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What Does *Height* Really Mean? Part III: Height Systems

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Part III: Height Systems

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ABSTRACT: This is the third paper in a four-part series considering the fundamental question, "what does the word "height" really mean?" The first paper reviewed reference ellipsoids and mean sea level datums. The second paper reviewed the physics of heights culminating in a simple development of the geoid and explained why mean sea level stations are not all at the same orthometric height. This third paper develops the principle notions of height, namely measured, differentially deduced changes in elevation, orthometric heights, Helmert orthometric heights, normal orthometric heights, dynamic heights, and geopotential numbers. We conclude with a more in-depth discussion of current thoughts regarding the geoid.

Introduction

There are two general visions of what the word "height" means—a geometric separation versus hydraulic head. For Earth mensuration, these visions are not the same thing, and this discrepancy has lead to many formulations of different types of heights. In broad strokes there are orthometric heights, purely geometric heights, and heights that are neither. None of these are inferior to the others in all respects. They all have strengths and weaknesses, so to speak, and this has given rise to a number of competing height systems. We begin by introducing these types of heights, then examine the height systems in which they are measured, and conclude with some remarks concerning the geoid.

Heights

Uncorrected Differential Leveling

Leveling is a process by which the geometric height difference along the vertical is transferred from a reference station to a forward station. Suppose a leveling line connects two

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stations A and B as depicted in Figure III.1 (c.f. Heiskanen and Moritz 1967, p. 161). If the two stations are far enough apart, the leveling section will contain several turning points, the vertical geometric separation between which we denote as δv_i. Any two turning points are at two particular geopotential numbers, the difference of which is the potential gravity energy available to move water between them; hydraulic head. We also consider the vertical geometric separation of those two equipotential surfaces along the plumb line for B, $\delta H_{B,i}$.
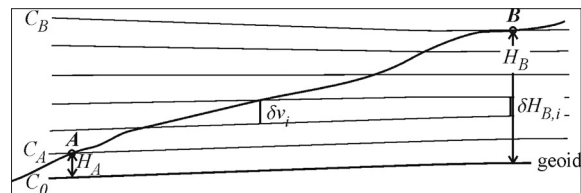
Figure III.1. A comparison of differential leveling height differences δv_i with orthometric height differences $\delta H_{B,i}$. The height determined by leveling is the sum of the δv_i whereas the orthometric height is the sum of the $\delta H_{B,i}$. These two are not the same due to the non-parallelism of the equipotential surfaces whose geopotential numbers are denoted by C.

We will now argue that differential leveling does not, in general, produce orthometric heights. Figure III.1 depicts two stations A and B, indicated by open circles, with geopotential numbers C_A and C_B, and at orthometric heights H_A and H_B, respectively. The geopotential surfaces, shown in cross section as lines, are not parallel; they converge towards the right. Therefore, it follows that $\delta v_i \neq \delta H_{B,i}$. The height difference from A to B as

determined by differential leveling is the sum of the δv_i. Therefore, because $\delta v_i \neq \delta H_{B,i}$ and the orthometric height at B can be written as $H_B = \sum \delta H_{B,i}$, it follows that $\sum \delta v_i \neq H_B$.

We now formalize the difference between differential leveling and orthometric heights so as to clarify the role of gravity in heighting. In the bubble "*gedanken* experiment" in the second paper of this series (Meyer et al. 2005, pp. 11-12), we argued that the force moving the bubble was the result of a change in water pressure over a finite change in depth. By analogy, we claimed that gravity force is the result of a change in gravity potential over a finite separation:

$$g = -\delta W / \delta H \qquad \text{(III.1)}$$

where g is gravity force, W is geopotential and H is orthometric height. Simple calculus allows rearranging to give $-\delta W = g\,\delta H$. Recall that δv_i and $\delta H_{B,i}$ are, by construction, across the same potential difference so $-\delta W = g\,\delta v_i = g'\,\delta H_{B,i}$, where g' is gravity force at the plumb line. Now, $\delta v_i \neq \delta H_{B,i}$ due to the non-parallelism of the equipotential surfaces but δW is the same for both, so gravity must be different on the surface where the leveling took place than at the plumb line. This leads us to Heiskanen and Moritz (1967, p. 161, Equation (4-2)):

$$\delta H_{B,i} = \frac{g}{g'}\delta v_i \neq \delta v_i \qquad \text{(III.2)}$$

which indicates that **differential leveling height differences differ from orthometric height differences by the amount that surface gravity differs from gravity along the plumb line at that geopotential**. An immediate consequence of this is that two different leveling lines starting and ending at the same station will, in general, provide different values for the height of final station. This is because the two lines will run through different topography and, consequently, geopotential surfaces with disparate separations. **Uncorrected differential leveling heights are not single valued**, meaning the result you get depends on the route you took to get there.

In summary, heights derived from uncorrected differential leveling:

- Are readily observed by differential leveling;
- Are not single valued by failing to account for the variability in gravity;
- Will not, in theory, produce closed leveling circuits; and
- Do not define equipotential surfaces. Indeed, they do not define surfaces in the mathematical sense at all.

Orthometric Heights

According to Heiskanen and Moritz (1967, p. 172), "Orthometric heights are the natural 'heights above sea level,' that is, heights above the geoid. They thus have an unequalled geometrical and physical significance." National Geodetic Survey (1986) defines **orthometric height** as, "The distance between the geoid and a point measured along the plumb line and taken positive upward from the geoid" (*ibid.*), with plumb line defined as, "A line perpendicular to all equipotential surfaces of the Earth's gravity field that intersect with it" (*ibid.*).

In one sense, orthometric heights are purely geometric: they are the length of a particular curve (a plumb line). However, that curve depends on gravity in two ways. First, the curve begins at the geoid. Second, plumb lines remain everywhere perpendicular to equipotential surfaces through which they pass, so the shape of the curve is determined by the orientation of the equipotential surfaces. Therefore, orthometric heights are closely related to gravity in addition to being a geometric quantity.

How are orthometric heights related to geopotential? Equation (III.1) gives that $g = -\delta W/\delta H$. Taking differentials instead of finite differences and rearranging them leads to $dW = -g\,dH$. Recall that geopotential numbers are the difference in potential between the geoid W_0 and a point of interest A, W_A: $C_A = W_0 - W_A$, so:

$$\int_{W_0}^{W_A} dW = - \int_0^{H_A} g\,dH$$

$$W_A - W_0 = - \int_0^{H_A} g\,dH$$

$$W_0 - W_A = \int_0^{H_A} g\,dH$$

$$C_A = \int_0^{H_A} g\,dH \qquad \text{(III.3)}$$

in which it is understood that g is not a constant. Equation (III.3) can be used to derive the desired relationship:

$$C_A = \overline{g}H_A \qquad \text{(III.4)}$$

meaning that a geopotential number is equal to an orthometric height multiplied by the average acceleration of gravity along the plumb line. It was argued in the second paper that geopotential is single valued, meaning the potential of any particular place is independent of the path

taken to arrive there. Consequently, orthometric heights are likewise single valued, being a scaled value of a geopotential number.

If orthometric heights are single valued, it is logical to inquire whether surfaces of constant orthometric height form equipotential surfaces. The answer to this is, unfortunately, no. Consider the geopotential numbers of two different places with the same orthometric height. If orthometric heights formed equipotential surfaces, then two places at the same orthometric height must be at the same potential. Under this hypothesis, Equation (III.4) requires that the average gravity along the plumb lines of these different places *necessarily* be equal. However, the acceleration of gravity depends on height, latitude, and the distribution of masses near enough to be of concern; it is constant in neither magnitude nor direction. There is no reason that the average gravity would be equal and, in fact, it typically is not. **Therefore, two points of equal orthometric height need not have the same gravity potential energy, meaning that they need not be on the same equipotential surface and, therefore, not at the same height from the perspective of geopotential numbers.**

Consider Figure III.2, which is essentially a three-dimensional rendering of Figures II.9 and III.1, and which shows an imaginary mountain together with various equipotential surfaces. Panel (b) shows the mountain with just one gravity equipotential surface. Everywhere on gravity equipotential surface is the same gravity potential, so water would not flow along the intersection of the equipotential surface with the topography without external influence. Nevertheless, the curve defined by the intersection of the gravity equipotential surface with the topography would *not* be drawn as a contour line on a topographic map because a contour line is defined to be, "An imaginary line on the ground, all points of which are at the same *elevation* above or below a specified reference surface" (National Geodetic Survey 1986). This runs contrary to conventional wisdom that would define a contour line as the intersection of a horizontal plane with the topography. In panels (c) and (d), one can see that the equipotential surfaces undulate. In particular, notice that the surfaces do not remain everywhere the same distance apart from each other and that they "pull up" through the mountains. Panel (d) shows multiple surfaces, each having less curvature than the one below it as a consequence of increasing distance from the Earth.

Now consider Figure III.3, which is an enlargement of the foothill in the right side of panel III.2(c). Suppose that the equipotential surface containing A and D is the geoid. Then the orthometric height of station B is the distance along its plumb line to the surface containing A and D; the same for station C. Although neither B's nor C's plumb line is shown—both plumb lines are inside the mountain—one can see that the separation from B to the geoid is different than the separation from C to the geoid, even though B and C are on the same equipotential surface. Therefore, they have the same geopotential number but have different orthometric heights. This illustrates why orthometric heights are single valued but do not create equipotential surfaces.

How are orthometric heights measured? Suppose an observed sequence of geometric height differences δv_i has been summed together for the total change in geometric height along a section from station A to B, $\Delta v_{AB} = \sum \delta v_i$. Denote the change in orthometric height from A to B as ΔH_{AB}. Equation (III.4) requires knowing a geopotential number and the average acceleration of gravity along the plumb line but neither of these are measurable. Fortunately, there is a relationship between leveling differences Δv and orthometric height differences ΔH. A change in orthometric height equals a change in geometric height plus a correction factor known as the **orthometric correction** (for a derivation see Heiskanen and Moritz 1967, pp.167-168, Equations (4-31) and (4-33)):

$$\Delta H_{AB} = \Delta v_{AB} + OC_{AB} \qquad (III.5)$$

where OC_{AB} is the orthometric correction and has the form of:

$$OC_{AB} = \sum_A^B \frac{g_i - \gamma_0}{\gamma_0} \delta v_i + \frac{\overline{g}_A - \gamma_0}{\gamma_0} H_A - \frac{\overline{g}_B - \gamma_0}{\gamma_0} H_B \text{ (III.6)}$$

where g_i is the observed force of gravity at the observation stations, $\overline{g}_A, \overline{g}_B$ are the average values of gravity along the plumb lines at A and B, respectively, and γ_0 is an arbitrary constant, which is often taken to be the value of normal gravity at 45° latitude.

Although Equation (III.6) stipulates gravity be observed at every measuring station, Bomford (1980, p. 206) suggested that the observation stations need to be no closer than two to three km in level country but should be as close as 0.3 km in mountainous country. Others recommended observation station separations be 15 to 25 km in level country and 5 km in mountainous
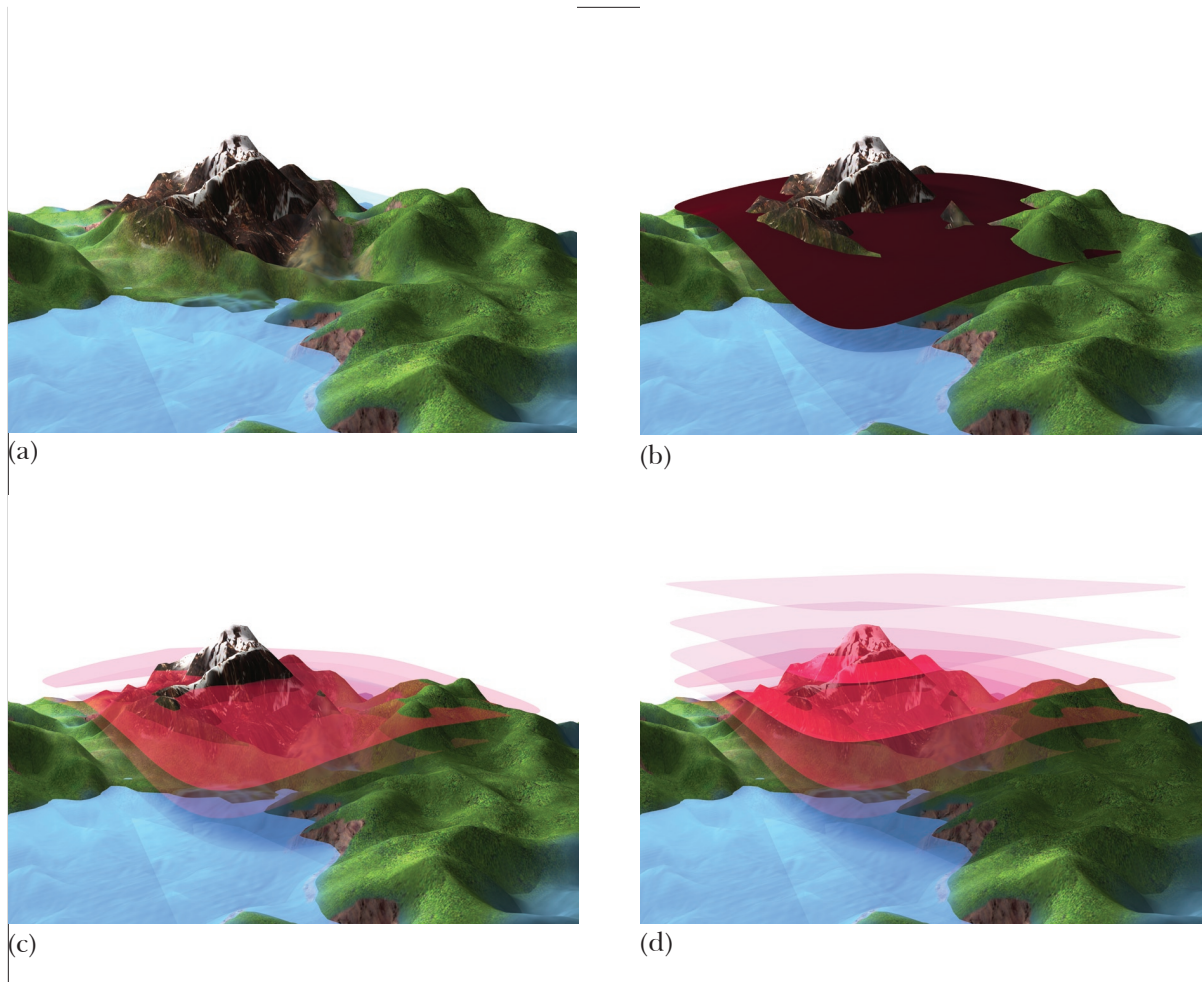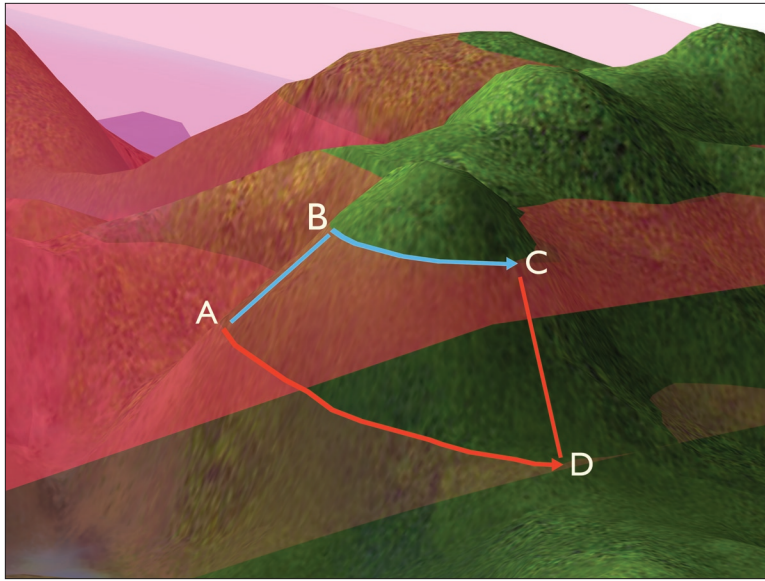
Figure III.2. Four views of several geopotential surfaces around and through an imaginary mountain. (a) The mountain without any equipotential surfaces. (b) The mountain shown with just one equipotential surface for visual simplicity. The intersection of the surface and the ground is a line of constant gravity potential but *not* a contour line. (c) The mountain shown with two equipotential surfaces. Note that the surfaces are not parallel and that they undulate through the terrain. (d) The mountain shown with many equipotential surfaces. The further the surface is away from the Earth, the less curvature it has. (Image credit: Ivan Ortega, Office of Communication and Information Technology, UConn College of Agriculture and Natural Resources).

country (Strang van Hees 1992; Kao et al. 2000; Hwang and Hsiao 2003).

There is a fair amount of literature on practical applications of orthometric corrections, of which the following is a small sample: Forsberg (1984), Strang van Hees (1992), Kao et al. (2000), Allister and Featherstone (2001), Hwang (2002), Brunner (2002), Hwang and Hsiao (2003), and Tenzer et al. (2005). The work described in these reports was undertaken by institutions with the resources to field surveying crews with gravimeters. Although there has been progress made in developing portable gravimeters (Faller and Vitouchkine 2003), it remains impractical to make the required gravity measurements called

for by Equation (III.6) for most surveyors. For first-order leveling, National Geodetic Survey (NGS) has used corrections that depend solely on the geodetic latitude and normal gravity at the observation stations, thus avoiding the need to measure gravity (National Geodetic Survey 1981, pp. 5-26), although if leveling is used to determine geopotential numbers, such as in the NAVD 88 adjustment, orthometric corrections are not used. The Survey's data sheets include modeled gravity at benchmarks, which provide a better estimate of gravity than normal gravity and are suitable for orthometric correction.

Although exact knowledge of $\bar{g}$ is not possible at this time, its value can be estimated either

Figure III.3. B and C are on the same equipotential surface but are at difference distances from the geoid at A-D. Therefore, they have different orthometric heights. Nonetheless, a closed leveling circuit with orthometric corrections around these points would theoretically close exactly on the starting height, although leveling alone would not.

- Are not directly measurable from their definition. Orthometric heights can be determined by observing differential leveling-derived geometric height differences to which are applied a small correction, the **orthometric correction**. The orthometric correction requires surface gravity observations and an approximation of the average acceleration of gravity along the plumb line.

Ellipsoid Heights and Geoid Heights

Ellipsoid heights are the straight-line distances normal to a reference ellipsoid produced away from (or into) the ellipsoid to the point of interest. Before GPS it was practically impossible for anyone outside the geodetic community to determine an ellipsoid height. Now, GPS receivers produce three-dimensional baselines (Meyer 2002) resulting in determinations of geodetic latitude, longitude, and ellipsoid height. As a result, ellipsoid heights are now commonplace.

Ellipsoid heights are almost never suitable surrogates for orthometric heights (Meyer et al. 2004, pp. 226-227) because equipotential ellipsoids are not, in general, suitable surrogates for the geoid (although see Kumar 2005). Consider that nowhere in the conterminous United States is the geoid closer to a GRS 80-shaped ellipsoid centered at the ITRF origin than about two meters. Confusing an ellipsoid height with an orthometric height could not result in a blunder less than two meters but would typically be far worse, even disastrous. For example, reporting the height of an obstruction in the approach to an airport runway at New York City using ellipsoid heights instead of orthometric heights would apparently lower the reported height by around 30 m, with a possible result of causing a pilot to mistakenly believe the aircraft had 30 m more clearance than what is real.

Ellipsoid heights have no relationship to gravity; they are purely geometric. It is remarkable, then, that ellipsoid heights have a simple (approximate) relationship to orthometric heights, namely:

$$H \approx h - N \qquad \text{(III.7)}$$

using a **free-air correction** (Heiskanen and Moritz 1967, pp. 163-164), or by the reduction of Poincaré and Prey (*ibid.*, p 165). The former depends on knowledge of normal gravity only by making assumptions regarding the mean curvature of the potential field outside the Earth. Orthometric heights that depend upon this strategy are called **Helmert orthometric heights**. The National Geodetic Survey publishes NAVD 88 Helmert orthometric heights. The Poincaré and Prey reduction, which requires a remove–reduce–restore operation, is more complicated and only improves the estimate slightly (*ibid.*, pp. 163-165).

In summary, orthometric heights:

- Constitute the embodiment of the concept of "height above sea level;"
- Are single valued by virtue of their relationship with geopotential numbers and, consequently, will produce closed leveling circuits, in theory;
- Do not define equipotential surfaces due to the variable nature of the force of gravity. This could, in principle, lead to the infamous situation of water apparently "flowing uphill." Although possible, this situation would require a steep gravity gradient in a location with relatively little topographic relief. This can occur in places where subterranean features substantially affect the local gravity field but have no expression on the Earth's surface; and

where H is orthometric height, h is ellipsoid height, and N is the ellipsoid height of the geoid itself, a **geoid height** or **geoid undulation**.

This relationship is not exact because it ignores the deflection of the vertical. Nevertheless, it is close enough for most practical purposes. According to Equation (III.7), ellipsoid heights can be used to determine orthometric heights if the geoid height is known. As discussed in the previous paper, geoid models are used to estimate N, thus enabling the possibility of determining orthometric heights with GPS (Meyer et al. 2005, p.12). We will explore these relationships in some detail in the last paper in the series on GPS heighting.

In summary, ellipsoid heights:

- Are single valued (because a normal gravity potential field satisfies Laplace's equation and is, therefore, convex);
- Do not use the geoid or any other physical gravity equipotential surface as their datum;
- Do not define equipotential surfaces; and
- Are readily determined using GPS.

Geopotential Numbers and Dynamic Heights

Geopotential numbers C are defined from Equation (II.6) (c.f. Heiskanen and Moritz 1967, p. 162, Equation (4-8)) which gives the change in gravity potential energy between a point on the geoid and another point of interest. The geopotential number for any place is the potential of the geoid W_0 minus the potential of that place W (recall the potential decreases with distance away from the Earth, so this difference is a positive number). Geopotential numbers are given in **geopotential units** (g.p.u.), where 1 g.p.u. = 1 kgal-meter = 1000 gal meter (Heiskanen and Moritz 1967, p. 162). If gravity is assumed to be a constant 0.98 kgal, a geopotential number is approximately equal to 0.98 H, so geopotential numbers in g.p.u. are nearly equal to orthometric heights in meters. However, geopotential numbers have units of energy, not length, and are therefore an "unnatural" measure of height.

It is possible to scale geopotential numbers by dividing by a gravity value, which will change their units from kgal-meter to meter. Doing so results in a **dynamic height**:

$$H^{dyn} = C / \gamma_0 \qquad \text{(III.8)}$$

One reasonable choice for γ_0 is the value of normal gravity (Equation (I.2)) at some latitude, conventionally taken to be 45 degrees. Obviously,

scaling geopotential numbers by a constant does not change their fundamental properties, so dynamic heights, like geopotential numbers, are single valued, produce equipotential surfaces, and form closed leveling circuits. They are not, however, geometric like an orthometric height: two different places on the same equipotential surface have the same dynamic height but generally do not have the same orthometric height. Thus, dynamics heights are not "distances from the geoid."

Measuring dynamic heights is accomplished in a manner similar to that for orthometric heights: geometric height differences observed by differential leveling are added to a correction term that accounts for gravity thus:

$$\Delta H_{AB}^{dyn} = \Delta v_{AB} + DC_{AB} \qquad \text{(III.9)}$$

where Δv_{AB} is the total measured geometric height difference derived by differential leveling and DC_{AB} is the **dynamic correction**. The dynamic correction from station A to B is given by Heiskanen and Moritz (1967, p. 163, Equation (4-11)) as:

$$DC_{AB} = \sum_A^B \frac{g_i - \gamma_0}{\gamma_0} \delta v_i \qquad \text{(III.10)}$$

where g_i is the (variable) force of gravity at each leveling observation station, $\gamma_0 = \gamma_0(45°)$, and the δv_i are the observed changes in geometric height along each section of the leveling line.

However, DC typically takes a large value for inland leveling conducted far from the defining latitude. For example, suppose a surveyor in Albuquerque, New Mexico (at a latitude of around 35 N), begins a level line at the Route 66 bridge over the downtown railroad tracks at an elevation of, say, 1510 m, and runs levels to the Four Hills subdivision at an elevation of, say, 1720 m, a change in elevation of 210 m.

From Equation (III.10), DC = $\Delta v(g - \gamma_0)/\gamma_0$. So taking $\gamma_0 = \gamma_{45°} = 980.62$ gal and $g_{35°} = 979.734$ gal, then

$$DC = 210 \text{ m} (979.734 \text{ gal} - 980.62 \text{ gal}) / 980.62 \text{ gal} =$$

-0.189775 m, a correction of roughly two parts in one thousand.

This is a huge correction compared to any other correction applied in first-order leveling, with no obvious physical interpretation such as the refraction caused by the atmosphere. It is unlikely that surveyors would embrace a height system that imposed such large corrections that would often affect even lower-accuracy work. Nonetheless, dynamics heights are of practical use wherever water levels are needed, such as

at the Great Lakes and also along ocean shores, even if they are used far from the latitude of the normal gravity constant. The geoid is thought to be not more than a couple meters from the ocean surface and, therefore, shores will have geopotential near to that of the geoid. Consequently, shores have dynamic heights near to zero regardless of their distance from the defining latitude. Even so, for inland surveying, DC can have a large value, on the order of several meters at the equator.

The dynamics heights in the International Great Lakes Datum of 1985 are established by the "Vertical Control–Water Levels" Subcommittee under the Coordinating Committee on Great Lakes Basic Hydraulics and Hydrology Data (CCGLBHHD).

In summary, dynamic heights:

- Are a scaling of geopotential numbers by a constant to endow them with units of length;
- Are not geometric distances;
- Are single valued by virtue of their relationship with geopotential numbers and, consequently, will produce closed-circuits, in theory;
- Define equipotential surfaces; and
- Are not measurable directly from their definition. Dynamic heights can be determined by observing differential leveling-derived geometric height differences to which are applied a correction, the **dynamic correction**. The dynamic correction requires surface gravity observations and can be on the order of meters in places far from the latitude at which γ_0 was defined.

Normal Heights

Of heights defined by geopotential (orthometric and dynamic) Heiskanen and Moritz (1967, p. 287) write:

> The advantage of this approach is that the geoid is a level surface, capable of simple definition in terms of the physically meaningful and geodetically important potential W. The geoid represents the most obvious mathematical formulation of a horizontal surface at mean sea level. This is why the use of the geoid simplifies geodetic problems and makes them accessible to geometrical intuition.
>
> The disadvantage is that the potential W inside the earth, and hence the geoid $W =$ const., depends on [a detailed knowledge of the density of the Earth]…Therefore, in order to determine or to use the geoid,

the density of the masses at every point between the geoid and the ground must be known, at least theoretically. This is clearly impossible, and therefore some assumptions concerning the density must be made, which is unsatisfactory theoretically, even though the practical influence of these assumptions is usually very small.

These issues led Molodensky in 1945 to formulate a new type of height, a **normal height**, which supposed that the Earth's gravity field was normal, meaning the actual gravity potential equals normal gravity potential (Molodensky 1945). The result of this postulate allowed that the "physical surface of the Earth can be determined from geodetic measurements alone, without using the density of the Earth's crust" (Heiskanen and Moritz 1967, p. 288). This conceptualization of heights allowed a fully rigorous method to be formulated for their determination, a method without assumptions. The price, however, was that "This requires that the concept of the geoid be abandoned. The mathematical formulation becomes more abstract and more difficult" (*ibid.*). Normal heights are defined by:

$$C = \int_0^{H^*} \gamma \, dH^* \qquad \text{(III.11)}$$

and

$$C = \bar{\gamma} H^* \qquad \text{(III.12)}$$

where H* is normal height and γ is normal gravity. These formulae have identical forms to those for orthometric height (c.f. Equations (III.3) and (III.4)), but their meaning is completely different. First, the zero used as the lower integral bound is not the geoid; it is a reference ellipsoid. Consequently, normal heights depend upon the choice of reference ellipsoid and datum. Second, normal gravity is an analytical function, so its average may be computed in closed form; no gravity observations are required. Third, from its definition one finds that a normal height H* is that ellipsoid height where the normal gravity potential equals the actual geopotential of the point of interest. Regarding this, Heiskanen and Moritz (1967, p. 170) commented, "…but since the potential of the Earth is evidently not normal, what does all this mean?"

Like orthometric and dynamic heights, normal heights can be determined from geometrical height differences observed by differential leveling and applying a correction. The correction term has the same structure as that for orthometric correction, namely:

$$NC_{AB} = \sum_A^B \frac{g_i - \gamma_0}{\gamma_0} \delta v_i + \frac{\bar{\gamma}_A - \gamma_0}{\gamma_0} H^*_A - \frac{\bar{\gamma}_B - \gamma_0}{\gamma_0} H^*_B$$

(III.13)

with $\bar{\gamma}$ being the average normal gravity from A to B and other terms defined as Equation (III.6). Normal corrections also depend upon gravity observations g_i but do not require assumptions regarding average gravity within the Earth. Therefore, they are rigorous; all the necessarily quantities can be calculated or directly observed. Like orthometric heights, they do not form equipotential surfaces (because of normal gravity's dependence on latitude; recall that dynamic heights scale geopotential simply by a constant, whereas orthometric and normal heights' scale factors vary with location). Like orthometric heights, normal heights are single valued and give rise to closed leveling circuits. Geometrically, they represent the distance from the ellipsoid up to a surface known as the telluroid (see Heiskanen and Moritz 1967 for further discussion).

In summary, normal heights:
- Are geometric distances, being ellipsoid heights, but not to the point of interest;
- Are single valued and, consequently, produce closed-circuits, in theory;
- Do not define equipotential surfaces; and
- Are not measurable directly from their definition. Normal heights can be determined by observing differential leveling-derived geometric height differences to which are applied a correction, the **normal correction**. The normal correction requires surface gravity observations only and, therefore, can be determined without approximations.

Height Systems

The term "height system" refers to a mechanism by which height values can be assigned to places of interest. In consideration of what criteria a height system must satisfy, Hipkin (2002b) suggested two necessary conditions:

(i. Hipkin)   Height must be single valued.

(ii.Hipkin)   A surface of constant height must also be a level (equipotential) surface.

Heiskanen and Moritz (1967, p. 173) held two different criteria, namely:

(i.H&M)   Misclosures must be eliminated.

(ii.H&M)   Corrections to the measured heights must be as small as possible.

The first two criteria (i.Hipkin and i.H&M) are equivalent: if heights are single valued, then leveling circuits will be closed, and vice versa. The second two criteria form the basis of two different philosophies about what is considered important for heights. Requiring that a surface of constant height be equipotential requires that the heights be a scaled geopotential number and excludes orthometric and normal heights. Conversely, requiring the measurement corrections to be as small as possible precludes the former, at least from a global point of view, because dynamic height scale factors are large far from the latitude of definition. No height meets all these criteria. This has given rise to the use of (Helmert) orthometric heights in the United States, dynamic heights in Canada, and normal heights in Europe (Ihde and Augath 2000). Table III.1. provides a comparison of these height systems.

NAVD 88 and IGLD 85

Neither NAVD 88 nor IGLD 85 attempts to define the geoid or to realize some level surface which was thought to be the geoid. Instead, they are based upon a level surface that exists near the geoid but at some small, unknown distance from it. This level surface is situated such that shore locations with a height of zero in this reference frame will generally be near the surface of the ocean. IGLD 85 had a design goal that its heights be referenced to the water level gauge at the mouth of the St. Lawrence River. NAVD 88 had a design goal that it minimize recompilation of the USGS topographic map series, which was referred to NGVD 29. The station at Father Point/Rimouski met both requirements. NAVD 88 was realized using Helmert orthometric heights, whereas IGLD 85 employs dynamic heights. Quoting from IGLD 85 (1995):

Two systems, orthometric and dynamic heights, are relevant to the establishment of IGLD (1985) and NAVD (1988). The geopotential numbers for individual bench marks are the same in both height systems. The requirement in the Great Lakes basin to provide an accurate measurement of potential hydraulic head is the primary reason for adopting dynamic heights. It should be noted that dynamic heights are basically geopotential numbers scaled by a constant of 980.6199 gals, normal gravity at sea level at 45 degrees latitude.

	Single valued	Defines Level Surfaces	No misclosure	Small Correction	Physically Meaningful	Rigorous Implementation
Uncorrected Dif. Leveling	No	No	No	n/a	Yes	Yes
Helmert Orthometric	Yes	No	Yes	Yes	Yes	No
Ellipsoidal	Yes	No	Yes	n/a	Yes	Yes
Dynamic	Yes	Yes	Yes	No	Yes	Yes
Normal	Yes	No	Yes	Yes	No	Yes

Table III.1. A comparison of height systems with respect to various properties that distinguish them.

Therefore, dynamic heights are also an estimate of the hydraulic head.

Also, "IGLD 85 and NAVD 88 are now one and the same… The only difference between IGLD 85 and NAVD 88 is that IGLD 85 benchmark values are given in dynamic height units, and NAVD 88 values are given in Helmert orthometric height units. The geopotential numbers of benchmarks are the same in both systems" (Pfeifer 2001). The United States covers a large area North-to-South within which is a considerable variety of topographic features. Therefore, dynamic heights would not be entirely acceptable for the U.S., because the dynamic corrections in the interior of the country would often be unacceptably large. The U.S. is committed now and for the future to orthometric heights, which in turn implies a commitment to geoid determination.

Geoid Issues

The geoid is widely accepted as the proper datum for a vertical reference system, although this perspective has challengers (Hipkin 2002b). Conceptually, the geoid is the natural choice for a vertical reference system and, until recently, its surrogate, mean sea level, was the object from which the geoid was realized. However, no modern vertical reference system, in fact, uses the geoid as its datum, primarily because the geoid is difficult to realize (although Canada has recently proposed re-defining their vertical datum using GPS and a geoid model). An exact, globally satisfactory definition of the geoid is not straightforward. Both of these issues will be explored in turn.

The reasons that the geoid is not realizable from a mean sea level surrogate were given in the second paper in the discussion regarding why the mean sea surface is not a level surface.

Quoting Hipkin (2002b, p. 376), the "…nineteenth century approach to establishing a global vertical datum supposed that mean sea level could bridge regions not connectable by leveling. The 'geoid' was formalized into the equipotential [surface] best fitting mean sea level and, for more than a century, the concepts of mean sea level, the geoid, and the leveling datum were used synonymously." We now know this use of "geoid" for "mean sea level," and vice versa, to be incorrect because the mean sea surface is not an equipotential surface. Therefore, the mean sea surface is questionable as a vertical datum.

Furthermore, Hipkin argues that measuring changing sea levels is one of the most important contributions that geodesy is making today. For this particular application, it does not make sense to continually adjust the vertical datum to stay at mean sea level and, thus, eliminate the phenomena to be observed. In contrast, chart makers, surveyors, and mappers, who define flood planes and subsidence zones, would probably require that the vertical datum reflect changes in sea level to ensure their products are up-to-date and not misleading. Although a valid scientific point, Hipkin's argument does not override the need for NGS to determine the geoid, or a level surface near the geoid, in order to provide a well defined datum for orthometric heights.

The second issue asserts that it is not straightforward to produce a globally acceptable definition of the geoid. If one searches for a physics-based definition of the geoid, one finds that, according to Smith (1998, p.17), "The Earth's gravity potential field contains infinitely many level surfaces… The geoid is one such surface with a particular potential value, W_0." W_0 is a fundamental geodetic parameter (Burša 1995; Groten 2004), and its value has been estimated by using sea surface topography models (also called dynamic ocean topography models) and spherical harmonic expansions of satellite altimetry data (e.g., Burša 1969; Burša 1994; Nesvorny and Sima 1994; Burša et al. 1997; Burša et al. 1999), as well as GPS + orthometric height observations (Grafarend and Ardalan 1997).

More recently (summer 2005, January/February 2006), research conducted in a joint effort between NGS, the National Aeronautics and Space Administration Goddard Flight Center, and Naval Research Laboratory personnel has attempted to model the geoid by coupling sea surface topography model results with airborne gravimetry and Light Detection And Ranging (LIDAR) measurements in a manner similar to the aforementioned, space-based altimetry efforts. If successful, this work will result in another solution to the ongoing problem of determining W_0 with particular focus on the coastal regions of the U.S. (*c.f.* Smith and Roman 2001, p. 472). The National Geodetic Survey is also examining earth gravity models (EGMs) derived from the satellite-based Gravity Recovery and Climate Experiment (GRACE) (Tapley et al. 2004) and (soon) Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) data (Rebah et al. 2000) in order to establish higher confidence in the long wavelengths in EGMs (i.e., macroscopic scale features in the geoid model). Aerogravity data are being collected to try and bridge the gaps at the shorelines between terrestrial data and the deep ocean and altimeter-implied gravity anomalies. Earth gravity models and aerogravity data are being used to cross-check each other, existing terrestrial data.

Even so there is no consensus as to which value for W_0 should be chosen. Smith (1998) suggested W_0 could be chosen at least two ways: pick a "reasonable" value or adopt a so-called "best fitting ellipsoid." Hipkin (2002b) has argued for the first approach: "To me it seems inevitable that, in the near future, we shall adopt a vertical reference system based on adopting a gravity model and one that incorporates $W = W_0 \equiv U_0$ to define its datum," with the justification that, "Nowadays, when observations are much more precise, their differences [between mean sea surface heights at various measuring stations] are distinguishable and present practice leads to confusion. It is now essential that we no longer associate mean sea level with any aspect of defining the geoid" (*ibid.*).

In fact, G99SSS and GEOID99 were computed by choosing to model a specific $W = W_0$ surface (Smith and Roman 2001). Defining $W_0 \equiv U_0$ is unnecessary because it is computable as the zero-order geoid undulation (Smith 2006, personal communication). Other researchers have explored the second alternative by using the altimetry and GPS + leveling methods mentioned above. However, different level surfaces fill the needs of different user groups better than others. Moreover, it is probably unsatisfactory to define a single potential value for all time because mean sea level is constantly changing due to, for example, the changing amount of water in the oceans, plate tectonics changing the shape and volume of the ocean basins and the continents, and "thermal expansion of the oceans changing ocean density resulting in changing sea levels with little corresponding displacement of the equipotential surface" (Hipkin 2002b). The geoid is constantly evolving, which leads to the need for episodic datum releases, as is done in the U.S. with mean sea level. If a global vertical datum is defined, it will only be adopted if it meets the needs of those who use it. With the United States' commitment to orthometric heights comes a need to define the geoid into the foreseeable future.

Summary

Heights derived through differential spirit leveling, ellipsoid and geoid heights, orthometric heights, geopotential numbers, dynamic heights, and normal heights were defined and compared regarding their suitability as an engineering tool and to reflect hydraulic head. It was shown that differential leveling heights provide neither single valued heights nor an equipotential surface, resulting in theoretical misclosures of leveling circuits. Orthometric heights are single valued but do not define level surfaces and require an approximation in their determination. Geopotential numbers are single valued and define level surfaces but do not have linear units. Dynamics heights are single valued, define level surfaces, are not intrinsically geometric in spite of having linear units, and often have unacceptably large correction terms far away from the latitude at which they are normalized. Normal heights are geometric, single valued, have global applicability, and can be realized without assumptions, but they do not define level surfaces. There is, in fact, no single height system that is both geometric and honors level surfaces simultaneously because these two concepts are physically incompatible due to the non-parallelism of the equipotential surfaces of the Earth's gravity field. Two modern vertical datums in use in North America (NAVD 88 and IGLD 85) express heights as either Helmert orthometric heights or dynamic heights. It was

shown that this difference is, in one sense, cosmetic because these heights amount to different scalings of the same geopotential numbers. Nevertheless, Helmert orthometric heights and dynamic heights are incommensurate. The fact that there are disparate height systems reflects the needs and, to some extent, the philosophies behind their creation. No one height system is clearly better than the others in all aspects.

Different organizations and nations have chosen various potentials to be their geoids for reasons that suit their purposes best. Others have argued that the gravity potential value $W = W_0 = U_0$ could be adopted to be the geoid's potential, which is attractive for some scientific purposes, though the U_0 of GRS 80 is no better or worse choice than any other U_0. However, the United States is committed to orthometric heights, and NGS is actively engaged in measurements to locate the geoid based on LIDAR observations, gravimetric geoid models, and sea surface topography models.

REFERENCES

Allister, N.A., and W. E. Featherstone. 2001. Estimation of Helmert orthometric heights using digital barcode leveling, observed gravity and topographic mass-density data over part of Darling Scarp, Western Australia. *Geomatics Research Australia* 75:25-52.

Bomford, G. 1980. *Geodesy* 4th ed. Oxford, U.K.: Clarendon Press. 561p.

Brunner, F. K. 2002. The role of local quasi-dynamic heights in engineering geodesy. In: *INGEO2002, 2nd Conference of Engineering Surveying*, Bratislava, Slovakia. pp. 21-4.

Burša, M. 1969. Potential of the geoidal surface, the scale factor for lengths and Earth's figure parameters from satellite observations. *Studia Geophysica et Geodaetica* 13:337-58.

Burša, M. 1994. Testing geopotential models. *Earth, Moon and Planets* 64 (3):293-9.

Burša, M. 1995. Report of Special Commission SC3, Fundamental constants. Presented at the 21st General Assembly of the International Association of Geodesy, Boulder, Colorado, 2-14 July.

Burša, M., K. Radej, Z. Sima, S. A. True, and V. Vatrt. 1997. Determination of the geopotential scale factor from TOPEX/POSEIDON satellite altimetry. *Studia Geophysica et Geodaetica* 41(3):203-16.

Burša, M., J. Kouba, M. Kumar, A. Müller, K. Radej, S.A. True, V. Vatrt, and M. Vojtíšková. 1999. Geoidal geopotential and world height system. *Studia Geophysica et Geodaetica* 43(4):327-37.

Faller, J.E., and A. L. Vitouchkine. 2003. Prospects for a truly portable absolute gravimeter. *Journal of Geodynamics* 35(4-5):567-72.

Forsberg, R. 1984. A study of terrain reductions, density anomalies and geophysical inversion methods in gravity field modeling. Report 355, Department of Geodetic Science and Surveying. The Ohio State University, Columbus, Ohio.

Grafarend, E.W., and A.A. Ardalan. 1997. W_0: an estimate in the Finnish Height Datum N60, epoch 1993.4, from twenty-five GPS points of the Baltic Sea Level Project. *Journal of Geodesy* 71(11):673-9.

Groten, E.2004.Fundamental parameters and current (2004) best estimates of the parameters of common relevance to astronomy, geodesy, and geodynamics. *Journal of Geodesy* 77(10-11):724-97.

Heiskanen, W.A., and H. Moritz. 1967. *Physical Geodesy*. New York, New York: W. H. Freeman & Co. 364p.

Hipkin, R.G. 2002a. Is there a need for a Geodetic Datum 2000? Discussion of a "Heiskanen & Moritz' Proposition." In: Adam, J. and K.-P. Schwarz (eds), *Vistas for geodesy in the New Millennium*. Heidelberg, Germany: Springer-Verlag Telos. pp.124-7.

Hipkin, Roger. 2002b. Defining the geoid by W = W0 = U0: Theory and practice of a modern height system. In: *Gravity and Geoid 2002, 3rd Meeting of the International Gravity and Geoid Commission*, Thessaloniki, Greece. pp. 367-77.

Hwang, C. 2002. Adjustment of relative gravity measurements using weighted and datum-free constraints. *Computers & Geosciences* 28(9):1005-15.

Hwang, C., and Y.S. Hsiao. 2003. Orthometric corrections from leveling, gravity, density and elevation data: A case study in Taiwan. *Journal of Geodesy* 77(5-6):279-91.

IGLD 85.1995. Establishment of International Great Lakes Datum (1985) .Coordinating Committee on Great Lakes Basic Hydraulic and Hydrologic Data, Chicago, Illinois. 48p.

Ihde, J., and W. Augath, J.A. Torres, and H. Hornik (eds). 2000. Report on the Symposium of the IAG Subcommission for Europe (EUREF). pp. 99-101.

Kao, S., R. Hsu, and F. Ning. 2000. Results of field test for computing orthometric correction based on measured gravity. *Geomatics Research Australia* 72:43-60.

Kumar, M. 2005. When ellipsoidal heights will do the job, why look elsewhere? *Surveying and Land Information Science* 65(2):91-4.

Meyer, T.H. 2002. Grid, Ground, and Globe: Distances in the GPS Era. *Surveying and Land Information Science* 62(3):179-202.

Meyer, T.H., D. Roman, and D. B. Zilkoski. 2004. What does *height* really mean? Part I: Introduction. *Surveying and Land Information Science* 64(4):223-34.

Meyer, T.H., D. Roman, and D. B. Zilkoski. 2005. What does *height* really mean? Part II: Physics and Gravity. *Surveying and Land Information Science* 65(1):5-15.

Molodensky, M.S.1945. Fundamental problems of geodetic gravimetry (in Russian).*TRUDY TsNIIGAiK* 42, Geodezizdat, Moscow.

NGS (National Geodetic Survey). 1981. Geodetic Leveling. *NOAA Manual NOS NGS 3*. National Oceanic and Atmospheric Administration, Rockville, Maryland.

NGS (National Geodetic Survey). 1986. *Geodetic glossary*. National Oceanic and Atmospheric Administration, Rockville, Maryland. 274p.

Nesvorny, D., and Z. Sima. 1994. Refinement of the geopotential scale factor R0 on the satellite altimetry basis. *Earth, Moon and Planets* 65(1):79-88.

Pfeifer, L. 2001. Eighteen yards and nine metres. *Survey Review* 36(281):175-91.

Rebhan, H., M. Aguirre, and J. Johannessen. 2000. The gravity field and steady-state Ocean Circulation Explorer Mission—GOCE. *ESA Earth Observation Quarterly* 66:6-11.

Smith, D.A. 1998. There is no such thing as "The" EGM96 geoid: Subtle points on the use of a global geopotential model. *IGeS Bulletin* 8:17-28.

Smith, D.A., and D.R. Roman. 2001. GEOID99 and G99SSS: 1-arc-minute geoid models for the United States. *Journal of Geodesy* 75:469-90.

Strang van Hees, G.L. 1992. Practical formulas for the computation of the orthometric and dynamic correction. *Zeitschrift fur Vermessungswesen* 117.

Tapley, B.D., S. Bettadpur, M. Watkins, and C. Reigber. 2004. The Gravity Recovery and Climate Experiment: Mission overview and early results. *Geophysical Research Letters* 31, L09607.

Tenzer, R., P. Vanicek, M. Santos, W. E. Featherstone, M. and Kuhn. 2005. The rigorous determination of orthometric heights. *Journal of Geodesy* 79(1-3): 82-92.