower's return in the event that he pays off the loan, while the second term is his return in the event of default. Using (1), we can simplify this expression to

$$R_1 = V(e) - L - \left(\frac{1-p(e)}{1+r}\right)T - e. \tag{5}$$

The borrower therefore fully internalizes the present value of the loan as of $t=0$, including the expected transaction costs, $(1-p(e))T/(1+r)$.

Renegotiation

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[5] Harding and Sirmans (2002) assume that the deadweight loss is proportional to the value of the asset. Our assumption of an additive cost is made purely for simplicity.
[6] Presumably, the lender, in using the asset as collateral, would not have made the original loan unless this condition was met.

The second option in the event of default is for the borrower and lender to renegotiate the loan balance. In the current model, this amounts to the lender's forgiving the loan in the low income state. The advantage of this option, as noted above, is that it avoids the transaction costs of a foreclosure sale. As a consequence, however, the lender accepts a discounted payoff (zero) in the bad state and foregoes any claim to future income from the asset.[7] Obviously, however, the interest rate will have to adjust to ensure zero profits. Specifically, we can write the lender's expected return from the loan as of $t=0$ to be

$$\pi = -L + p(e)L(1+i)/(1+r). \tag{6}$$

Zero expected profit requires that

$$1+i = (1+r)/p(e), \tag{7}$$

which implies that $i>r$ in this case, given $p(e)<1$.

The expected return of the borrower under the renegotiation option is

$$R_2 = \frac{1}{1+r} [p(e)(y-L(1+i)+ V(e))+(1-p(e))V(e)] - e,$$

which can be rewritten

$$R_2 = V(e) - p(e)L(1+i)/(1+r). \tag{8}$$

Substituting for $(1+i)$ from (7) yields

$$R_2 = V(e) - L - e. \tag{8'}$$

Note that, for a given e, this differs from (5) only by the absence of the transaction costs associated with foreclosure.

Extended Maturity

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[7] There may be transaction costs of renegotiation, but since these are almost certainly lower than the costs of foreclosure, there is no loss in generality in assuming that they are zero.

The third and final option in the event of default is for the borrower to refinance the loan for one additional period, or, what amounts to the same thing, to extend the maturity on the initial loan one period. We assume, however, that if the borrower defaults on this extension, foreclosure and forced sale will occur with certainty.[8] Note that it does not matter for our purposes whether the loan extension is made by the same or a new lender, as long as both the initial and the new loan each promises zero expected profits.

Consider first the extended loan, assuming that the borrower defaulted on the initial loan. Since the initial loan obligated the borrower to repay $L(1+i)$ at $t=1$, this amount becomes the principal for the extension. Thus, in $t=2$ the borrower owes $L(1+i)(1+i_1)$ at $t=2$, where i_1 is the new interest rate (which may or may not be equal to i). If the good state occurs in $t=2$, the borrower fully repays this loan (given that $y > L(1+i)(1+i_1)$, which we assume is true), but if zero income is realized, the asset is sold with certainty to cover the balance. As before, we assume that the value of the asset as of $t=2$, net of the transaction costs, is sufficient to cover this amount; that is, $V(e)-T \geq L(1+i)(1+i_1)$. Thus, as under the foreclosure option, the lender expects to be fully repaid for the extended loan regardless of the outcome in $t=2$. By implication, the lender who made the initial loan in $t=0$ (whether the same or a different lender) also expects to be fully repaid in present value terms, regardless of the outcome in $t=1$. It follows that $i_1=i=r$.

Given these results, we can write the borrower's expected return under the refinancing (extended maturity) option as

$$R_3 = \frac{1}{1+r} \{p(e)(y-L(1+r)+V(e)) +$$

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[8] If foreclosure could be continually forestalled, this option would become equivalent to renegotiation. This is why we need to assume that the parties are able to commit at $t=0$ to carry out the foreclosure in $t=2$ if the borrower defaults.

$$(1-p(e))\left(\frac{1}{1+r}\right)[p(e)(y-L(1+r)^2+V(e)) + (1-p(e))(V(e)-T-L(1+r)^2)]\} - e.$$

Simplifying this expression yields

$$R_3 = V(e) - L - \left(\frac{1-p(e)}{1+r}\right)^2 T - e, \tag{10}$$

which reflects the present value of the loan, net of expected transaction costs. Note that this differs from (5) in that the expected transaction cost of foreclosure is delayed an additional period.

Comparison of Default Options

We now turn to a comparison of the three default options. Since lenders earn zero expected profits under each of them, we can rank the options simply by comparing the borrower's expected returns.

Fixed Probability of Default

Assume initially that $p(e)$ is fixed and equal under the three options (i.e., the borrower has no control over the probability of default). In that case, the comparison depends only on the expected transaction costs. Comparison of (5), (8'), and (10) therefore implies the following ranking for all $T>0$:

$$R_2 > R_3 > R_1. \tag{11}$$

Renegotiation is the best option because it completely eliminates the transaction costs associated with default. (This, of course, is the conventional argument in favor of renegotiation.) The next best option is extended maturity because, by extending the loan one period in the event of default in $t=1$, there is a chance that the borrower will be able

to repay it in $t=2$ without the need for a foreclosure sale.[9] Finally, immediate foreclosure

is the least desirable option because the transaction costs associated with a forced sale are

incurred with certainty if the borrower defaults in $t=1$.

Figure 1 graphs the expected returns as functions of T. Note that the relative

attractiveness of renegotiation increases with the magnitude of the transaction costs.

[Figure 1 here]

Endogenous Probability of Default

We now take into account the impact of the borrower's choice of effort on the

expected value of the loan. As noted above, this reflects the idea that borrowers have the

ability to influence the profitability of the asset by increasing the likelihood of the high

income state. This effort could involve the provision of a labor input, maintenance of the

asset, or monitoring of workers. For simplicity, we assume that the effort choice is one-

time. Thus, immediately after securing the loan at time $t=0$, the borrower chooses e to

maximize the present value of his expected return, taking as given the default option.

This choice of e then determines the value of p in all subsequent periods.

Under the foreclosure option, the borrower therefore chooses e to maximize (5),

yielding the first-order condition:[10]

$$p'(e)\left(\frac{y}{r}+\frac{T}{1+r}\right)=1. \tag{12}$$

Note that the resulting level of effort, $e_1{}^*$, is increasing in T, reflecting the borrower's

desire to minimize the expected transaction costs.[11]

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[9] This conclusion is true for any finite extension.

[10] The second-order conditions for this and all subsequent cases are satisfied given $p''<0$.

[11] The effort level implied by (12) is second best because of the borrower's need for a loan to finance purchase of the asset. In a world in which the borrower could purchase the asset without a loan, he would

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Next, under renegotiation, the borrower chooses e to maximize (8), taking as given the interest rate, i, as determined by (7).[12] The resulting effort level, e_2^*, therefore solves the first-order condition

$$p'(e)\left[\frac{y}{r} - L\left(\frac{1+i}{1+r}\right)\right] = 1. \tag{13}$$

Comparison of (12) and (13) immediately reveals that $e_1^* > e_2^*$. The borrower exerts less effort under renegotiation compared to foreclosure for two reasons. First, under renegotiation, the borrower perceives that he can escape a portion of the debt in the zero-income state, so he has less incentive to work hard to avoid that state. This is the moral hazard problem associated with renegotiation. Second, as noted above, part of the borrower's effort under foreclosure is aimed at avoiding the transaction costs associated with default, which are not present under renegotiation.

Finally, under the refinancing (extended maturity) option, the borrower chooses e to maximize (10). The optimal effort level in this case, e_3^*, solves the first-order condition

$$p'(e)\left[\frac{y}{r} + 2\frac{(1-p(e))T}{(1+r)^2}\right] = 1. \tag{14}$$

The borrower's effort in this case again exceeds that under renegotiation for the same two reasons note above (that is, $e_3^* > e_2^*$). Comparing (14) and (12), however, shows that the borrower may exert more or less effort under refinancing as compared to foreclosure. Specifically, $e_1^* > e_3^*$ if $1+r > 2(1-p)$, but $e_1^* < e_3^*$ if the reverse is true.

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simply maximize $V(e)-L-e$, yielding the first-order condition $p'y/r=1$. The borrower's actual effort is higher than is implied by this condition due to the potential transaction cost associated with default on the loan.
[12] Thus, although the borrower knows that the interest rate adjusts in equilibrium to ensure zero profits for lenders, he does not perceive the dependence of i on his choice of effort.

The foregoing results concerning borrower effort show that renegotiation is no longer necessarily the best default remedy because it gives borrowers the least incentive to avoid default in the first place. A complete comparison of the three options therefore requires consideration of both the transaction costs of default and the impact of borrower effort. For purposes of this comparison, we define R_i^* to be the maximized value of the borrower's return under default option i (i=1,2,3). In what follows, we consider how these returns vary with T, taking account of borrower effort.

Note first that when T=0 and $p(e)$ is fixed, the three options are equivalent (see Figure 1). If we then allow effort to be endogenous while holding T fixed at zero, foreclosure and extended maturity remain equivalent because the borrower chooses the same effort under each of these options (compared (12) and (14) with T=0). However, the moral hazard problem results in too little effort under renegotiation (i.e., e_2^* is independent of T according to (13)). Thus, when T=0, $R_1^*=R_3^*>R_2^*$.

Now consider what happens as T becomes positive. Differentiating (5), (8), and (10) with respect to T, and applying the Envelope Theorem where appropriate, yields

$$\frac{\partial R_1^*}{\partial T} = -(1-p(e_1^*))/(1+r) < 0, \tag{15}$$

$$\frac{\partial R_2^*}{\partial T} = 0, \tag{16}$$

$$\frac{\partial R_3^*}{\partial T} = -[(1-p(e_3^*))/(1+r)]^2 < 0. \tag{17}$$

Thus, the maximized returns are decreasing in T under foreclosure and refinancing (though the latter is decreasing more slowly given $(1-p)/(1+r)<1$). In contrast, the return under renegotiation is independent of T. Combining these result with those for the case where T=0 yields the relationships shown in Figure 2.

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[Figure 2 here]

Note that there are two ranges, separated by the critical value, T'. For $T<T'$, refinancing (extended maturity) is the optimal choice, while for $T>T'$, renegotiation is the optimal choice. These results reflect the trade-off between transaction costs and moral hazard. Specifically, when transaction costs are low, refinancing is preferred because the greater incentive for borrower effort under this option dominates the expected costs of a foreclosure sale. However, when transaction costs are high, renegotiation is preferred because the gain from avoiding the costs of foreclosure dominates the diminished incentives for effort. Note finally that immediate foreclosure is never the preferred option because it is everywhere dominated by refinancing. (Only in the extreme case where $T=0$ are the two options equivalent.)

Conclusion

This paper has reinforced the conclusion of Harding and Sirmans (2002) that the observed preference of borrowers and lenders for extended maturity rather than renegotiation in the case of loan default can be attributed to the superior incentive properties of the former. Specifically, extended maturity gives borrowers a greater incentive to avoid default, as compared to renegotiation, because it reduces the likelihood that they will be able to escape paying off some of the loan balance. Thus, although extended maturity leaves open the possibility of foreclosure, it will nevertheless be preferred to renegotiation as long as the deadweight costs of a foreclosure sale are not too large.

Ideally, one could test this conclusion empirically using a sample of firms facing default. As Mann's (1997) study shows, the responses of lenders vary (though the majority prefer maturity extension). Thus, one could theoretically examine the characteristics of those firms for which one or the other type of response was adopted and determine whether the agency costs tend to be higher in those cases resolved by maturity extension, as predicted by the theory. The difficulty, of course, is finding a firm-specific proxy for the importance of these costs. This challenge is left for future work.

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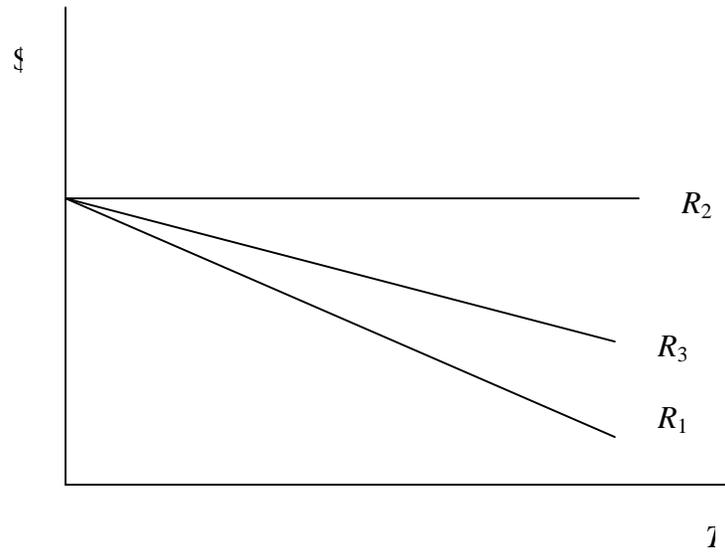
Figure 1. Comparison of default options as a function of T when the probability of default is fixed.
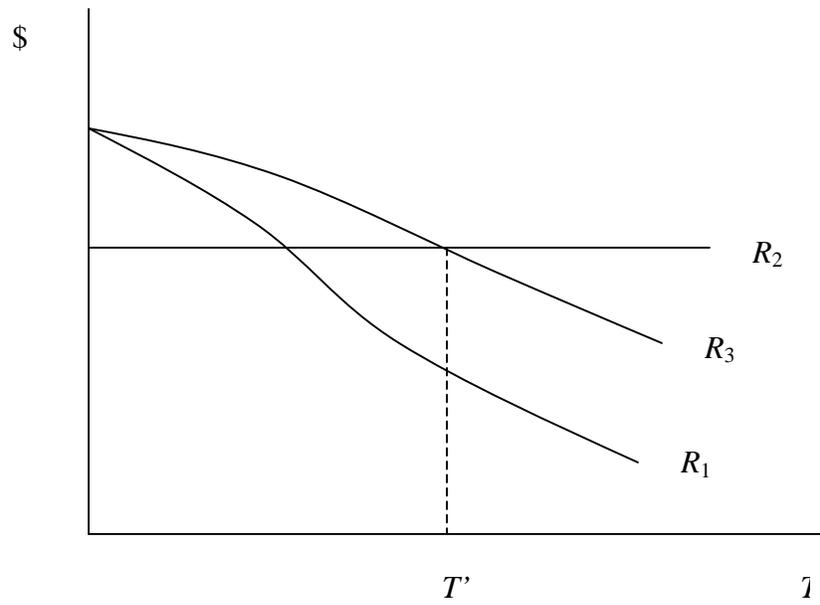
Figure 2. Comparison of default options when borrower effort is endogenous.