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# Empirical Analysis of Credit Risk Regime Switching and Temporal Conditional Default Correlation in Credit Default Swap Valuation: The Market liquidity effect

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## Abstract

In this paper, we extend the debate concerning Credit Default Swap valuation to include time varying correlation and co-variances. Traditional multi-variate techniques treat the correlations between covariates as constant over time; however, this view is not supported by the data. Secondly, since financial data does not follow a normal distribution because of its heavy tails, modeling the data using a Generalized Linear model (GLM) incorporating copulas emerge as a more robust technique over traditional approaches.

This paper also includes an empirical analysis of the regime switching dynamics of credit risk in the presence of liquidity by following the general practice of assuming that credit and market risk follow a Markov process. The study was based on Credit Default Swap data obtained from Bloomberg that spanned the period January 1st 2004 to August 08th 2006. The empirical examination of the regime switching tendencies provided quantitative support to the anecdotal view that liquidity decreases as credit quality deteriorates. The analysis also examined the joint probability distribution of the credit risk determinants across credit quality through the use of a copula function which disaggregates the behavior embedded in the marginal gamma distributions, so as to isolate the level of dependence which is captured in the copula function. The results suggest that the time varying joint correlation matrix performed far superior as compared to the constant correlation matrix; the centerpiece of linear regression models.

**Keywords:** Credit Default Swaps, Market Liquidity, Copulas, Joint conditional distributions, Markov process, Regime Switching, Illiquidity, and Correlation.

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## 1. Introduction

One of the emphases of this article is an examination of the multivariate outcomes of the various marginal distributions interacting simultaneously with each other in the risk-neutral correlated hazard process of the study's referenced entities. Initial observation of the study's credit and market risk data would tend to suggest that dependencies vary over time and are not constant. Following Sun *et al* (2006) a copula is a multivariate distribution with uniform marginal distributions on the interval [0, 1]. Copulas allow for the construction of time varying joint conditional distributions (Patton (2006a, b)), which permits the evolution and subsequent evaluation of conditional correlation between financial asset's credit quality and their explanatory variables. Incorporating time variation in the joint conditional default correlation may be achieved through time varying conditional marginal distributions[3] (transitional matrices) and through variations in the evolution of the copula parameters over time.

This paper models the copula dynamics of the transition between credit risk, market risk and market liquidity conditions which will enable us to derive the implied future (*n*-step ahead) credit default swap (CDS) premia distribution from the historical CDS prices. The study's main innovations are (a) exploring and

[3] The study used two matrix structures in the fitted t-copula. We used both a uniformed correlation (*EX*) and a time-varying (*AR-1*) structure to test changes in the covariates over the sample period.

introducing a copula based valuation approach for pricing credit risk which includes a liquidity measure for the credit default swap premium, and (b) examining and using the joint credit risk and liquidity regime switching dynamics of credit risk premia to explain investors' behavior in investing in credit risky products. The paper develops and implements a multifactor Markov chain model, using time varying historical transition matrices and a set of latent credit risk explanatory variables. Unlike earlier studies that focused solely on using copulas to evaluate correlated default processes, this paper shows how to calibrate future credit default swap (CDS) prices using copula theory that includes a financial liquidity proxy. Further, the paper looks at credit risk and liquidity migration to determine whether higher credit and liquidity risk exhibits higher levels of volatility.

The remainder of the paper is organized into four sections. Section 2 reviews the literature concerning the use of copulas in financial applications. Section 3 introduces the theoretical foundations of the models and discusses the Markov switching model and the transition parameters. Section 3 also discusses the application of copula theory to credit risk analysis, parameterization of the study's t-copula and the *n*-step a-head calibration procedure is discussed and presented. Section 4 gives a brief description of the CDS data and the various explanatory variables. Section 5 presents the main empirical findings regarding the copula model and the regime switching analysis. Section 6 summarizes the finding and proposes areas of future research.

## 2. Literature Review

Quite recently there has been increased interest in the use of copula analytics in credit risk analyses (Li (2000), Das and Geng (2004), Burtschell *et al* (2005), Luciano (2005), Bandreddi *et al* (2006)). Copulas are a tool for understanding relationships among multivariate outcomes. It is a function that links univariate marginals to their full multivariate distribution. Copula functions, which were introduced to probability theory in 1959[4], measure correlation dependencies, or association, embedded in the underlying marginal distributions.

The normal distribution assumption has long dominated the study of multivariate distributions. Frees and Valdez (1998) suggest that the normal distribution assumptions were ideal in probability analysis because (a) their underlying marginal distributions are also assumed to be normal, and (b) dependencies between random variables can be fully described by the model's implied correlation coefficient and underlying marginal distributions. This assumption of normality is widespread throughout the finance literature, as evidenced in a number of asset pricing theories that assume normally distributed asset returns. However, Sun *et al* (2006) states that the distributions of financial asset prices exhibit heavy tails, which calls into question the assumption of a normal distribution assumed in most pricing theories.

Copulas have been used extensively in survival analysis and actuarial analytics. Since the assumption of marginal normality is not consistent with observed

[4] See Sklar, A. (1959) "Fonctions de répartition a n dimensions et leurs marges," Publication Inst. Stat. Univ. Paris, 8, 229-231.

financial data, given the data's asymmetric properties, a number of recent studies have attempted to use copulas to model dependent risk in *n*-variate credit risk distributions (Embrechts *et al* (1999), Das *et al* (2004), Sun *et al* (2006)). The copula method for understanding multivariate distributions has a relatively short history in credit risk analysis, with most of the credit risk applications arising during the last five years. Further, Das *et al* (2004) suggests that an important feature of copulas is its ability to permit varying degrees of tail dependence (i.e., the extent to which the correlation between random variables arises from extreme observations).

One of the earlier researchers to introduce credibility theory to financial data was Li (2000), who in using copula functions to determine joint probabilities, introduced the use of copula functions to collateralized debt obligations (CDO) credit risk valuation analysis. Sun *et al* (2006) used a multivariate model, based on a copula function to address the drawback of the normality assumption in financial asset pricing. Hull and White (2006) used a one factor copula model to derive implied CDO quotes. Das *et al* (2004) used copula analysis to evaluate default risk at the portfolio level (multivariate distribution) and more importantly evaluated default dependencies among issuers using a copula function to separate the marginal behavior from the correlated dependencies.

The choice of copula function to utilize in any study is critical given the number of copula's available to researchers. Kole *et al* (2006) found some issues with the Gaussian copula in that it did not capture the dependence among extreme events, which was also shown in Embrechts *et al* (1999). In general, the choice of copula function depends on the statistical fit to the data. Estimation can be accomplished using either a parametric or semi-parametric approach.

The application of copula functions to credit markets has increased in recent years. Abid and Naifar (2005) applied copulas in their analysis of the impact of equity market volatility on credit default swap rates. They found that the dependence structure between credit default swap rates and stock return volatility was asymmetric with positive skew and displayed right tail dependence best modeled using a Gumbel copula. Further, they found that high rated entities (AAA) have a weaker dependence on equity volatility than lower rated entities. Frey *et al* (2001) analyzed the use of copulas in modeling credit portfolio losses. They show that the copula of the latent variables determine higher order joint default probabilities for groups of obligors, illustrating the extreme risk of multiple defaults present in the referenced portfolio. This illustrated that traditional, linear correlation is not adequate when seeking to describe the dependence between defaults in a portfolio.

Cherubini and Luciano (2002) used copulas to evaluate default puts and credit switch contracts. They found that by using copulas to represent the dependence structure, one can accurately devise so-called super-hedging strategies for counterparty risk. They also found that the choice of copula function can have a significant impact on the resulting evaluation of counterparty risk; overvaluation in some cases, undervaluation in others. Cherubini (2004) later applied copulas in evaluating counterparty risk in swap transactions. He found that dependence affects both the level and slope of credit spreads particularly for institutions paying fixed premiums. Das and Geng (2004) used copula functions to model, simulate and assess the joint default process of their referenced set of issuers. They found that it imperative to capture the interdependence of marginal distributions and copula to achieve the best joint distribution depicting default.

Mashal and Naldi (2002) examined the estimation, simulation and pricing of multi-name contingent instruments. Multi-name instruments have payoffs that are contingent upon the default realization in a portfolio of names. They find that some copulas allow for an accurate estimation of the tail-dependencies for joint extreme events. In particular, their findings suggested that the t-copula has non-trivial tail dependence and therefore allows for more extreme joint events. Mashal and Zeevi (2002) found that the fit of the t-copula is generally superior to that of the Gaussian copula due to the ability of the t-copula to better capture extreme values often observed in financial data. They found that the t-copula captures extreme co-movements regardless of the marginal behavior of the individual assets.

The choice of the t-copula for the analysis was fairly straightforward for two main reasons; (a) Das and Geng (2004) found from fitting a number of copulas (Frank, Gumbel, Clayton and the t-copula) to CDS data that the t-Copula had the best fit., (b) The analysis of the study showed that the data exhibits symmetric tail dependencies, which is best suited for a t-copula. The fitting of the t-copula with the symmetric tail behavior makes it possible to test whether times of increased dependency can also be characterized by changes in both tails of the distribution. However following Rodriquez (2006) in order to capture these dependence structures, the copula that describes it must be time varying. Following the pioneering work by Patton (2006a, b) in time varying copula structures we introduce a AR-1 matrix structure to the copula model and a uniform dependence matrix structure to test if a time varying approach best predicts future CDS premia.

Schönbucher and Schubert (2001) provided insights into the connection between default dependencies and the joint dynamics of default intensities which are implicitly specified by specifying the default dependency. They illustrate the use of copula functions to specify the dependency structures between individual obligors and the portfolio of obligors without affecting the calibration of the other obligors or the dependency structure.

## 3. Model Specification

This section lays out the general model structure and the component parts of the models used in the study. The methodology used will abstract from actuarial credibility theory to estimate credit risk transition matrices in a multivariate Markov framework for valuing credit risk. Model one evaluates the regime switching transitions of credit risk, market liquidity conditions as represented by the bid-ask asymmetries and market risk. A Markov chain model is fitted to observed credit default swap (CDS) prices, market risk (spot rate) and the market liquidity proxy to determine the transitional matrices needed for the regime switching analysis.

Model two presents the multivariate *t*-copula function that will be implemented as part of a Generalized Linear Model (GLM)[5] framework to evaluate the joint conditional default probabilities. As discussed in section 3.2, using a t-copula function, we separate the estimation of the marginal distribution from the estimation of the joint distribution. A Gamma marginal distribution is considered for each issuer's hazard rates, combined with a t-copula. The t-copula is fitted with 2 different correlation structures to capture the differences in the dependency structure of the CDS premia. In additional the model is extended from the traditional literature to include a liquidity measure.

We begin by establishing the complete probability space which is represented by a filtered probability space $(\Omega, \mathbf{F}, \mathbf{P})$, where $\Omega$ is the state space of all possible credit states, $\mathbf{F}$ is the σ-algebra representing measurable events, and $\mathbf{P}$ is the empirical probability measure. Information evolves over the trading interval according to the augmented right-continuous complete filtration $\{\mathbf{F}_t : t \in [0, \tau]\}$. We let $\mathbf{E}(\bullet)$ denote expectation with respect to the probability measure $\mathbf{P}$. Further, we will let $\tau$ represent the time period $(0, 1, 2, 3, \ldots, \infty)$ of all economic activities, for a set of *n* correlated corporate credit risk of varying credit qualities[6]. Table 2 shows that credit risk has a strong positive correlation across credit qualities.

[5] The general linear model can be seen as an extension or generalization of linear multiple regression (OLS). Generalized linear models (GLMs) are used to do regression modeling for non-normal data such as financial and actuarial data. GLM assumes a parametric distribution family for the dependent variable but then allows the mean parameter to be a function of the covariates.

[6] The credit default swap qualities vary across the spectrum of reference entities, these ranges from AAA to C on the S&P rating scale.

### 3.1 Model One – Markov switching Model

A number of studies suggest that market and credit risk are positively correlated so as market risk increases the likelihood of default of the firm also increases (Jarrow *et al* (2001), Dunbar (2007)). Similarly, as illustrated by Dunbar (2007) market liquidity plays a unique role in enhancing credit markets. High credit risk products[7] tend to be less liquid than low credit risk assets because of investors risk appetite or levels of risk aversion. Both of these phenomena should generate differing regimes (states) of credit risk, thereby differing extremes in the hazard functions of the underlying referenced entities. This external bifurcation of credit risk into 2 regimes assigns state 1 to investment grade securities (S&P AAA, AA, A, BBB) and state 2 to high yield securities with a S&P rating of BB and B.

To determine credit risk regime switching, a Markov regime switching framework with time varying transition probabilities is specified, like that of Das *et al* (2004), where a regime switching model was used to capture the regime switching effects of high and low probability of default in credit risk products. Markov switching models exhibit a number of useful statistical properties such as their treatment of nonlinearities, non-stationary distributions, serial correlation and the fat-tailed distributions of volatile equity prices.

The study's Markov switching model was used to capture the probability effects of liquidity changes along the lines of credit quality. The methodology assumes that the distribution of market credit default swap (CDS) prices is governed by a two state, first order Markov Switching process. Each state is characterized by

[7] Assets of poor credit quality such as with a S&P credit rating of B or C.

high or low variances and means that correspond to either regime. As stated above, State 2 is characterized by high market risk and low market liquidity, this obligor class includes reference entities with credit quality BB and B on the (Standard and Poor's) S&P credit scale. Economic theory suggests that credit risk and market risk are positively correlated; hence high market risk implies high credit risk. Similarly credit risk is inversely related to market liquidity (Dunbar (2007)), so as credit risk increases (deterioration of credit quality) market liquidity disappears. State 1 is the opposite of state 2, with low market risk and high levels of market liquidity.

The probability that CDS prices and volatility is either in state 1 or 2 at time *t* is a function of the earlier state in *t-1*. Regime (state) 1 of the Markov model indicates a situation of high variance and high mean whereas regime (state) 2 implies low variance and low mean. This analysis should supplement the financial literature on the behavior of rational investors in financial markets. Rational investors are presumed to invest in highly volatile assets only if the risk premium of the volatile asset is high to compensate for the investors' investment risk. Given the preceding discussion, the historical distribution of observed CDS market prices is assumed to be collection of both states 1 and 2, where the collection is determined by a probabilistic transition between the two states.

The hazard rate process across all reference entities are assumed to follow a diffusion volatility model which can be represented as follows:

$$\lambda_{r_t^i, l_t^i} = \kappa_{r,l}^i [\theta_{r,l}^i - \lambda_{r,l}^i] dt + \sigma_{r,l}^i \sqrt{\lambda_{r,l}^i(t)} d(t) \in (t), \qquad r,l = \{Hi, Lo\} \qquad (1)$$

Where $\theta^i_{r,l}$, $\kappa^i_{r,l} \neq 0$, and $\sigma^i_{r,l} > 0$ are positive constants. $\kappa^i_{r,l}$ is the rate of mean reversion, $\theta^i_{r,l}$ represents the mean premia returns to investors, and $\sigma^i_{r,l}$ is the volatility parameter. In determining regimes from the sample data the average hazard rates across all issuers is first computed.

As discussed earlier the regimes or states are indexed by $r_t$ (the spot interest rate process) and $l_t$ (the market liquidity process) which is either high or low representing the two levels of economic conditions. The process $r_t$ gives knowledge of the macro economy, implying information of future macroeconomic conditions. Whereas $l_t$ on the other hand describes the evolution of the level of liquidity available to the various credit qualities in the market at given points in time. A logistic model is then used to generate a transition matrix which is then used to determine the probability of switching between regimes.

The transition matrix is presented below

$$\begin{pmatrix} p_{kij}(r,l)_{Lo} & 1-p_{kij}(r,l)_{Lo} \\ 1-p_{kij}(r,l)_{Hi} & p_{kij}(r,l)_{Hi} \end{pmatrix} \tag{2}$$

Where $p_{kij}(r,l) = \dfrac{e^{(\alpha_{r,l})}}{1+e^{(\alpha_{r,l})}}$, $r,l \in \{Lo, Hi\}$

$p_{ij}$ - denotes the probability of state *i* transitioning to state *j* after *k* steps, and where $p_{kij} \geq 0 \quad \forall \quad i \ \& \ \ j$ [8]. Alternatively, the *n*-step transition shown in expression 3 below satisfies the Chapman-Kolmogorov equations for any 0<*k*<*n*.

$$p_{ij}^{n}(r,l) = P\sum_{r \in S} p_{ir}^{k} p_{rj}^{(n-k)} \tag{3}$$

All parameters are estimated using maximum-likelihood, and the transition probabilities defined in expression 2 are the observed conditional risk-neutral transition probabilities for the process. This regime switching model is then fitted across all $\tau$ risk classes using the stochastic process outlined in the diffusion process of equation 1. Expression 4 below presents the regime shifting model for the risk classes:

$$\lambda_{n,r_t^i,l_t^i} = \kappa_{n,r,l}^{i}[\theta_{n,r,l}^{i} - \lambda_{n,r,l}^{i}(t)]dt + \sigma_{n,r,l}^{i}\sqrt{\lambda_{n,r,l}^{i}(t)}d(t) \in (n,t), \quad r,l = \{Hi, Lo\} \tag{4}$$

Where $n = \{1,...,6\}$

### 3.2 Model Two – Copula Dynamics under the Generalized Linear Model[9]

In this section we show how copulas can be applied to incorporate the dependence structure of credit risk. Before fitting the Copula model, a Hierarchal Linear Model (HLM) is first fitted to examine the inter-credit class and intra-credit

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[8] $\sum_{j=1}^{\tau} p_{ij} = 1 \ \forall i$

[9] For a discussion on GLM see Sun, J. et al (2006) "Heavy-Tailed Longitudinal Data Modeling Using Copulas" *Working Paper – Department of Actuarial Science, Risk management and Insurance, School of Business, University of Wisconsin*

class dependencies. The HLM framework makes use of a nested structure that allows effects to vary from one context to another. In hierarchical data, entities in the same group (credit class) are also likely to be more similar than entities in different groups. Due to this, the variations in outcome may be due to differences between groups, than to individual differences within a group. Thus, variance component models, where disturbance may have both a group and an individual component, can be of help in analyzing data of this nature. Within these models, individual components are independent, but while group components are independent within groups, they are perfectly correlated within the groups.

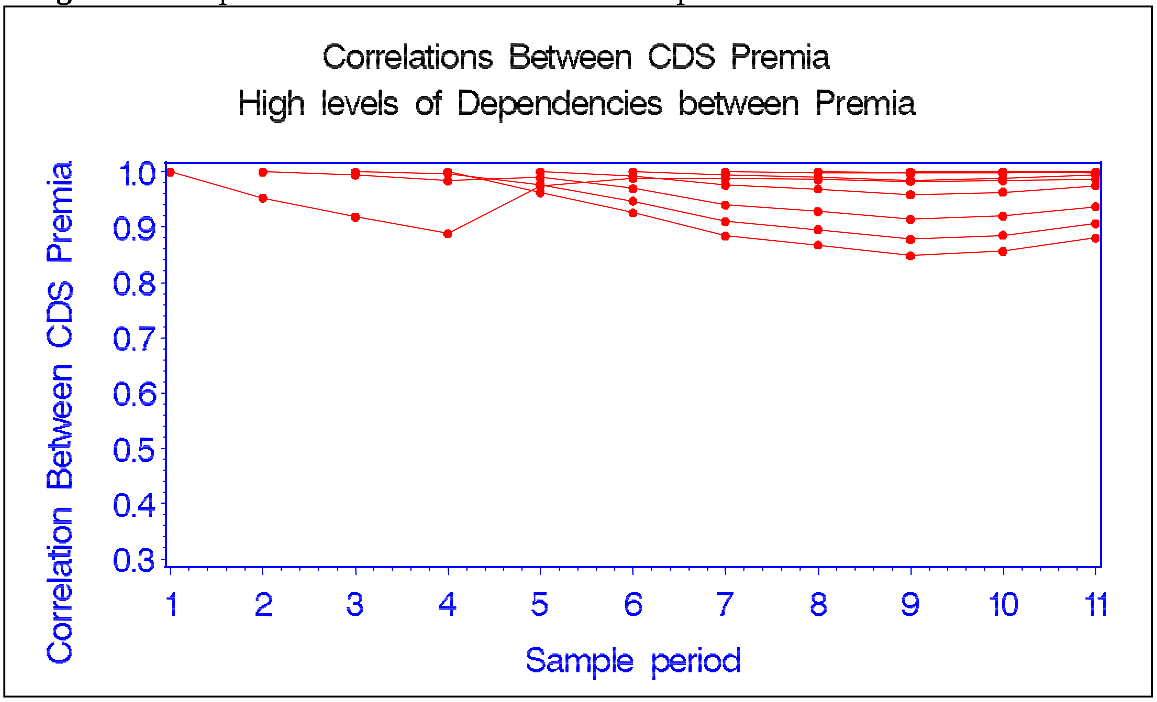
Model 2 looks at the normality assumption that is widely used in the analysis of financial data. As illustrated in table 4, credit risk data exhibits heavy tails, suggesting that extreme values in the data are more likely to occur than in normally distributed data. Further, the preceding discussion in section 1 suggests that the use of copulas in finance allows researchers to model the effects of the underlying marginal distribution separately from their dependency structures, i.e. the joint probability distributions.

Figure 1 shows dependency structure of the CDS premia across the sample period for several distinct entities. So, from our proposed dataset of $\tau$ risk classes of referenced entities credit risk, we can determine each issuer's marginal historical distribution and using a *t*-copula[10] construct the joint distribution with a desired correlation structure. Given the implied marginal distribution of the issuers, time

[10] Sklar (1959) suggests that every continuous joint distribution may be represented as a unique copula and the marginal distributions, which should make this technique very useful to finance in general and multivariate credit risk analysis in particular.

varying joint conditional probabilities can be computed by choosing copulas with time varying parameter values.

**Figure 1:** Temporal Correlation Structure of CDS premia




In finance the pricing of a risky asset involves the determination of the asset's joint risk-neutral default probabilities and the historical marginal distribution of the referenced entities market price of credit risk. Under the study's risk neutral measure, the historical price of credit risk, $\eta_t$ linking both the risk-neutral probability distribution  and the historical distribution Υ of $x_i$ depend only on the explanatory variables $r_t$ and $l_t$. In setting up the obligor's joint risk-neutral probability distribution functions we begin by specifying $\tau$ potential outcomes $x_1, x_2, \ldots, x_\tau$ for the $i^{th}$ obligor class[11] of credit risk. This joint probability distribution function can be described as follows

$$C_i\left(x_1, x_2, ..., x_\rho\right) = \Pr\left(X_1 \le x_1, X_2 \le x_2, ..., X_\rho \le x_\rho\right) \quad (5)$$

Where;

(a) The function C is a copula.

(b) X is a uniform random variable - $X \in R^n = \{X_1, ..., X_n\}$

(c) x is the observed corresponding historical distribution.

From equation 5 the copula probability distribution function[12] can be derived in a straightforward manner from the probability density function through integration and can be represented as follows:

$$C\left[F_1(x_1), F_2(x_2), ..., F_\tau(x_\tau)\right] = \Pr\left(X_1 \le x_1, X_2 \le x_2, ..., X_\tau \le x_\tau\right) = F\left(x_1, x_2, ..., x_\tau, \lambda_\tau^\theta\right) \quad (6)$$

Where

(a) Risk class $n = 1, \ldots, \tau$.

(b) $\lambda_t^Q = y_i \beta$

(c) $y_i$ is a *K x 1* vector of known explanatory variables (short rate and market liquidity proxy).

(d) β is a *K x 1* vector of unknown parameters.

(e) Each member of the *n*-variate distribution has its own marginal distribution - $F_i(x_i) \ \ i = 1, 2, ..., n$

[11] Obligor class refers to the Standard & Poor's credit risk ratings AAA, AA, A, BBB, BB and B. Where AAA is most secure and B is least secure

[12] The corresponding probability density function is $f_i(x_i, ..., x_\tau) = c[F_1(x_1), ..., F_\tau(x_\tau), \lambda_\tau^Q] \prod_{t=1}^{\tau} f_t(x_t)$.

Equation 6 defines a copula enhanced multivariate distribution function evaluated at the observed historical distribution $x_1, x_2, \ldots, x_\tau$ with marginal distributions $F_1, F_2, \ldots, F_\rho$. The risk-neutral default process in the reduced form model framework used by a number of researchers such as Jarrow *et al* (2001) can be specified as $\lambda_t^Q$ [13], which following Das *et al* (2004) is a joint probability stochastic process. For this study, the cumulative hazard function used to generate the hazard rates is derived as an integral of a hazard function, represented as:

$$\lambda_{kij}^Q = -\ln(1 - P_{kij}) \geq 0 \tag{7}$$

Where, $P_{kij}$ is the probability of default for the various credit ratings of the study. $P_{kij}$ is derived from the transition matrix in expression 2.

Equation 6 is established under the assumption that the marginal distribution function $F$ (.) for the observations of market price of credit risk is common up to a systematic component $\lambda_t^Q$ that is known up to $n$ parameters. Credit risk default is assumed to be random following a Poisson process, hence it is further assumed that $F$ (.) is from the natural exponential family of distributions, which encompasses the Normal, Poisson and Gamma distributions, such that the probability density function for the $i^{th}$ obligor class at time $t$ can be expressed as

$$f(y_i, \lambda_i^Q) = e^{\left(\frac{(y_i \lambda_i) - \alpha(\lambda_i)}{\phi} + \psi(y_i, \phi)\right)} \tag{8}$$

Where the functions $\alpha$ and $\psi$ are chosen to represent a particular distribution and $\phi$ is a known dispersion factor. Following Frees *et al* (2005) we use a canonical link

[13] $\lambda_t^Q = \lambda_0 + \lambda_1 r_t + \lambda_2 l_t$

function to link $\lambda_t^Q$ to the systematic component so that $\alpha(\lambda_i^Q) = E(y_i)$ and $\lambda_i^Q = x_i^{'}\beta = g(Ey_i)$. Now that equation 8 has been specified, the generalized linear model uses these along with the copula covariates as input. Assuming independence among credit risk class, the copula dependence can be estimated parametrically using maximum likelihood. Given the copula marginal distribution function $F_{it}(y_{it}, x_{it}, \beta, \gamma)$ with an accompanying density function $f_{it} = f(y_{it}, x_{it}, \beta, \gamma)$ with parameter vector $\theta$, then the log-likelihood function of the $i^{th}$ credit risk class can be written as follows;

$$l_i(y_i, x_{i1}, \ldots x_{i\tau}, \beta, \gamma, \theta) = \sum_{t=1}^{T_i} \ln f_{it}(y_{i,} \lambda_i^Q) + \ln c(F_1, F_2, \ldots, F_i, \theta) \tag{9}$$

Where $\gamma$ is the scale and shape parameter; $\beta$ Is the marginal parameters estimated, and $x_{it}$ is the explanatory credit risk data.

The log likelihood function using the Generalized Linear Model (GLM) framework can thus be written as

$$l_i = \alpha_0 + \sum_{t=1}^{T_i} \frac{y_i x_i^{'}\beta - \alpha(x_i^{'}\beta)}{\phi} + \ln c(F_1, F_2, \ldots, F_{iT}) \tag{10}$$

Substituting the copula density function into the GLM framework illustrated in equation 9 yields an expression for the log-likelihood of the $i^{th}$ credit risk class. Following Sun *et al* (2006) the parameters $\beta, \gamma, and \theta$ were estimated by estimating the sum of the likelihood function.

### 3.2.1 Implied CDS premia from the Copula Predictive distribution

Following Frees *et al* (2005), the *t*-copula used in this paper is parameterized by the degree of freedom, *r*, and the correlation matrix $-\Sigma_{\tau}$. Frees *et al* (2005) showed that the number of parameters to be evaluated is dependent on the matrix structure adopted. Suppose the correlation matrix associated with the multivariate distribution $\{Y_1, Y_2, Y_3, \ldots, Y_\tau, Y_{\tau+1}\}$ is given by

$$\begin{pmatrix} \Sigma_{\tau} & P_{\tau+1|\tau} \\ P_{\tau+1} & 1 \end{pmatrix} \tag{11}$$

Where;

(a) $\sum_t$ describes the correlation between $\{Y_1, Y_2, Y_3, \ldots, Y_\tau\}$

(b) $P_{\tau+1|\tau}$ describes the correlation between the implied $Y_{\tau+1}$ and the observe credit premia $\{Y_1, Y_2, Y_3, \ldots, Y_\tau\}$.

(c) $Y_t$ is a multivariate *t*-distribution with *r* degrees of freedom; the accompanying marginal distribution is a *t*-distribution with *r* degrees of freedom denoted by $G_r$.

So as to formulate the joint conditional density function, we first derive the conditional variance which is expressed as

$$\sigma^2_{\tau+1|\tau} = 1 - P'_{\tau+1|\tau}\Sigma_{\tau}^{-1}P_{\tau+1|\tau} \tag{12}$$

Using expression 12 the joint conditional density function for the implied CDS premia distribution is therefore given as

$$f(Y_{\tau+1} \mid Y_1, Y_2, \ldots, Y_\tau) = g_r\left(\frac{V_{\tau+1} - P_{\tau+1|\tau}\Sigma_{\tau}^{-1}v}{\sigma_{\tau+1|\tau}}\right)\frac{f\left(Y_{\tau+1}, \theta_{\tau+1}\right)}{g_r(V_{\tau+1})\sigma_{\tau+1|\tau}} \tag{13}$$

Where

(a) $V_t = G_r^{-1}(F_t(Y_t)), t = 1,2,3,...,\tau+1$; $G_r$ is a distribution function of credit premia

(b) $F_t(Y_t)$ *and* $f(Y_t)$ are both cumulative mass functions.

(c) $Y_t$ is a multivariate *t*-distribution with *r* degrees of freedom; the accompanying marginal distribution is a *t*-distribution with *r* degrees of freedom denoted by $G_r$.

Given the preceding discussion the marginal premia distribution will be modeled as a two parameter Gamma distribution with density

$$f(Y;\alpha,\gamma) = \frac{\gamma^{\alpha-1}}{\gamma^{\alpha}\Gamma(\alpha)} e^{(-\frac{Y}{\gamma})} \tag{14}$$

The dependence structure will be modeled by a *t*-copula with *r* degrees of freedom and the log-likelihood function for firm *i* over τ years expressed as

$$l_i(\alpha,\gamma) = \alpha_0 + \sum_{t=1}^{T_i} \frac{y_i x_i^{'}\beta - \alpha(x_i^{'}\beta)}{\phi} + \ln c(F_1, F_2, ..., F_{iT}) \tag{15}$$

The study assumes that the Gamma distribution parameters are constant across obligors in our sample, so there are four parameters to be estimated for the fitted *t*-copula. These are $\rho_1, \alpha, \gamma$ *and* $r$.

## 4. Description of the Data

This section provides a description of the data used in this paper. Credit default swap bid-ask[14] and mid price data for the pricing estimates were obtained from Bloomberg. In addition, Bloomberg was also used to obtain weekly U.S

[14] The bid-ask prices are consensus quotes among market participants regarding the value of the CDS.

Treasury note and bill prices that were needed for the parameter estimation of the spot rate process. The Bloomberg credit default swap dataset is comprised of quotes for contracts of maturities 1 through 15 years. The sample period covered in the study was comprised of 135 weeks of default swap quotes per reference entity.

The data analyzed is based on weekly observations from January 2nd, 2004 to August 08th, 2006, where $t = \frac{1}{365},...,\bar{t}$. One observation made during this stage of the exercise is the fact that prior to 2002 the CDS market was not as liquid and active as it is currently. Hence there is not an abundance of reasonable data prior to 2003. The data is comprised of a mixture of 29 US dollar denominated AAA, AA, A, BBB, BB and B credit default swaps issued by 29 fortune 500 companies, across several industries chosen to stratify the various industry groupings such as cable/media, financial, insurance, U.S banks, telecom, energy, retail, technology and manufacturing.

Quotations are available only on days when there is some level of liquidity in the market as evidenced either through trades or by active market making by a dealer. Bloomberg was also used to obtain CDS characteristics such as maturity dates, coupon percentages and seniorities. In reality, since most of the credit default swap trading activity is within the 5-year time to maturity group, the price quotes on the 5 year CDS premia will be used in the study's pricing analysis. For an issuer to be included in the sample, it must have at least 130 weekly observations of its 5-year CDS data points. As a result of this selection technique, the CDS dataset used in this study covers 29 issuers with an average of 133 weekly observations per issuers, for maturities of 1, 3, 5, 7 and 15 years respectively. Additionally, the sample period was subdivided into 11 sub-periods of 13 weeks of daily observations so as to allow the copulas to determine changes in the covariates over the sub-periods of the larger sample period.

As stated earlier, the market liquidity proxy will be derived from bid-ask spreads. In any market that is in equilibrium, there will generally be a difference between the best quoted ask price and the best quote bid price. That difference is called the bid-ask spread (or bid-offer spread). For the market liquidity proxy, this study follows the approach in Dunbar (2007) and uses the percentage bid-ask spread, which is the bid-ask premia divided by the mid price. Tang and Yan (2006) suggest that bid-ask spreads measure trading costs that compensate market makers for the risk of adverse selection and hedging costs. Depending upon the market bid-ask quotes may be expressed as actual prices, yields, implied volatilities, etc. The average of the bid and ask prices is called the mid-offer price.

## 5. Discussion of Empirical Results

Descriptive statistics for the 29 firms included in this study are presented in table 1. Daily market observations from January 2, 2004 to April 10, 2006 were broken into 11 subgroups each having 13 weeks of daily observations; resulting in 174 observations of market premia. Thirteen weeks of observations from April 11 2006 to August 08, 2006 were held back for the out-of sample portion for the copula prediction model. From table 1, the average CDS premia varies from year to year. The table also show that variability moves inversely with credit quality.

**Table 1: Summary statistics of the 5 year CDS premia showing credit ratings average premia and variability among credit classes**. The Mean, Minimum, Maximum and Standard Deviation cover the period of the study

| Issuer | Ticker | Industry | S&P Rating | Moodys Ratings | Mean CDS Premia | Maximum CDS Premia | Minimum CDS Premia | Standard Deviation |
|---|---|---|---|---|---|---|---|---|
| General Electric | GE | Industrial | AAA | Aaa | 24.47 | 34.66 | 15.99 | 5.28 |
| Altria | MO | Consumer | BBB+ | Baa2 | 118.11 | 177.19 | 54.46 | 45.18 |
| Aetna | AET | HealthCare | A- | A3 | 33.29 | 44.26 | 21.81 | 8.49 |
| Ace Insurance | ACE | Insurance | A- | A3 | 44.69 | 67.87 | 27.09 | 11.54 |
| Alcan | AL | Mining | BBB+ | Baa1 | 33.77 | 51.07 | 24.20 | 8.78 |
| Alcoa | AA | Mining | A- | A2 | 26.94 | 41.20 | 18.67 | 6.93 |
| Altell | AT | Telecom | A- | A2 | 38.54 | 54.80 | 24.70 | 9.91 |
| American Express | AXP | Financial Services | A+ | A1 | 23.63 | 29.76 | 18.22 | 4.13 |
| American International Group | AIG | Insurance | AA | Aa2 | 23.05 | 37.74 | 17.75 | 5.60 |
| Arrow Electronics | ARW | Electronics/Wholesale | BBB- | Baa3 | 87.41 | 128.93 | 60.06 | 27.02 |
| Bristol-Myers Squibb | BMY | Pharmaceutical | A+ | A1 | 24.73 | 39.33 | 13.94 | 8.51 |
| Cendant | CD | Rental & Leasing | BBB+ | Baa1 | 59.54 | 97.40 | 43.70 | 18.45 |
| Caterpillar | CAT | Industrial | A | A2 | 21.99 | 28.88 | 16.31 | 4.12 |
| Cingular | AT&T | Telecom | A | Baa1 | 99.52 | 295.99 | 22.83 | 86.17 |
| CapitalOne | COF | Financial Services | BBB | A3 | 46.11 | 68.19 | 26.24 | 15.43 |
| IBM | IBM | Computer | A+ | A1 | 20.47 | 26.93 | 14.60 | 4.04 |
| WalMart | WMT | Consumer | AA | Aa2 | 13.84 | 17.86 | 9.20 | 2.79 |
| Target | TGT | Consumer | A+ | A2 | 18.98 | 28.83 | 11.52 | 6.97 |
| Dow Chemical | DOW | Manufacturing | A- | A3 | 33.57 | 51.16 | 22.57 | 10.09 |
| Washington Mutual Bank | WAMU | Financial Services | A | A2 | 39.06 | 51.06 | 29.84 | 6.23 |
| Viacom | VIA | Cable | A | Baa3 | 49.02 | 75.49 | 24.00 | 15.00 |
| Carnival Corporation | CCL | Entertainment | A- | A3 | 32.62 | 46.51 | 24.64 | 8.41 |
| Lucent | LU | Manufacturing | B | B1 | 245.29 | 408.75 | 29.58 | 105.04 |
| Starwood Resorts | HOT | Hotel | BB+ | Ba2 | 122.70 | 202.50 | 63.35 | 35.39 |
| Unum Provident Group | UNM | Insurance | BB+ | Ba1 | 148.47 | 265.73 | 76.12 | 62.78 |
| Nordstrom | JWN | Consumer | A | Baa1 | 34.17 | 50.90 | 23.78 | 7.16 |
| Haliburton | HAL | Industrial | BBB+ | Baa1 | 42.57 | 75.23 | 24.31 | 17.82 |
| Marriott | MAR | Hotel | BBB+ | Baa2 | 40.40 | 54.10 | 27.57 | 8.58 |
| XL Capital | XL | Insurance | AAA | A3 | 44.53 | 52.35 | 35.75 | 5.42 |

Table 2 summarizes the Hierarchical Linear Model (HLM) marginal analysis correlations among CDS prices across credit qualities and cross-sectional residuals over time. As discussed in section 3.2, the HLM has a nested structure that allows effects to vary from one context to another. In hierarchical data, entities in the same group (credit class) are also likely to be more similar than entities in different groups. Due to this, the variations in outcome may be due to differences between groups, than to individual differences within a group. As a result of this tendency, table 2 looks at variations between groups rather than within groups.

In table 2, the results in the upper right corner above the diagonal represent the correlations of the residuals of the predicted marginal model. The results in the lower left of the diagonal represent the correlation of actual CDS premia observations over the sample sub-periods. The observed premia show strong correlations, while correlations of the predicted residuals vary from period to period. The results show that any model that ignores temporal dependencies does not provide an appropriate fit to the relationships being explored. Further, though temporal dependencies from period to period are high, they are not constant across the sample period. Hence, as stated earlier the dependencies among variables over time cannot be ignored, neither can we underscore the importance of time-varying covariates.

**Table 2: Credit Default Swap Correlations -** The table displays correlations of CDS prices for the 11 subperiods of the study. From the tble it is apparent that the multivariate average CDS prices are not independent.

| | CDS Period 1 | CDS Period 2 | CDS Period 3 | CDS Period 4 | CDS Period 5 | CDS Period 6 | CDS Period 7 | CDS Period 8 | CDS Period 9 | CDS Period 10 | CDS Period 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| CDS Period 1 | **1.0000** | -0.0512 | -0.3678 | -0.5442 | -0.5520 | -0.3837 | 0.1833 | -0.5132 | -0.0814 | 0.6858 | -0.4028 |
| CDS Period 2 | 0.9528 | **1.0000** | -0.3957 | 0.0139 | -0.2621 | -0.2824 | -0.2825 | -0.0107 | -0.6088 | -0.1529 | -0.3259 |
| CDS Period 3 | 0.9188 | 0.9947 | **1.0000** | 0.4118 | 0.4145 | 0.6810 | -0.1119 | 0.6181 | 0.4550 | -0.3024 | 0.5071 |
| CDS Period 4 | 0.8883 | 0.9835 | 0.9968 | **1.0000** | 0.8609 | 0.7058 | -0.2283 | 0.4883 | 0.0820 | -0.8563 | 0.4631 |
| CDS Period 5 | 0.9747 | 0.9895 | 0.9773 | 0.9627 | **1.0000** | 0.8178 | -0.0114 | 0.5047 | 0.2133 | -0.8014 | 0.6731 |
| CDS Period 6 | 0.9882 | 0.9696 | 0.9471 | 0.9258 | 0.9931 | **1.0000** | 0.1430 | 0.6336 | 0.4450 | -0.5329 | 0.8624 |
| CDS Period 7 | 0.9878 | 0.9403 | 0.9100 | 0.8839 | 0.9765 | 0.9949 | **1.0000** | -0.1272 | 0.2667 | 0.2232 | 0.2286 |
| CDS Period 8 | 0.9856 | 0.9284 | 0.8955 | 0.8675 | 0.9691 | 0.9909 | 0.9992 | **1.0000** | 0.5162 | -0.6216 | 0.7340 |
| CDS Period 9 | 0.9829 | 0.9140 | 0.8780 | 0.8481 | 0.9591 | 0.9850 | 0.9973 | 0.9992 | **1.0000** | 0.0728 | 0.6069 |
| CDS Period 10 | 0.9845 | 0.9202 | 0.8854 | 0.8564 | 0.9634 | 0.9876 | 0.9983 | 0.9997 | 0.9999 | **1.0000** | -0.4355 |
| CDS Period 11 | 0.9867 | 0.9372 | 0.9067 | 0.8805 | 0.9752 | 0.9940 | 0.9998 | 0.9996 | 0.9979 | 0.9988 | **1.0000** |

Table 3 presents the results of the regime switching model that incorporates the liquidity proxy variable in the hazard functions of the firms comprising the study. Following Das *et al* (2004) average hazard rates across all issuers were first computed using the square root drift diffusion process in exhibit 4 of section 3.1. On one hand, the results were as expected and consistent with prior findings that did not include a measure of market liquidity. The data from the high regime model indicates that a higher average return is required in order to compensate investors for the higher levels of risk. Conversely, investors are prepared to take a lower average return on investments bearing lower levels of volatility.

While on the other hand, the inclusion of liquidity as an explanatory variable in the CDS valuation model adds an entirely new dimension to the regime switching model. In table 3 we see that while investors are being compensated for assuming higher levels of credit risk with higher average returns, this average return declines with credit quality. This is because as credit risk increases the credit risk premium paid by protection seekers increases, so the cost of this added credit risk protection reduces any potential returns to the investor.

**Table:3** Results of Regime Switching Model indexed by market risk and liquidity - Regime 1 represents high risk/high returns and Regime 2 represents low risks/low returns

| | *Parameters* | | | | | |
|---|---|---|---|---|---|---|
| **Credit Quality** | **Regime 1** | | | **Regime 2** | | |
| | $\kappa_{HI}$ | $\theta_{HI}$ | $\sigma_{HI}$ | $\kappa_{LO}$ | $\theta_{LO}$ | $\sigma_{LO}$ |
| AAA | 0.740 | 2.623 | 0.020 | 0.824 | 0.075 | 0.010 |
| *t-stat* | *2.39* | *178.39* | *4.48* | *2.53* | *68.09* | *4.47* |
| AA | 0.718 | 2.614 | 0.089 | 0.766 | 0.077 | 0.043 |
| *t-stat* | *1.93* | *40.03* | *4.47* | *2.01* | *15.63* | *4.47* |
| A | 0.699 | 2.056 | 0.091 | 0.674 | 0.139 | 0.052 |
| *t-stat* | *2.57* | *34.66* | *4.47* | *2.44* | *14.95* | *4.47* |
| BBB | 0.065 | 1.915 | 0.112 | 0.116 | 0.216 | 0.090 |
| *t-stat* | *0.27* | *0.58* | *4.47* | *0.53* | *0.51* | *4.47* |
| BB | 0.680 | 1.283 | 0.029 | 0.733 | 0.325 | 0.020 |
| *t-stat* | *7.77* | *78.52* | *4.47* | *8.62* | *60.54* | *4.48* |
| B | 0.142 | 1.941 | 0.059 | 0.595 | 0.126 | 0.013 |
| *t-stat* | *0.58* | *8.55* | *4.47* | *4.21* | *48.30* | *4.47* |

From the high levels of dependencies between variables shown by Table 2 a further attempt was made to look at the upper and lower tail dependencies. Table 4 shows that there is a high level of tail dependencies in the upper and lower quartiles of the computed default probabilities. This characteristic best suits a t-copula which is then fitted to the sample data.

Table 4: Tail Dependencies in the CDS Premia Data. Both Upper and Lower Percentile shows high levels of Dependencies among credit ratings.

**Lower Percentile Tail Dependency**

| | AAA | AA | A | BBB | BB | B |
|---|---|---|---|---|---|---|
| AAA | 1.000 | 0.971 | 0.998 | 0.801 | 0.970 | 0.918 |
| AA | 0.971 | 1.000 | 0.983 | 0.634 | 1.000 | 0.795 |
| A | 0.998 | 0.983 | 1.000 | 0.766 | 0.982 | 0.894 |
| BBB | 0.801 | 0.634 | 0.766 | 1.000 | 0.632 | 0.973 |
| BB | 0.970 | 1.000 | 0.982 | 0.632 | 1.000 | 0.794 |
| B | 0.918 | 0.795 | 0.894 | 0.973 | 0.794 | 1.000 |

**Upper Percentile Tail Dependency**

| | AAA | AA | A | BBB | BB | B |
|---|---|---|---|---|---|---|
| AAA | 1.000 | 0.925 | 0.787 | 0.965 | 0.822 | 0.995 |
| AA | 0.925 | 1.000 | 0.962 | 0.992 | 0.977 | 0.958 |
| A | 0.787 | 0.962 | 1.000 | 0.922 | 0.998 | 0.844 |
| BBB | 0.965 | 0.992 | 0.922 | 1.000 | 0.943 | 0.986 |
| BB | 0.822 | 0.977 | 0.998 | 0.943 | 1.000 | 0.875 |
| B | 0.995 | 0.958 | 0.844 | 0.986 | 0.875 | 1.000 |

Table 5 presents the results of the covariates fitting *t*-copula model which used 2 differing correlation matrix structures to show the effects of a constant, versus time-varying joint covariance structure in credit risk forecasting. The results show that the correlation coefficients ($\rho$) for both specification of the predictive model were statistically strong with *p*-values of less than 1 percent. The fact that $\rho$ is statistically significant, provides strong statistical evidence that the correlation structure of the financial data is not independent. Further, these findings compliment the earlier analysis presented in table 2 which suggested that the CDS data displayed high levels of dependencies across the sample period. With the exception of $R_t$ – the spot rate on interest which proxies the macro economy, all coefficients were statistically significant at the less than 1% level of significance.

**Table 5**: Summary of Maximum Likelihood Estimates of *t*-Copula Parameters.

| *Parameter* | **AR1 Correlation Matrix** | | | **EX Correlation matrix** | | |
|---|---|---|---|---|---|---|
| | *Coefficient* | *Std Error* | *P Value* | *Coefficient* | *Std Error* | *P Value* |
| $\rho$ | 0.9839 | 0.0061 | 0.0000 | 0.9719 | 0.0108 | 0.0000 |
| Intercept | 6.8919 | 2.1232 | 0.0006 | 6.6754 | 1.9886 | 0.0004 |
| Rt | 0.0020 | 0.0536 | 0.4851 | 0.0101 | 0.0374 | 0.3934 |
| Liq | -1.5678 | 0.5076 | 0.0010 | -1.5085 | 0.4790 | 0.0008 |
| α | 23.1196 | 8.0865 | 0.0021 | 23.7059 | 8.0765 | 0.0017 |
| r | 2.1486 | 0.7722 | 0.0027 | 2.0129 | 0.6916 | 0.0018 |
| AIC | 974.70 | | | 1008.87 | | |

These results strengthen the view that the spot rate and liquidity are useful predictors of credit risk. Economic theory suggests that credit risk and market risk are positively correlated so an increase in credit risk will lead to an increase in market risk. Similarly, the negative coefficient for liquidity indicates that as credit risk increases, market liquidity falls off. We see this phenomenon in the market for securities, with high yield securities being less liquid than their investment grade counterparts. Comparing the goodness of fit of both models the AIC[15] technique is

[15] Defined as AIC = -2*log (maximum likelihood) + 2*(number of estimated parameters). A smaller AIC value indicates a better statistical fit to the data, or a better model.

used. The result shows that the AR-1[16] time-varying correlation matrix gave a much better fit to the data than the constant correlation (EX)[17] matrix model. These findings support the hypothesis that a model that best captures time-varying joint conditional correlations across time will give a better fit to financial data than one that assumes constant covariates.

Traders, speculators, portfolio managers are interested in predictive methodologies that produce accurate and reliable forecasts of CDS premium. Current multivariate techniques provide point estimates and predictive intervals that are most appropriate for normal distributions. Since CDs data is not normal, copulas were used to obtain the predictive distribution (see discussion in section 3.2.1).

Table 6 presents the simulation results of the predictive *t*-copula model used in the study. The table presents the results of the *t*-copula fitted with a constant joint correlation matrix, consistent with the assumption of traditional multivariate analysis. The table also presents the results of a time varying joint correlation matrix which shows that the predictive capability of the model is by far superior to the constant correlation matrix. For the model utilizing a constant correlation matrix the mean absolute difference between actual was approximately 7.8, whilst the model fitted with time-varying correlations had a mean absolute difference of approximately 3.2. Model 2 of table 6 also had a standard deviation of 30.1 which was very close to the 27.6 of the actual CDS data. So given the results of table 5 and

[16] This is a time series representation of temporal relationships, which shows that credit risk at $t_0$ has diminishing influence on credit risk in later periods; $t_n$.
[17] The Exchange structure or uniform correlation matrix (discussed at length in Frees (2004)) indicates that covariates within a credit class does not change over time, or exhibits constant correlation.

table 6, the joint time-varying conditional correlation model performed better than the constant correlation model often assumed in multivariate analyses.

These findings regarding the use of copulas to model dependencies over time are consistent with results obtained by Sun *et al* (2006) who found that long tailed longitudinal data are best fitted with a copula that is capable of separating the multivariate joint distribution into interdependent probabilities and marginal distributions.

**Table 6**: Empirical Results of CDS Predictive Estimates from the t-Copula model. Model one was estimated using a constant correlation (*EX*) framework, while model 2 used time-varing correlations (*AR-1*) to derive its outsample predictions.

| Issuers | Actual CDS Premia | Predicted CDS Premia | |
|---|---|---|---|
| | | ***Model 1*** *Constant Correlations* | ***Model 2*** *Time Varying Correlations* |
| Altria | 54.4610 | 68.4822 | 63.7125 |
| GE | 15.9900 | 30.8940 | 20.5989 |
| Aetna | 29.6240 | 30.7100 | 29.3459 |
| ACE | 36.4660 | 40.6982 | 36.2411 |
| ALCAN | 30.9690 | 31.1159 | 33.1364 |
| ALCOA | 21.0450 | 23.3363 | 24.8489 |
| ALTELL | 53.5330 | 41.7641 | 45.1203 |
| American Express | 18.2170 | 22.8554 | 19.5336 |
| AIG | 18.2250 | 20.4799 | 17.4940 |
| Arrow Electronics | 62.8980 | 67.1101 | 64.5184 |
| Bristol-Myers-Squibb | 13.9390 | 12.3316 | 13.5419 |
| Cendant | 54.9590 | 62.0015 | 78.1635 |
| Caterpillar | 16.7910 | 16.7296 | 17.3426 |
| Cingular | 22.8330 | 35.8996 | 23.7369 |
| CapitalOne | 26.2750 | 25.7546 | 24.2826 |
| IBM | 14.6020 | 12.2117 | 14.2246 |
| WalMart | 9.1980 | 5.9703 | 8.5244 |
| Target | 11.5190 | 10.9676 | 11.8339 |
| Dow Chemical | 22.5740 | 26.9606 | 26.9012 |
| Viacom | 49.1890 | 65.0839 | 65.5109 |
| Carnival Corporation | 29.5780 | 33.0708 | 32.0097 |
| Lucent | 138.9580 | 213.1035 | 131.4094 |
| Starwood Resorts | 81.6660 | 124.4306 | 108.9638 |
| Nordstrom | 23.7830 | 27.5370 | 26.1198 |
| Haliburton | 30.6820 | 29.0880 | 31.7721 |
| Marriott | 30.9250 | 35.1615 | 33.9705 |
| XL Capital | 45.1770 | 44.5439 | 41.9858 |
| Washington Mutual Bank | 29.8390 | 37.0511 | 33.8988 |
| Unum Provident Group | 81.6090 | 104.9778 | 89.5346 |
| **Mean Absolute Difference** | - | **7.752** | **3.198** |
| **Std Deviation** | *27.6* | *42.0* | *30.1* |

## 6. Conclusion

Dependency structures vary over time and are not constant as in traditional multivariate analyses. In this paper we incorporated a copula function to model the "heavy-tail" dynamics of credit risk data. The copula was used to model the dependencies over time. To test the multivariate normality assumption we explored two different specifications of the covariance matrices across the reference entities of the study. In addition, to test the time-varying joint conditional and constant correlation hypotheses we developed alternative constant and time-varying correlation matrices that were fitted to the copula methodology. Recent empirical work by Duffie *et al* (2007) on term structures of conditional probabilities of corporate default used covariate estimates that assumed normality in the underlying data, however our findings clearly illustrate that allowing dependency structures to vary over time is superior to fixed correlation parameters.

Secondly, several studies have looked at the regime switching characteristics of credit risk, but none looked at the effects of liquidity on the Markov switching dynamics of the credit default swaps. The study shows that as credit risk deteriorates, investors demand a higher average return for assuming greater levels of risk. However, this return moves inversely with credit quality due to the CDS premium investors pay to protection sellers which increase at a magnitude that is greater than the corresponding increase in expected returns (because of increasing illiquidity as credit quality decreases).

Prior work by Das *et al* (2004) and others indicate that investors require a higher average return to take on higher levels of risk; however these results ignored the level of returns across credit classes in the presence of a liquidity measure. So an important extension to both the markov switching and copula dynamics discussion is the inclusion of liquidity as a determinant of financial asset pricing and investor psyche. From prior research we know that investors take on higher levels of risks because of the higher levels of average returns, but our findings now suggests that these average levels of returns tend to change with the levels of liquidity of financial assets.

Further, credit risk and liquidity migration across credit classes suggests that higher credit and liquidity risk generally exhibits higher levels of volatility. The regime switching analysis indicates that the increase in credit risk across credit classes is met with an increase in illiquidity and a simultaneous increase in volatility. This increased volatility or risk level as credit quality falls off partly explains the need for credit risk insurance or credit default swap protection, across credit quality, to make these investments attractive to potential investors.

## REFERENCES


Abid, F., and Naifar, N. (2005) "The Impact of Stock Returns Volatility on Credit Default Swap Rates: A Copula Study" University of Sfax, Tunisia

Bandreddi, S., Sanjiv Das and Rong Fan (2006), "Correlated Default Modeling with a Forest of Binomial Trees", working paper, Santa Clara University

Burtschell, X., J. Gregory and J. P. Laurent, (2005). "A Comparative Analysis of CDO Pricing Models", working paper, BNP *Paribas*.

Cheng, W., (2001), "Recent Advances in Default Swap Valuation," *Journal of Derivatives*, 1, pp. 18-27.

Chen, Long, David Lesmond, and Jason Wei, (2004), "Corporate Yield Spreads and Bond Liquidity" Working paper, Michigan State University Dept of Finance.

Cox, J., Johnathan, E.I. and S. A. Ross (1985), "A theory of the term structure of interest Rates", *Econometrica* vol 53, 385-407.

Cherubini, Umberto (2004) "Pricing Swap Credit Risk with Copulas" Department of Mathematical Economics, University of Bologna, EFMA 2004 Basel Meetings Paper. Available at SSRN

Cherubini, U., and Luciano, E. (2002) "Pricing and Hedging Vulnerable Credit Derivatives with Copulas", EFA 2003 Annual Conference Paper No. 738. Available at SSRN

Das, Sanjiv and Gary Geng (2004). "Correlated Default Processes: A Criterion Based Copula Approach," *Journal of Investment management* 2 (2), 44-70.

Das, Sanjiv, Paul Hanouna, and Atulya Sarin, (2006), Fundamentals-Based vs Market-Based Cross-Sectional models of CDS Spreads. *DefaultRisk.com*

Dunbar, K. (2007), "U.S. Corporate Default Swap Valuation: The Market Liquidity Hypothesis and Autonomous Credit Risk" *forthcoming The Journal of Quantitative Finance*.

Duffie, D., Leandro Saita, Ke Wang (2007), "Multi-period Corporate default prediction with Stochastic Covariates", *Journal of Financial Economics*, 83, pp 635-665.

Embrechts, P., McNeil, A., and Straumann, D. (1999) "Correlation: Pitfalls and Alternatives" Journal of Risk pp 69 – 71.

Fermanian, J., and Scaillet, O. 2004 “Some Statistical Pitfalls in Copula Modeling for Financial Applications,” International Center for Financial Asset Management and Engineering

Frey, R., Nyfeler, M., and McNeil, A. (2001) “Copulas and Credit Models” Journal of Risk pp 111 – 114.

Frees, E. and Wang, P. (2005), “Credibility using Copulas”, *North American Actuarial Journal*, Volume 9 (2), 31-48.

Frees, E. and Valdez (1998), “Understanding Relationships using Copulas”, *North American Actuarial Journal*, Volume 2, no 1, 1-25.

Hull, J. and A White, (2006). “The perfect Copula”, University of Toronto Working papers.

Jarrow, R. and Y. Yildirim , (2001), “A Simple Model for Valuing Default Swaps when both Market and Credit Risks are correlated” Cornel University (December): pp1-29.

Kole, E., Koedijk, K., and Verbeek, M. (2006) ‘Selecting Copulas for Risk Management’, CEPR Discussion Paper No. 5652 Available at SSRN

Li, David (2000). “On default Correlation : A Copula Function Approach,” *Journal of Fixed Income* 9(4), 43-54.

Luciano, Elisa (2005). “Calibrating Risk-Neutral Default Correlation,” Working paper no 12/2005, International Center for Economic Research, University of Turin.

Madan, D. and H. Unal (2000), “A two-factor hazard rate model for Pricing Risky Debt and the Term Structure of Credit Spreads,” *Journal of Financial and Quantitative Analysis*.

Mashal, R., and Naldi, M. (2002) “Pricing Multi-name Credit Derivatives: Heavy Tailed Hybrid Approach,” Working Paper, Columbia University

Mashal, R., and Zeevi, A. (2002) “Beyond Correlation: Extreme Co-movements Between Financial Assets,” Working Paper, Columbia University

Merton, R.C. (1974), “On the Pricing of Corporate Debt: The Risk Structure of Interest Rates*” Journal of Finance*, vol 29, no 2 (May): pp449-470.

Patton, A., (2006a) "Modeling Asymmetric Exchange Rate Dependence", *International Economic Review* **47**, pp. 527–556.

Patton, A., (2006b) "Estimation of Multivariate Models for Time Series of Possibly Different Lengths", *Journal of Applied Econometrics* **21**, pp. 147–173.

Rodriguez, J. (2006) "Measuring financial Contagion: A Copula Approach", *Journal of Empirical Finance"*

Schönbucher, P., and Schubert, D. (2001) "Copula-Dependent Default Risk in Intensity Models," Department of Statistics, Bonn University

Sklar, A. (1959) "Fonctions de repartition a n dimensions et leurs marges," Publication Inst. Stat. Univ. Paris, 8, 229-231.

Sun, J., Frees, E., and Rosenberg, M. (2006), "Heavy-Tailed Longitudinal Data Modeling Using Copulas", *Working Paper – Department of Actuarial Science, Risk management and Insurance, School of Business, University of Wisconsin*

Tang, D., and Hong Yan (2006), Liquidity, Liquidity Spillover, and Credit Default Swap Spreads., Financial Management Association Working papers.