October 2006

Interbank Markets under Currency Boards

Marius Jurgilas
University of Connecticut

Interbank Markets under Currency Boards

Marius Jurgilas
University of Connecticut

Abstract

This paper analyzes interbank markets under currency boards. Under such an environment, problematic endogeneity issues common to other monetary regimes do not arise. Using daily data from the interbank markets in Bulgaria and Lithuania we show, that contrary to the existing literature, overnight interest rates tend to decrease towards the end of the reserve holding period. Empirical results are supported by a finite horizon heterogeneous agents model showing that interest rates tend to decrease in the case of excess aggregate reserves in the banking system. Results contrast with Quir'os and Mendiz'abal (2006) who find that interest rates should be increasing regardless of the outstanding aggregate liquidity in the market. We also show that responsiveness of banks to interest rate changes diminishes as the end of reserve holding period approaches. Under certain circumstances this could lead to multiple equilibria with increasing or decreasing interest rates.

1 Introduction

This paper investigates interbank money markets under currency boards. The empirical evidence provided indicates that overnight interest rates are decreasing throughout the reserve holding period. A theoretical model links this to excess reserves in the market.

According to the market efficiency hypothesis, interbank market interest rate fluctuations within the bank reserve holding period can not be systemic. The empirical literature argues that the reserve requirement (besides other factors, such as holding operational balances or transaction costs) commercial banks face plays a major role in explaining interbank interest rate determination (Hamilton (1996), Prati et al. (2003), Quirós and Mendizábal (2006)). In most cases, banks have the flexibility of meeting this requirement by maintaining an average required balance at the central bank. Thus, if interest rates do fluctuate in some predictable fashion, banks would hold high required reserve balances in periods with low interest rates and lower balances when the interest rates are high. This would eliminate any observed systemic fluctuations in the interbank interest rate and make it a martingale.

The empirical evidence does not support the market efficiency story. Starting with Hamilton (1996), there has been a growing literature showing that interbank interest rates *do* fluctuate in a predictable fashion. As the outstanding liquidity in the banking system increases, interest rates tend to be lower, and vice versa. Literature calls a relationship between the outstanding liquidity and the interest rates the "liquidity effect". Thornton (2001) challenges Hamilton (1996) results and shows that US interbank market data does note support a liquidity effect when one accounts for several large reserve fluctuations. Carpenter and Demiralp (2006), on the other hand, provide empirical evidence of a strong liquidity effect towards the end of the reserve holding period. Thornton (2006) points out that the liquidity effect reported by Carpenter and Demiralp (2006) is valid only for the period after, but not before, August 1995, and that the estimated liquidity effect is very small and of questionable economic importance. The nonexistence of a liquidity effect is justified by the central bank targeting interest rates through *open mouth operations* using terminology of Guthrie and Wright (2000). They argue that public announcements by the central bank, and clearly communicated changes in policy are the actual movers of the market interest rates.

Theoretical literature also provides equivocal explanations. For example, in a recent study of the Eurozone interbank markets, Ewerhart et al. (2004) develops a theoretical model that allows for the possibility of a liquidity effect *reversal*. In a market with banks behaving strategically, one might observe situations in which the banking system's outstanding reserves and interest rates are rising or falling at the same time. Thus, it appears that there is no

consensus in the literature as to the existence or even the sign of a liquidity effect.

Empirical studies on the liquidity effect must account for the fact that changes in the interest rate induce central banks to respond by altering the outstanding liquidity in the market, to bring the effective interest rate into the target zone. Such central bank behavior introduces a simultaneity problem, and requires the use of instrumental variable estimation. This problem would not arise if the central bank was not an active player in the interbank market. Actually, Prati et al. (2003) among others, reports that although violations of the martingale hypothesis are observed across countries, patterns of violation are heterogeneous. Namely, in some countries interest rates tend to shoot up towards the end of the reserve holding period indicating market tightness, while in other countries no market tightness is observed.

The analysis of the US interbank market by Bartolini et al. (2001) shows that settlement day federal funds rates exceed non-settlement day rates by 18 basis points (in the period of 1994-1998). In general, however, settlement day tightness is not a robust finding across countries. Hartmann et al. (2001) shows that peaks in the short term interest rates around the end of reserve holding period occur during periods of deficient aggregate liquidity. Thus, it may be argued, that we could not observe such peaks if central banks intervened in the interbank market to fine-tune aggregate liquidity around the end of reserve maintenance period.

In this paper we take a different approach than past studies because we examine the interbank market under a currency board. A currency board is an attractive setup to examine interbank market interest rate fluctuations. Under a currency board, a central bank cannot intervene in the interbank market, and thus there is no endogeneity problem. Banks should not expect any support from the central bank during periods of tight liquidity, and therefore, this eliminates the central bank responsiveness problem as discussed by Prati et al. (2003).

A currency board is a particular form of a fixed exchange rate regime under which all liabilities of a central bank (including money in circulation) have to be fully backed by foreign reserves. Thus, the central bank surrenders active monetary policy and control of the interbank market interest rates.

Using daily data from the interbank markets in Bulgaria and Lithuania, we show that, contrary to the existing literature, overnight interest rates tend to decrease towards the end of the reserve holding period. These empirical results are supported by a theoretical model showing that interest rates tend to decrease in the case of excess aggregate reserves in the banking system. This contrasts with the results of Quirós and Mendizábal (2006) who find that interest rates should be increasing regardless of the outstanding aggregate liquidity

in the market. We also show that under certain circumstances this could lead to multiple equilibria with increasing or decreasing interest rates.

The rest of the paper is structured as follows. Section 2 provides an overview of the institutional structure of the interbank market under a currency board. Sections 3 and 4 feature empirical investigations of two currency board countries, Lithuania and Bulgaria. Section 5 develops a general equilibrium heterogeneous agents model of optimal reserve management and provides simulation results. Section 6 concludes.

2 Institutional framework

The following analysis is based on examining two currency board countries - Lithuania and Bulgaria. As is discussed, both countries have very similar institutional frameworks and the same reference currency (euro).

2.1 Interbank market

An interbank market under a currency board differs from an interbank market with an independent central bank in many important ways. Most importantly, a currency board arrangement does not allow the central bank to engage in active intervention in the reserves market. Therefore, the only policy instrument under the currency board is the required reserves policy. In some currency board countries, central banks still have a possibility to intervene in the interbank market, but only in some limited circumstances.[1] However, central banks under a currency board make explicitly clear that such cases are undesirable and banks should avoid them.

Another important distinction of a currency board is that all the liabilities of the central bank have to be fully backed by foreign reserves. The relevance of this 100% backing for the interbank market is that the central bank has to exchange the base currency on demand. Therefore, domestic money market interest rates fluctuate around the base currency money market interest rate. How wide are these fluctuations?

Under usual circumstances, independent central banks (e.g. the European Central Bank) limit the fluctuations of the money market interest rates by introducing standing deposit and lending facilities. These facilities provide commercial banks with an opportunity to

---

[1]According to the Law on Bank of Lithuania (2004), the Bank of Lithuania has a legal possibility to engage in repurchase agreements and provide secured liquidity loans. Such a loan was taken only once (March 23, 2001) over a 10 year period. Similarly, according to the Law on Bulgarian National Bank (2005a), the Bulgarian National Bank may not extend credits to banks in general, but may provide secured emergency liquidity loans.

make unsecured overnight deposits at the deposit facility rate (the floor rate) and unsecured overnight loans at the lending facility rate (the ceiling rate). These interest rates bound market interest rate fluctuations.

There are no such standing facilities available under a currency board. If we were to ignore the base currency interest rate risk, however, commercial bank's free exchange of the base currency at the central bank may be viewed as a standing facility under a currency board. Whenever there is a shortage of domestic currency liquidity, commercial banks can borrow in the base currency money market abroad and utilize the foreign exchange window of the central bank to gain liquidity in domestic currency. Similarly, in the case of excess liquidity in the domestic currency, banks can exchange funds into base currency and deposit funds abroad. Domestic market interest rate deviations from the base currency interest rates are therefore bounded by the transaction costs of utilizing the foreign exchange window of the central bank.

There is a fee to deter commercial banks from overusing the foreign exchange window and to cover the costs of the central bank. However, even without a fee, banks incur costs due to the possible lag in settling of these transactions. For example, the Bank of Lithuania may buy the base currency (euro) with same day or T+2 settlement, but sells the base currency with settlement at T+2 only. Up until March 2004, a fee of 0.025-0.125% of the transaction value was charged. Currently, a minuscule fixed fee of 50 Litas per transaction is charged. As Figure 1(a) shows, the decrease in the transaction costs of utilizing the foreign exchange window decreased the variability of overnight interest rates. In Bulgaria, according to the Law on Bulgarian National Bank (2005a), up until June 2004, a fee of 0.5% was charged. Petrov (2000) reports that the Bulgarian National Bank had been buying foreign currency with the settlement at T+3 until June 2003, when currency could be exchanged with the same day settlement. The decrease in the foreign exchange transaction costs by the Bulgarian National Bank in June 2004 had the same effect on the Bulgarian interbank market as described for Lithuania (see Figure 1(b)).

2.2 Required reserves

As mentioned previously, under the currency board, the only monetary policy tool of the central bank is the required reserves policy. Banks have to maintain required reserves on average over the maintenance period. In Bulgaria[2] the minimum required reserves rate is

---

[2]Starting August 2005, Bulgarian banks may be required (depending on the growth of the loan portfolio) to maintain additional minimum required reserves based on the growth rate of the extended loans (Bulgarian National Bank, 2005b).
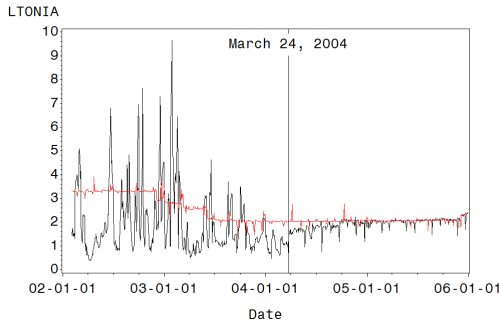
(a) Lithuania: Litas overnight index average
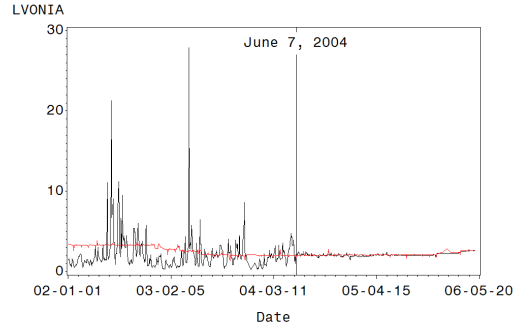
(b) Bulgaria: Leva overnight index average

Figure 1: Interbank interest rates: domestic market interest rate with respect to base currency interest rate fluctuations (EONIA)

currently set at 8%; in Lithuania at 6%. Historically, the required reserve rate has been gradually decreasing in both countries. The Bank of Lithuania remunerates required but not excess reserves, while the Bulgarian National Bank has an option to pay interest on the required reserves. In Bulgaria, the base period (a period when the base of the required reserves is estimated) coincides with the calendar month, while the maintenance period (a period when the required reserves must be held at the central bank) starts on the fourth day of the reporting base period and ends on the third day of the following period. In Lithuania, the required reserves base is estimated based on the bank balance sheet for the month preceding the month on which the reserve maintenance period begins. The maintenance period starts on the 24th day of the calendar month and ends on the 23rd day of the next month (matching the required reserve policy of the European Central Bank).

2.3   Exogenous liquidity shocks

Treasury flows, and thus changes in the account balance at the central bank, are major sources of exogenous liquidity shocks to the banking system. Independent central banks try to predict these liquidity shocks and then offset them with appropriate market interventions. Thus, only unpredicted changes in Treasury account balances can be thought of as exogenous liquidity shocks. In the case of a currency board, however, central banks can not offset changes in the Treasury account.

In both Bulgaria and Lithuania, the Treasury maintains an account with the central bank. Vetlov (2004) shows that since 1998 about 90% of fluctuations in the official foreign reserves of the Bank of Lithuania can be explained by the activity in the Treasury account. Garbaravicius (2004) reports that net Treasury flows constitute up to 40% of the required

reserves, thus greatly affecting the interbank market. Similarly, Petrov (2000) reports that cash flows from and to the budget account at the Bulgarian National Bank are the major source of exogenous shocks to liquidity in the Bulgarian banking system.

3 Data and methodology

All the estimations are performed using daily data on overnight interest rates and bank reserve management provided by the Bulgarian National Bank and the Bank of Lithuania. Changes in the Treasury account balance are also provided by respective central banks. Developing nature of the banking sector in both countries and the institutional changes that took place recently limit the time series used for analysis. For Lithuania, we choose to analyze the period after the adoption of the euro peg.[3] Availability of Treasury account data is the limiting factor in choosing a sample period for Bulgaria. Overall, the full data set for Bulgaria spans the period from 2002:01 to 2006:04 with 1041 observations. The sample period for Lithuania is 2002:02-2005:12 with 990 observations.

As Figure 1 and Figure 2 show, the variability of overnight interest rates is higher in Bulgaria. In both countries, interest rate fluctuations drop after foreign exchange fee decreases (March 24, 2004 in Lithuania and June 7, 2004 in Bulgaria). From that point onward domestic market interest rates follow base currency (euro) interest rate much closer.

Fluctuations in the Treasury account at the central bank throughout the reserve maintenance period (Figure 3) are higher in Bulgaria and, on average, constitute up to 60% of the required reserves. Note that major Treasury account outflows take place around the end of the reserve maintenance period in Bulgaria, while in Lithuania it happens in the middle of the maintenance period.

Figure 4 shows the differences in Bulgarian and Lithuanian bank reserve maintenance practices. While excess reserves are generally positive on all days, Bulgarian banks maintain high excess reserves at the beginning of the reserve maintenance period, while Lithuanian banks hold large excess reserves towards the end of the reserve maintenance period.

We test the following hypotheses: (i) is there an end of the reserve holding period effect on the level of the overnight interest rates? and (ii) do exogenous shocks to liquidity affect interbank interest rates? Following the current literature (Hamilton (1997), Prati et al. (2003), Gaspar et al. (2004), Carpenter and Demiralp (2006)), the empirical model

---

[3]Extending the sample period into the period of a peg to the US dollar would make the analysis susceptible to the parameter instability criticism.

estimated for each country is of EGARCH (γ, q) form:

$$\imath_t \;=\; \mu_t + h_t\,\epsilon_t$$

where

$$\mu_t \;=\; \imath_{t-1} + b'\,X_t$$

$$\ln h_t = \lambda' V_t + \sum_{j=1}^{q} d_{j1}\left(\ln h_{t-1} - \lambda' V_{t-1}\right) + \sum_{i=1}^{\gamma} d_{i2}\,\frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}} + d_{i3}\left(\frac{|\epsilon_{t-1}|}{\sqrt{h_{t-1}}} - \sqrt{\frac{2}{\pi}}\right).$$

and $\imath_t$ is the overnight interest rate measured as the overnight interest rate index average. X_t is a vector of the explanatory variables included in the mean equation and V_t is a vector of variables in the variance equation[4]. The error term ϵ_t is assumed to be distributed according to a mixture of two normal distributions:

$$\epsilon_t \sim p\,N[0,1] + (1-p)\,N[0,\sigma^2]$$

Following the literature EGARCH $(1,1)$ is chosen. The EGARCH specification has been introduced by Nelson (1991) and allows the model variance to respond asymmetrically to positive and negative error terms. In our formulation, we would expect variance to increase when interest rates are rising and variance to decrease when interest rates are falling. The mixture of distributions is introduced to capture periods of frequent small changes in interest rates and occasional large fluctuations. Hamilton (1996) argues that this formulation makes the estimated parameters robust with respect to large outliers and has become the standard formulation in high frequency financial models.

Besides the usual list of explanatory variables found in the empirical interbank market literature, we include interest rate developments of the base currency into the mean equation. It is expected that the interbank market of a currency board has to follow, to some extent, the base currency interest rate movements. Day of the week dummies are included, both in the mean and variance equation, to test for weekday effects. The end of reserve maintenance period dummy $(RMPE)$ takes the value of one for the last three days of the reserve maintenance period.

Motivated by observations in Siaudinis (2003), that interest rates are increasing if banks are deficient in maintaining the required reserves, we also include two measures of

---

[4]The full list of explanatory variables is provided in Table 1 and 2.
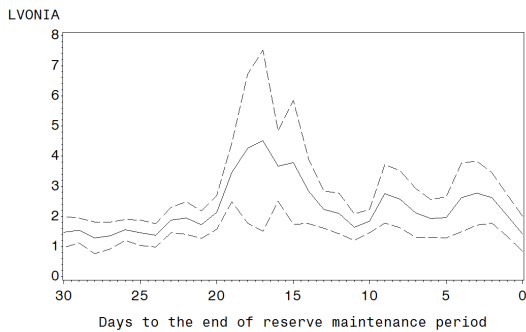
(a) Bulgaria
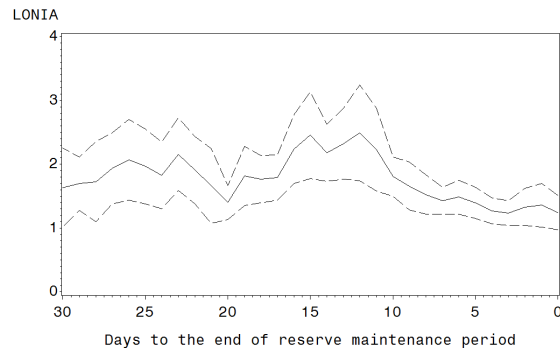
(b) Lithuania

Figure 2: Mean overnight interest rates with 95% confidence intervals.
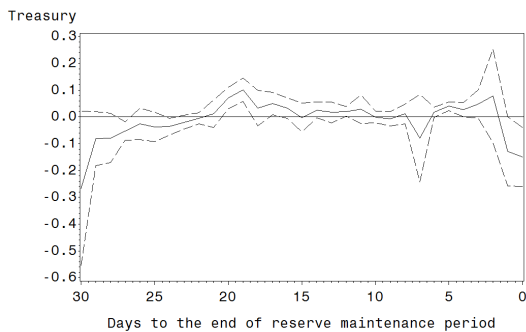
(a) Bulgaria
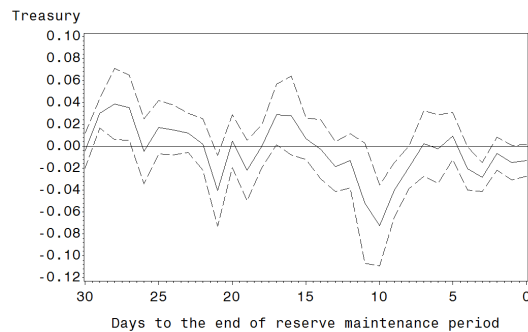
(b) Lithuania

Figure 3: Mean changes in Treasury account with 95% confidence intervals. Treasury accounts inflows and outflows are measured with respect to the size of the reserve requirement.
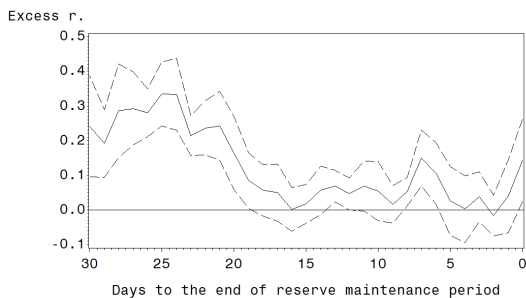
(a) Bulgaria
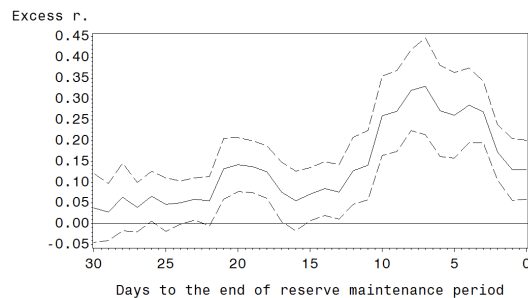
(b) Lithuania

Figure 4: Average daily excess reserves throughout the reserve maintenance period with 95% confidence intervals. Note that reserves are increasing (decreasing) as Treasury balance is decreasing (increasing).

9

reserve deficiency. We define *average deficiency* as the difference between the average reserve requirement and average current account holdings at the central bank to date. In the ideal case this measure would be zero throughout the period. *Current deficiency* measures the difference in average current account holdings and current account holdings, and is meant to capture the change in accumulated average reserves. These two measures of reserve deficiency allow us to distinguish the effects of falling balances at the central bank from reserve deficiency. Both deficiency measures are scaled by the current reserve requirement.

As mentioned above, the left hand side variable is an effective interbank rate. On some days no contracts took place. In that case two options were explored: (1) dropping the days where no contracts were signed or (2) using the average of the bid and ask rates. No significantly different results were obtained, thus only the first case is reported. The model is estimated maximizing the sample conditional log likelihood function, as described by Hamilton (1997).

4 Estimation results

Estimation results of the empirical model are provided in Table 1 and Table 2. According to the martingale hypothesis, overnight interest rates should not fluctuate in a systemic fashion over the reserve maintenance period. For both of the countries examined, this is not the case. In Lithuania, as well as Bulgaria, overnight interest rates are higher or lower on some particular reserve maintenance period days or weekdays. Since both countries are transition economies, violation of the martingale hypothesis could be attributed to the developing nature of the interbank market. Nonetheless, studies of interbank markets in developed economies also report violations of the martingale hypothesis (Bartolini et al. (2001), Hartmann et al. (2001), Quirós and Mendizábal (2006)). The major concern in the literature interpreting violations of the martingale hypothesis is the endogeneity issue - the reaction of the central bank to developments in the interbank market. In the case of an independent central bank, it is not clear if the observed fluctuations in the interest rates are market driven or stem from the actions of the central bank. Such a problem does not arise in our case, since central banks can not intervene, meaning that all observed overnight interest rate fluctuations are market-driven.

In Table 1 the striking similarity in mean parameter estimates for Bulgaria and Lithuania, shows the close similarity of the two countries in terms of the evolution of the interbank market interest rates. Such similarity could not be discerned from the raw data (Figures 2 through 4). Estimated model fits the data for Lithuania much better than for Bulgaria. The

Table 1: Estimation results: mean parameter estimates

Parameter	Lithuania		Bulgaria	
p	0.3301	***	0.1910	***
	(0.0340)		(0.0252)	
$\sigma_1{}^2$	0.2519	***	0.2014	***
	(0.0150)		(0.0148)	
Mean constant	0.0101	**	0.0064	**
	(0.0043)		(0.0031)	
End of reserve maintenance	-0.0539	***	-0.0070	*
	(0.0132)		(0.0038)	
First day of the reserve maintenance	0.1089	**	0.0113	
	(0.0424)		(0.0088)	
Change in EONIA†	0.0873	***	0.0717	***
	(0.0176)		(0.0164)	
Treasury	0.0403		-0.0011	
	(0.0282)		(0.0074)	
Average deficiency	0.1088	***	-0.0022	
	(0.0224)		(0.0159)	
Current deficiency	0.1222	***	0.0347	**
	(0.0162)		(0.0156)	
Average deficiency x End of reserve maintenance	-0.2438	***	-0.1262	*
	(0.0464)		(0.0658)	
Current deficiency x End of reserve maintenance	-0.0455		-0.0387	**
	(0.0724)		(0.0168)	
Tuesday	-0.0061		-0.0087	**
	(0.0056)		(0.0041)	
Wednesday	-0.0076		-0.0078	**
	(0.0056)		(0.0036)	
Thursday	-0.0080		-0.0049	
	(0.0053)		(0.0038)	
Friday	0.0035		-0.0064	*
	(0.0055)		(0.0039)	

†Euro overnight index average.
* - 10% significance. ** - 5% significance. *** - 1% significance.

adjusted R^2 is 0.77 and 0.37 for Lithuania and Bulgaria, respectfully.

4.1 End and beginning of the reserve maintenance period effect

For both of the countries, interest rates are estimated to be significantly lower at the end of reserve maintenance period (Table 1). This is quite surprising, since to date empirical studies report various degrees of interbank market tightness and thus higher interest rates towards the end of the reserve holding period.

It has to be acknowledged though, that tightness at the end of the reserve maintenance period is not a well established fact in the literature and varies across countries. Prati et al. (2003) argues that interbank market tightness around the settlement day depends upon the way in which central banks manage liquidity in the market. Some central banks induce market participants to manage their funds prudently, thus allowing for settlement day tightness, others are supportive and accommodate market demand for reserves. Thus, banks may be comfortable with accruing reserve deficiency in anticipation that a central

Table 2: Estimation results: variance parameter estimates

Parameter	Lithuania		Bulgaria	
Variance constant	2.2457	***	5.5836	***
	(0.5326)		(0.8531)	
End of reserve maintenance	1.8586	***	-0.0672	
	(0.2337)		(0.3226)	
First day of reserve maintenance	1.4184	***	1.0204	***
	(0.3088)		(0.2738)	
Average deficiency	4.4001	***	3.2446	***
	(0.8714)		(0.7665)	
Current deficiency	0.6613	*	3.6508	***
	(0.3638)		(0.6001)	
Average deficiency x End of reserve maintenance	5.8120	***	-1.4050	
	(1.0309)		(1.8657)	
Current deficiency x End of reserve maintenance	0.0370		0.2866	
	(1.1291)		(1.1232)	
No fee	-4.3104	***	-5.3781	***
	(0.2926)		(0.3586)	
Tuesday	0.1849		0.0940	
	(0.1880)		(0.1512)	
Wednesday	0.2136		-0.3694	**
	(0.2042)		(0.1810)	
Thursday	0.0260		-0.0679	
	(0.1844)		(0.1671)	
Friday	0.1624		-0.2189	
	(0.1854)		(0.1882)	
d_{11}	0.8069	***	0.7817	***
	(0.0383)		(0.0350)	
d_{12}	0.3040	***	-0.6272	***
	(0.0926)		(0.1372)	
d_{13}	1.1154	***	1.6994	***
	(0.1679)		(0.2428)	
N	990		1041	
R^2 adj.	0.7685		0.3700	

* - 10% significance. ** - 5% significance. *** - 1% significance.

bank will intervene in the market, trying to dampen increasing interest rates.

Although the end of the reserve maintenance period effect is very strong for Lithuania, it is only marginal for Bulgaria (p-value of 6%). As will be discussed later, this is explained by the leniency of the Bulgarian National Bank that appears to tolerate (or fails to deter) persistent violations of the reserve requirement.

Moreover, the estimation results indicate that the Lithuanian interbank market experiences higher volatility at the end of the reserve maintenance period (Table 2) although there is no such effect for Bulgaria. Once again, this could be explained by different attitude of the central banks towards the violations of minimum reserve requirements. Usually increased interest rate volatility is expected to be observed in countries with small or no carry over provisions, and both Lithuania and Bulgaria lack carry over provisions for reserve maintenance.

Furthermore, estimation results indicate that interest rates are significantly higher on the first day of the reserve maintenance period for Lithuania, but not for Bulgaria. A surge

in funds demand, e.g. banks trying to build up reserve positions early in the period, could explain higher interest rates. Another hypothetical explanation could be a liquidity effect reversal, as in Ewerhart et al. (2004): with the abundance of funds in the market some banks act strategically forcing up the interest rates.

4.2   Reserve deficiency

In a recent study, Siaudinis (2003) raises a hypothesis that the deficiency or surplus of funds in the interbank market can explain the fluctuation in the domestic interest rates. He argued that whenever the outstanding reserves plunge to less than 25% of the required reserves, an increase in interest rates is observed. Empirical estimation carried out in the current paper quantifies this proposition. The results reported in Table 1 show that, in Lithuania, the overnight interest rates tend to rise with an increase in average and current deficiency (both defined above). In the case of Bulgaria, interest rates rise whenever current account holdings at the central bank fall below the average account holdings (current deficiency increases). Interestingly, as the parameters on the interactions of deficiency measures with a dummy for the end of reserve maintenance period indicate, falling deficiency is associated with lower interest rates on the last days of the reserve holding period for both countries. This looks peculiar, but can be explained by the developing nature of the interbank markets.

Figure 4 shows that in both countries commercial banks are reluctant to end the reserve maintenance period with minimal excess reserves. Banks hold excess reserves almost every day of the maintenance period. Interestingly, Lithuanian banks hold more excess reserves on the second half of the reserve maintenance period, while Bulgarian banks maintain large excess reserves at the start of the maintenance period. Therefore, it is apparent that banks approach the end of the reserve holding period with high average balances and find it perfectly safe to cut excess reserves on the very last days. Interest rates go down, while "deficiency" increases. Note that in Lithuania excess reserves decrease at the very end of the maintenance period.

We also observe that variability of the interest rates increases in both countries with the increase in deficiency. It is expected that higher deficiency will have a stronger impact towards the end of maintenance period. This does prove to be the case for Lithuania - the effects of the deficiency in the reserve market are exacerbated on the last days of the reserve maintenance period (Table 2).

Why are the end of the reserve maintenance period effects weaker in Bulgaria than those in Lithuania? In Lithuania, there was only one case throughout the sample period when some bank did not meet the reserve requirement. The Bank of Lithuania appears to

have communicated clearly to the banks that failure to comply with the reserve requirement can result in higher scrutiny from the central bank as well as fines. This is not the case in Bulgaria. Every year, the Bulgarian National Bank reports[5] that, although with an improvement lately, there were some banks that did not comply with the minimum reserve requirement (15 in 2002, 14 in 2003, 16 in 2004 and 2 in 2005). Despite abundant aggregate liquidity, banks "face difficulty" in maintaining required balances. These phenomena indicate that "fear of being short" is much weaker in Bulgaria than it is in Lithuania, thus yielding much lower or insignificant end of the reserve holding period effects in Bulgaria.

4.3   Liquidity effect

Fluctuations in the treasury account are used in the literature to identify the exogenous shocks to liquidity in case of independent central bank. If exogenous changes in outstanding liquidity in the market are shown to move the interest rate, this is interpreted as evidence of central bank being able to manage market interest rates through open market operations. As discussed above, there is no consensus in the empirical literature as to the existence of such a liquidity effect.

Although under the currency board the central bank does not engage in open market operations, it is still possible to measure how exogenous changes in liquidity (although not initiated by the central bank) affect the interbank interest rates. As reported by Garbaravicius (2004) for Lithuania and Petrov (2000) for Bulgaria, fluctuations in the Treasury account are major shocks to the liquidity of these countries' banking systems. Average Treasury account inflows and outflows throughout the reserve maintenance period are shown in Figure 3. For example, it can be seen that some sizable outflows take place on the last days of the maintenance period in Bulgaria. By including changes in the Treasury account into the mean equation, we try to capture the equivalent of a liquidity effect in the case of an independent central bank. We expect that an outflow from a Treasury account (a positive liquidity shock) would decrease interbank market interest rates, whereas an inflow would increase overnight interest rates.

However, we do not find any empirical evidence that changes in Treasury account holdings at the central bank have a significant impact on the interbank interest rates. Despite sizable fluctuation (with respect to the size of the reserve requirement), changes in the Treasury account have no significant impact on the interbank market interest rate in currency board countries. It may be the case that Treasury account fluctuations are highly predictable, and accordingly have limited impact on the interbank market interest rates.

---

[5]Annual reports of the Bulgarian National Bank.

5 A model of optimal reserve management

Empirical evidence shows that in the case of a currency board there is no "settlement day tightness." What theoretical justification can be provided to explain this observation? We build on the model developed by Gaspar et al. (2004) and Quirós and Mendizábal (2006) to incorporate interbank market specifics of the currency board. The model constructed is solved numerically showing that interest rates could be decreasing or increasing depending on the aggregate bank liquidity.

5.1 Model formulation

Assume that the economy is populated by an infinite number of banks $j \in [0, 1]$ with the aggregate mass of one. Each bank is of mass zero, does not engage in any strategic interaction, and thus takes market conditions as given. Banks are heterogeneous and differ in individual reserve requirements as well as liquid assets available. Every bank has to meet the minimum reserve requirement over a period of time called the reserve maintenance period.

Bank j begins the reserve maintenance period with an amount of liquid assets a_1^j and reserve deficiency d_1^j. Reserve deficiency d_t^j is defined as the total amount of reserves that has to be maintained at the central bank over the reserve maintenance period, which is assumed to be T days long. This way d_1^j/T would be the average reserve requirement faced by bank j. The determination of the required reserves is not specified, and banks take it as given. The required reserves are met by maintaining an average required balance at the central bank. At any particular day, banks may decide to maintain higher or lower balances, but the average balance throughout the period should not be lower than the average reserve requirement.

During any day t, bank j observes payment activity on its account at the central bank. This activity is treated as liquidity fluctuation and is summarized by a liquidity shock λ_t^j assumed to be i.i.d. $\sim N(0, \sigma_\lambda^2)$. After payments clear (after the liquidity shock is realized), bank j observes a balance at the central bank (b_t^j) of the size:

$$b_t^j = a_t^j + \lambda_t^j. \tag{1}$$

Given this information, banks may decide to obtain extra liquidity or lend excess reserves in the domestic interbank market. Due to the currency board arrangement, banks can also participate in the foreign currency interbank market.

Next assume that the foreign money market interest rate ι_t^* is constant. This does not
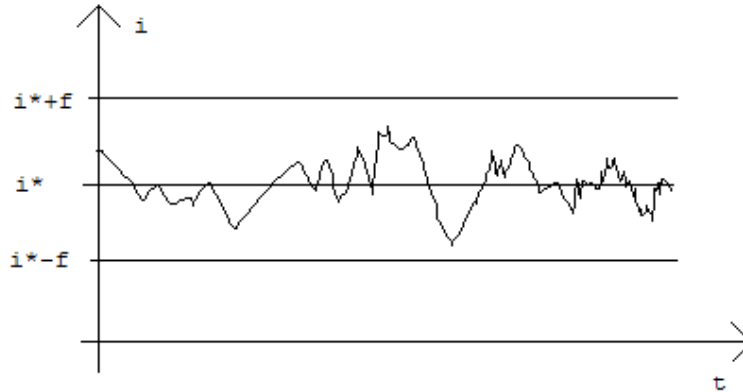
Figure 5: Stylized diagram of an interbank market under the currency board: domestic interest rate deviations from the base currency interest rate are limited by the foreign exchange transaction costs f.

seem to be an unreasonable assumption given the relative variability of domestic and foreign overnight interest rates. Participation in the foreign money market is possible, since under the currency board the central bank stands ready to exchange domestic currency to/from the base currency at the fixed exchange rate. Thus, despite the foreign transaction costs, banks may find it optimal to exchange domestic currency to the base currency (e.g. euro) and make an overnight deposit with a foreign counterpart. Similarly, banks could borrow in foreign currency, exchange the funds at the central bank and obtain liquidity in domestic currency. Note that under the currency board, the open foreign exchange position in the base currency is not limited.

Let f denote the proportional foreign exchange transaction costs that are incurred whenever commercial banks make a foreign exchange transaction at the central bank. These costs create a wedge between the base currency money market interest rate i_t^* and the actual interest rate $\hat{\imath}_t$ faced by the bank: $\hat{\imath}_t = i_t^* \pm f$ depending if the bank is borrowing or lending. This is illustrated by a simple diagram in Figure 5.

Note that no bank will be making deposits abroad if the domestic money market interest rate $\imath_t > \hat{\imath}_t$, and no bank will borrow abroad if $\imath_t < \hat{\imath}_t$, that is, if the interest rates stay within the bounds as in Figure 5. Similar to the standing facilities present in the case of an independent central bank, the opportunity to utilize the foreign exchange window at the central bank appears to be creating *de facto* standing facilities.

After observing the liquidity shock (λ_t^j) and the realized mid-day balance $(b_t^j = a_t^j + \lambda_t^j)$ at the central bank, bank j decides if it wants to lend some of its reserves in the domestic (l_t)

16

or the foreign market (lf_t). After lending decisions are made, the contracts are signed and executed. After the market closes, some late payments may still realize, altering the bank's balance at the central bank. As in Quirós and Mendizábal (2006), this activity is modeled as another liquidity shock, summarized by ϵ_t, which is assumed to be i.i.d. $\sim N(0, \sigma_\epsilon^2)$. Bank j's end of the day balance at the central bank (ae_t) is then:

$$ae_t^j = b_t^j - l_t^j - lf_t^j + \epsilon_t^j \tag{2}$$

What tradeoffs does a bank face? The benefit of making a loan is the interest rate collected, while the cost is that the bank's expected overnight balance at the central bank diminishes and the bank may have difficulty in meeting the reserve requirement later. In addition, if a bank finishes trading with a very low balance, there is a possibility that this balance will be insufficient to cover large late payments that could realize after the market is closed. If this happens, the central bank will extend an automatic liquidity loan to the bank at a penalty rate $\imath_{cb}$. Note that the bank observes λ_t, but does not observe ϵ_t until after making its decisions.

If the balance maintained overnight at the central bank is ae_t^j, bank j's reserve deficiency in the next period is:

$$d_{t+1}^j = \max(0, d_t^j - \max(0, ae_t^j)) \tag{3}$$

Assuming no compounding of interest and that the loans have to be repaid by the start of the next day, bank j's balance at the beginning of the next day is:

$$a_{t+1}^j = a_t^j + \lambda_t^j + \epsilon_t^j \tag{4}$$

If the end of the day balance at the central bank becomes negative, a bank is forced to take a liquidity loan from the central bank at the penalty rate $\imath_{cb}$. Recall that the central bank under the currency board does not intervene in the money market, but it wants to sustain an operational payment system. Thus, the penalty rate is set high to deter banks from running negative balances. Similarly, if the bank fails to meet the required reserves by the end of the reserve maintenance period, the deficient amount has to be borrowed from the central bank at the same penalty rate.

Clearly the decisions of a particular bank will depend upon, among other things, the observed market interest rate and expected future interest rates. Assuming rational expectations, the reserve management problem faced by bank j can be written as the following

dynamic programming problem:

$$V_t(s_t^j) = \max_{l_t^j, lf_t^j} \left(\iota_t l_t^j + \hat{\iota}_t l f_t^j - \underset{\epsilon_t^j}{E} c_t^j + \underset{\{\epsilon_t^j, \lambda_{t+1}^j\}}{E} V_{t+1}\left(s_{t+1}^j\right) \right) \text{ for } t < T \qquad (5)$$

$$V_T(s_T^j) = \max_{l_T^j, lf_T^j} \left(\iota_T l_T^j + \hat{\iota}_T l f_T^j - \underset{\epsilon_T^j}{E} c_T^j \right)$$

subject to

$$c_t^j = -I(ae_t^j < 0)ae_t^j \iota_{cb}, \text{ if } t < T$$
$$c_T^j = I(ae_T^j < d_T^j)(d_T^j - ae_T^j)\iota_{cb}$$
$$s_t^j = (b_t^j, d_t^j)$$
$$b_t^j = a_t^j + \lambda_t^j$$
$$b_{t+1}^j = b_t^j + \epsilon_t^j + \lambda_{t+1}^j$$
$$d_{t+1}^j = \max(0, d_t^j - \max(0, ae_t^j))$$
$$ae_t^j = b_t^j - l_t^j - lf_t^j + \epsilon_t^j$$

Where $\hat{\iota}_t = \iota_t^* + f$, if $lf_t < 0$, and $\hat{\iota}_t = \iota_t^* - f$, if $lf_t \geq 0$. $I(.)$ is an index function taking the value of one if the argument in the parenthesis is true.

Bank j maximizes its expected interest earnings net of the penalties over the reserve maintenance period by choosing optimal lending in domestic and foreign money markets. If reserves become negative ($ae_t < 0$) on any day before the last one, the bank incurs a cost of $c_t = -I(ae_t < 0)ae_t\iota_{cb}$. On the last day ($t = T$), in addition to maintaining a nonnegative balance, the bank has to meet the reserve requirement. Thus, if the end of the day balance is insufficient ($ae_T < d_T$), the bank incurs a cost of $c_T = I(ae_T < d_T)(d_T - ae_T)\iota_{cb}$.

Note that bank j's problem does not depend upon the distribution of banks across different states, since there is no aggregate uncertainty and it takes interest rates as given. Nonetheless, individual shocks matter for the resulting distribution of banks on any particular day of the reserve maintenance period ($f_t(s_t)$, $t = 1...T$). Thus, it is necessary to obtain the aggregate law of motion for the resulting distribution in order to be able to check for market clearing conditions.

Given some initial distribution $f_1(s_1)$ and the optimal decision rules $\left(l_t^*(s_t^j), lf_t^*(s_t^j)\right)_{j \in [0,1]}$ that solve the problem defined in Equation 5, the resulting distribution of banks on any par-

ticular day of the reserve maintenance period can be defined recursively:

$$f_{t+1}(s_{t+1}) = \int_0^1 f_t(s_t^j) h_t \left[s_{t+1} | s_t^j, l_t^*(s_t^j), l f_t^*(s_t^j) \right] dj \qquad (6)$$

where $h[s_{t+1}|s_t^j, l^*(s_t^j), lf^*(s_t^j)]$ is a conditional probability density function, that after taking the optimal decisions $\left(l_t^*(s_t^j), lf_t^*(s_t^j) \right)$ bank j currently at $s_t^j = (b_t^j, d_t^j)$ will find itself at $s_{t+1} = (b_{t+1}, d_{t+1})$ next period. Due to implicit truncation of variables it is not possible to provide an explicit expression for $h[.]$. Intuitively, given s_t^j and $\left(l_t^*(s_t^j), lf_t^*(s_t^j) \right)$, b_{t+1} (mid-day balance) will be distributed as:

$$b_{t+1} \sim N[b_t^j, \sigma_\epsilon^2 + \sigma_\lambda^2].$$

Note, that next period assets are not affected by the bank's decisions and vary only due to liquidity shocks.

Given s_t^j, next day reserve deficiency d_{t+1} has a truncated normal distribution of the following form:

$$
\begin{aligned}
d_{t+1} &\sim N[d_t - (b_t^j - l_t^*(s_t^j) - lf_t^*(s_t^j)), \sigma_\epsilon^2], \text{ for } 0 < d_{t+1} < d_t \\
d_{t+1} &= d_t, \text{ with } Prob(ae_t < 0) \\
&\quad ae_t \sim N[b_t^j - l_t^*(s_t^j) - lf_t^*(s_t^j), \sigma_\epsilon^2]) \\
d_{t+1} &= 0, \text{ with } Prob(d_t - ae_t < 0) \\
&\quad (d_t - ae_t) \sim N[d_t - (b_t^j - l_t^*(s_t^j) - lf_t^*(s_t^j)), \sigma_\epsilon^2]
\end{aligned}
$$

Thus, $h[s_{t+1}|s_t^j, l^*(s_t^j), lf^*(s_t^j)]$ is a conditional multivariate distribution function of (b_{t+1}, d_{t+1}), univariate distribution properties of which are provided above.

Since there is no aggregate uncertainty (idiosyncratic shocks cancel out), equilibria with deterministic aggregate dynamics are considered.

Definition. *Reserve maintenance equilibrium is a deterministic sequence of interest rates $\{\iota_t\}_{t=1}^T$ and a collection of contingent plans $(\{l^*(s_t^j), lf^*(s_t^j)\}_{t=1}^T)_{j \in [0,1]}$ such that, given the initial distribution of banks $f_1(s_1)$, penalty rate ι_{cb}, and foreign money market rates $(\iota_t^*)_{t=1}^T$: (i) contingent plans $(\{l_t^*(s_t^j), lf_t^*(s_t^j)\}_{t=1}^T)_{j \in [0,1]}$ solve every bank j's problem defined by Equation 5; and (ii) given the resulting distribution $f_t(s_t)$, defined by Equation 6, the domestic money market clears:*

$$\int_0^1 f_t(s_t^j) l_t^*(s_t^j) dj = 0, \text{ for } t = 1...T.$$

5.2 Characterization of reserve maintenance equilibrium

Quirós and Mendizábal (2006) argues that interest rates should be increasing throughout the reserve maintenance period. The intuitive explanation provided is based on the assertion that the "probability of going to the deposit facility should be increasing over time" (page 112). We show that, depending on the aggregate liquidity in the market, the probability of ending the reserve holding period with deficient reserves can be increasing or decreasing over time.

Define bank j's non-borrowed reserves at the end of the day t as $na_t = a_t + \lambda_t + \epsilon_t$. The *average non-borrowed reserves* over the maintenance period ($average(T)$) can then be defined as:

$$average(T) = a_1 + \sum_{t=1}^{T} \frac{T-t+1}{T}(\lambda_t + \epsilon_t).$$

Note that the average is a function of the days towards the end of the reserve maintenance period. Define $PD(T)$ as bank j's probability of ending the reserve maintenance period with average non-borrowed reserves below the reserve requirement (or having *deficient non-borrowed reserves*):

$$PD(T) = Prob[average(T) < d_1/T]$$

Proposition 1. *The probability $PD(T)$ of ending the reserve maintenance period with deficient non-borrowed reserves is: (i) decreasing in T, if $a_1 - d_1/T < 0$ (average deficient reserves); (ii) increasing in T, if $a_1 - d_1/T > 0$ (average excess reserves); and (iii) constant in T, if $a_1 - d_1/T = 0$.*

Proof. Let x stand for the average excess reserves at the beginning of the period: $x = a_1 - d_1/T$. The question is, how does the probability of actually ending the period with deficient reserves change when the number of days until the end of the period decreases, holding the average excess reserves constant. Note that all shocks are i.i.d. and there is no aggregate uncertainty.

Express the probability of deficiency as $PD(T) = Prob[\sum_{i=1}^{T} \frac{T-i+1}{T}(\lambda_i + \epsilon_i) < -x]$. Denote $\xi(T) = \sum_{i=1}^{T} \frac{T-i+1}{T}(\lambda_i + \epsilon_i)$, thus $PD(T) = 1 - F_{\xi(T)}[x]$. Note that $\xi(T) \sim N[0, \sigma_{\xi(T)}^2]$, where $\sigma_{\xi(T)}^2 = \sum_{i=1}^{T} \frac{(T-i+1)^2}{T^2}(\sigma_\lambda^2 + \sigma_\epsilon^2)$, since $\sigma_{\lambda_i}^2 = \sigma_\lambda^2$ and $\sigma_{\epsilon_i}^2 = \sigma_\epsilon^2$. Thus, $\sigma_{\xi(T)}^2$ is increasing in T. Since

$$\frac{\partial F_{\xi(T)}[x]}{\partial \sigma_{\xi(T)}} = -\frac{x \cdot e^{-\frac{x^2}{2\sigma_{\xi(T)}^2}}}{\sqrt{2\pi}\sigma_{\xi(T)}}$$

is greater than 0 if $x < 0$, the probability of deficiency $PD(T)$ is decreasing as T in-

creases[6]. Accordingly, the probability of ending the period with deficient reserves is decreasing throughout the reserve maintenance period if $x > 0$, and it is constant if $x = 0$. $\quad\square$

Intuitively, if banks start the reserve maintenance period with average excess reserves, most probably they will not be deficient. Still, there is a possibility for a specific bank to end the period with deficient reserves. As the end of the reserve holding period approaches, the probability of this occurring diminishes. Thus, following the argumentation of Quirós and Mendizábal (2006), interest rates should be decreasing. If, in aggregate, banks have deficient reserves $(a_1 - d_1/T < 0)$ it may be expected that interest rates should be increasing throughout the reserve holding period. Similarly, interest rates should be constant if $a_1 - d_1/T = 0$.

5.3 Numerical simulation

Due to nonlinearities, non-differentiability and the heterogeneous nature of the economy a closed form solution to this problem cannot be provided. Thus, we have to resort to numerical methods. Note that we are solving a stochastic general equilibrium model with heterogeneous agents in case of finite horizon.

The usual methods for solving infinite horizon stochastic general equilibrium models with heterogeneous agents cannot be applied in the case of a finite horizon. The usual practice in the case of infinite horizon models boils down to obtaining a numerical approximation of the steady state distribution function. Even with the numerical approximation, it is not a straightforward task since the distribution function is an infinite dimensional object. This is a serious problem, since it means keeping track of an infinite dimensional object. Krusell and Smith (1998) suggest approximating the distribution function by a finite number of moments as a solution to this problem. This is the usual path taken by those dealing with heterogeneity.

Unfortunately, in the case of finite horizon, the distribution function must be kept track of in every period, since there is no reason to assume that the initial distribution will reach a steady state within the reserve holding period. Thus, the usual numerical methods cannot be applied.

The literature on stochastic general equilibrium with heterogeneous agents in case of finite horizon is not well developed. There is, however, an extensive literature on overlapping generations models with heterogeneous agents, which in a sense deals with an infinite number of finite horizon problems. It seems that finite horizon problems have been avoided, generally

---

[6]Note that T is number of days until the end of the reserve maintenance period and decreases with time.

due to the curse of dimensionality. The method used to solve this model is not presented as a cure-all tactic, but it may prove practical in cases of a reasonable state space size.

5.3.1 Proposed numerical solution methodology

In the absence of a suitable numerical technique to solve stochastic general equilibrium models in case of a finite horizon we propose the following solution algorithm[7]:

Step 1: Discretize the problem by choosing a grid over the individual state space and liquidity shocks.

Step 2: Specify the initial density $f_0(s_0)$ over the grid.

Step 3: Make an initial guess for the interest rate sequence $\{i_t\}_{t=1...T}$

Step 4: Given the interest rates, solve backwards every bank j type (grid point) problem for the optimal decision rules $\{l_t^*(s_t), lf_t^*(s_t)\}_{t=1}^T$.

Step 5: Given the solution to *Step 4* and the initial distribution specified in *Step 1*, iterate forward to obtain the resulting distribution over the grid in every period.

Step 6: Check if market clear given the the resulting distribution in *Step 5* and optimal decision rules in *Step 4*. If market do not clear, adjust interest rates for all periods and repeat the algorithm starting with *Step 4*.

5.3.2 Multiplicity of equilibria in case of discrete choices

As explained above, this model is solved numerically discretizing the choice set. Discreteness of the loan size can be justified by the standardizes size of the interbank loans. We will show, that in case bank choices are not continuous, there could be multiple equilibrium interest rate paths. To do so we need to establish some properties.

Proposition 2. *Uncertainty faced by banks decreases as the end of the reserve holding period approaches.*

Proof. Assets on the last day of reserve holding period are $a_T = a_1 + \sum_{i=1}^{T} \lambda_i + \epsilon_i$. Thus, a_T inherits the distributional properties of the liquidity shocks. Namely, a_T is $\sim N[0, \sigma_{a_T}^2]$. Since all liquidity shocks are i.i.d., $\sigma_{a_T}^2 = T(\sigma_\lambda^2 + \sigma_\epsilon^2)$ and, as can be seen, is increasing in T. □

---

[7]The proposed method is similar to the "shooting method" described in Heer and Maussner (2005) and Ríos-Rull (1999) for solving transitional dynamics.

Intuitively, the higher is T (more days towards the end of the reserve holding period), the fatter tailed is the distribution of a_T. As banks move towards the end of the reserve holding period, although possibly having different assets and reserve deficiencies, the uncertainty over realized liquid assets and reserve deficiency at the end of the reserve maintenance period diminishes, since there are fewer opportunities left to get a negative or a positive liquidity shock.

Next we show that banks respond differently to changes in the interest rate at the beginning and the end of the reserve maintenance period.

Proposition 3. *Bank j's optimal lending decision* $l_t^*(s_t^j)$ *at some day t is a function of the interest rate sequence* $\{\imath_t\}_{t=t...T}$ *and has the following property:*

$$\frac{\partial l_t^*(s_t^j, \imath_t, ..., \imath_T)}{\partial \imath_t} > \frac{\partial l_{t+1}^*(s_{t+1}^j, \imath_{t+1}, ..., \imath_T)}{\partial \imath_{t+1}}$$

Proof. The increase in the interest rate on day t raises the marginal benefit of lending on that day increasing $l_t^*(s_t^j, \imath_t, ..., \imath_T)$. The associated increase in marginal costs (due to higher deficiency) depends on the change in the probability of actually being deficient. The change in this probability is equal to $F_{a_T}[d_1/T - \Delta l_t^*] - F_{a_T}[d_1/T]$, which, under Proposition 2, is increasing in T. Thus increasing lending at t will increase the probability of deficiency less than it would increase at $t + 1$. As the uncertainty over a_T is higher at the beginning of the period, banks will respond more to changes in the interest rate early in the period and less to changes in the interest rate at the end of the period. $\square$

Discretization of the problem combined with the diminishing responsiveness to interest rate changes may lead to multiple equilibrium interest rate paths.

Proposition 4. *There exist multiple reserve maintenance equilibria in case of discrete choices*

Proof. Assume all banks start a two day $(T = 2)$ reserve maintenance period with $a_1 = d_1/2$, so that there is no aggregate deficiency or excess reserves in the market. Proposition 1 shows that in the case when there is no excess aggregate reserves or aggregate deficiency, the probability of actually ending the period with deficient reserves is constant over time. Further, assume that there exists an interest rate sequence $\{\imath_t\}_{t=1..T} = \bar{\imath}$, that is constant throughout the reserve maintenance period, by which optimal bank lending decisions lead to market equilibrium. Since bank decisions are discrete, there exists some small change in the

Table 3: Numerical simulation parameters

T	$\imath_{cb}$	$\imath^*$	f	b_1	d_1	ϵ	λ
10	5%	2.5%	2.5%	21	207-213	{-1,0,1}	{-1,0,1}

T - days in the reserve maintenance period; $\imath_{cb}$-liquidity loan rate at the central bank; $\imath^*$ - foreign interbank interest rate; f- foreign exchange transaction costs; b_1-beginning of the period liquid assets; d_1 - reserve deficiency; λ_t and ϵ_t - liquidity shocks.

interest rates ξ_1 and ξ_2 such that: (i) $\Delta i_2 < \xi_1$, $\Delta l_2^*(s_2^j, \imath_2)/\Delta\imath_2 = 0$, and $\Delta l_1^*(s_1^j, \imath_1, \imath_2)/\Delta\imath_2 = 0$; (ii) $\Delta i_1 < \xi_1$ and $\Delta l_1^*(s_1^j, \imath_1, \imath_2)/\Delta\imath_2 = 0$.

Since, as argued in Proposition 3, a bank's response to interest rate changes early in the period is higher, than its responses to interest changes later in the period, it must be the case that $\xi_2 > \xi_1$. Thus, there exists some other equilibrium interest rate sequence (i_1', i_2'), such that $\imath_1' = \imath_1 + \xi_1$, $\imath_2' > \imath_2 + \xi_2$, and $\xi_1 < \xi_2$. $\qquad\square$

Intuitively, due to discreteness, banks do not change their choices smoothly due to small changes in the interest rate. Thus there is a band of "indifference" where banks do not react to changes in the interest rate. This band is wider as the end of the reserve holding period is approaching and uncertainty is resolved.

5.3.3 Parametrization

To simplify the exposition of solution results, assume that all banks are identical on the first day of the reserve maintenance period.[8] We solve the model with the parametrization summarized in Table 3, implementing the proposed algorithm. Liquidity shocks λ_t and ϵ_t are discretized to take values $\{-1, 0, 1\}$ with equal probability. The length of the reserve holding period is 10 days. Thus, a bank with $b_1 = 21$ liquid assets on the first day and required reserve deficiency of $d_1 = 210$, has just enough non-borrowed reserves on average $(21 = 210/10)$. Situations with aggregate deficiency and aggregate excess reserves are also investigated. Recall that with this parametrization, domestic market interest rates are bounded in the interval $[\imath^* - f, \imath^* + f] = [0, 5]$.

5.3.4 Simulation results

Simulation results are presented in Figures 6 and 7. Expecting the possibility of multiple equilibria, the search algorithm is started with different initial search values of the interest

---

[8]The results are qualitatively similar for different initial distributions investigated.

(a) $b_1 = 21, d_1 = 211$ (b) $b_1 = 21, d_1 = 210$ (c) $b_1 = 21, d_1 = 209$
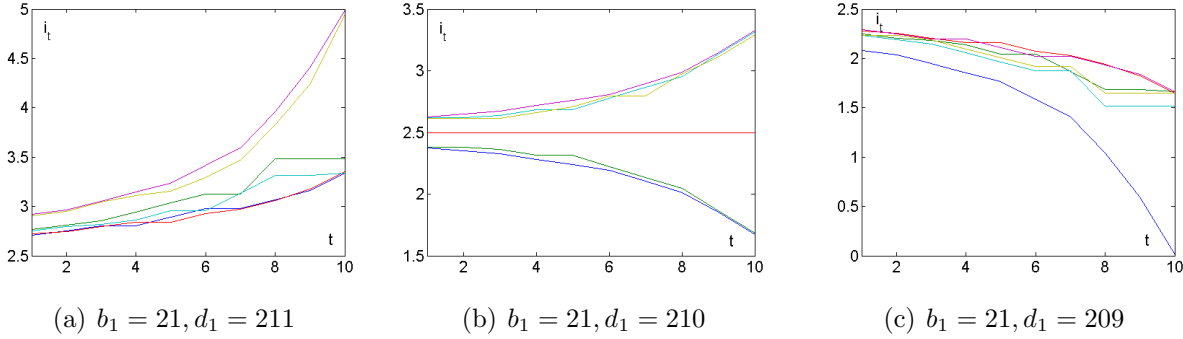
Figure 6: Equilibrium interest rates within the reserve holding period: (a) increasing, if there is aggregate deficiency; (b) could be increasing or decreasing if aggregate reserves equal the requirement; (c) decreasing, if there is aggregate excess reserves. Multiple interest rate sequences result in market equilibrium.

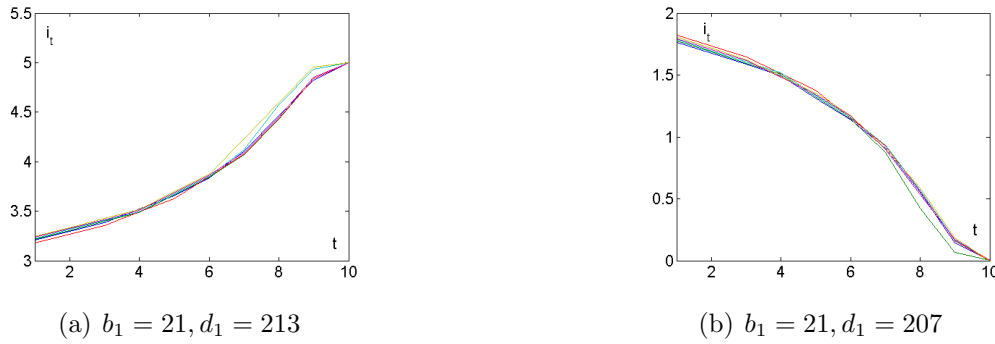(a) $b_1 = 21, d_1 = 213$ (b) $b_1 = 21, d_1 = 207$

Figure 7: Equilibrium interest rates within the reserve holding period: (a) Interest rates are increasing if there is aggregate deficiency (b) Interest rates are decreasing if there is aggregate excess reserves.

rate sequence. Convergent equilibrium interest rate sequences resulting from different initial search values (but the same parametrization) are shown on the same graph.

As expected, we find multiple equilibrium interest rate paths. Different interest rate sequences $\{i_t\}_{t=1...10}$ and resulting optimal lending decisions lead to different equilibria in the domestic interbank market. In a probabilistic sense, if all possible equilibria are equally likely, interest rate variability is higher toward the end of the reserve holding period. Note that this result does not depend on risk averseness of banks, since banks are assumed to be risk neutral. Acknowledging the fact that this result stems from the discreteness of the choice set, it matches the empirical results discussed in Section 4.

If, on average, a banking system has reserves exactly as required (Figure 6(b)), interest rates can be increasing or decreasing depending upon the self-fulfilling expectations of the banks. In that sense, the variability of interest rates over the reserve holding period could

25

be diminished if banks could coordinate on some target.

In the case of deficient aggregate reserves (Figure 6(a)), interest rates are found to be increasing. Since banks are deficient in aggregate, they utilize the foreign exchange window at the central bank to obtain needed liquidity. Since the cost of foreign funds is the upper bound of domestic interbank interest rates throughout the reserve holding period, banks find it optimal to delay borrowing until the last moment, since from the individual bank's perspective, there is still a possibility of having an unexpected increase in liquidity. As argued above, under these circumstances, the probability of actually ending the period with deficient reserves is increasing, and so does the interest rate. Note that the interest rates may not hit the upper bound since aggregate liquidity is obtained as some banks are forced to take liquidity loans due to negative balances at the end of the day.

Interest rates are *decreasing* if there is excess aggregate liquidity in the market, and this is shown in Figure 6(c). Since banks have excess reserves on average, they will find it optimal to lend some reserves at the foreign interbank market. Since the interest rate of a loan at the foreign interbank market is the lower bound of domestic interbank interest rates, banks choose to hold off until the last day of the reserve holding period. Unlike in the case of excess reserves, it is shown that the probability of actually ending the period with deficient reserves is decreasing over the reserve maintenance period, as is the interest rate. Thus, unlike in Quirós and Mendizábal (2006), here it is shown that interest rates can be decreasing over the reserve maintenance period. We also highlight the possible multiplicity of equilibria in solutions utilizing discretization, a result not yet reported in the literature.

If aggregate excess reserves are high enough (Figure 7(b)) or low enough (Figure 7(a)), interest rates hit the upper or lower bound at the end of the period, leaving less room for alternative expectations.

How do these implications of the model relate to the empirical results presented above? Theoretically, interest rates should be decreasing if there are aggregate excess reserves in the market. As Figure 4 shows, this is actually the case. Average excess reserves are substantial both for Lithuania and Bulgaria ($\sim$20% of the required reserves on a daily basis). It also confirms the comments received from practitioners pointing out that reserves become a "hot potato" on the last days of the maintenance period. As predicted by the model, one may also observe that the variability of interest rate is higher at the end of the period (as in Lithuania).

6 Conclusions

This paper investigates interbank markets under currency boards. Empirical evidence for Lithuania and Bulgaria shows that in both countries overnight interest rates fluctuate in a similar systemic fashion. Contrary to empirical results in the literature for other countries, interest rates are decreasing towards the end of the reserve holding period.

We develop a theoretical model illustrating that interest rates can be decreasing or increasing depending upon the aggregate excess reserves in the market. It is also shown that self-fulfilling expectations could lead to different equilibrium interest rate paths.

The proposed solution method could prove practical in solving other general equilibrium finite horizon heterogeneous agents models. Although we acknowledge, that discretization of choice set implemented may lead to multiplicity of equilibria, that is not present in the underlying continuous choice model. Further research is needed in this respect.

Empirical results for Lithuania and Bulgaria match the implications of the model. Interest rates are found to be rising if there is an increase in aggregate deficiency. Since there is abundant liquidity in both of the countries, interest rates are lower at the end of the reserve maintenance period as predicted by the model.

Adding to the debate on the existence of a liquidity effect, no empirical evidence was found that exogenous liquidity shocks, measured as changes in the Treasury account balance at the central bank, have an impact on the interbank market interest rates. It may be the case that Treasury account fluctuations are anticipated, suggesting that a better measure of exogenous liquidity shocks is needed.

References

Bank of Lithuania. (2004). *Law on the Bank of Lithuania.* National Law.

Bartolini, L., Bertola, G., & Prati, A. (2001, July). Banks' reserve management, transaction costs, and the timing of federal reserve intervention. *Journal of Banking & Finance*, *25*(7), 1287-1317. (available at http://ideas.repec.org/a/eee/jbfina/v25y2001i7p1287-1317.html)

Bulgarian National Bank. (2005a). *Law on the Bulgarian National Bank.* National Law.

Bulgarian National Bank. (2005b). *on the Minimum Required Reserves Maintained with the Bulgarian National Bank by Banks* (Ordinance No. 21). Bulgarian National Bank.

Carpenter, S., & Demiralp, S. (2006, June). The Liquidity Effect in the Federal Funds Market: Evidence from Daily Open Market Operations. *Journal of Money, Credit, and Banking*, *38*(4), 901-920.

Ewerhart, C., Cassola, N., Ejerskov, S., & Valla, N. (2004, July). Liquidity, information, and the overnight rate. *European Central Bank Working Paper*(378).

Garbaravicius, T. (2004, June). Impact of treasury transfers on the Lithuanian money market. *Pinigu Studijos, Bank of Lithuania*(2), 5-33.

Gaspar, V., Quiroz, P., & Mendizabal, H. (2004, April). Interest rate determination in the interbank market. *European Central Bank Working Paper*(351).

Guthrie, G., & Wright, J. (2000, October). Open mouth operations. *Journal of Monetary Economics*, *46*(2), 489-516.

Hamilton, J. D. (1996, June). *Measuring the liquidity effect* (University of California at San Diego, Economics Working PaperSeries). Department of Economics, UC San Diego.

Hamilton, J. D. (1997). Measuring the liquidity effect. *American Economic Review*, *87*(1), 80-97.

Hartmann, P., Manna, M., & Manzanares, A. (2001, November). The microstructure of the euro money market. *Journal of International Money and Finance*, *20*(6), 895-948.

Heer, B., & Maussner, A. (2005). *Dynamic General Equilibrium Modeling.* Springer.

Krusell, P., & Smith, A. A. (1998, October). Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, *106*(5), 867-896.

Nelson, D. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica*, *59*, 347-370.

Petrov, B. (2000, December). *Bank reserve dynamics under currency board arrangement for Bulgaria* (Discussion Paper 15). Bulgarian National Bank.

Prati, A., Bartolini, L., & Bertola, G. (2003). The overnight interbank market: Evidence from the G-7 and the euro zone. *Journal of Banking and Finance*, *27*, 2045-2083.

Quirós, G. P., & Mendizábal, H. R. (2006). The daily market for funds in europe: What has changed with the EMU? *Journal of Money, Credit and Banking, 38*, 91-117.

Ríos-Rull, J.-V. (1999). Computation of equilibria in heterogeneous-agent models. In R. Marimon & A. Scott (Eds.), *Computational methods for the study of dynamic economics.* Oxford university Press.

Siaudinis, S. (2003). Effectiveness of modern currency board operational frameworks: discussioncontinued. *Pinigu Studijos*(2), 5-22.

Thornton, D. L. (2001, July/August). Identifying the liquidity effect at the daily frequency. *Federal Reserve Bank of St. Louis Review*, 59-78.

Thornton, D. L. (2006). *The daily liquidity effect* (Working Papers No. 2006-020). Federal Reserve Bank of St. Louis.

Vetlov, I. (2004). Monetary transmission mechanism in Lithuania. In D. G. Mayes (Ed.), *The monetary transmission mechanism in Baltic States* (p. 61-107).