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The Impact of the IRS Retirement Option Relative Value

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Relative Value Regulations and Reporting

My thesis will evaluate the effects of the new Government regulations in regards to the relative value notification of retirement plan options. I will look at how these new regulations will affect retirement plan option utilization and how retirement plan providers will change options in order to minimize risk.

The new Government regulations for retirement plans stipulate that retirement plan managers and pension companies must provide plan participants with information about the Relative Value (RV) of each of the different retirement benefit options which are available to them. Prior to the implementation of these regulations pension plan managers did not need to provide any more information than the benefits which would be due upon retirement to retirees and their beneficiaries. Although most pension plan options set benefit amounts for each option equal to that amount which would create equality in the present value of each option, the new regulations require that each defined benefit plan must show the present value of each option relative to the single life annuity for single participants or to the 50% joint and survivor option for married participants, but that the present values of each option must be calculated under 417(e) assumptions.

These assumptions are:

- The discount rate used shall be the 30 year treasury rate in the month that is one month prior to expected retirement date.
- If the expected retirement date is more than one month hence, the rate used shall be the most recent 30 year treasury rate.
- The mortality table used shall be the 1994 Group Annuity Reserving table, with adjustments stipulated within the SOA report.

Retirement companies are never alike. Retirement fund managers invest in many different ways, and in many different commodities. They have differing returns and have different option structures. Having to report the RV of each different option under different assumptions than those used to calculate payments will lead to relative values which may fall outside the required range and therefore will be reported as either higher or lower than the present value of the standard option (that being either the 50% Joint and Survivor option or the Single Life Annuity option.)

Since retirement funds are managed differently from one another and because different methods are employed by fund managers and retirement plan companies my methodology in this study has been simplified to show the risks associated with reporting the RV of each retirement option to prospective retirees. I have, for the sake of simplicity and conforming to the assumptions set by the IRS, chosen to use the above assumptions when calculating different present

values for different scenarios. I have also used data from a real life retirement plan, with benefits and relative values which were set at the time of retirement for each option. Before discussing the results I will outline the methodology I have used.

Methodology

This study uses a hypothetical retirement plan with several different options. For the purpose of this study I will only look at the how the regulations affect married retirees in order to examine the effect on all retirement optional forms. The first option is the Single Life Annuity (SLA). This option is the most common option available to retirement plan participants and is usually the default option for retirement plans. The present value of the SLA is calculated using actuarial methods. Since the SLA is the default option it is usually the option which has the "benchmark" benefit, or the benefit defined under the plan specifications of the Defined Benefit plan. Other optional form benefits are calculated from this optional form's benefit amount.

The inputs required to calculate the present value of the SLA are:

- Age of retiree at retirement (w)
- Date of benefit commencement (BCD) (set at the first day of the retirement month if retirement occurs during the first 15 days of that month, or at the first date of the month following retirement is retirement occurs after the first 15 days of the retirement month)

- Sex of retiree (male = 1, female = 2)
- Interest rate (i)
- Benefit amount (B)

From these inputs the present value of the total expected retirement benefit is calculated as follows.

1. From the retirement date the interest rate is set at the preceding month's 30 year treasury rate
2. Using the sex value of the retiree, the appropriate table is used to calculate the probability of the retiree receiving payments each year.
3. The total payment for each year is multiplied by the probability of survival of the retiree
4. The total payment for each year and probability is discounted back to the valuation date.
5. Each discounted benefit is summed to provide a Present Value of future benefits.

Actuarially this can be represented by the following formula:

$$\mathbf{SLA = B^* \Sigma\ {}_xp_w * v^x}$$

Where the sum is from $x = 1$ to $120 - w$
${}_xp_w$ = probability of retiree surviving year x
$v^x = (1+i)^{-x}$

The second optional form is the Y% Joint and Survivor (Y% J&S) annuity. This is set at the default in this study to conform to the IRS regulations. The calculation of the J&S optional form annuity uses the same inputs as the SLA option, plus the following.

- Age of beneficiary at retiree's BCD (s)
- Sex of beneficiary (male = 1, female = 2)
- A percentage rate (which is used to calculate the benefit after the death of the retiree provided the beneficiary is still alive) (Y)

The present value of the J&S option is calculated as follows:

1. Follow the first four steps in calculating the PV of SLA
2. Using the value of the beneficiary sex, use appropriate mortality table to find the probability of survival through each year
3. Using both the retirees mortality table and the beneficiaries mortality table calculate the probability of both surviving each year
4. Multiply each of these probabilities by the discount factor back to the present value at the date of valuation
5. Subtract the discounted joint probability from the discounted beneficiary probability
6. Multiply the above by the benefit amount and by the percentage of the beneficiary benefit
7. Add this amount to the amount found for the same benefit under the SLA procedure

This can be written actuarially as follows:

$$Y\%\ J\&S = B^* \Sigma\ {}_xp_w * v^x + B^* Y * (\Sigma\ {}_xp_s * v^x - \Sigma\ {}_xp_s^*\ {}_x\ p_w * v^x)$$

Where Y is the percentage of beneficiary benefit.

The Term Certain optional form pays the benefit for the period specified regardless of whether the retiree lives to the end of the specified period or not. If the retiree does live beyond the period of the term then the annuity will continue to be paid until the death of the retiree. If the retiree does not survive to the end of the period the beneficiary will receive the full benefit until the end of the period and at that point the benefit will stop.

To evaluate the term certain present value the following are required:

- Age of retiree at benefit commencement date
- Sex of retiree
- Number of years of term certain benefit (m)
- Interest rate

To calculate the PV of this annuity the following procedure is carried out:

1. apply the discount rate to the probability of survival of the retiree for each year beyond the certain period
2. Multiply this by the benefit amount
3. add the above to the PV of an annuity of the benefit amount for the certain period

Actuarially this is:

$$\mathbf{M\ CC = B * a_n + B * \Sigma\ {}_x p_w * v^x}$$

Where the sum is from M to 120 – w – M.

By using these methods we find the present values for different optional forms and compare them to the standard option. The relative value of each optional form is the present value of that optional form divided by the present value of the 50% Joint and Survivor optional form.

For example:

RV of SLA = PV SLA / PV 50% J&S

RV of 10 CC = PV 10 CC / PV 50% J&S

The following table is an example of a set of relative values for a male retiring at the age of 65, with a female beneficiary who is 58 years old at the benefit commencement date (BCD). The BCD is January 1st 2005. The benefits are a sample from an actual retiree (the BCD was changed for this example).

	Participant		Beneficiary		Participant Benefit				Relative Value Benefit Bounds	
Data	**age1**	**sex1**	**age2**	**sex2**	**benefit**	**Liability**	**Option**	**RV**	**Lower Bound**	**Upper Bound**
PP1	65	1	58	2	1000	218289.27	SLA	101.93%	932.01	1005.58
BCD	Jan-05				925	214154.52	50% J&S	100.00%	-	-
Intrate	4.73				870	207175.68	75% J&S	96.74%	854.34	921.79
					825	201916.71	100% J&S	94.29%	831.25	896.88
					990	220499.92	10 CC	102.96%	913.43	985.55
					975	214784.87	5 CC	100.29%	923.53	996.44

The relative value benefit bounds are the upper and lower bounds for the benefit for each optional form which would ensure the RV would be within the acceptable range.

In this example we can see that the relative values of the 100% J&S option will be identified as being less than 95% of the present value of the 50% J&S option. The 10 CC option will be identified as being more than 102.5% of the present value of the 50% J&S option. The new regulations will require the retirement plan provider to inform the retiree that these two options are outside the range of acceptable relative value as defined by the IRS. With this information the participant would probably opt to select the 10 CC option as it is worth more than the 50% J&S annuity at its present value. However, under the retirement plan provider's assumptions these plans are all of equal relative value. This will affect how retirement plan providers evaluate benefits for each of the different options available under their retirement plan. The 10 CC option would be worth opting for if the likelihood of the retiree dying within the next 10 years is higher than the mortality tables and experience ratings the retirement provider uses suggest, and if the beneficiary has some form of supplemental income other than the benefits from the annuity. The 10 CC option would also be worth opting for if the beneficiary and the retiree both died within 10 years of the BCD. This would allow a second beneficiary to collect the remaining payments from the annuity.

The present value of each plan is also dependant on the interest rate, which under the assumptions of the IRS 417(e), are to be set at the 30 year treasury rate.

Since the interest rate used to discount all payments for the annuity is not the same as the difference between the inflation rate at the market return (or the return of any investment which might be used by the retiree) the present value of each of the optional forms is different from the actual value. In fact, all annuities rely on actuarial assumptions, and are not actually valued at the correct value (unless the actuary valuating the annuities can foresee the future.) For the IRS to set the interest rate and the mortality tables used for these evaluations means that the retirees may choose options which would not be beneficial for them, or may not be worth more than other options. Many different factors need to be considered when evaluating a retirement plan and no present value will be accurate (it is nice to know that with every evaluation you are always wrong!). For the above reasons I feel that the new government regulations will actually disadvantage retirees who are not knowledgeable on retirement plan design and actuarial evaluation, the complete opposite affect of what the regulations are designed for.

I also think that the new regulations may cause retirement plan designers to manipulate their evaluation practices to ensure that all options fall within the acceptable range so that retirees only look at the benefits that they would be entitled to and make judgments on which plan would be most beneficial to them based on their health and lifestyle rather than a percentage which is reported to them by their retirement plan provider which is calculated under several variable assumptions.

The assumptions that the IRS requires (417(e) assumptions) will mean that retirees with the exact same life expectancy and exact same variables, other than their BCD will have different relative values. People with the same circumstances retiring about six months apart, will have a differing relative value at any one point in time.

The following example shows how a retiree with exactly the same mortality rate, and the same mortality rate for the beneficiary and the same benefits has a different relative value for each of his retirement plan options simply because his BCD is seven months before the second example.

PP1 benefit	BCD Liability	1/1/2005 Option	RV
1000	218289.3	SLA	101.93%
925	214154.5	50% J&S	100.00%
870	207175.7	75% J&S	96.74%
825	201916.7	100% J&S	94.29%
980	218272.6	10 CC	101.92%
975	214784.9	5 CC	100.29%

PP2 benefit	BCD Liability	6/1/2004 Option	RV
1000	199393.1	SLA	102.58%
925	194387.6	50% J&S	100.00%
870	187508	75% J&S	96.46%
825	182246	100% J&S	93.75%
980	199221.1	10 CC	102.49%
975	196150.5	5 CC	100.91%

Notice that not only are the Present values different, but the Relative values are different as well. Both retirees would be informed that the 100% J&S option is worth less than 95% relative to the 50% J&S option. The second retiree would be inclined to select the single life annuity option as it would be reported to be worth more than 102.5% relative to the 50 % J&S option. However, the SLA and the 10 CC options differ in worth by only $172 (from the liability column), but the retiree would be told that the 10 CC option is worth approximately the same as the 50%

J&S option because it is within the acceptable range. These results differ simply because the first retiree's options were valued on June 1st 2004, and the second retiree's options were valued on January 1st 2005. Because the retiree must select which optional form they wish to receive upon retirement the first retiree would probably choose a form different to that chosen by the second retiree.

At the point in time that the second retiree chooses the optional form he would like to receive, both retirees' options are worth the same under the 417(e) assumptions. These discrepancies in the relative value reporting required by the IRS will cause some retirees to choose their options based on inaccurate (since present values are not actually equal to the real value of each optional form) valuations and varying assumptions.

There are several other hypothetical cases which show how different variables affect the relative values of each optional form, and may influence retiree's in their decision as to which optional form they will receive. The first is the age of retirement. With all other variables held constant, changing the retiree's age will change the relative value of each optional form. Obviously changing the retiree's age will change the present value of each form, but the relative values change at varying rates, causing some optional forms to increase in relative value and others to decrease.

PP1	Age	65	
benefit	Liability	Option	RV
1000	218289.3	SLA	101.93%
925	214154.5	50% J&S	100.00%
870	207175.7	75% J&S	96.74%
825	201916.7	100%	94.29%

PP2	Age	62	
benefit	Liability	Option	RV
1000	224305.8	SLA	103.16%
925	217424.7	50% J&S	100.00%
870	209172.1	75% J&S	96.20%
825	202786.4	100%	93.27%

		J&S	
990	220499.9	10 CC	102.96%
975	214784.9	5 CC	100.29%

		J&S	
990	225802.1	10 CC	103.85%
975	220345.4	5 CC	101.34%

The relative values of the SLA and both CC options increased, while the relative values of the J&S options decreased. If the benefit amounts had been different this may have caused the retiree to choose different options. If, in the first case the benefits were such that they within the lower and upper bounds of the relevant range so as not to be identified as being either above or below the relative value of the 50% J&S option, relative values in the second case could have pushed the SLA and CC options above the 102.5% range, and the J&S options below the 95% range. This would, therefore, influence the decision the retiree would make even though the actual relative values may be within the range.

Changing the age of the beneficiary has a similar effect. If the beneficiary is younger the joint and survivor relative values will increase, while the SLA relative, and the CC relative values will decrease. This is, however, misleading. The present value of the SLA option will remain unchanged with all other variables held constant if the only change is the beneficiary's age. This is due to the fact that the single life annuity option has nothing to do with the beneficiary at all as the single life annuity is paid to the retiree from the BCD until death, regardless of whether the beneficiary dies before or after the retiree. The relative value changes because the present value of the 50% J&S optional form increases, causing the value of the SLA option to decrease in relative terms. Again, if the

benefits are such that the SLA and J&S options are all within the required range the retiree is not to know that the J&S options may be more valuable in relative terms.

Beneficiary 1		Age	70	Beneficiary 2		Age	58
benefit	Liability	Option	RV	benefit	Liability	Option	RV
1000	218289.3	SLA	103.83%	1000	218289.3	SLA	101.93%
925	210228.6	50% J&S	100.00%	925	214154.5	50% J&S	100.00%
870	201637	75% J&S	95.91%	870	207175.7	75% J&S	96.74%
825	194913.8	100% J&S	92.72%	825	201916.7	100% J&S	94.29%
990	220499.9	10 CC	104.89%	990	220499.9	10 CC	102.96%
975	214784.9	5 CC	102.17%	975	214784.9	5 CC	100.29%

The differences in relative values from those found under the retirement plan provider's assumptions compared to the IRS assumptions can also stem from differences in mortality tables. Many retirement valuation firms (including retirement plan providers and human resource consulting firms) choose to use their own mortality tables. They often have experience data which improves the accuracy of their actuarial calculations because the mortality data more accurately represents the retirees within a certain company or within an industry. However, the IRS is requiring that all valuations for the relative value regulations must be made using one table. That table is the 1994 Group Annuity Reserving table with yearly mortality improvement adjustments. This table was complied by the SOA to represent the entire population. This does not mean that it is accurate for the entire population. For example the mortality rates for blue collar retirees are very different from those of white collar retirees. So then why are all retirement plans required to use this one table? If the IRS wants all retirees to gain an accurate

picture of the relative values of each retirement option available to them it would be more accurate for evaluators to use appropriate tables. Experience ratings should also be used, especially when there is substantial evidence to suggest that a certain company's retiree mortality rates are different from those found in the 1994 GAR table. Retirees from certain industries who have a shorter life expectancy and a greater mortality rate would be better off choosing a CC option or the 100% J&S option because this would provide their beneficiaries with a greater pension after their death. Those retirees with a longer life expectancy should opt for a lower percentage J&S option or even the SLA option.

Retirement plan providers may also change benefit rates to make sure certain options fall within the required range (i.e. between 95% and 102.5% of the PV of the 50% J&S option.) In the attached appendix I have made some hypothetical benefit ranges which ensure each plan option would be within the acceptable range to avoid having to inform the retiree.

Each variable affects the relative value of each retirement option. Some variables have been generalized too much (such is the case with the mortality tables) while other generalizations can be warranted (the interest rate variable can be generalized to bring every plan to a standard discount rate rather than allowing retirement plan evaluators decide what interest rate to use.) The problem with the new regulations is that retirees may not benefit from the information that they are receiving. In fact they may be disadvantaging themselves by making uniformed decisions. They may choose solely on the relative values of each option, rather

than choosing an option based on their own situation. If retirees could have more information about which option would be best for them with regards to their own personal health and their own financial situation then they would be able to make better decisions. I feel that all the relative value regulations are doing is creating more work for plan providers and at the same time taking away from the process of making the best decision for each retiree. The cost of these regulations is also increasing the risk for retirement plan providers by taking up employee time and employer money, without increasing any returns. The regulations will also increase the risks associated with each plan option. If one option is influenced more by the IRS assumptions, increasing its relative value above the upper bound limit of 102.5% then it will be utilized more by retirees. This increases the risk for the retirement plan provider if any assumptions are incorrect as it will reduce the risk reduction properties which are seem with diversification. If all retirees pick one option then the risk of an incorrect assumption cannot be spread over the other options.

Appendix

These tables show the upper and lower bounds of benefits for several different retiree variables. The 50% J&S option is set at $950.

Relative Value Benefit Bounds

Option	Lower Bound	Upper Bound		
SLA	957.19	1032.76	**Age Retiree**	65
50% J&S	950.00	950.00	**Age Benef**	58
75% J&S	877.43	946.70	**Sex retiree**	M
100% J&S	853.72	921.12	**Sex Benef**	F
10 CC	938.12	1012.18	**BCD**	1/1/2005
5 CC	948.49	1023.37	**Intrate**	4.73

Option	Lower Bound	Upper Bound		
SLA	950.07	1025.07	**Age Retiree**	65
50% J&S	950.00	950.00	**Age Benef**	58
75% J&S	880.46	949.97	**Sex retiree**	F
100% J&S	859.47	927.32	**Sex Benef**	M
10 CC	938.94	1013.07	**BCD**	1/1/2005
5 CC	945.09	1019.70	**Intrate**	4.73

Option	Lower Bound	Upper Bound		
SLA	945.74	1020.41	**Age Retiree**	62
50% J&S	950.00	950.00	**Age Benef**	58
75% J&S	882.33	951.99	**Sex retiree**	M
100% J&S	863.04	931.17	**Sex Benef**	F
10 CC	930.08	1003.51	**BCD**	1/1/2005
5 CC	938.67	1012.78	**Intrate**	4.73

Option	Lower Bound	Upper Bound		
SLA	939.45	1013.62	**Age Retiree**	62
50% J&S	950.00	950.00	**Age Benef**	58
75% J&S	885.09	954.97	**Sex retiree**	F
100% J&S	868.35	936.90	**Sex Benef**	M
10 CC	930.43	1003.89	**BCD**	1/1/2005
5 CC	935.47	1009.33	**Intrate**	4.73

Option	Lower Bound	Upper Bound		
SLA	939.65	1013.83	**Age Retiree**	65
50% J&S	950.00	950.00	**Age Benef**	70
75% J&S	885.01	954.88	**Sex retiree**	M
100% J&S	868.18	936.72	**Sex Benef**	F
10 CC	920.92	993.63	**BCD**	1/1/2005
5 CC	931.10	1004.61	**Intrate**	4.73

Option	Lower Bound	Upper Bound		
SLA	931.40	1004.93	**Age Retiree**	65
50% J&S	950.00	950.00	**Age Benef**	70
75% J&S	888.71	958.88	**Sex retiree**	F
100% J&S	875.34	944.45	**Sex Benef**	M
10 CC	920.49	993.16	**BCD**	1/1/2005
5 CC	926.52	999.66	**Intrate**	4.73

Option	Lower Bound	Upper Bound		
SLA	930.98	1004.48	**Age Retiree**	62
50% J&S	950.00	950.00	**Age Benef**	70
75% J&S	888.90	959.08	**Sex retiree**	M
100% J&S	875.71	944.85	**Sex Benef**	F
10 CC	915.56	987.84	**BCD**	1/1/2005
5 CC	924.02	996.97	**Intrate**	4.73

Option	Lower Bound	Upper Bound		
SLA	924.01	996.95	**Age Retiree**	62
50% J&S	950.00	950.00	**Age Benef**	70
75% J&S	892.12	962.55	**Sex retiree**	F
100% J&S	881.97	951.60	**Sex Benef**	M
10 CC	915.13	987.38	**BCD**	1/1/2005
5 CC	920.09	992.73	**Intrate**	4.73

Microsoft Excel Visual Basic code

SLA option

```
Function liability(age1, age2, sex1, sex2, benefit, intrate, BCD)

Sheets("sheet1").Select


intr = intrate / 100

adjust = Year(BCD) - 1994

x = 1

For x = 1 To (120 - age1)

    If sex1 = 1 Then

        APV = APV + (1 + intr) ^ (1 - x) * (1 - (Sheets("sheet2").Cells(age1 + x +

1, 2).Value) * (1 - Sheets("sheet2").Cells(age1 + x + 1, 3).Value) ^ (adjust - 1 +

x))

    Else: APV = APV + (1 + intr) ^ (1 - x) * (1 - (Sheets("sheet2").Cells(age1 + x

+ 1, 4).Value) * (1 - Sheets("sheet2").Cells(age1 + x + 1, 5).Value) ^ (adjust - 1 +

x))

    End If

Next x


liability = 12 * benefit * APV
```

End Function

J&S Function

```
Function JS(age1, age2, sex1, sex2, benefit, intrate, percent, BCD)
Sheets("sheet1").Select

intr = intrate / 100
adjust = Year(BCD) - 1994
x = 1
For x = 1 To (120 - age1)
    If sex1 = 1 Then
        APV1 = APV1 + (1 + intr) ^ (1 - x) * (1 - (Sheets("sheet2").Cells(age1 + x
+ 1, 2).Value) * (1 - Sheets("sheet2").Cells(age1 + x + 1, 3).Value) ^ (adjust - 1 +
x))
    Else: APV1 = APV1 + (1 + intr) ^ (1 - x) * (1 - (Sheets("sheet2").Cells(age1
+ x + 1, 4).Value) * (1 - Sheets("sheet2").Cells(age1 + x + 1, 5).Value) ^ (adjust -
1 + x))
    End If
Next x
j = 1
For j = 1 To (120 - age2)
    If sex2 = 1 Then
```

```
        apv2 = apv2 + (1 + intr) ^ (1 - j) * (1 - (Sheets("sheet2").Cells(age2 + j + 1,
2).Value) * (1 - Sheets("sheet2").Cells(age1 + j + 1, 3).Value) ^ (adjust - 1 + j))
    Else: apv2 = apv2 + (1 + intr) ^ (1 - j) * (1 - (Sheets("sheet2").Cells(age2 + j
+ 1, 4).Value) * (1 - Sheets("sheet2").Cells(age1 + j + 1, 5).Value) ^ (adjust - 1 +
j))
    End If
Next j
i = 1
If age1 > age2 Then
  For i = 1 To (120 - age1)
    apv3 = apv3 + (1 + intr) ^ (1 - i) * (1 - (Sheets("sheet2").Cells(age1 + i + 1,
2).Value)) * (1 - (Sheets("sheet2").Cells(age2 + i + 1, 4).Value) * (1 -
Sheets("sheet2").Cells(age1 + i + 1, 3).Value) ^ (adjust - 1 + i))
  Next i
Else
  For i = 1 To (120 - age2)
    apv3 = apv3 + (1 + intr) ^ (1 - i) * (1 - (Sheets("sheet2").Cells(age1 + i + 1,
2).Value)) * (1 - (Sheets("sheet2").Cells(age2 + i + 1, 4).Value) * (1 -
Sheets("sheet2").Cells(age1 + i + 1, 3).Value) ^ (adjust - 1 + i))
  Next i
End If
```

```
JS = 12 * benefit * APV1 + 12 * benefit * percent * (apv2 - apv3)

End Function
```

Certain Function

```
Function CC(age1, sex1, benefit, intrate, years, BCD)

Sheets("sheet1").Select


intr = intrate / 100

adjust = Year(BCD) - 1994

x = 1

For x = 1 To (120 - age1 - years)

    If sex1 = 1 Then

        APV = APV + (1 + intr) ^ (1 - (x + years)) * (1 -
(Sheets("sheet2").Cells(age1 + x + 1 + years, 2).Value) * (1 -
Sheets("sheet2").Cells(age1 + x + 1, 3).Value) ^ (adjust + years - 1 + x))

    Else: APV = APV + (1 + intr) ^ (1 - (x + years)) * (1 -
(Sheets("sheet2").Cells(age1 + x + 1 + years, 4).Value) * (1 -
Sheets("sheet2").Cells(age1 + x + 1, 5).Value) ^ (adjust + years - 1 + x))

    End If

Next x

Certain = (1 - (1 + intr) ^ (-years)) / (intr / (1 + intr))


CC = 12 * benefit * (APV + Certain)
```

End Function

Resources

http://www.irs.gov/irb/2004-02_IRB/ar06.html#d0e169

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