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CONTENTS

A POSITIVE SPATIAL EQUILIBRIUM MODEL OF BROILER MARKETS: A SIMULTANEOUS-EQUATION APPROACH*

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I. Introduction

The purposes of this study are (1) to present a model of spatial equilibrium that may be simultaneously and empirically estimated and tested from time series data, and (2) to empirically study the effect of predetermined variables such as income, factor price, or transportation costs on spatial equilibrium in a simple three-market model for broilers.

A spatial equilibrium model is a competitive economic model in which the state of the economic system may be described by a set of simultaneous equations including the aggregate demand for each good by consumers for each market, the aggregate supply of each good by producers for each market, the distribution activities over space and the equilibrium conditions. The equilibrium process in the comparative static economy is that profit, which results from the excess price differential over transportation costs, will cause commodities to flow among markets so that supply equals demand in every spatially separated but not economically isolated market. It is assumed that in each market each consumer acts so as to maximize his utility, each producer acts so as to maximize his profit, and perfect competition prevails.

In recent years, spatial equilibrium analysis has been formulated as a quadratic programming problem. Takayama and Judge [12] derived the model by maximizing the so called "net social payoff" subject to linear demand and supply functions in the various markets. Their study is largely conceptual due to the assumption that linear demand and supply functions are known or given. When applying this model, many researchers [7, 8, 14] have estimated demand and supply functions based on

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methods that are exogenous to the structure of spatial equilibrium. The supply function of each market may be estimated as a single regression equation independent of the supply functions in other markets and of the demand function in the same market [14]. Or market supply functions may be estimated by the micro-to-macro build-up approach from the linear programming step supply function [8]. The demand functions are usually derived [7, 8] from Brandow's demand elasticities or slopes that were estimated in 1961 [2].

Normative approaches of two types are often used in spatial equilibrium analysis. One combines the exogenously and separately estimated equations and considers them simultaneously [8, 14]. This method permits an examination of the perfection of a group of markets and provides reference for efficient marketing. Another method uses estimated demand functions in an activity model and provides reference for farmers in production or adjustment decisions [7]. However, a positive model is required, if the purpose is to produce the quantitive economic statements that explain behavior of existing spatially separated competitive markets and to predict future courses of economic variables.

Within the context of positive economics, equilibrium conditions and definitional equations place a restriction on the parameters which are being estimated. It should be noted that if a simultaneous solution of exogenously estimated structural equation were intended as a positive analysis, it would be subject to Haavelmo's criticism. He emphasized in a 1943 article [6]:

"A most dangerous procedure in estimating parameters in systems of stochastic equations is to fit each equation separately without regard to the fact that the variables involved are, usually, assumed to satisfy, simultaneously, a number of other stochastic relations."

Broiler markets, in our analysis, are considered spatially competitive. Broiler production in the southern states has been increasing tremendously since 1957 while the northeast region has changed from a surplus to a deficit area. It is believed that the change is associated with decreasing farm price of broilers and increasing feed price in the northeastern states relative to that of the southern states. In view of these changes we are interested in studying structural changes of the broiler economy. Specifically, the objective is to analyze the effect of a change in feed price in each region on broiler production, price of broilers, shipment of broilers among regions and derived demand for feed.

In this paper, a general simultaneous-equation model of spatial equilibrium is presented and discussed. In addition a specific model of spatial equilibrium is proposed to study the structural change of broiler markets in the aggregate with emphasis on regional interrelationships. There is no intention of studying the broiler industry at the producing or retail firm level.

II. The Simultaneous-Equation Model

Assuming n spatially separated markets or regions, the linear demand and supply functions are hypothesized for a single commodity in the i-th region as

$$D_i = \alpha_i p_i + a_{io} z_o + a_{i1} z_1 + \dots + a_{ik} z_k + u_i \quad (1)$$

and

$$S_i = \beta_i p_i + b_{io} z_o + b_{i1} z_1 + \dots + b_{ik} z_k + v_i \quad (2)$$

where p_i is the price and z's are predetermined variables such as income, factor price, etc., and u_i and v_i are stochastic variables which may be assumed to have zero means, constant variances over time and zero autocorrelations, but may not be independent of price. The parameters are α, β, a's and b's. There are n such demand equations.

The spatial equilibrium models consider the interregional flow of commodities and demand and supply may be decomposed into shipments:

$$S_i = \sum_j x_{ij} \qquad i = 1, 2, \ldots, n \qquad (3)$$

and

$$D_j = \sum_i x_{ij} \qquad j = 1, 2, \ldots, n \qquad (4)$$

where x_{ij} denotes quantity shipped from i-th region to the j-th region. Thus the quantity produced in the i-th region S_i may not be the same as the quantity consumed in the same region D_i. There are n equations for each type of (3) and (4).

Before reaching equilibrium, commodity flow is considered possible whenever the price differential between any pair of regions exceeds the transportation costs t_{ij}. Thus, whether the product in the i-th region should be sold in j-th region depends on the following per unit loss (or per unit gain):

$$r_{ij} = t_{ij} - (p_j - p_i) \qquad \text{all } i, j \qquad (5)$$

If r_{ij} is positive, shipping one unit from i-th region to j-th region will result in a loss of r_{ij}. If r_{ij} is negative, a per unit profit results from shipping one unit from i to j. There are n^2 such definitional equations. Since in the same region the transportation cost is zero, r_{ii} is always assured to be zero. Thus only n(n-1) equations are required to determine n(n-1) interregional flows of commodities.

If equilibrium of the competitive economy is disturbed by an exogenous factor such as increasing demand due to increased population, the process of restoring equilibrium requires the existence of a profit ($r_{ij} < 0$). A profit will continue to induce commodity flow ($x_{ij} > 0$) until the price structure changes through equations (3) and (4) and then (1) and (2), which in turn eliminates the profit ($r_{ij} = 0$).

For a competitive equilibrium to hold, either there is no profit ($r_{ij} = 0$) or there is no flow of commodities ($x_{ij} = 0$)[1]. The above statement may be translated into mathematical language as:[2]

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[1] These two assertions are equivalent to the Kuhn-Tucker conditions of the quadratic programming formulation of spatial equilibrium. See [12].

[2] In practice, due to error in measurement and imperfect competition, the sample data do not necessarily fulfill this condition, especially in the supply-demand adjustment over space in a short period of time. Under these conditions the per unit loss function may be redefined as

(5a) $\quad r^*_{ij} = (t_{ij} + c_{ij}) - (p_j - p_i) + d_{ij}$

where c_{ij} denotes opportunity cost defined as the possible profit that the producers would earn if they engaged in other economic activities with the same effort as shipping one unit of the commodity from i to j, and d_{ij} is the possible error in measuring prices. Thus c_{ij} is due to imperfect competition and d_{ij} to errors of observation which may have zero expectation. The equilibrium condition is then

(6a) $\quad r^*_{ij}\, x_{ij} = 0.$

$$r_{ij}x_{ij} = 0 \qquad \text{all } i, j \qquad (6)$$

Equation (6) is the equilibrium condition. There are n^2 such equations. Since within the same region r_{ii} is always assured to be zero, only $n(n-1)$ equations are required for the interregional flow activities.

In total there are $2n+2n^2$ equations with the same number of jointly dependent variables which specifies a complete system of equations.[3]

The model specified is for a single product case, but it may be generalized into a multiproduct case of general equilibrium. Supply and demand functions may be formulated as functions of many commodity prices and some predetermined variables. All variables are jointly determined with possible direct and cross elasticities.

In summary, the spatial equilibrium model involves equations for demand and supply (1) and (2), distribution activities (3) and (4), and equilibrium conditions (6), in which the loss variable is defined in (5).

III. Identification and Estimation Procedures

The demand and supply functions of the model are identifiable as long as transportation costs t_{ij}'s do not appear in the demand and supply functions. In each equation, there will be two endogenous variables and at most k+1 exogenous variables, and at least $n(n-1)/2$ transportation costs which are exogenous not entering the equation. If there are two or more markets, the number of exogenous variables not appearing in the equation but appearing in the system is larger than or equal to one. Thus the necessary condition for identification is met.

To estimate the demand and supply parameters which are identifiable, the simultaneous equation approach should be used such as two-stage or three-stage least squares. If the system of equations is large, as will obtain where many markets or many products are involved, and if available time series data are rather short, difficulties will arise since the estimation method used requires estimates of the moments of reduced form disturbances. The reduced form equation estimation requires that price be regressed on all predetermined variables including all transportation costs. With a large number of regions, multicollinearity may be encountered because (a) transportation costs for each pair of regions may be correlated and (b) the number of predetermined variables may easily grow in excess of the number of observations. This is a serious problem, for it implies that the moment matrix of predetermined variables is singular or near singular, and the inverse of this matrix, which is needed in the estimation process, does not exist or is uncertain. As a result the usual procedure of estimating simultaneous equations breaks down.

However, a solution to the foregoing problem is still possible. The large number of predetermined variables in the reduced form equation estimation may be

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3 An augmented model may contain n^2 equations for each set of (5) and (6) including r_{ii}'s and the demand price p_i may be different from supply price (say p^i). Thus there will be $4n+2n^2$ equations and $4n+2n^2$ variables, and the system is still complete. An abbreviated model may be obtained if equations (3) and (4) are substituted into equations (1) and (2). In this case, there will be only $2n^2$ equations and $2n^2$ variables, and the model is still complete (e.g., see Table 6).

condensed into their principal components. That is, one may use relatively few principal components of predetermined variables as instrumental variables in the two-stage least-squares estimation. For the detail alternative treatments and discussion, see Kloek and Mennes[9].

IV. Spatial Equilibrium and Static Analysis

A. Existence and Uniqueness

In the model specified above, there are $2n+2n^2$ equations in which n(n-1) equations are bilinear. There will be $2^{n(n-1)}$ solutions satisfying equations (1) through (6) including any possible real and complex number solutions. The complex number solution is not possible because the quadratic equations are homogeneous and may be factorized into two linear equations. However, among $2^{n(n-1)}$ real solutions, most of the basic solutions have some variable or variables which take negative values and are not economically feasible solutions. Thus, to obtain an economically meaningful equilibrium solution, a nonnegativity restriction must be imposed on all jointly dependent variables. As a consequence, questions about existence and uniqueness of spatial equilibrium arise. However, in the light of the theorem of Arrow and Debreu [1], Debreu [3], and McKenzie [10], we may tentatively conclude that if the economy has only one equilibrium, then the system of equations specified above will have an unique nonnegative solution. This tentative conclusion will be further confirmed when the iteration method of solving the system of equations is introduced.

B. The Reduced Form Equations Associated with Equilbirium

(1) Abbreviated Structural Equations

To solve for the reduced form equations, the model may be simplified by equating equations (1) and (3), and (2) and (4) to reduce the number of variables and equations to obtain

$$\sum_{j=1}^{n} x_{ij} - \alpha_i p_i = \sum_{m=0}^{k} a_{im} z_m + u_i \qquad \text{all } i \qquad (7)$$

and

$$\sum_{i=1}^{n} x_{ij} - \beta_j p_j = \sum_{m=0}^{k} b_{jm} z_m + v_j \qquad \text{all } j \qquad (8)$$

Then, equations (7) and (8) together with equation (5) may be expressed by compact matrix notations as

$$\begin{bmatrix} O & H & I \\ G & B & O \end{bmatrix} \begin{bmatrix} X \\ P \\ R \end{bmatrix} = \begin{bmatrix} T \\ \Gamma Z \end{bmatrix} + \begin{bmatrix} O \\ U \end{bmatrix} \qquad (9)$$

and the whole set of equation (6) may be expressed as

$$R'X_0 = 0 \qquad (10)$$

under the nonnegative conditions:

$$X, R, P \geqslant 0 \qquad (11)$$

where:

$$X = (X_o' \mid X_{oo}')'$$
$$= (x_{12}\, x_{13} \dots x_{1n}\ x_{21}\, x_{23} \dots x_{n1} \dots x_{n\,n-1} \mid x_{11}\, x_{22} \dots x_{nn})'$$
$$R = (r_{12}\, r_{13} \dots r_{1n}\, r_{21}\, r_{23} \dots r_{n1} \dots r_{n\,n-1})'\ n(n-1)x1$$
$$T = (t_{12}\, t_{13} \dots r_{1n}\, r_{21}\, r_{23} \dots r_{n1} \dots r_{n\,n-1})'\ n(n-1)x1$$
$$P = (p_1\, p_2 \dots p_n)'\ nx1$$
$$Z = (z_1\, z_2 \dots z_k)'\ kx1$$
$$U = (u_1\, u_2 \dots u_n\, v_1\, v_2 \dots v_n)'\ 2nx1$$

B is a (2n x 2n) diagonal matrix with elements slopes of demand and supply equations,
Γ is a (2n x k) matrix of a's and b's,
G is a (2n x n^2) matrix with elements 0's and 1's permuting for the distribution of commodities,
H is a (n(n-1) x n) matrix of 0's, 1's and (-1)'s permuting for price differentials, and
I is a (n(n-1) x n(n-1)) identity matrix.

Equation (9) consists of equations (5), (7), and (8). Equation (10) is the sum of the n(n-1) equations of (6) and provides the same function as (6) because all the variables are restricted to be nonnegative by (11) and hence no cancellation or offsetting of negative and positive values in equation (10).

(2) The Solution Method

The procedure for obtaining a solution when equations have been simultaneously estimated and the predetermined variable z's are given is first to assign n(n-1) variables out of n(n-1) r_{ij}'s and n(n-1) x_{ij}'s, with value zero. Next eliminate variables assigned zero from equation (9) and solve the system for remaining $n+n^2$ variables. As previously mentioned, not all $2^{n(n-1)}$ solutions will fulfill the restraints (11), and only the one which satisfies (11) will be chosen.

In practice, the Gauss elimination or pivoting method may be used for solving simultaneous linear equations. Nonnegative restraints are imposed and controlled by choosing the smallest nonnegative pivot ratio as if solving a linear programming problem. Condition (10) will be controlled by watching the counterparts r_{ij} and x_{ij} so that both do not have positive solutions. In linear programming terminology, if r_{ij} is in the basis, do not introduce x_{ij} and vice versa. Thus, the procedure is equivalent to Wolf's quadratic programming algorithm [15], which was also adopted by Takayama and Judge [12]. Thus solving the 2nd order simultaneous equations (1) through (6) is equivalent to solving Takayama and Judge's quadratic programming problem.[4] Since the solution will be identical with Takayama and Judge's, the existence and uniqueness of a spatial equilibrium is confirmed.

(3) The Reduced Form Equations

When an equilibrium solution is obtained those variables taking value zero from (10) may be deleted and a set of reduced form equations obtained. If we let asterisks denote the submatrix after the deletion of such n^2 variables and the

[4] The exactly identical Takayama-Judge simplex tableau may be obtained if demand and supply prices are independently set up. We prefer to have the simplex tableau set up in the most compact form and yet containing the necessary information.

corresponding n^2 column of coefficients, then (9) associated with an equilibrium solution may be written as

$$\begin{bmatrix} O & H & I_* \\ & & \\ G_* & B & O \end{bmatrix} \begin{bmatrix} X_* \\ P \\ R_* \end{bmatrix} = \begin{bmatrix} T \\ \\ \Gamma Z \end{bmatrix} + \begin{bmatrix} O \\ \\ U \end{bmatrix} \quad (12)$$

The reduced form equations are then

$$\begin{bmatrix} X_* \\ P \\ R_* \end{bmatrix} = \begin{bmatrix} O & H & I_* \\ & & \\ G_* & B & O \end{bmatrix}^{-1} \begin{bmatrix} T \\ \\ \Gamma Z \end{bmatrix} + V \quad (13)$$

where V is a ($n^2+2n \times 1$) vector of the reduced form disturbances. Equation (13) is known only when direction of commodity flows are known, that is, those x_{ij}'s which are zero are known and may be deleted. However, the precise quantity of flow is not necessarily known. Direction of flow is primarily determined by the magnitude of exogenous variables T and Z. Although the quantity of shipment may be different, the direction of shipment may remain unchanged when the exogenous variable falls within certain ranges. Therefore, coefficients of reduced form equations are rather stable in the neighborhood of equilibrium. It is reasonable to study the effect of infinitesimally small changes in T or Z on X_*, R_* and P by taking the partial derivative of say p_i with respect to z_j from reduced form equations.

Locally stable reduced form coefficients with respect to the infinitesimal change of an exogenous variable does not imply globally stable. If the change in Z or T is significantly large, the direction of commodity flow may change and a new equilibrium must be relocated to determine the new flow pattern and the associated new set of reduced form equations.

V. Applications of the Model to the Broiler Markets

A. A Model for Broilers

Alternative models were formulated to estimate supply functions of broilers in the Northeast, the South[5] and the rest of the U.S. The variables used in explaining supply include: current price, feed per pound of broilers, feed price, lagged broiler price, lagged regional production, and lagged national broiler production and consumption (see Tables 1-3).[6] More than 20 combinations of the above variables were used as regressors but most of the results were unsatisfactory. With successive modification of the economic models, it was postulated that the supply of broilers never depended on current price. At the national level, it has been shown that change in broiler supply from the preceding year is positively related to lagged profitability [13]. Thus, current supply of broilers is related to the preceding year's supply and profitability. If spatial flow of broilers is also considered, lagged supply in other regions may affect current regional supply. In the final analysis, therefore,

[5] The Northeast region consists of New England, Mid-Atlantic states and Virginia. The Southern region includes the following states: N.C., S.C., Ga., Fla., Ala., Miss., La., and Ark. Hereafter the North will be region one, the South two and the rest of the U. S. three.

[6] See Appendix A for method of computation and sources of data.

Table 1. Basic Endogenous Variables for Ready-to-Cook Broilers

	Broiler Farm Price			Available for Consumption			Consumption		
	North	South	Rest of the U.S.	North	South	Rest of the U.S.	North	South	Rest of the U.S.
	(cents per lb.)			(millions of lbs.)					
1956	27.5	25.8	28.9	877	1207	792	867	413	1596
1957	27.1	24.8	27.5	939	1496	807	904	466	1812
1958	26.6	24.1	26.9	1018	1820	931	1125	544	2100
1959	24.0	20.8	23.6	972	2063	962	1189	579	2229
1960	24.5	22.2	24.5	998	2221	926	1222	605	2318
1961	21.1	17.8	21.5	986	2609	1020	1359	677	2579
1962	22.9	19.7	22.1	1011	2712	972	1396	699	2659
1963	21.9	18.9	21.6	1066	2931	964	1462	734	2765
1964	21.5	18.5	21.1	1081	3062	974	1505	758	2854
1965	22.7	19.6	22.5	1151	3437	1033	1649	838	3134
1966	22.5	19.9	21.9	1226	3880	1085	1816	925	3450
1967	20.6	17.0	20.1	1237	4038	1101	1861	953	3562

Table 2. Predetermined Variables

	Lagged U.S. Broiler Production	National Disposable Income	Transportation Costs			Lagged Profitability		
			South to West	South to North	North to West	North	South	Rest of the U.S.
	(millions of lbs.)	(billions of dollars)	(per cwt)	(per cwt)	(per cwt)			(cents per lb.)
1956	2248	290792	1.57	1.82	1.78	16.40	13.48	15.98
1957	2876	306510	1.55	1.79	1.75	8.80	6.45	10.74
1958	3242	315476	1.52	1.77	1.73	9.52	6.56	10.51
1959	3769	334935	1.50	1.75	1.70	7.48	4.90	9.04
1960	3997	346113	1.47	1.73	1.68	6.36	3.23	6.90
1961	4145	360677	1.45	1.71	1.66	8.61	6.31	9.12
1962	4615	380430	1.45	1.71	1.63	5.50	2.56	6.46
1963	4695	398251	1.45	1.71	1.61	6.48	3.80	6.58
1964	4961	427524	1.45	1.71	1.58	4.92	2.64	5.92
1965	5117	462030	1.40	1.68	1.56	4.62	2.40	5.58
1966	5621	501928	1.35	1.66	1.54	6.45	4.19	7.38
1967	6191	536815	1.30	1.65	1.51	4.56	2.59	4.69

regional supply is hypothesized as a function of lagged national supply Q_{t-1} and lagged profitability $R_{i\ t-1}$. Hence:

$$S_{it} = b_{io} + b_{i1}Q_{t-1} + b_{i2}R_{i\ t-1} + v_{it}$$

where v_{it} is the disturbance term. Lagged regional profitability in this study is defined as the lagged broiler price $(p_{i\ t-1})$ minus the major cost of production. The major cost is calculated as regional lagged feed price $(p_{fi\ t-1})$ multiplied by lagged feed per pound of broilers $(F_{i\ t-1})$.[7] In equation form, the profitability relation is

[7] The cost analysis would be in detail if other costs were included such as chick costs, medication and grower payments. However, from the regional cost advantage point of view, only relative profitability between regions is necessary. Hence only feed costs are included in the regional profitability function. Other components of total costs vary between regions in almost the same manner as do feed costs. Feed costs represent over 60% of the total cost of a finished bird and are approximately 60% of total costs even after adjusting feed price for integrated operations.

$$R_{i\,t-1} = p_{i\,t-1} \cdot F_{i\,t-1}\, p_{fi\,t-1}$$

It implies producers will respond to profitability but are indifferent as to whether the increase in profitability is due to the increase in broiler price or decrease in feed price, or improvement in technology.

Table 3. Variables Used in Computing Profitability

	Lagged Broiler Farm Price			Lagged Feed per Pound of Broilers	Lagged Feed Price		
	North	South	West		North	South	West
1956	35.2	33.6	34.7	3.90	4.82	5.16	4.80
1957	27.5	25.8	28.9	3.84	4.87	5.04	4.73
1958	27.1	24.8	27.5	3.67	4.79	4.97	4.63
1959	26.6	24.1	26.9	3.84	4.98	5.00	4.65
1960	24.0	20.8	23.6	3.63	4.86	4.84	4.60
1961	24.5	22.2	24.5	3.44	4.62	4.62	4.47
1962	21.1	17.8	21.5	3.32	4.70	4.59	4.53
1963	22.9	19.7	22.1	3.45	4.76	4.61	4.50
1964	21.9	18.9	21.6	3.43	4.95	4.74	4.57
1965	21.5	18.5	21.1	3.41	4.95	4.72	4.55
1966	22.7	19.6	22.5	3.23	5.03	4.77	4.68
1967	22.5	19.9	21.9	3.49	5.14	4.96	4.93

On the demand side, aggregate quantity demanded in each region is postulated as a linear function of current broiler price p_{it} and current total national disposable income y_t:

$$D_{it} = a_{i0} + a_{i1} p_{it} + a_{i2} y_t + u_{it}$$

where u_{it} is the disturbance term. The total national disposable income may be thought of as average per capita disposable income multiplied by population and thus per capita disposable income and populations are considered as factors causing increased demand.[8]

The definitional equations include 6 equations of the decomposition of supply and demand into shipments (3) and (4) and 6 equations of the per unit loss (5). The 6 equilibrium conditions are defined to be of type (6). Thus there are in total 18 equations in which 12 equations are linear and 6 equations are bilinear (see Table 6).

B. The Estimated Structural Equations

Although current regional broiler prices are in fact not used in explaining supply, they are simultaneously determined by the regional demand, shipments, and predetermined quantity of supply in each region. That is, the broiler farm price that fulfills the demand equation should also fulfill per unit loss equation (5) which in turn will be considered in the equilibrium condition (6). Using the two-stage least squares procedure, we first regress the regional broiler farm prices on all the

[8] Regional disposable income was used in the initial analysis and then deleted in the final analysis because, (1) regional disposable income is highly correlated with national disposable income, and alternative results are very close, and (2) if the regional disposable income is used, the reduced form equations suffer from multicollinearity and is very expensive in terms of degree of freedom.

Table 4. Estimated Regional Supply and Demand Relations for Broilers (1956-1967)
By Two-Stage Least Squares

Region	Supply or Demand	Constant Term	National Disposable Income	Lagged National Supply	Regional Farm Price	Lagged Regional Profit-ability	R^2	Computed F Statistics[a]
		(Millions of lbs.)	(Billions of dollars)	(Millions of lbs.)	(Cents per Pound)			
North	S_1	536.1950		0.1066 (0.0202)[b]		7.1109 (7.0915)	0.8772	32.134
South	S_2	−1436.6000		0.8879 (0.0475)		50.9301 (17.4122)	0.9879	367.7505
Rest of the U.S.	S_3	523.4655		0.0909 (0.0223)		6.1178 (8.2029)	0.8614	27.9649
North	D_1	902.8352	0.00312356 (0.00030828)		−31.7407 (10.2671)		0.9781	201.4295
South	D_2	288.2400	0.00173915 (0.00015478)		−13.5484 (4.3070)		0.9804	225.5702
Rest of the U.S.	D_3	1197.217	0.006235600 (0.00079306)		−44.9175 (22.2277)		0.9649	123.6707

[a] The value in the F-table for degrees of freedom (2,9) at the 1% significance level is 8.02.
[b] The figures in parentheses are standard errors.

predetermined variables including lagged regional profitabilities, transportation costs of broilers among three regions, lagged national broiler production, and national disposable income to obtain a set of reduced form equations. This is done in order to obtain the "calculated value" of regional farm prices. These prices are then used in the 2nd stage as regressors,[9] combining other predetermined variables to estimate regional demand and supply functions.

The estimated structural equations are given in Table 4. The figures in parentheses are standard errors of the coefficients. T-value for significant tests may be calculated by dividing the estimated coefficient by the standard error. The results show that lagged quantity as well as profitability are very significant variables in explaining current supply at the one percent significance level. Also, all the F-tests show highly significant association between regressand and regressors at the one percent significance level. In the demand equations, the variables are highly associated; coefficients are significant; and signs are compatible with theory.

C. The Predicted Competitive Spatial Equilibrium

When values of the predetermined variables are given in each year, changes in supply and shifts in demand may be calculated. The results of these calculations are given in Table 5. Estimated regional supplies are very close to actual observations given in Table 1. The increasing values for the intercept of demand functions indicates that demand functions are shifting upward due to an increase in per capita disposable income and population.

Given the fixed supply of broilers and a downward sloping demand function in each year, the competitive process of equilibrium results in the distribution of a given quantity of supply to consumers in each region so that producers' revenue is a maximum and consumer expenditure a minimum. The equilibrium solution may be

Table 5. Estimated Changes of Supply and Demand

	Supplies			Demands		
				$D_1=a_1-31.7407p_1$	$D_2=a_2-13.5484p_2$	$D_3=a_3-44.9175p_3$
	North	South	West	a_1	a_2	a_3
1956	892.45	1245.94	825.57	1811.14	793.97	3010.48
1957	905.35	1445.50	850.60	1860.24	821.31	3108.49
1958	949.49	1776.97	882.46	1888.24	836.90	3133.22
1959	991.16	2159.45	921.37	1949.03	870.74	3285.74
1960	1007.50	2276.84	929.01	1983.94	890.18	3355.44
1961	1039.28	2565.11	956.04	2029.43	915.51	3446.25
1962	1067.26	2791.44	982.49	2091.32	949.87	3569.43
1963	1082.76	2925.62	990.50	2146.80	980.86	3680.55
1964	1100.02	3102.73	1010.64	2238.23	1031.77	3863.06
1965	1114.52	3229.02	1022.74	2346.01	1091.78	4078.25
1966	1181.26	3767.68	1079.56	2470.64	1161.17	4327.04
1967	1217.85	4192.30	1114.92	2579.61	1221.84	4544.58

[9] The terms regressand and regressors are used instead of conventional dependent and independent or explanatory variables because (1) the regressors are not necessarily independent but are jointly dependent, and (2) some of the regressors in the second stage estimation are "calculated" values of jointly dependent variables. For distinction among terms, see Goldberger [5; pp. 213-215].

Table 6. The Spatial Simplex Tableau for the Three-Market Model

			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	Basis	B_0	X_{12}	X_{13}	X_{21}	X_{23}	X_{31}	X_{32}	X_{11}	X_{22}	X_{33}	p_1	p_2	p_3	r_{12}	r_{13}	r_{21}	r_{23}	r_{31}	r_{32}
1	13	t_{12}										−1	1		1					
2	14	t_{13}										−1		1		1				
3	15	t_{21}										1	−1				1			
4	16	t_{23}											−1	1				1		
5	17	t_{31}										1		−1					1	
6	18	t_{32}											1	−1						1
7	0	$D_1(z)$			1		1		1			$-\alpha_1$								
8	0	$D_2(z)$	1					1		1			$-\alpha_2$							
9	0	$D_3(z)$		1		1					1			$-\alpha_3$						
10	0	$S_1(z)$	1	1					1			$-\beta_1$								
11	0	$S_2(z)$			1	1				1			$-\beta_2$							
12	0	$S_3(z)$					1	1			1			$-\beta_3$						
13 … 18			$x_{ij} = 0$ if $r_{ij} > 0$						$x_{ii} \geqslant 0$			$p_i \geqslant 0$			$r_{ij} = 0$ if $x_{ij} > 0$					

predicted by solving simultaneously the estimated structural equations, the set of definitional equations and the spatial equilibrium conditions.[10] Following the methodology described previously, the simplex tableau used with counterparts control is given in Table 6. In the tableau, the variables x_{ij} and r_{ij} are counterparts, and both cannot be in the basis at the same time.

The results of equilibrium solution for each year is given in Table 7. It is interesting to note that in 1956 the northeast region was a self-sufficient area with no receipts of broilers from the southern states. After 1957, the northeast region became a deficit area. About 30% of all broilers consumed in the northeast region came from the southern states in 1967. The differences in prices are small comparing equilibrium and actual prices.

Table 7. Spatial Equilibrium Solutions

	p_1	p_2	p_3	x_{11}	x_{21}	x_{33}	x_{23}	x_{22}
1956	28.94	28.43	30.00	892.45	0	825.57	837.20	408.73
1957	29.08	27.29	28.84	905.35	31.73	850.60	962.26	451.73
1958	25.34	23.57	25.09	949.49	134.54	882.46	1123.92	517.61
1959	22.93	21.18	22.68	991.16	230.05	921.37	1345.62	583.78
1960	21.63	19.90	21.37	1007.50	289.83	929.01	1366.46	620.54
1961	20.67	18.96	20.43	1039.28	334.02	956.04	1572.48	658.61
1962	20.00	18.29	19.74	1067.26	389.20	982.49	1700.20	702.05
1963	21.55	19.84	21.29	1082.76	379.95	990.50	1733.64	712.03
1964	21.67	19.96	21.41	1100.02	450.48	1010.64	1890.87	761.38
1965	24.22	22.54	23.94	1114.52	462.63	1022.74	1980.04	786.35
1966	21.80	20.14	21.49	1181.26	597.34	1079.56	2282.07	888.27
1967	20.61	18.96	20.26	1217.85	707.63	1114.92	2519.69	964.98

Table 8. Estimated "Partial: Price and Income Elasticities for Broilers at Farm Level, By Regions, 1956-1967

Year	Price Elasticities			Income Elasticities		
	North	South	Rest of the U.S.	North	South	Rest of the U.S.
1956	−1.0292	−0.9422	−0.8103	1.0176	1.2371	1.0904
1957	−0.9848	−0.8188	−0.7145	1.0215	1.1805	1.0542
1958	−0.7420	−0.6170	−0.5617	0.9091	1.0601	0.9650
1959	−0.5960	−0.4916	−0.4494	0.8567	0.9979	0.9213
1960	−0.5292	−0.4345	−0.4007	0.8333	0.9700	0.9009
1961	−0.4777	−0.3900	−0.3629	0.8203	0.9524	0.8894
1962	−0.4358	−0.3530	−0.3305	0.8160	0.9424	0.8842
1963	−0.4676	−0.3775	−0.3510	0.8504	0.9727	0.9116
1964	−0.4436	−0.3552	−0.3315	0.8613	0.9766	0.9188
1965	−0.4874	−0.3883	−0.3581	0.9150	1.0218	0.9594
1966	−0.3890	−0.3072	−0.2871	0.8814	0.9827	0.9310
1967	−0.3399	−0.2662	−0.2504	0.8711	0.9675	0.9210

[10] Equilibrium solutions are computed merely for the purpose of forecasting and no normative connotation is attached [11; p.8]. Deviations of the data from equilibrium solutions are only an indication of the goodness-of-fit and not a normative critique of the competitive markets.

D. The Direct Effect of Price and Income on Demand and "Partial" Elasticities

The conventional way of studying consumer behavior is via the price and income elasticities from demand equations. However, the slope of the demand equation only indicates a direct effect within a single sector of the economy especially when price or quantity is simultaneously determined with other sectors. Thus, elasticities computed from a single structural equation may be considered as "partial" elasticities.

Other things being equal (hardly so in a model where the endogenous variables are changing simultaneously), "partial" price and income elasticities evaluated at equilibrium prices and quantities are given in Table 8. The results indicate that elasticities are changing from time to time because of simultaneous shifting of demand and supply: broiler price is decreasing, income is increasing and quantity demanded is increasing. Price elasticities in the northeast decreased in absolute figures from 1.0292 in 1956 to 0.3399 in 1967. Price elasticities in the other regions follow the same changing pattern.[11] Income elasticities are rather stable changing from 1.0176 in 1965 to 0.8711 in 1967 in the northeast. Income elasticities in the southern states are a little higher than in the north.

E. Spatial Equilibrium Multipliers and "Total" Elasticities

To study the effect of a change in any predetermined variable (say income) on endogenous variables, after taking account of interdependencies among current endogenous variables, requires solving the set of structural equations for reduced form equations. Reduced form coefficients indicate "total" effect and may be called "multipliers." Since the equilibrium solution in 1956 is different from other years in the series (Table 7), the pattern of the derived reduced form equations for 1956 is different from that of other years. Our interest is in the most current situation and therefore impact multipliers are computed for the shipping pattern of the period 1957 through 1967 and are given in Table 9.

Since supply equations do not depend on current prices the model may be considered a recursive system. The figures in Table 9 are interpreted as follows: if profitability in the northeast region is increased by one cent per lb., other predetermined variables being constant, supply in the northeast region will increase by 7.1109 million pounds. However, this increase in the northeast supply will cause a decrease in supply price of 0.0789 cents per pound. The flow of commodities will result in the South's decreasing the shipment of broilers to the North by 4.6086 million pounds. The South will increase shipments to the rest of the nation by 3.5405 million pounds and increase by 1.0681 million pounds the quantity of broilers sold in the South. As a result, the South and the rest of the nation must decrease the price by 0.0789 cents in order to stimulate more consumption. The consumption in each region will increase by 2.5023, 1.0681, and 3.5405 million pounds respectively for the North, the South, and the rest of the U. S.

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[11] The elasticities in 1956 may be compared with the average monthly elasticities of the period 1953-1963 given by Farris and Darley [4]. In making comparison, however, one should bear in mind that their elasticities are converted from flexibilities computed at mean prices and quantities for 1953-1963. In addition, all data were on a per capita basis, and estimates were obtained by single equation ordinary least squares.

Table 9. Spatial Equilibrium Multipliers For Broilers, 1957-1967

Total effect of / on	Lagged Profitability					Transportation Costs			
	North	South	West	Lagged National Supply	National Disposable Income	South to North	South to Rest of U.S.	North to Rest of U.S.	Constant Term
P_1	−0.0789	−0.5653	−0.0679	−0.0120	0.0001232	0.6481	−0.4979	0	30.6941
P_2	−0.0789	−0.5653	−0.0679	−0.0120	0.0001232	−0.3519	−0.4979	0	30.6941
P_3	−0.0789	−0.5653	−0.0679	−0.0120	0.0001232	−0.3519	0.5021	0	30.6941
X_{11}	7.1109	0	0	0.1066	0	0	0	0	536.1950
X_{21}	−4.6086	17.9223	2.1141	0.2754	−0.0007819	−20.572	15.805	0	−606.4448
X_{22}	1.0681	7.6497	0.9189	0.1630	0.0000722	4.7672	6.7463	0	−127.0948
X_{23}	3.5405	25.3581	−3.0717	0.4495	0.0007097	15.805	−22.551	0	−703.0574
X_{33}	0	0	6.1178	0.0909	0	0	0	0	523.4655
S_1	7.1109	0	0	0.1066	0	0	0	0	536.1950
S_2	0	50.9301	0	0.8879	0	0	0	0	−1436.6000
S_3	0	0	6.1178	0.0909	0	0	0	0	523.4655
D_1	2.5023	17.9223	2.1141	0.3820	−0.0007819	−20.572	15.805	0	−70.2498
D_2	1.0681	7.6497	0.8189	0.1630	0.0000722	4.7672	6.7463	0	−127.0978
D_3	3.5405	25.3581	3.0464	0.5404	0.0007097	15.805	−22.551	0	−179.5919

"Total" elasticities which take into account the interdependence of the current endogenous variables, are evaluated at equilibrium in 1967 and are given in Table 10. The results indicate that elasticities are all small especially the "total" income elasticities in the regions. Other predetermined variables being equal, if the national disposable income increases by one percent, demand for broilers in the Northeast region increases by 0.02 percent, which is far smaller than the partial income elasticity 0.8711.

Table 10. "Total" Elasticities, 1967

Elasticity of \ With Respect to	Lagged Profitability					Transportation Costs	
	North	South	Rest of the U.S.	Last National Supply	National Disposable Income	South to North	South to Mid-west
P_1	−0.0174	−0.0710	−0.0155	−0.3605	0.0032	0.0409	−0.0399
P_2	−0.0190	−0.0772	−0.0168	−0.3918	0.0035	−0.0241	−0.0021
P_3	−0.0178	−0.0723	−0.0157	−0.3667	0.0033	−0.0226	0.0001
X_{11}	0.0266	0	0	0.0542	0	0	0
X_{21}	−0.0297	0.0656	0.0140	0.2409	0.0006	−0.0378	0.0369
X_{22}	0.0050	0.0205	0.0045	0.1046	0.0000	0.0064	0.0115
X_{23}	0.0064	0.0261	−0.0057	0.1104	0.0002	0.0082	−0.0148
X_{33}	0	0	0.0257	0.5048	0	0	0
S_1	0.0266	0	0	0.0542	0	0	0
S_2	0	0.0315	0	0.1311	0	0	0
S_3	0	0	0.0257	0.5048	0	0	0
D_1	0.0059	0.0241	0.0051	0.1228	0.0002	0.0139	0.0135
D_2	0.0050	0.0205	0.0045	0.1046	0.0000	0.0064	0.0115
D_3	0.0043	0.0176	0.0038	0.0895	0.0001	0.0055	−0.0100

F. The Effect of Change in Feed Price on Broiler Production and Its Effect on Derived Demand of Feed

Increased profitability may result from either an increase in output price or decrease in input cost. The effect of a change in feed price on broiler production may be evaluated by

$$\frac{\partial S_{it}}{\partial pf_{i\ t-1}} = \frac{\partial S_{it}}{\partial R_{i\ t-1}} \cdot \frac{\partial R_{i\ t-1}}{\partial pf_{i\ t-1}} = - \frac{\partial S_{it}}{\partial R_{i\ t-1}} \cdot F_{i\ t-1}$$

With 1967 technology, if the feed price decreased by one cent per pound, production in the Northeast will increase by 24.8170 million pounds. Production in the South will increase 177.7460 million pounds, the rest of the U.S. will increase 21.3511 million pounds. Total increase in U.S. broiler production will be 223.5305 million pounds.

The percentage change in supply with respect to the percentage change in feed price is −0.1047. Since the requirement for feed is 3.49 multiplied by the total change in supply in absolute figures, the elasticity of derived demand for feed with respect to price is also −0.1047. The percentage change in profitability with respect to percentage change in feed price is −3.9339.

G. The Stability of the Dynamic Spatial Equilibrium

The spatial equilibrium model presented is a dynamic recursive model. Regional supplies are functions of lagged national supply (which is equal to national demand) and lagged regional profitability, which is a function of lagged broiler price. Thus given the quantity produced, prices are determined by the demand functions and transportation costs, and generated current prices determine next year's production. The resulting cobweb model involves three markets with one commodity.

When the supply functions are expressed as functions of lagged price and major costs, the reduced form equations may be written in the compact matrix notations as

$$y_t = Ay_{t-1} + Bz_t + v_t$$

where y_t denotes the vector of current endogenous variables, y_{t-1} the vector of lagged endogenous variables, z_t is the vector of predetermined variables, v_t is the vector of disturbances, and A and B are parameter matrices. The necessary condition for the system to be stable is that the matrix A^k approaches a null matrix as k increases [5]. The matrix A^k will approach a null matrix if the characteristic roots of the matrix A are all less than unity in absolute value.

Table 11. The Reduced Form Coefficients for Lagged Endogenous Variables

Equations	p_{1t-1}	p_{2t-1}	p_{3t-1}	x_{11t-1}	x_{21t-1}	x_{22t-1}	x_{23t-1}	x_{33t-1}
p_{1t}	−0.0789	−0.5653	−0.0679	−0.0120	−0.0120	−0.0120	−0.0120	−0.0120
p_{2t}	−0.0789	−0.5653	−0.0679	−0.0120	−0.0120	−0.0120	−0.0120	−0.0120
p_{3t}	−0.0789	−0.5653	−0.0679	−0.0120	−0.0120	−0.0120	−0.0120	−0.0120
x_{11t}	7.1109	0	0	0.1066	0.1066	0.1066	0.1066	0.1066
x_{21t}	−4.6086	17.9223	2.1141	0.2754	0.2754	0.2754	0.2754	0.2754
x_{22t}	1.0681	7.6497	0.9189	0.1630	0.1630	0.1630	0.1630	0.1630
x_{23t}	3.5405	25.3581	−3.0717	0.4495	0.4495	0.4495	0.4495	0.4495
x_{33t}	0	0	6.1178	0.0909	0.0909	0.0909	0.0909	0.0909

For the empirical model, such an A matrix is given in Table 11. If shipments are aggregated into one national supply variable the equivalent A matrix for the abbreviated system is given by

	$P_{1\,t-1}$	$P_{2\,t-1}$	$P_{3\,t-1}$	Q_{t-1}
P_{1t}	-0.9789	-0.5653	-0.0679	-0.0120
P_{2t}	-0.0789	-0.5653	-0.0679	-0.0120
P_{3t}	-0.0789	-0.5653	-0.0679	-0.0120
Q_t	7.1109	50.9301	6.1178	1.0854

The matrix is singular and has rank 2. The characteristic roots are found to be −0.0112, 0, 0, 0.3845, in which none is larger than 1 in absolute value. Thus the system is stable. If we raise the power of the A matrix, A^k may be considered as a null matrix when k equals approximately 10.

An alternative way to determine the stability of the system is to recursively compute successive equilibria by holding other predetermined variables constant. This may be accomplished by recursive quadratic programming iterations. One should note that without imposing the nonnegative restrictions, the same answer will be obtained as from raising the power of A. The equilibrium solutions will converge in about 10 recursive cycles for the problem at hand. However, if the prices and quantities are restricted to be nonnegative, the situation will be different. In this case, use of 1967 national supply will permit the projection of supply in 1968. If demands are assumed to be unchanged, the price in each region would be 5.49¢, 3.84¢ and 5.14¢ respectively for the North, South and the rest of the U.S. Another recursive solution for 1969 would be over production and the price would be negative in the South. In other words, no feasible solution results. However, if the time trends of the predetermined variables are known or predictable, then more realistic values of the predetermined variables may be assigned to the model to obtain a reasonable time path of jointly determined variables in each succeeding spatial equilibria.

Summary and Conclusions

Spatially competitive markets may be positively analyzed. If one will accept a certain level of significance, the simultaneously estimated supply and demand functions along with the equilibrium conditions permits a description of how interregional competition has developed and, therefore, provides a basis for forecasting, assuming past structure prevails in the future. In addition, the model provides equilibrium multipliers which answer questions arising from comparative static analysis.

The empirical applicability of the model is demonstrated by a study of the spatial markets for broilers. The variables used in explaining supply included current price, feed per pound of broilers, feed price, lagged broiler price, lagged regional production and lagged national broiler production and consumption. Many combinations of the above variables were used as regressors with most of the results unsatisfactory. It was finally postulated that the supply of broilers never depended upon current price. Demand, however, was postulated as a linear function of current broiler price and current total national disposable income.

When the two-stage procedure is used, regional broiler farm prices are first regressed on all the predetermined variables to obtain a set of reduced form equations. This procedure was necessary to obtain the "calculated value" of regional farm prices. The "calculated values" are then used in the 2nd stage as regressors combining other predetermined variables to estimate regional demand and supply functions.

The estimated regional supplies are very close to the actual observations. The increasing values obtained for the demand functions intercept indicates these functions are shifting upward due to increases in per capita disposable income and population.

Given a fixed supply of broilers and a downward sloping demand function the equilibrium solution may be predicted by solving simultaneously the estimated structural equations, definitional equations and the spatial equilibrium conditions.

It was found that if profitability in the northeast region is increased by one cent per pound, supply in the northeast region will increase by 7.1109 million pounds. This increase in the Northeast supply will cause a decrease in supply price of 0.0789 cents per pound. The flow of commodities will result in the South decreasing shipment of broilers to the North by 4.6086 million pounds. The South will increase shipments to the rest of the nation by 3.5405 million pounds and increase by 1.0681 million pounds the quantity of broilers sold in the South. The South and the rest of the nation must decrease price by 0.0789 cents and consumption in each region will increase by 2.5023, 1.0681, and 3.5405 million pounds respectively for the North, South, and the rest of the U. S.

A considerable difference was found in "total" elasticities which take into account the interdependence of the current endogenous variables, and the partial elasticities which only indicates a direct effect within a single sector of the economy. This is especially true when price or quantity is simultaneously determined with other sectors. Other predetermined variables being equal, if the national income increases by one percent, demand for broilers in the Northeast increases by 0.02 percent which is far smaller than the partial income elasticity of 0.8711.

Broiler production is quite sensitive to changes in feed prices. If feed price decreases by one cent per pound, production in the Northeast will increase by 25 million pounds. Production in the South will increase by 178 million pounds and in the rest of the U. S. 21 million pounds. The percentage change in supply with respect to the change in feed price is –0.1047.

APPENDIX A. Sources of Data and Explanation of Calculation

Farm Price of Broilers

Calculation. Prices for each region were determined by weighting the state prices received by pounds of broilers produced. Farm prices were converted to ready-to-cook basis by multiplying by 1.37. The conversion factor 1.37 was obtained from *Farm Retail Spread for Food Products*, Miscellaneous Publication No. 741, November 1957, p. 113.

Sources of Data. U. S. Department of Agriculture, ERS, SRS, AMS, *Egg and Poultry Statistics Through Mid-1961*, Statistical Bulletin No. 305 (Washington, D.C., March 1962). (For 1956-60 prices)

U. S. Department of Agriculture, CRB, SRS, *Prices Received by Farmers for Chickens, Turkeys, and Eggs*, Statistical Bulletin No. 357 (Washington, D.C., May 1965). (For 1961-63 prices)

U. S. Department of Agriculture, CRB, SRS, *Chickens and Eggs–Production, Disposition, Cash Receipts and Gross Income,* (Washington, D.C., yearly issues). (For 1963-67 prices)

Pounds Available for Consumption

Calculation. The total pounds of live broilers produced in each region was first multiplied by the appropriate dressing percentage for each year. This was 70.3 for 1956, 71.0 for 1957 and 72.0 percent from 1958-67. This was obtained by simply dividing total pounds produced by the pounds edible, using national figures.

The ratio between total production and total civilian consumption was next determined, also using national figures. These series of data take account of changes in storage holdings during the year as well as imports and exports.

An example may help. In 1967 the North produced 1,782,000,000 pounds of broilers with a dressing percentage of 72 or 1,283,000,000 ready-to-cook pounds. Allowing for changes in storage and imports and exports the total available for consumption was 96.4 percent of total production. Multiplying 96.4 times 1,283,000,000 results in 1,237,000,000 available for consumption in the North in 1967. (See column 4 Table 1).

Sources of Data. U. S. Department of Agriculture, *Agricultural Statistics - 1968,* (Washington, D. C.: Government Printing Office, 1968), pp. 413 and 416.

U. S. Department of Agriculture, *Agricultural Statistics - 1967,* (Washington, D.C.: Government Printing Office, 1967), pp. 488 and 491.

Consumption

Calculation. The relationship between total production of chickens and broilers and broiler production was determined. The percentage broilers were of total production was then multiplied by per capita total consumption to obtain per capita broiler consumption.

The civilian resident population was obtained from the census reports for each year. Civilian resident population for each region was multiplied by per capita broiler consumption to obtain total consumption.

An example—the per capita consumption of chickens and broilers in 1967 was 37.1 pounds. Total production of chickens and broilers was 7.5 billion pounds and broilers only 6.6 billion pounds. Broiler consumption was 88 percent of total consumption. This percent was multiplied by 37.1 to obtain per capita broiler consumption of 32.6. Total civilian population in 1967 in the North region was 57,091,000. Multiplying 32.6 times 57,091,000 results in total consumption of 1,861,000,000 pounds of broilers reported in column 7 Table 1.

Sources of Data. U. S. Department of Agriculture, *Agricultural Statistics, - 1967,* (Washington, D.C.: Government Printing Office, 1967), page 491.

National Disposable Income

Calculation. For the period 1956 to 1963 disposable income was obtained directly from published reports.

For the years 1964-1967 total disposable income was estimated by a regression fitted to total disposable income (Y) and total personal income (X) for the period 1953-1963. The regression equation is

$$Y = 11887.84 + 0.8444 X. \qquad (R^2 = 0.9983)$$

Sources of Data. U. S. Department of Commerce, Office of Business Economics, *Survey of Current Business,* Vol. 45, No. 4, April 1965, page 21.

Transportation

Calculation. Transportation rates were determined from given points within each region. The south to north movement was assumed to take place from Gainesville, Georgia to New York City, and the south to west (remainder of U. S.) from Gainesville, Georgia to Chicago. The north to west movement was assumed from New York City to Chicago. No rates were calculated for the north to south movement because little or no broilers moved in this direction after 1956.

The 1954 level of rates for the three movements were determined by the regression equation determined by Henry and Bishop and reported in a North Carolina publication.

The 1961 level of rates was based upon regression equations determined by Martin and reported in a Maryland publication.

For the north to west movement a straight line connecting the years 1954 and 1961 was drawn and rates for the period 1956 to 1967 determined from this straight line.

For the south to north and south to west movement a straight line connecting 1954 and 1961 levels was the basis for determination of rates for that time period.

From 1961-64 no evidence of any change in rates could be found.

The 1968-69 rates from south to north and south to west were based upon reports from processors in Georgia as determined by Harold Jones.[12] He reported a rate of $1.25 per cwt. for movement from south to west and $1.50 to $1.75 for movement south to north. A straight line was drawn connecting 1964 and 1968 and rates for 1965-67 determined.

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[12] Harold Jones is employed by the USDA—ERS and is stationed at the University of Georgia in Athens.

Sources of Data. William R. Henry and Charles E. Bishop, *North Carolina Broilers in Interregional Competition,* A.E. Information Series No. 56 (Raleigh, North Carolina: North Carolina State College, February 1957).

James E. Martin, *The Effects of Changes in Transportation Rates on the Delmarva Poultry Industry*, Agricultural Experiment Station Miscellaneous Publication No. 515 (College Park, Maryland: University of Maryland, May 1964).

Profitability

Calculation. Profitability is a simple measure of the difference between feed cost per pound and price per pound. Feed required per pound of live broilers was multiplied by 1.37 to convert to a ready-to-cook basis. This was then multiplied by feed price per pound and the results subtracted from farm price per pound reported in the first three columns of Table 1. Profitability was then lagged one year in the analysis.

Sources of Data. U. S. Department of Agriculture, ERS, *Poultry and Egg Situation,* PES-255, (Washington, D.C.: Government Printing Office, February 1969).

Lagged Feed per Pound of Broilers

Calculation. The feed per pound of live broilers as reported in the February 1969 *Poultry and Egg Situation* is multiplied by 1.37 to convert to ready-to-cook basis.

Lagged Feed Price

Calculation. Broiler mash feed price for each state was weighted by production of broilers to obtain a weighted average for each of the three regions.

Sources of Data. U. S. Department of Agriculture, ERS, SRS, AMS, *Egg and Poultry Statistics Through Mid-1961,* Statistical Bulletin No. 305 (Washington, D.C., March 1962).

U. S. Department of Agriculture, CRB, SRS, *Agricultural Prices - 1967 - Annual Summary,* (Washington, D.C.: Government Printing Office, June 1968).

APPENDIX B. Table 1. The B Matrix for the Estimated Structural Equations (BY + ΓX = u) for the Flow Pattern of the Period 1957-1967

	p_1	p_2	p_3	x_{11}	x_{21}	x_{22}	x_{23}	x_{33}	r_{12}	r_{13}	r_{31}	r_{32}
Demand												
1	31.7407			1	1							
2		13.5484				1						
3			44.9175				1	1				
Supply												
1				1								
2					1	1	1					
3								1				
Equilibrium Condition												
1	−1	1							1			
2	−1		1							1		
3	1	−1										
4		−1	1									
5	1		−1								1	
6		1	−1									1

APPENDIX B. Table 2. The Γ Matrix for the Estimated Structural Equations (BY + ΓX = u), 1956-1967

	R_{1t-1}	R_{2t-1}	R_{3t-1}	Q_{t-1}	y_t	t_{21}	t_{23}	t_{13}	Const.
Demand									
1					0.003123562				902.8352
2					0.001739151				288.2400
3					0.006235600				1197.217
Supply									
1	7.1109			0.1066					536.1950
2		50.9301		0.8879					−1436.6000
3			6.1178	0.0909					523.4655
Equilibrium Condition									
1						1			
2								1	
3						1			
4							1		
5								1	
6							1		

APPENDIX B. Table 3. The Inverse Matrix of B for the Flow Pattern of the Period 1957-1967

	p_1	p_2	p_3	x_{11}	x_{21}	x_{22}	x_{23}	x_{33}	r_{12}	r_{13}	r_{31}	r_{32}
Demand												
1	0.0111	0.0111	0.0111	−0.0111	−0.0111	−0.0111			0.6481	−0.4979		
2	0.0111	0.0111	0.0111	−0.0111	−0.0111	−0.0111			−0.3519	−0.4979		
3	0.0111	0.0111	0.0111	−0.0111	−0.0111	−0.0111			−0.3519	−0.5021		
Supply												
1				1								
2	0.6481	−0.3519	−0.3519	−0.6481	0.3519	0.3519			−20.572	15.805		
3	−0.1502	−0.8498	−0.1502	0.1502	0.1502	0.1502			4.7672	6.7463		
Equilibrium Condition												
1	−0.4979	−0.4979	0.5021	0.4979	0.4979	−0.5021			15.805	−22.551		
2						1						
3							1		1			
4								1	1	−1		
5									−1	1	1	
6										1		1

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