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Using Concept Maps: What do Students Know About Sequences and Series?

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Study purpose

There is a gap in the literature on sequences and series, and it is important to study these topics as students tend to struggle with them (Earls, 2017). These topics are further used in higher mathematical courses, such as but not limited to real analysis. They are used in additional subjects as well, such as physics and computer science. Within calculus, students often struggle with broad concepts such as the limit concept and convergence (Cottrill et al., 1996; Oehrtman et al., 2014).

This paper seeks to add to the existing literature by investigating how students think about sequences and series. In particular, we look to address the research question, "How do students' concept maps of sequences and series differ from an optimal map based on experts' expectations of second semester calculus students?" This paper begins with a look at literature pertaining to sequences and series. We then explain our rationale for using concept maps before describing the study and the results. This paper is an extension of the poster proposal presented in Earls et al. (2022).

Literature Review

When researching student knowledge of sequences and series, it is first important to recognize difficulties they have with understanding prerequisite topics**.** Przenioslo (2006) discusses how sequences taught in relation to other mathematical subjects can negatively alter how students learn and understand them. This issue is much more prevalent in calculus when it comes to the teaching of sequences and limits. The overlap in the ideas of limits and sequences, especially when taught closely together, can cause students to confuse these topics (Roh, 2008). Cottrill et al. (1996) note that students need to look at limits as different processes. Teaching limits in multiple parts will ease confusion and build greater mathematical understanding amongst the students. Williams (1991) argues that students struggle to understand the formal definition of a limit and are restrained by their pre-existing beliefs in mathematics. Difficulties in solving limits are caused by a student´s lack of initial abilities, logical abilities and/or mathematical connection skills (Isnani et al., 2021). Thus, not only the teaching of second semester Calculus must be looked at when figuring out why a student could not correctly solve a sequence or series problem, but also previous mathematics courses such as first semester Calculus or Algebra.

For sequences and series, rather than just applying the procedure to a problem, it is important for students to visualize the concepts to have a greater understanding of the connections between concepts**.** Alcock and Simpson (2005) discuss how understanding high level mathematics relies on visualization of the concepts. Furthermore, if students cannot visualize concepts, they may struggle with understanding difficult topics in mathematics. For example, many students struggle with comprehending sequences and series as they cannot visualize the concept of "infinity." Along with this difficulty surrounding infinity, Martinez-Planell et al. (2012) discuss how students struggle to comprehend how an infinite series is a sequence of partial sums. Genc and Akinci (2020) discuss that having a balance between procedural and conceptual knowledge is essential in reducing errors or learning difficulties when learning of the convergence of series.

Another issue found with student understanding of sequences and series is the confusion in previous mathematical concepts. With a lack of understanding of concepts comes common misconceptions within mathematical topics. Nardi and Iannone (2001) discuss how students are resistant to the fact that tests for convergence can be inconclusive. While performing such convergence tests, Earls and Demeke (2016) found common students' errors in solving series problems such as algebraic manipulation, failure to check assumptions, use of the wrong test, and arriving at the wrong conclusion. Earls (2017) discusses how students have difficulties identifying the contrapositive of the nth term test. Further, Earls (2017) mentions how students may not differentiate between sequences and series because they are interchangeable words in the English language.

Theoretical Framework

As defined by Novak and Canas (2006), “concept maps are graphical tools for organizing and representing knowledge”. Concept maps help us gain insight into a student's concept image, revealing “the organization and structure of an individual’s knowledge within a particular domain” (Williams, 1998, p.414). Williams (1998) states that a concept map will provide some insight into a student's conceptual understanding.

Coutinho (2014) discusses how concept maps have become an insight view into the thought process of students and as a result, have been used as an educational resource because they allow students to organize their thoughts and connect general concepts. Furthermore, Baroody and Bartels (2000) agree that concept maps are a great tool for allowing students to connect concepts and gain a stronger understanding of their ideas.

Concept maps provide information about how individual students relate concepts to form organized conceptual frameworks (Hasemann and Mansfield,1995). Knowing more about how students connect different mathematical concepts can also help researchers in understanding students’ and we conjecture overall learning behavior.

Concept maps can also help researchers in understanding student’s responses as the study conducted by Bolte (1999) reveals. When comparing scores from concept maps and interpretive essays from the same students, Bolte (1999) concluded that concept maps are viable tools for assessing student’s mathematical understanding of concepts. Finally, concept maps allow researchers to see into the thought process of a student but being able to quantify in a clear and consistent way is the challenging part of the process (Moni et al., 2005).

When reviewing both literature on sequences and series and the conceptual framework of concept maps, it is important to note that through concept maps, researchers can see the gap in understanding of students in sequences and series. Common misconceptions studied by Nardi and Iannone (2001) and Earls and Demeke (2016) may be apparent in the concept maps reviewed through a disconnect of concepts.

Methodology

This study was undertaken during the Spring and Summer of 2021 by a research group focused on addressing the research question, “How do students’ concept maps differ from an optimal map based on experts’ expectations of second semester calculus students?” This study builds on the work of Earls (2017) which used concept maps in order to better understand students’ mathematical knowledge of sequences and series. This paper is the result of a pilot study, which will be used as the basis for a more extensive project that will include a larger group of participants and broader data collection effort.

Participants in the preliminary data collection round, used in this paper, included six participants from a small, private college in a Northeastern urban center. All participants had completed their Calculus II courses during the COVID-19 pandemic. Four of the participants completed Calculus II during Spring 2020 while still in high school and two completed Calculus II during the Spring 2021 semester. Both groups of students experienced some online instruction; however, those who completed the course in Spring 2020 experienced a shift from in-person to online instruction, while those who completed the course in Spring 2021 experienced the entire course in the online setting.

In this study, we have used concept maps as our primary mode of data collection. Concept maps are designed to help understand the conceptual knowledge students have of a topic (Williams, 1998). Data collection occurred remotely. Participants were provided with an example concept map in mathematics and then asked if they had any questions. Following this introduction, they constructed their own concept maps. The concept maps were collected from participants.

Once the concept maps were collected, we compared them to an optimal concept map. The optimal concept map was developed using experts in sequences and series, as well as textbooks to provide a picture of what the “ideal” calculus II student would know about sequences and series following their course (Earls, 2017). The optimal concept map served as a foundation for the scoring system used in this project.

In order to compare the optimal and student maps, we developed a scoring scheme using Cronin, Decker, and Dunn (1982) and Bartels (1995). In this study, points were calculated across four categories: concepts, linking words, mathematical definitions, and holistic assessment. Concepts included mathematical objects and properties. Concepts were scored on presence of substantive series and sequence ideas as well as their connections to each other. Across all categories, deductions were applied for incorrect concepts and connections. For this round of coding, each map was coded by the team of six researchers. The team then met to reconcile the scoring of each map until all scores reached a consensus. The rubric used for scoring concept maps can be found in Table 1, and an example of how the optimal map was scored using this rubric is given in Figure 1.

Results

The preliminary data for this study were seven concept maps - one map created by experts to be an optimal map, and six created by students who had taken second semester calculus. Each map was assigned a score based on the method given above, with sub scores for concepts and mathematical definitions, linking words, presentation and deductions for incorrect concepts or links.

The optimal expert map was assigned a total of 63.5 points. Of these points, 48.5 were assigned for concepts and mathematical definitions, 13 for linking words and 2 points in presentation for exhibiting a high level of understanding and being clearly drawn. No deductions were taken from the optimal map. A score of 1 for presentation indicates that the map does suggest some understanding, and is somewhat clearly drawn, while a score of 0 indicates a map that is unclearly drawn or shows little-to-no understanding by the participant. The scores from the six student maps are given in Table 2.

The maps were then characterized on a scale from poor to excellent. The scores for the maps ranged from 11 to the optimal score of 63.5, giving a range of 52.5. Dividing this range into three sections, we get the following score ranges for excellent, good, average, and poor maps, also shown in Table 3.

Excellent: Score greater than or equal to 63.5 (optimal map score)
Good: Score between 46 and 63.5 (including 46)
Average: Score between 28.5 and 46 (including 28.5)
Poor: Score between 0 and 28.5 (including 0)

Using this scale, one of the student participants, or around 16.67% of the participants, produced an excellent map, and another one of the students (again, around 16.67%) produced a good map. On the other hand, three of the student participants, or 50%, produced an average map, and one student participant, or around 16.67%, produced a poor map.

Conclusions

The map rated “poor,” shown in Figure 2, was missing several key concepts, as can be seen from the low concepts score. There were no mentions of any tests for series convergence. This could mean that tests for series convergence were forgotten, or that this student did not view convergence tests as a concept. Linking words were not used, indicating that this student struggled to tell us how concepts were related. It is also worth noting that this student had sequences and series as one concept bubble. This could mean that the student does not see the difference between the two topics. Failing to see the difference could be a result of the terms being synonyms in the English language, as mentioned in Earls (2017).

The “average” maps (an example is in Figure 3) showed more concepts than the poor maps, but still struggled a bit to join all the concepts with linking words. This suggests that these students were familiar with many of the concepts and may understand how certain concepts are related, but not others. Many of the missing linking words were related to tests for convergence, indicating that students may not see how the tests relate back to convergence. One of the average maps was similar to the poor map in that one concept bubble was used for both sequences and series.

The “good” and “excellent” maps both were consistent in meeting experts’ expectations for concepts as well as linking words between the concepts. This indicates that both of these students were familiar with the concepts taught as well as how the concepts were linked together. It is also worth noting that the “excellent” map had a higher score than the optimal map. This suggests that this student may have learned more than was expected in the course, either through additional problems or reading. The “excellent” map is shown in Figure 4.

Educational implications

Once we have more knowledge of how students think about sequences and series, new curriculum materials and teaching strategies can be developed to help students better understand the concepts and how they link together, as mentioned in Earls (2017). Also, it is worth investigating if student concept maps improve after taking more proof based advanced mathematics courses, such as real analysis, also mentioned in Earls (2017). Since previous courses play such a vital role in determining sequence and series convergence, it is worth investigating changes to prerequisite courses to better help students when they take second semester calculus.

Table and Figures

Table 1
Rubric for grading concept maps

Concepts	Concepts are objects, events, situations, or properties of things Important concepts are: "Series," "Sequences," "Divergence," "Convergence"	• Score 2 points for each concept that is connected to at least one other concept by a linking word • Score 1 point for each concept that is connected to at least one other concept without a linking word • Score 5 points for each important concept that is present	
Linking Words	Relationships between concepts are represented by connecting word(s) and phrases written on the line joining any two concepts. • A Simple Proposition is a simple English word or phrase • · A Scientific Proposition is a phrase or statement that is composed of technical or scientific word(s)	• Simple Propositions score 1 point for each word or phrase ○ give 0.5 points for repeated use of Simple Propositions • Scientific Propositions score 2 points for each proposition ○ give 1 point for repeated use of Scientific Proposition	
Mathematical definitions/ examples	Mathematical definitions or examples are not considered to be concepts, but they go more into detail about a concept	• 0.5 points for each mathematical definition or example	
Presentation	Concept map clarity and showing an understanding of topics	• 2 points: high level of understanding and clear to read • 1 point: some understanding and somewhat clear to read • No points: not clear to read, does not present understanding of the concepts	
Deductions	Deductions are applied for every objectively incorrect concept or connection	• -1 point for incorrect concept • -0.5 points for incorrect connection	

Figure 1
Scoring of the optimal concept map

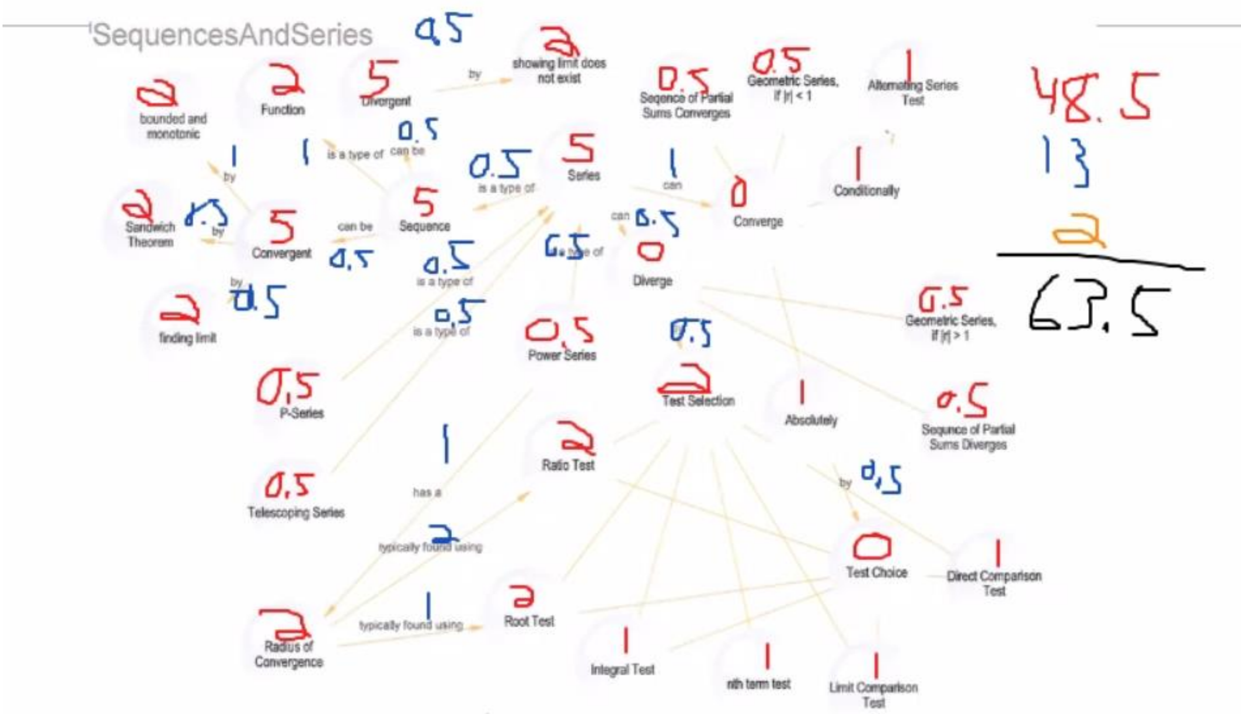


Table 2
Concept map and score breakdown

Map Number	1	2	3	4	5	6	Optimal
Concepts and Mathematical Definitions	48.5	40.5	37.5	31	11.5	28	48.5
Linking Words	16	10.5	3	7	0	9	13
Presentation	2	1	1	1	0	1	2
Deductions	0	-1	-3	-3	-0.5	-3.5	0
Total	66.5	51	38.5	36	11	34.5	63.5

Table 3
Score ranges for Likert-like scale

Name	Range
Excellent	Greater than or equal to 63.5 (optimal map score)
Good	Score between 46 and 63.5 (including 46)
Average	Score between 28.5 and 46 (including 28.5)
Poor	Score between 0 and 28.5 (including 0)

Figure 2
Concept map rated as "poor"

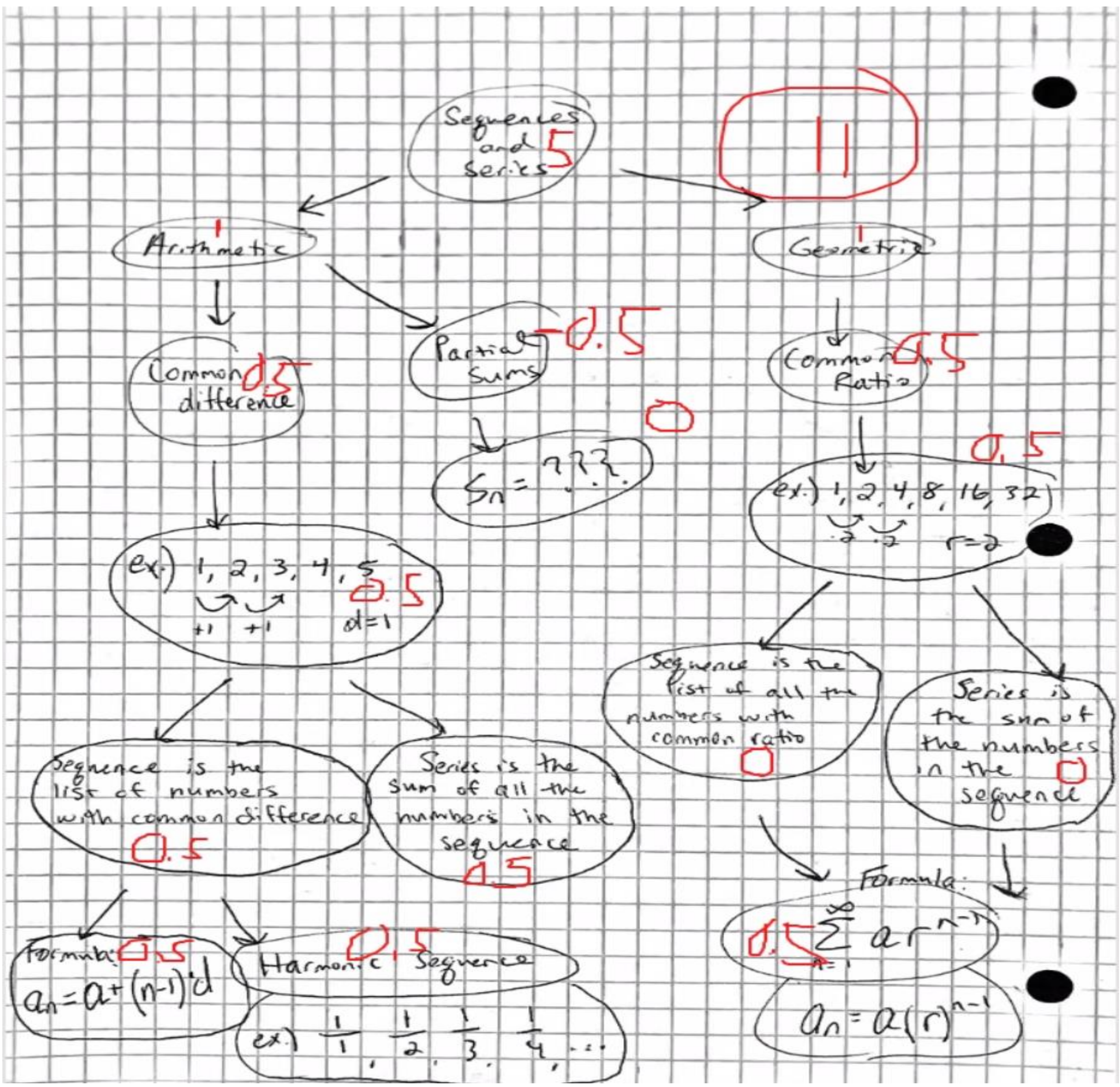


Figure 3

Concept map rated as "average"

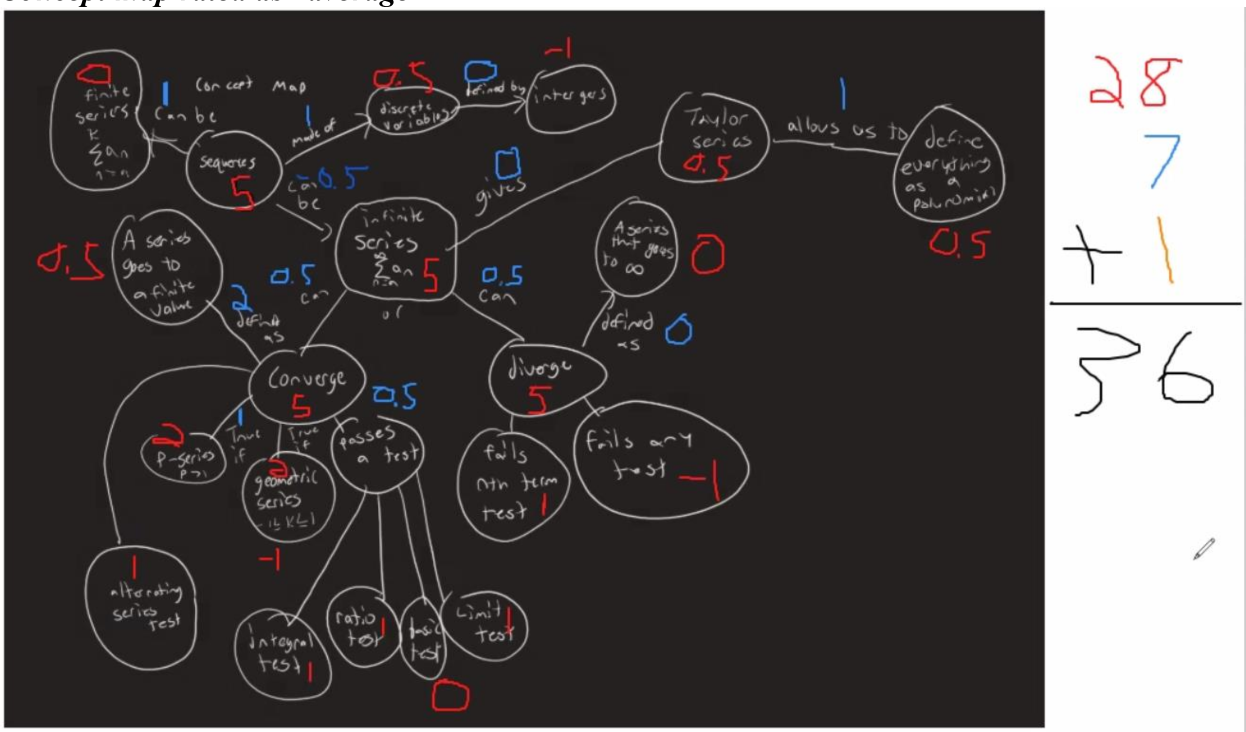
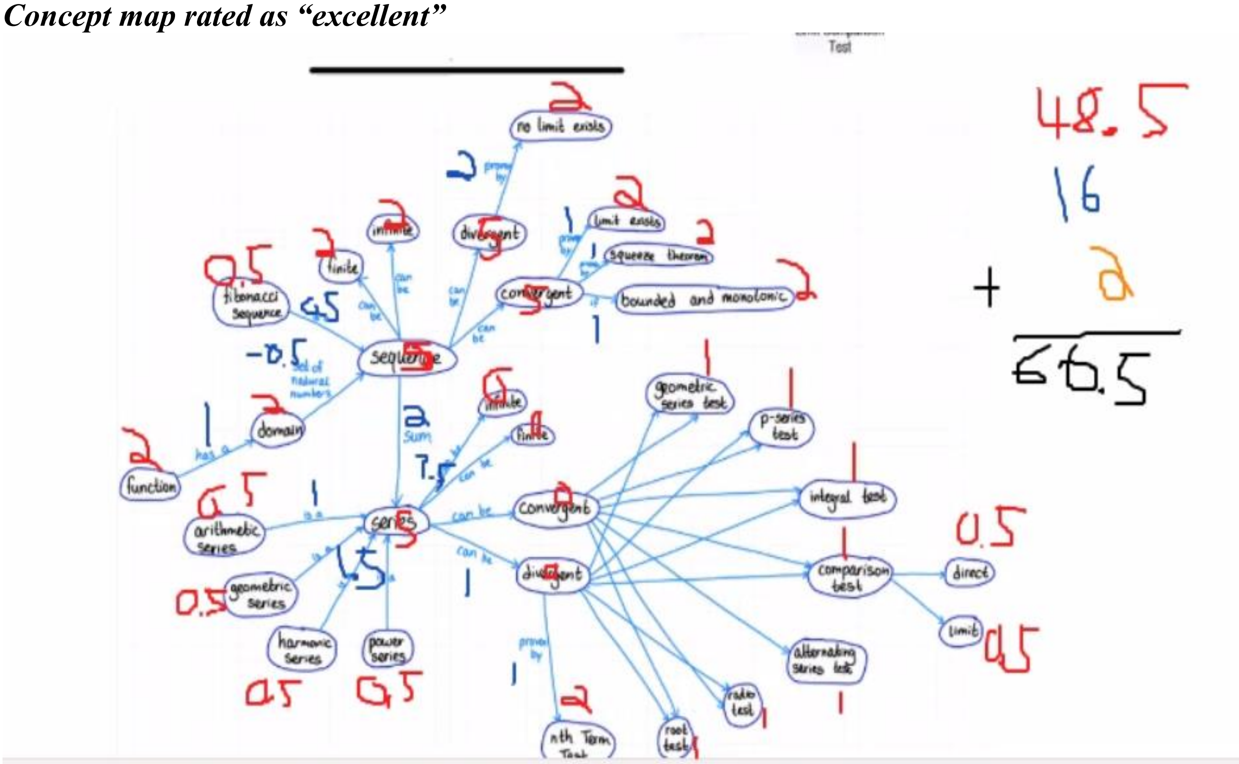


Figure 4

Concept map rated as "excellent"



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