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Abstract

Determining the profit maximizing input-output bundle of a firm requires data on prices. This paper shows how endogenously determined shadow prices can be used in place of actual prices to obtain the optimal input-output bundle where the firm.s shadow profit is maximized. This approach amounts to an application of the Weak Axiom of Profit Maximization (WAPM) formulated by Varian (1984) based on shadow prices rather than actual prices. At these prices the shadow profit of a firm is zero. Thus, the maximum profit that could have been attained at some other input-output bundle is a measure of the inefficiency of the firm. Because the benchmark input-output bundle is always an observed bundle from the data, it can be determined without having to solve any elaborate programming problem. An empirical application to U.S. airlines data illustrates the proposed methodology.

Journal of Economic Literature Classifi cation: C61, D21

Keywords: DEA, Shadow Prices, Non-radial Efficiency

SHADOW PROFIT MAXIMIZATION AND A GENERALIZED MEASURE OF INEFFICIENCY

In Data Envelopment Analysis (DEA) the efficiency of a firm is measured by comparing its observed input-output bundle with a reference point on the frontier or the graph of the technology. Standard measures of technical efficiency are either input- or output-oriented. In a radial input-oriented model one seeks the maximum equi-proportionate reduction in all the inputs of a firm that would be possible without violating the feasibility of its output bundle. In the output-oriented approach, on the other hand, the objective is to expand all outputs by the same factor without using any additional input. When the technology exhibits variable returns to scale, the two approaches yield different measures of efficiency. In the case of constant returns to scale, although the efficiency measures are identical, the reference bundles for comparison are different. In any empirical application one has to choose between an input-oriented and an output-oriented model.

When input and output prices are available, the reference bundle is one that maximizes profit and an inefficient firm attains full efficiency by simultaneously altering its inputs and outputs as needed. There are, indeed, several approaches in the DEA literature that allow changes in both inputs and outputs in order to obtain the efficient projection of an inefficient input-output bundle without the benefit of prices. Färe, Grosskopf, and Lovell (FGL) (1985) introduced the concept of graph efficiency and the corresponding hyperbolic distance function. It is measured by the maximum scalar by which all outputs can be expanded and all inputs can be contracted at the same time. Chambers, Chung, and Färe (CCF) (1996) introduced the directional distance function and the corresponding Nerlove-Luenberger measure of efficiency. Here one seeks to increase all outputs and reduce all inputs by the same proportion. In both these approaches, however, a single parameter determines how the output bundle is expanded and the input bundle is contracted. Also, as in the case of the oriented radial models, the graph hyperbolic and the directional distance functions alter all outputs and all inputs equi-proportionately. As a result, slacks may exist in individual inputs and/or outputs at the optimal projection.

The non-radial Russell efficiency measures defined by Färe and Lovell (1978) do not suffer from the problem of slacks. But like the radial measures, they also are either input- or output-oriented. Pastor, Ruiz, and Sirvent (1999) introduced a generalized Russell measure that allows individual outputs to increase and the individual inputs to decrease by different scale factors. As a result, there is no slack in any input or any output at the optimal projection.

In a related strand in the literature, Briec (1997, 1998), Frei and Harker (1998), and Briec and Leleu (2003) explore alternative ways to project an observed input-output bundle on to the frontier allowing inputs as well as outputs to change simultaneously.

In none of these approaches, however, does the reference bundle show an increase in any input or a decrease in any output compared to observed input-output bundle of the firm. Yet, when the firm maximizes profits the optimal bundle can show either an increase or a decrease in any input or output so long as the resulting profit is higher. Determining the profit-maximizing bundle of inputs and outputs requires data on the prices faced by the firm under evaluation. This paper shows how endogenously determined shadow prices of inputs and outputs of a firm can be used in place of actual prices to obtain the optimal projection of its observed input-output bundle where its shadow profit is maximized. It can be see from the dual of the relevant DEA model, the overall inefficiency of a firm is measured by the difference between the average proportionate change in outputs and the average proportionate change in inputs. As shown below, our approach amounts to an application of the Weak Axiom of Profit Maximization (WAPM) formulated by Varian (1984). But our evaluation is based on the shadow prices rather than actual prices. Moreover, the benchmark input-output bundle is always an observed bundle from the data and can be determined without having to solve any elaborate programming problem.

The rest of the paper unfolds as follows. In section 2 we present the basic nonparametric methodology proposed in this paper. Section 3 provides a geometric interpretation of the model using a small numerical example for the 1-output, 1-input case. An application with a data set due to Caves, Christensen, and Tretheway (1984) is provided in section 4. Section 5 concludes.

2. The Nonparametric Methodology

Consider a data set for N firms from an industry. Let y^j be the m-element output vector and x^j the corresponding n-element input vector of firm j $(j = 1,2,\ldots,N)$. Under the standard assumptions of convexity of the technology, free disposability of inputs and outputs, and variable returns to scale, an inner approximation to the unobserved production possibility set of this industry is

$$S = \{(x, y) : x \geq \sum_{1}^{N} \lambda_j x^j ; y \leq \sum_{1}^{N} \lambda_j y^j ; \sum_{1}^{N} \lambda_j = 1; \lambda_j \geq 0; (j = 1,2,\ldots,N)\}. \quad (1)$$

The efficient input-oriented projection of any observed input-output bundle (x^0, y^0) is $(\theta^0 x^0, y^0)$, where

$$\theta^0 = \min \theta : (\theta x^0, y^0) \in S. \quad (2)$$

The corresponding input-oriented measure of technical efficiency is

$$\tau^x{}_0 = \theta^0. \qquad (2a)$$

Similarly, the output-oriented efficient projection is $(x^0, \varphi^0 y^0)$, where

$$\varphi^0 = \max\ \varphi : (x^0, \varphi y^0) \in S. \qquad (3)$$

The output-oriented measure of technical efficiency is

$$\tau^y{}_0 = \tfrac{1}{\varphi^0}. \qquad (3a)$$

Note that employing either *(2)* or *(3)* involves a prior judgment on whether expanding outputs or contracting inputs is more important in a given context.

For FGL's graph efficiency measure, the efficient projection of (x^0, y^0) is $(\tfrac{1}{\delta^0} x^0, \delta^0 y^0)$ obtained from the hyperbolic distance function

$$\delta^0 = \max \delta : (\tfrac{1}{\delta} x^0, \delta y^0) \in S. \qquad (4)$$

As can be seen for the 1-input, 1-output case, the actual and the projected input-output bundles lie on a rectangular hyperbola. Hence, for an efficient projection, δ^0 must be greater than or equal to unity. Note that here inputs are reduced and outputs are increased simultaneously.

Another measure of graph efficiency based on simultaneous change in inputs and outputs is the Nerlove-Luenberger efficiency measure that is derived from CCF's directional distance function

$$\beta^0 = \max \beta : \left((1-\beta)x^0, (1+\beta)y^0\right) \in S. \qquad (5)$$

In both (4) and (5), however, a single parameter determines how both inputs and outputs change.

Now suppose that we had information on the output and input prices for the firm under review. Specifically, assume that p^0 and w^0 were the output and input price vectors, respectively. In that case, the optimal projection of the observed input output bundle would be (x_*^0, y_*^0) satisfying the inequality

$$p^{0\prime} y_*^0 - w^{0\prime} x_*^0 \geq p^{0\prime} y - w^{0\prime} x \forall (x, y) \in S. \qquad (6)$$

Define $\pi_*^0 \equiv p^{0\prime} y_*^0 - w^{0\prime} x_*^0$ and $\pi^0 \equiv p^{0\prime} y^0 - w^{0\prime} x^0$. A measure of the *inefficiency* of the firm is $\Delta^0 = \pi_*^0 - \pi^0$. Note that in order to get to the profit-efficient projection, the firm does not increase all of its outputs or decrease all of its inputs by the same proportion. In fact, it may increase or reduce individual inputs or outputs appropriately so long as the resulting bundle maximizes profit.

It is interesting to note that the maximum profit can be easily obtained as

$$\pi_*^0 = \max_j \{p^{0\prime} y^j - w^{0\prime} x^j\} (j = 1,2,..., N). \qquad (7)$$

Suppose that for a given data set

$$p^{0\prime} y^k - w^{0\prime} x^k \geq p^{0\prime} y^j - w^{0\prime} x^j \text{ for } j = 1,2,...,N. \qquad (7a)$$

Then, for any set of non-negative λ_js adding up to unity,

$$p^{0\prime} y^k - w^{0\prime} x^k \geq p^{0\prime}\left(\sum_1^N \lambda_j y^j\right) - w^{0\prime}\left(\sum_1^N \lambda_j x^j\right).$$

But, by assumption, for any $(x, y) \in S$, there exist some non-negative λ_js adding up to unity such that

$$y \leq \sum_1^N \lambda_j y^j \text{ and } x \geq \sum_1^N \lambda_j x^j .$$

Hence,

$$p^{0\prime} y^k - w^{0\prime} x^k \geq p^{0\prime} y^j - w^{0\prime} x^j \text{ for all } (x, y) \in S. \qquad (7b)$$

Varian's Weak Axiom of Profit Maximization (WAPM) argues that if the input-output bundle of a particular firm evaluated at the prices it faces yields a lower profit than what could be earned if it had chosen the observed input-output bundle of some other firm in the sample, then the firm under consideration could not be maximizing profit. An implication of *(7a-b)* above is that if the firm k does satisfy *WAPM* then it actually is maximizing profit over the production possibility set S. Further, *(7)* can also be expressed as

$$\begin{aligned} &\pi_*^0 = \min \pi \\ &s.t. \pi \geq p^{0\prime} y^j - w^{0\prime} x^j (j = 1,2,..., N). \end{aligned} \qquad (8)$$

Lacking the necessary price information we cannot take this approach. We may, however, use endogenously determined shadow prices to look for the input-output bundle that maximizes profit over the entire production possibility set S at these prices. Consider some output price vector u^0 and input price vector v^0 such that at these prices the input-output bundle (x^0, y^0) yields zero profit. That is

$$u^{0\prime} y^0 - v^{0\prime} x^0 = 0. \qquad (9)$$

We now look for the optimal bundle (x^*, y^*) such that

$$P^* \equiv u^{0\prime} y^* - v^{0\prime} x^* \geq u^{0\prime} y - v^{0\prime} x \forall (x, y) \in S. \qquad (10)$$

The maximum profit P^* provides a measure of the overall inefficiency of the firm producing y^0 from x^0. One problem that remains, however, is that one can change the shadow prices of inputs and output by the same proportion and P^* also changes by the same proportion without violating the requirement of zero profit at the observed input-output bundle. As a result, the maximum shadow profit P^* would be unbounded. One way to overcome this problem is to normalize the shadow prices separately[1] so that

$$u^{0\prime} y^0 = v^{0\prime} x^0 = 1. \qquad (11)$$

Following *(8)*, the shadow profit maximization for the firm under review can be specified as

$$\begin{aligned} &\min \; P \\ \text{s.t.} \quad & u^{0\prime} y^j - v^{0\prime} x^j \le P; (j = 1,2,.., N) \\ & u^{0\prime} y^0 = 1; \\ & v^{0\prime} x^0 = 1; \\ & u^0 \ge 0; v^0 \ge 0; \\ & P \text{ unrestricted.} \end{aligned} \qquad (12)$$

The dual of this linear programming problem is

$$\begin{aligned} &\max \; \varphi - \theta \\ \text{s.t.} \quad & \sum_1^N \lambda_j y^j \ge \varphi y^0; \\ & \sum_1^N \lambda_j x^j \le \theta x^0; \\ & \sum_1^N \lambda_j = 1; \\ & \lambda_j \ge 0; \phi, \; \theta \text{ unrestricted.} \end{aligned} \qquad (13)$$

Note that (13) combines the features of both the output- and the input-oriented radial models for a variable returns to scale technology. In fact, setting θ equal to unity, we get the measure of the firm's output-oriented inefficiency, (ϕ- *1*). Similarly, when ϕ is preset at unity, the model yields

[1] In the standard DEA models in the multiplier form, shadow prices of either the inputs or the outputs (but not both) are normalized so that the shadow cost or the shadow revenue of the firm under evaluation equals unity.

the firm's input-oriented inefficiency, ($1-\theta$). Thus, the optimal value of the objective function in *(13)* is the sum of its output- and input-oriented technical inefficiencies.

One problem with the normalization of the shadow prices of output and inputs specified in (11) above is that when multiple outputs or multiple inputs are involved, individual shadow prices may be zero at the optimal solution of *(12)* leading to positive output or input slacks at the optimal projection of (x^0, y^0) obtained from *(13)*. One possible way to resolve this problem is to replace the restrictions in *(11)* by

$$u_r^0 y_r^0 = \tfrac{1}{m} \text{ for each output } r\ (r = 1,2,...,m) \qquad (14a)$$

and

$$v_i^0 x_i^0 = \tfrac{1}{n} \text{ for each input } i\ (i = 1,2,...,n) \qquad (14b)$$

What (14a) implies is that outputs are so priced that each shares equally in the shadow value of the output. Similarly, (14b) ensures that the share of each input is equal in the total shadow cost. For the 1-output, 1-input case, of course, (14a-b) and (11) are equivalent. The revised form of the problem *(12)* is

$$\begin{aligned}
&\min \ P \\
&\sum_1^m u_r^0 y_r^j - 1\sum_i^n v_i^0 x_i^j \le P; (j = 1,2,..,N) \\
\text{s.t.}\quad & u_r^0 y_r^0 = \tfrac{1}{m}; (r = 1,2,...,m); \qquad\qquad (15)\\
& v_i^0 x_i^0 = \tfrac{1}{n}; (i = 1,2,...,n); \\
& u_r^0 \ge 0; v_i^0 \ge 0; (r = 1,2,...,m; i = 1,2,...,n) \\
& P \text{ unrestricted.}
\end{aligned}$$

Although the normalization in (14a-b) appears to be rather *ad hoc*, its intuitive meaning can be see from the following dual of this problem in (15):

$$\begin{aligned}
\max\ & \tfrac{1}{m}\sum_1^m \varphi_r - \tfrac{1}{n}\sum_1^n \theta_i \\
\text{s.t.}\quad & \sum_1^N \lambda_j y_r^j \ge \varphi_r y_r^0; (r = 1,2,...,m); \qquad\qquad (16)\\
& \sum_1^N \lambda_j x_i^j \le \theta_i x_i^0; (i = 1,2,...,n); \\
& \sum_1^N \lambda_j = 1; \\
& \lambda_j \ge 0;\ \phi_r,\ \theta_i\ (r=1,2,...,m;\ i=1,2,...,n) \text{ unrestricted.}
\end{aligned}$$

Clearly *(16)* combines the features of both output- and input-oriented non-radial models of measuring Russell efficiency. An implication of the way we restrict the shadow prices of outputs and inputs in *(14a-b)* is that each output specific scale factors (φ_j) is weighted equally in the objective function. Similarly, each input scale factor (θ_i) is also given the same weight. Problem *(16)* above has an apparent similarity with the arbitrarily weighted non-radial models considered by Zhu (1996) and Athanassopoulos, Lambroukos, and Seiford (1999). There are fundamental differences, however. First, the model introduced here allows inputs and outputs to increase or decrease so long as the average increase in output exceeds the average increase in inputs. More importantly, however, while the weights in the other models are arbitrarily specified offering no economic intuition, we start with the objective of shadow profit maximization and arrive at the dual problem shown in *(16)*.

Pastor, Ruiz, and Sirvent (1999) introduced an extended Russell measure of efficiency defined as

$$\Gamma = \frac{\frac{1}{n}\sum_{1}^{n}\theta_i}{\frac{1}{m}\sum_{1}^{m}\varphi_r}. \qquad (17)$$

As shown in Ray (2004), a linear approximation to Γ in *(17)* at all φ_r and θ_i equal to unity is , $\frac{1}{n}\sum_{1}^{n}\theta_i - \frac{1}{m}\sum_{1}^{m}\varphi_r$ .This is the objective function in *(16)*.

A remarkable feature of this model is its computational simplicity. The optimal solution can be obtained through a simple enumeration process without any need to solve any elaborate programming problem. As argued above, the optimal value of the objective function in *(15)* is attained at some observed input-output bundle (x^k, y^k) in the sample. Further, by virtue of the restrictions on the shadow prices,

$$u_r^0 = \frac{1}{my_r^0} \text{ for each output } r \text{ and} \qquad (17a)$$

$$v_i^0 = \frac{1}{nx_i^0} \text{ for each input } i. \qquad (17b)$$

Thus, $$\sum_{1}^{m} u_r^0 y_r^j - 1\sum_{i}^{n} v_i^0 x_i^j = \frac{1}{m}\sum_{1}^{m}\frac{y_r^j}{y_r^0} - \frac{1}{n}\sum_{1}^{n}\frac{x_i^j}{x_i^0}. \qquad (18)$$

Define $\varphi_r^j = \frac{y_r^j}{y_r^0}$ *(r=1,2,…,m)* and $\theta_i^j = \frac{x_i^j}{x_i^0}$ *(i=1,2,…,n)*. Then,

$$\sum_{1}^{m} u_r^0 y_r^j - \sum_{i}^{n} v_i^0 x_i^j = \tfrac{1}{m}\sum_{1}^{m} \varphi_r^j - \tfrac{1}{n}\sum_{1}^{n} \theta_i^j . \qquad (19)$$

Clearly, all we need to do is to evaluate the expression on the right hand side of *(18)* at the observed data points to obtain the optimal value of the objective functions in *(15)* and *(16).* The corresponding input-output bundle represents the relevant efficient projection of the bundle (x^0, y^0) on to the graph of the technology.

3. A Geometric Interpretation: The 1-input, 1-output Case

Consider a sample of firms from an industry producing a single output, y, from a single input, x. Evaluated at the (actual or imputed) input and output prices (w^0, p^0) the profit from any input-output bundle (x, y) is

$$\Pi = p^0 y - w^0 x.$$

Thus, all input-output bundles (x, y) satisfying

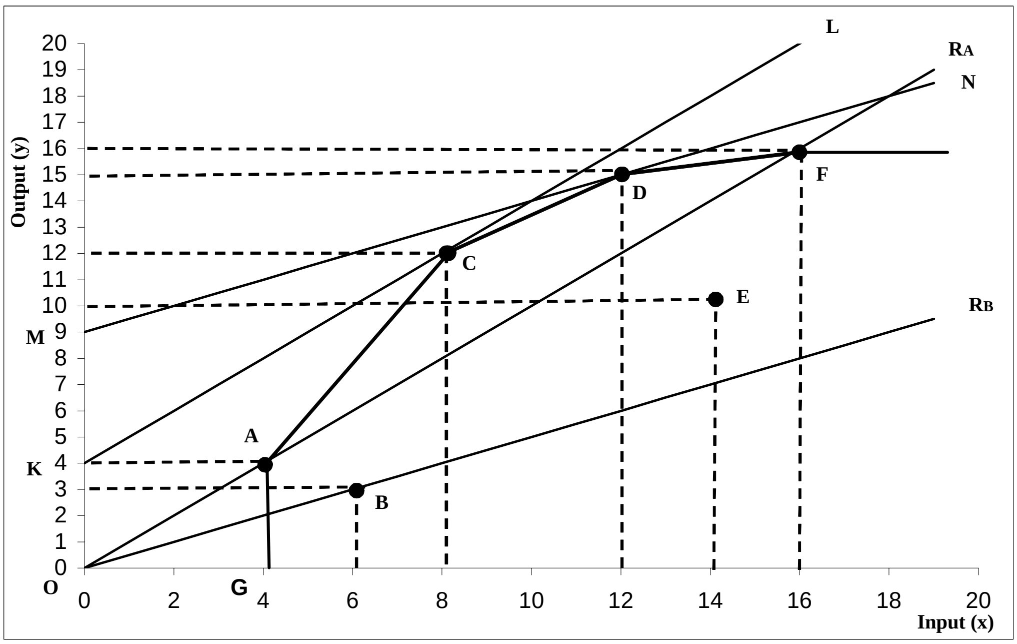
$$y = \frac{\Pi^0}{p^0} + \frac{w^0}{p^0} x$$

lead to the same profit Π^0 and, therefore, lie on the same iso-profit line. A firm maximizes profit at prices (w^0, p^0) by selecting an input-output bundle within its production possibility set that lies on the highest iso-profit line with slope $\frac{w^0}{p^0}$. Of course, when market prices are not available, one has to use the shadow prices for profit maximization as described above. This may be illustrated with the following example.

Figure 1 shows the input-output quantities of 6 firms represented by the points *A* through *F* and the broken line *GACDF*-extension is the frontier or graph of the technology empirically constructed from these observed input-output bundles. Points *B* and *E* lie below the frontier and are thus technically inefficient. All the other points *(A, C, D,* and *F)* are technically efficient by the conventional measure. But that is true only because we do not allow any increase in the input even if that results in an increase in the output. In the present case, however, we allow the input and output to change in any direction so long as then shadow profit of the firm increases. Consider, for example, the input-output bundle *A*. Shadow prices $(p^0=¼, w^0=¼)$ the shadow profit for this firm is 0 and the iso-profit line is the ray OR_A (with slope equal to unity) through the point *A*. The highest iso-profit line parallel to OR_A that can be attained at any point in the production possibility set is the line *MN* through the point *C.* Next we consider the point *B*. For this input-output bundle the zero profit line is the ray OR_B with slope equal to ½. This time the highest attainable iso-profit line is *KL* through the point *D.*

Table 1 shows the input-output quantities of the different firms used in this example along with their respective shadow-profit efficient projections on to the frontier. Several points may be underscored in this context. The only point with no overall inefficiency (i.e., zero shadow profit) is the point *C*.. The iso-profit line through *C* is also a tangent to the production frontier at that point. It is the *most productive scale size (MPSS)* characterized by locally constant returns to

Figure 1: Shadow Profit Maximization



scale. As one would expect, the tangent hyperplane at this point has a zero intercept (see Banker, Charnes, and Cooper (1984)). The overall efficient projection of the point *B* is *D* and not *C.* Being

` **Table 1. Data and Summary Results for Hypothetical Firms**

Firm	Input (x)	Output (y)	Reference	Φ^*	Θ^*	Π^*
A	4	4	C	2.4	2	0.4
B	6	3	D	5	2	3
C	8	12	C	1	1	0
D	12	15	C	0.8	0.67	0.13
E	14	10	D	1.5	0.8571	0.6429
F	16	16	C	0.75	0.5	0.25

the point of maximum average productivity, the point *C* would result in the maximum value of the *ratio of* φ and θ. But the objective function of *(16)* in this 1-input, 1-output example is the *difference between* φ and θ, which is maximized at the point *D.* If instead, *B* had been projected on to *C*, the resulting values of φ and θ would be 2 and 1.33, respectively, and the objective function value would be 0.67. Similar reasoning applies to point *E* as well. What is interesting is that the point *D* itself is not shadow-profit efficient when evaluated at its *own* shadow prices. As can be seen from Table 1, points *A* and *F* also are not shadow profit efficient even though they are Pareto efficient without any slack in either input or output.

4. A Study of U.S. Airlines

In this section we present an application of the procedure proposed in this paper to a data set for a number of U.S. airlines from the year 1984 analyzed before by Kumbhakar (1990) and Ray and Mukherjee (1996). A single output, five-input technology is considered at the DEA stage. The data form a subset of a larger data set constructed by Caves, Christensen, and Tretheway (1984). The output is a quantity index (QYI) constructed from the numbers revenue passenger miles flown, ton-miles of cargo flown, and ton-miles of mail flown. The inputs are quantity indexes of labor (QLI), fuel (QFI), materials (QMI), flight equipment (QFLI), and ground equipment (QGRI). Corresponding price indexes for the output and the individual input categories were constructed from the revenue and expenditure information for the individual airlines. Detailed accounts of the data were constructed are provided in Tretheway and Windle (1983) and Caves et al (1984). The input and output quantity and price data for the individual airlines are reported in Tables 2a and 2b, respectively. In this particular case, we *do have* the information on the prices of the inputs and the output for the individual firms and can directly find out the profit maximizing input-output bundle for each airline in our sample. Nevertheless, we follow the shadow profit maximization approach proposed above to obtain the corresponding efficient projection of the input-output bundles of the individual firms in our sample. These are then compared with the corresponding bundles that maximize the actual profits of these firms.

We first solved the profit maximization problem for each airline using the actual prices reported in Table 2b. Somewhat surprisingly, none of the airlines in the sample was found to be maximizing profit at the prices it faced. As noted before, the maximum profit over the free disposal convex hull of the data points is attained at the observed input-output bundle of one of the firms in the sample. In the present example, as shown in Table 3, for each airline in our sample, one out of three firms – North West (NW), People's Express (PE), and United (UN) – proved to be the profit maximizing reference point. Of these, the input-output bundle of North

West (NW) would maximize the profit for 13 of the 17 price vectors of the individual firms. United (UN) would maximize profit at the prices of three airlines- Piedmont (PI), Republic Hughes Air (RHA) and US Air (USA). People's Express (PE) maximizes profit at the prices of North West (NW). Interestingly, none maximizes profit at its own prices.

Table 4 reports the actual revenue, cost, and profit along with the maximum profit that an airline could earn for each of the airlines separately. We note in passing that the actual profit is negative for 15 out of the 17 firms in the sample. The maximum profit (shown in the column (Max-profit), however, would be positive for all but 2 airlines (North West (NW) and Pam American (PA)). The amount of profit lost due to inefficiency is measured by the difference between the actual and the maximum profit and is shown as (Lost Profit). We deflate this amount of lost profit by the actual cost of each airline to obtain the lost return on outlay shown in the last column of Table 4. Thus, even though North West shows a large amount of loss and would continue to incur losses even if selected the most profitable bundle at the applicable input-output prices, it actually is loss due to inefficiency is only 14 cents per dollar of outlay. On the other hand, although Piedmont (PI) and South West (SW) are actually earning positive profit, compared to what they could earn by selecting the profit maximizing bundles, the amounts of return per dollar of outlay lost due to inefficiency are \$2.59 for Piedmont (PI) and \$1.84 for South West (SW).

Results from the shadow profit maximization problem are reported in table 5. This time North West (NW) serves as the reference firm for 12 of the 17 airlines while United (UN) is the reference point for the remaining 5. In this case, North West (NW) is shadow profit efficient but United (UN) is not. The columns ϕ^* and θ^*_1 through θ^*_5 are the output and corresponding input scale (change) factors that would project the actual output and input bundle of a firm on to its shadow profit efficient reference point. The column P^* shows the shadow profit that is lost by a firm due to inappropriate choice of inputs and output. It should be noted in this context that by construction, the shadow cost of the actual input bundle employed is unity and the shadow profit from the observed input-output bundle of a firm is 0. This, P^* itself is a measure of the shadow profit lost per unit of the shadow cost of any airline. By this criterion, Ozark (OZ) is the most inefficient followed by Pacific South (PS). The next group of inefficient airlines consists of Piedmont (PI), Frontier (FR), and US Air (USA) in that order. Two other airlines with considerable inefficiency are South West (SW) and Republic Hughes Air (RHA). At the other end, North West (NW) shows no inefficiency. Among the others, United (UN), Pan American (PA), and American (AM) have the lowest levels of inefficiency, in that order.

For a head to head comparison of the (in)efficiency measures of the individual airlines we have pulled together the relevant columns from Tables 3, 4, and 5 in Table 6. In 13 out of the 17 cases, the same in airline was found to be the reference point on the frontier for both actual profit maximization and shadow profit maximization. In 11 of these cases, the reference firm was North West (NW). In the remaining 2, it was United (UN). There is thus a broad agreement between the findings from the alternative approaches. In fact, comparison of the lost (actual and shadow) returns on outlay further demonstrates the similarity in the findings. In fact, in no case are the two measures of inefficiency grossly inconsistent. Even in the case of North West (NW) which is found to be efficient by the shadow (but not by the actual) profit measure, the actual profit lost is a modest 14 cents per dollar of outlay. At the other end, the two most inefficient airlines Ozark (OZ) and Pacific South (PS) show comparable measures of lost returns (3.05 for the actual and 4.07 for the shadow profits lost) from broth criteria. In fact, the simple correlation between the two is as high as 0.928 and the coefficient of rank correlation is even higher at 0.985.

We conclude the section with a caveat. When reliable data on output and input prices are available, one should apply the criterion of actual profit maximization in order to determine the efficient projection of the actual input-output bundle of a firm on to the frontier. When prices are not known, the present approach of shadow profit maximization offers a reasonable alternative. In this application, we had an opportunity to compare the results from the alternative approaches because price data were actually available. While it is, in deed, reassuring that the results agree quite closely, there is no basis for generalization of this agreement.

5.Conclusion

This paper offers a measure of overall inefficiency of a firm based on the criterion of shadow profit maximization. Unlike in the existing radial or non-radial DEA models in the literature, in this approach inputs and outputs are allowed to either increase or decrease as appropriate for optimization. In fact, an input-output bundle that is Pareto efficient without any slack in either inputs or outputs may be evaluated as inefficient by the shadow profit criterion.

Table 2a. Input-Output Quantity Indexes: Selected US Airlines for the year 1984

Airline	Output QYI	Labor QLI	Fuel QFI	Material QMI	Flight Capital QFLI	Ground Capital QGRI
American (AM)	1.9365	1.2637	1.3036	2.209	1.5932	2.1644
Continental (COT)	0.5455	0.3078	0.3906	0.6076	0.4916	0.4303
Delta (DE)	1.3897	1.2116	1.123	1.7274	1.2238	1.7945
Eastern (EA)	1.5157	1.2891	1.1765	1.9574	1.6191	1.444
Frontier (FR)	0.2133	0.1723	0.1524	0.3031	0.2069	0.1961
Northwest (NW)	1.2485	0.4998	0.7906	1.224	1.2125	0.6194
Ozark (OZ)	0.1387	0.1347	0.1236	0.226	0.1826	0.1266
PanAmerican (PA)	1.5685	0.9195	0.9764	1.7506	1.4026	1.2589
Peoples(PE)	0.3277	0.1438	0.2154	0.3438	0.2924	0.2064
Piedmont(PI)	0.304	0.325	0.3004	0.4835	0.3228	0.2591
PacificSouth (PS)	0.155	0.124	0.1168	0.2176	0.1986	0.2274
Republic-HughesAir (RHA)	0.4332	0.4685	0.4369	0.6832	0.6375	0.3107
SouthWest (SW)	0.1997	0.131	0.1806	0.1987	0.1796	0.1587
TWA (TWA)	1.5134	0.8959	0.9349	1.7681	1.3134	1.5457
United (UN)	2.4424	1.484	1.5965	2.4479	2.1049	2.7084
USAir (USA)	0.4214	0.4134	0.374	0.6453	0.5468	0.4883
Western (WE)	0.4933	0.3509	0.3547	0.5251	0.4229	0.3141

Table 2b. Output and Input Price Indexes: Selected US Airlines for 1984

Airline	Output PY	Labor PL	Fuel PF	Materials PM	Flight Capital PFL	Ground Capital PGR
American(AM)	2419225	1252053	823391	780312.4	539857.9	86158.31
Continental(COT)	2098122	654243	843951.3	780097.8	526746.3	86153.19
Delta(DE)	3173110	1281531	821327.6	780109.4	520848.8	86154.19
Eastern(EA)	2781221	1060688	814236.9	780312.9	529472.4	86154.06
Frontier(FR)	2698588	1083174	844862.8	775926.3	532153.9	86159.31
Northwest(NW)	1952129	1117723	860378.3	780072.3	569012.9	86149.94
Ozark(OZ)	3302877	1137825	819571.5	776092.3	542749.3	86178.56
PanAmerican(PA)	2072299	1109509	867325.3	780098.3	570737.3	86154.19
Peoples(PE)	1782716	610426.1	825227.8	775971.6	534526.6	86165.94
Piedmont(PI)	3697954	938470.6	819421.1	775984	504924.9	86157.63
PacificSouth(PS)	3202013	1204671	834531.1	776089.4	537267.1	86145.75
Republic-HughesAir(RHA)	3482475	984645.1	828228.9	776019.8	538885.8	86161
SouthWest(SW)	2675433	916764.9	808326.4	775898.5	530197.8	86165.5
TWA	2212803	1259548	835999.5	780323	539058.2	86155.25
United(UN)	2364884	1246253	831375.1	780317.9	541706.6	86155.88
USAir(USA)	3781303	1285977	831339.5	775920.2	537154.9	86151.44
Western(WE)	2290557	1084819	844183.7	780061.1	534464.4	86144.31

Table 3. Profit Efficient Projections of Sample Firms

Airline	Reference Airline		
	NW	PE	UN
American (AM)	1	0	0
Continental (COT)	1	0	0
Delta (DE)	1	0	0
Eastern (EA)	1	0	0
Frontier (FR)	1	0	0
Northwest (NW)	0	1	0
Ozark (OZ)	1	0	0
PanAmerican (PA)	1	0	0
Peoples(PE)	1	0	0
Piedmont(PI)	0	0	1
PacificSouth (PS)	1	0	0
Republic-HughesAir (RHA)	0	0	1
SouthWest (SW)	1	0	0
TWA (TWA)	1	0	0
United (UN)	1	0	0
USAir (USA)	0	0	1
Western (WE)	1	0	0

Note: an entry 1 in any column identifies the relevant airline as the reference firm for the airline shown in the corresponding row.

Table 4. Actual and Optimal Profits and Related Data for Individual Airlines

Airline	Revenue	Cost	Profit	Max-Profit	Lost Profit	Lost Rate of Return
American (AM)	4684829	5425885	-741055	370485.3	1111541	0.204859
Continental (COT)	1144526	1301031	-156505	445493.1	601998.5	0.462709
Delta (DE)	4409671	4614633	-204962	1312839	1517801	0.32891
Eastern (EA)	4215497	4834342	-618846	991997.9	1610844	0.333208
Frontier (FR)	575608.8	677569.7	-101961	851018.1	952979	1.406466
Northwest (NW)	2437233	2936951	-499718	-86799.1	412918.9	0.140594
Ozark (OZ)	458109	539977.1	-81868.1	1564978	1646846	3.049844
Pan American (PA)	3250401	4141666	-891265	-14498.6	876766.1	0.211694
Peoples(PE)	584196	706392.6	-122197	90560.75	212757.3	0.301188
Piedmont(PI)	1124178	1111658	12519.54	2889013	2876493	2.587569
Pacific South (PS)	496312	542020.3	-45708.3	1383624	1429332	2.637045
Republic-Hughes Air (RHA)	1508608	1723646	-215038	2193609	2408646	1.397414
South West (SW)	534284	529149	5134.992	978680.9	973545.9	1.839833
TWA (TWA)	3348856	4130863	-782007	100000.3	882007.5	0.213517
United (UN)	5775993	6460453	-684460	300039.4	984499.5	0.152389
USAir (USA)	1593441	1679029	-85588.1	2437779	2523367	1.502873
Western (WE)	1129932	1342788	-212856	335861.8	548718	0.408641

Table 5. Shadow Price Efficient Projections and Overall Inefficiency of Sample Airlines

Airline	Reference Airlines								
	NW	UN	ϕ^*	θ^*_L	θ^*_F	θ^*_M	θ^*_{FL}	θ^*_{GR}	P^{**}
American (AM)	1	0	0.64472	0.606474	0.286176	0.395505	0.761047	0.554097	0.12406
Continental (COT)	1	0	2.288726	2.024066	1.439461	1.623782	2.466436	2.014483	0.37508
Delta (DE)	0	1	1.757502	1.421638	1.509278	1.224827	1.719971	1.417101	0.298939
Eastern (EA)	1	0	0.823712	0.671993	0.428947	0.387712	0.748873	0.625319	0.251143
Frontier (FR)	1	0	5.853258	5.187664	3.158593	2.900754	5.860319	4.038271	1.624138
Northwest (NW)	1	0	1	1	1	1	1	1	0
Ozark (OZ)	0	1	17.60923	12.91667	21.39336	11.01707	11.52738	10.83142	4.072048
Pan American (PA)	1	0	0.795983	0.809709	0.492017	0.543556	0.864466	0.699189	0.114196
Peoples(PE)	1	0	3.809887	3.670381	3.000969	3.475661	4.146717	3.560209	0.2391
Piedmont(PI)	0	1	8.034211	5.314581	10.45311	4.566154	6.520756	5.062875	1.650716
Pacific South (PS)	0	1	15.75742	13.66866	11.91029	11.96774	10.59869	11.24954	3.878434
Republic-Hughes Air (RHA)	1	0	2.882041	1.809567	1.993563	1.066809	1.901961	1.791569	1.169347
South West (SW)	1	0	6.251878	4.37763	3.902962	3.815267	6.751114	6.16004	1.250475
TWA (TWA)	1	0	0.824964	0.845652	0.400725	0.557875	0.923176	0.692269	0.141024
United (UN)	1	0	0.511178	0.495208	0.228696	0.336792	0.576037	0.50002	0.083827
USAir (USA)	0	1	5.795918	4.268717	5.54659	3.589744	3.849488	3.793429	1.586325
Western (WE)	1	0	2.530914	2.228926	1.971983	1.424337	2.867108	2.330985	0.366246

Note: See under Table 3.

Table 6. Comparison of Inefficiencies of Airlines based on Actual and Shadow Profit Maximization

Airline	At Acatual Prices Reference Airlines				At Shadow Prices Reference Airlines		
	NW	PE	UN	Lost Rate of Return (Actual)	NW	UN	Lost Shadow Return
American (AM)	1	0	0	0.204859	1	0	0.12406
Continental (COT)	1	0	0	0.462709	1	0	0.37508
Delta (DE)	1	0	0	0.32891	0	1	0.298939
Eastern (EA)	1	0	0	0.333208	1	0	0.251143
Frontier (FR)	1	0	0	1.406466	1	0	1.624138
Northwest (NW)	0	1	0	0.140594	1	0	0
Ozark (OZ)	1	0	0	3.049844	0	1	4.072048
PanAmerican (PA)	1	0	0	0.211694	1	0	0.114196
Peoples(PE)	1	0	0	0.301188	1	0	0.2391
Piedmont(PI)	0	0	1	2.587569	0	1	1.650716
PacificSouth (PS)	1	0	0	2.637045	0	1	3.878434
Republic-HughesAir (RHA)	0	0	1	1.397414	1	0	1.169347
SouthWest (SW)	1	0	0	1.839833	1	0	1.250475
TWA (TWA)	1	0	0	0.213517	1	0	0.141024
United (UN)	1	0	0	0.152389	1	0	0.083827
USAir (USA)	0	0	1	1.502873	0	1	1.586325
Western (WE)	1	0	0	0.408641	1	0	0.366246

Note: See under Table 3.

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